Model description and notes

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1. The ESO model framework

1.1. Conceptual model introduction

The Electricity Systems Optimisation (ESO) model framework is a mixed-integer linear program designed to perform cost-optimal power supply capacity expansion and unit commitment subject to technical, operability, economic, and environmental constraints. The framework contains several model versions which differ in their spatial and temporal level of granularity and horizon. Figure 1 visualises the different model types with respect to their spacial, temporal, and general complexity dimension. More documentation on the basic ESO model can be found in [1, 2].

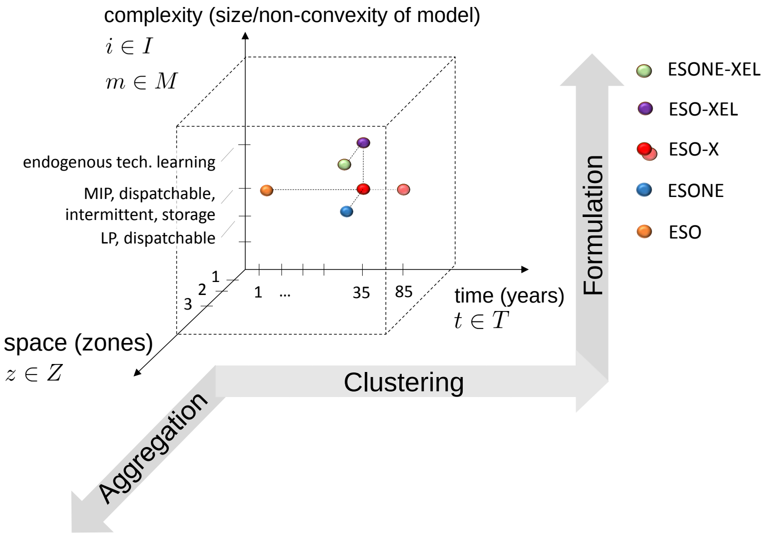

Figure 1: Facets of the ESO modelling framework.

1.2. Model Assumptions

The following list summarises some of the assumptions taken as part of the model building process:

- We take the perspective of a monopolistic system planner.
- The model is implemented under perfect foresight and imperfect foresight conditions over the planning horizon.
- We assume electricity demand and electricity import prices to be inelastic.
- Uncertainty in the input parameters is not considered. The model is deterministic.

- In model version ESONE and ESONE-XEL the electric transmission network is modelled as HVDC lines connecting the zone centroids. In the ESO, ESO-X, ESO-XEL model version, the electric transmission system is represented as a single-node network. Overall transmission losses are considered.
- In the ESO-XEL, ESONE-XEL model version, we assume that endogenous technological change is reflected in the capital cost of the power plant only and are based on global experience until today. Learning (endogenous change) of technology performance parameters is not assumed.
- CO_2 accounting of HVDC interconnection capacity is based on average carbon intensity levels for electricity generation in the connected power networks.

1.3. Mathematical Model Formulation

The following nomenclature and model formulation for constraints 7-34 and objective function 40 refer to the ESO-X model; constraints 7-39 and objective function 40 refer to the ESO-XEL model.

1.3.1. Nomenclature

Sets		
a	yrs	planning periods, $a \in A = \{1, ..., A_{end}\}$
t	h	time periods, $t \in T = \{1, ..., T_{end}\}$
c	-	clusters of representative days of each year, $c \in C = \{1, ..., C_{end}\}$
i	-	technologies, $i \in I = \{1, ..., I_{end}\}$
ig	-	power generating technologies, $ig \subseteq I$
ic	-	conventional generating technologies, $ic \subseteq I$
ir	-	intermittent renewable technologies, $ir \subseteq I$
is	-	storage technologies, $is \subseteq I$
il	-	technologies for which learning rate is applied, $il \subseteq I$
l	-	line segments for piecewise linear function
Parameters		
Δ_a	yrs	step width planning years
$DIni_i$	-	number of available units of technology i for $a = 1$
$DMax_i$	-	maximum number of available units of technology i for $a = 1$
Des_i	MW/unit	nominal capacity per unit of technology i
BR_i	unit/yr	build rate of technology i
$MA_{i,a}$	-	maturity parameter, availability of technology i in year a
$LTIni_i$	yrs	lifetime of initial capacity of technology i for $a = 1$
LT_i	yrs	lifetime of technology i

