1	Quantification of biochar content in a natural soil by
2	spectral induced polarization: a laboratory experiment
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• Abstract

Biochar garners significant attention due to its relevance in the realms of environmen-10 tal sustainability, archaeology, and agronomy. However, monitoring the fate of biochar 11 in soils remains challenging. Environmental geophysics characterizes the near surface non-12 intrusively to better understand the subsurface's hydro-bio-geo-chemical processes. Hence, 13 spectral induced polarization (SIP) has been proposed as a potential method to char-14 acterize biochar content in soils. Thus far, only highly controlled laboratory experiments 15 have been carried out with sand and biochar. Here, we present SIP measurements of a 16 natural soil, in which biochar was incorporated at different mass proportions: 0, 0.1, 1, 17 5, and 10%. Strong relationships between SIP derived parameters and biochar content 18 were found. Interestingly, we observe that simple curve descriptors such as the phase at 19 11.7 Hz and the shape parameter α (height of a triangle defined by a peak in the phase) 20 were enough and suitable to detect biochar content in the studied soil. Overall our study 21 suggests that SIP can quantify biochar content in the laboratory under controlled soil 22 moisture conditions. 23

²⁴ 1 Introduction

Biochar is the by-product of plant biomass pyrolysis (Roberts et al., 2010; Lehmann & Joseph, 2015). Biochar has been suggested as a mean to provide agronomic and environmental benefits including bioenergy production, increase in water retention, reduction of nutrient leaching, increase of crop productivity and mitigation of greenhouse gas emissions through its incorporation into soils (Woolf et al., 2010; J. Wang & Wang, 2019). The role of biochar to mitigate climate change is mainly attributed to its high degree

31	of aromaticity limiting biological and abiotic degradation favouring in turn, carbon stor-
32	age into soils (Xu et al., 2021). Although, persistence of biochar in soil depends on its
33	intrinsic properties (such as large specific surface area, abundant functional groups, and
34	developed pores, Lehmann et al., 2009), it also depends on soil properties (e.