TL	%	losses in transmission network
$TE_{i,*}$	various	features of technology i,
where * is:		
$Pmin$	%-MW	minimum power output
$Pmax$	%-MW	maximum power output
$Cmax$	%-MW	maximum capacity provision
RP	%-MW	reserve potential, ability factor to provide reserve capacity $\in= \{0,1\}$
IP	%-MW	inertia potential, ability factor to provide inertial services $\in= \{0,1\}$
Ems	$\mathrm{t_{CO_2}}$/MWh	emission rate.
$CAPEX_i$	£/unit	investment costs of technology i
$OPEX_{i,a}$	£/MWh	operational costs of technology i in year a
$OPEXSU_i$	£/MWh	start-up costs of technology i
$OPEXNL_i$	£/h	fixed operational costs of technology i when operating in any mode
$ImpElecPr_{c,t}$	£/MWh	electricity import price
UT_{ig}	h	minimum up-time for technology ig
DT_{ig}	h	minimum down-time for technology ig
$SEta_{is}$	%-MWh	storage round-trip efficiency
$SDur_{is}$	h	maximum storage duration
$SOCMin_{is}$	%-MW	minimum storage inventory level
$SOCMax_{is}$	%-MW	maximum storage inventory level
$AV_{ir,c,t}$	%-MW	availability factor of technology ir in cluster c at hour t
$SD_{c,t,a}$	MWh	system electricity demand in year a in cluster c at hour t
UD	MWh	maximum level of unmet electricity demand in any year a
PL_a	MW	peak load over time horizon T in each year a
CM	%-MW	capacity margin
RM	%-MW	absolute reserve margin
WR	%-MW	dynamic reserve for wind power generation
SI	MW.s	minimum system inertia demand
SE_a	$\mathrm{t_{CO_2}}$	system emission target in year a
$VoLL$	£/MWh	Value of Lost Load
$EPenalty$	£/$\mathrm{t_{CO_2}}$	penalty term for slack emissions
$Disc_a$	-	discount factor $(1+r)^a$ in year a
WF_c	-	weighting factor for clusters c
$Xlo_{il,l}$	MW	lower segment x-value of cumulative capacity of piecewise linear cost function
$Xup_{il,l}$	MW	upper segment x-value
$Ylo_{il,l}$	MW	lower segment y-value of cumulative CAPEX
$Yup_{il,l}$	MW	upper segment y-value

Variables

tsc	£	total system cost
$e_{ig,a,c,t}$	$\mathrm{t}_{\mathrm{CO}_2}$/MWh	emission caused by technology ig in year a at hour t of cluster c

Positive Variables

$p_{ig,a,c,t}$	MWh	energy output of technology i in year a in hour t of cluster c
$p2d_{ig,a,c,t}$	MWh	energy to demand
$p2s_{ig,a,c,t}$	MWh	energy to grid-level storage
$p2is_{is,a,c,t}$	MWh	energy to storage technology is
$r_{ig,a,c,t}$	MW	reserve capacity provided by technology ig
$s_{is,a,c,t}$	MWh	effective state of charge of technology is at the end of time period t
$s2d_{is,a,c,t}$	MWh	energy from storage to demand
$s2r_{is,a,c,t}$	MW	reserve capacity provided by technology is
$slak_{a,c,t}$	MWh	slack variable for lost load
$emslak_{a,c,t}$	$\mathrm{t}_{\mathrm{CO}_2}$/MWh	slack variable for emissions
$xs_{il,a,l}$	MW	position for technology i in year a on line segment l
$y_{il,a}$	£	cumulative CAPEX for technology i in year a

Integer Variables

$b_{i,a}$	-	number of new built units of technology i in year a
$d_{i,}$	-	number of units of technology i operational in year a, cumulative
$n_{ig,a,c,t}$	-	number of units of technology ig operating in year a at hour t of cluster c
$o_{is,a,c,t}$	-	number of units of storage technology is operating in year a at hour t of cluster c
$u_{ig,a,c,t}$	-	number of units of technology ig starting up in year a at time t of cluster c
$w_{ig,a,c,t}$	-	number of units of technology ig turning down in year a at time t of cluster c

Binary Variables

$\rho_{il,a,l}$	-	1, if cumulative CAPEX of technology il in year a on line segment l

1.3.2. System design constraints

$$d_{i,a} = DIni_i \qquad \forall i, a = 1 \tag{1}$$

$$b_{i,a} \leq BR_i \, MA_{i,a} \, \Delta_a \qquad \forall i, a > 1 \tag{2}$$

$$d_{i,a} \leq DMax_i \qquad \forall i, a \tag{3}$$

$$d_{i,a} = d_{i,a-1} - b_{i,a-\frac{LTIni_i}{\Delta_a}} + b_{i,a} \qquad \forall i, a \leq \frac{LTIni_i}{\Delta_a} + 1 \tag{4}$$

$$d_{i,a} = d_{i,a-1} + b_{i,a} \qquad \forall i, \frac{LTIni_i}{\Delta_a} + 1 < a \leq \frac{LT_i}{\Delta_a} + 1 \tag{5}$$

$$d_{i,a} = d_{i,a-1} - b_{i,a-\frac{LT_i}{\Delta_a}} + b_{i,a} \qquad \forall i, a > \frac{LT_i}{\Delta_a} + 1 \tag{6}$$

1.3.3. Unit commitment variables

$$n_{ig,a,c,t} \leq d_{ig,a} \qquad \forall ig, a, c, t \tag{7}$$

$$o_{is,a,c,t} \leq d_{is,a} \qquad \forall is, a, c, t \tag{8}$$

1.3.4. System wide power balance, security, and emission constraint

Constraint 13 is part of the model formulation only if enforced decarbonisation is desired. Otherwise, constraint 13 is relaxed and decarbonisation is driven by the carbon price (included in the parameter $OPEX_{ig,a}$) only.

$$\sum_{ig} p2d_{ig,a,c,t} + \sum_{is} s2d_{is,a,c,t} = SD_{c,t,a} \, (1 + TL) - slak_{a,c,t} \qquad \forall a, c, t \tag{9}$$

$$\sum_{i} d_{i,a} \, Des_i \, TE_{i,\mathsf{Cmax}} \geq PL_a \, (1 + CM) \qquad \forall a, t \tag{10}$$

$$\sum_{ig} r_{ig,a,c,t} \, TE_{ig,\mathsf{RP}} + \sum_{is} s2r_{is,a,c,t} \, TE_{is,\mathsf{RP}} \geq SD_{c,t,a} \, RM + \sum_{ir} p2d_{ir,a,c,t} \, WR \qquad \forall a, c, t \tag{11}$$

$$\sum_{ig} n_{ig,a,c,t} \, Des_{ig} \, TE_{ig,\mathsf{IP}} \geq SI \qquad \forall a, c, t \tag{12}$$

$$\sum_{ig,c,t} (e_{ig,a,c,t} - emslak_{a,c,t}) \, WF_c \leq SE_a \qquad \forall a \tag{13}$$

$$\sum_{c,t} slak_{a,c,t} \, WF_c \leq UD \sum_{t} SD_{c,t,a} \qquad \forall a \tag{14}$$

1.3.5. *Technology specific unit commitment, power and ancillary service constraints*

$$p_{ic,a,c,t} \geq n_{ic,a,c,t}\, Des_{ic}\, TE_{ic,\mathsf{Pmin}} \qquad \forall ic, a, c, t \tag{15}$$

$$p_{ig,a,c,t} + r_{ig,a,c,t} \leq n_{ig,a,c,t}\, Des_{ig}\, TE_{ig,\mathsf{Pmax}} \qquad \forall ig, a, c, t \tag{16}$$

$$p_{ig,a,c,t} = p2d_{ig,a,c,t} + p2s_{ig,a,c,t} \qquad \forall ig, a, c, t \tag{17}$$

$$p_{ir,a,c,t} \geq n_{ir,a,c,t}\, Des_{ir}\, TE_{ir,\mathsf{Pmin}}\, AV_{ir,c,t} \qquad \forall ir^{\setminus} InterImp, a, c, t \tag{18}$$

$$p_{ir,a,c,t} + r_{ir,a,c,t} \leq n_{ir,a,c,t}\, Des_{ir}\, AV_{ir,c,t} \qquad \forall ir^{\setminus} InterImp, a, c, t \tag{19}$$

$$e_{ig,a,c,t} = (p_{ig,a,c,t} + r_{ig,a,c,t})\, TE_{ig,\mathsf{Ems}} \qquad \forall ig, a, c, t \tag{20}$$

$$\sum_{c,t} p_{ic,a,c,t}\, WF_c / 8760 \geq TE_{ic,\mathsf{UtilMin}}\, d_{ic,a}\, Des_{ic} \qquad \forall ic, 1 < a < T_{end} \tag{21}$$

1.3.6. *Unit up-time and down-time constraints*

$$u_{ig,a,c,t} \geq n_{ig,a,c,t} - n_{ig,a,c,t-1} \qquad \forall ig, a, c, t \tag{22}$$

$$w_{ig,a,c,t} \geq n_{ig,a,c,t-1} - n_{ig,a,c,t} \qquad \forall ig, a, c, t \tag{23}$$

$$u_{ig,a,c,t} \leq n_{ig,a,c,\tau} \qquad \forall ig, a, c, \tau = t + t' - 1, t' \leq UT_{ig} \tag{24}$$

$$w_{ig,a,c,t} \leq d_{ig,a} - n_{ig,a,c,\tau} \qquad \forall ig, a, c, \tau = t + t' - 1, t' \leq DT_{ig} \tag{25}$$

1.3.7. *Storage operation constraints*

$$s2d_{is,a,c,t} + s2r_{is,a,c,t} \geq o_{is,a,c,t}\, Des_{is}\, TE_{is,\mathsf{Pmin}} \qquad \forall is, a, c, t \tag{26}$$

$$s2d_{is,a,c,t} + s2r_{is,a,c,t} \leq o_{is,a,c,t}\, Des_{is} \qquad \forall is, a, c, t \tag{27}$$

$$s2d_{is,a,c,t} + s2r_{is,a,c,t} \leq s_{is,a,c,t}\, SEta_{is} \qquad \forall is, a, c, t \tag{28}$$

$$s_{is,a,c,t} = Des_{is}\, SOCIni_{is}\, SDur_{is} \qquad \forall is, a, c, t = 1 \tag{29}$$

$$s_{is,a,c,t} \leq o_{is,a,c,t}\, Des_{is}\, SOCMax_{is}\, SDur_{is} \qquad \forall is, a, c, t \tag{30}$$

$$s_{is,a,c,t} \geq o_{is,a,c,t}\, Des_{is}\, SOCMin_{is}\, SDur_{is} \qquad \forall is, a, c, t > SDur_{is} \tag{31}$$

$$\sum_{ig} p2s_{ig,a,c,t} = \sum_{is} p2is_{is,a,c,t} \qquad \forall a, t \tag{32}$$

$$p2is_{is,a,c,t} \leq o_{is,a,c,t}\, Des_{is} \qquad \forall is, a, c, t \tag{33}$$

$$\begin{aligned} s_{is,a,c,t} = {} & s_{is,a,c,t-1} - s2d_{is,a,c,t} \\ & + \sum_{is} p2is_{is,a,c,t}\, SEta_{is} \qquad \forall is, a, c, t > 1 \end{aligned} \tag{34}$$

1.3.8. Endogenous cost reduction

$$\sum_{l} \rho_{il,a,l} = 1 \qquad \forall il, a \tag{35}$$

$$xs_{il,a,l} \geq Xlo_{il,l}\, \rho_{il,a,l} \qquad \forall il, a, l \tag{36}$$

$$xs_{il,a,l} \leq Xup_{il,l}\, \rho_{il,a,l} \qquad \forall il, a, l \tag{37}$$

$$\sum_{a'=1}^{a} b_{il,a'} = \sum_{l} xs_{il,a,l} \qquad \forall il, a, a' \leq a \tag{38}$$

$$\begin{aligned} y_{il,a} = \sum_{l} & Ylo_{il,l} \\ & + \rho_{il,a,l} Slope_{il,l} \left(xs_{il,a,l} - Xlo_{il,l}\, \rho_{il,a,l} \right) \qquad \forall il, a \end{aligned} \tag{39}$$

1.3.9. Objective function

$$\begin{aligned} & minimise\{tsc\} \\ & tsc = \sum_{i \in I \setminus il, a} CAPEX_i\, b_{i,a}\, Des_i / Disc_a \quad + \sum_{il,a} (y_{il,a} - y_{il,a-1}) / Disc_a \\ & \quad + \sum_{ig,a,c,t} (u_{ig,a,c,t}\, OPEXSU_{ig}\, WF_c) / Disc_a \\ & \quad + \sum_{ig,a,c,t} (OPEX_{ig,a}\, p_{ig,a,c,t}\, WF_c + OPEXNL_{ig}\, n_{ig,a,c,t}\, WF_c) / Disc_a \\ & \quad + \sum_{is,a,t} (OPEX_{is,a}\, s2d_{ig,a,c,t}\, WF_c + OPEXNL_{is}\, o_{is,a,c,t}\, WF_c) / Disc_a \\ & \quad + \sum_{i = InterImp,a,t} ImpElecPr_t\, p2d_{i,a,c,t}\, WF_c / Disc_a \\ & \quad + \sum_{a,c,t} slak_{a,c,t}\, WF_c\, VoLL \\ & \left(\quad + \sum_{a,c,t} emslak_{a,c,t}\, WF_c\, EPenalty \quad \right) \end{aligned} \tag{40}$$

1.4. Solution Strategies

The ESO models are implemented in GAMS 24.8.3 [3] and solved with CPLEX 12.3 [4]. On an Intel i7-4770 CPU, 3.4 GHz machine with 8 GB RAM using 8 threads, and an optimality gap of 3 % expected solution times are reported in Heuberger et al. [2].

1.4.1. Hourly Data Processing and Relaxation

In order to reduce computational solution times two main solution strategies are applied to the ESO model input data and formulation presented in section 1.3:

1. K-means data clustering [5] with "energy-preserving" profiling [2],
2. Relaxation of integer scheduling constraints [6, 7].

Heuberger et al. [1, 2] provides detail on both solution strategies and explicitly examines their implication on technology-specific and system-level results. The overall error in the objective function value (total system cost, tsc) ranges between -1.7 % to +2.5 % for 21 to 11 clusters with 24 hours each representing the 8760 hours of the year over a 35 year time period (2015-2050).

In all downloadable files the hourly time dependent data is compressed to 11 clusters and the integer scheduling variables $n_{ig,a,c,t}$, $o_{is,a,c,t}$, $u_{ig,a,c,t}$, and $w_{ig,a,c,t}$ are relaxed.

References

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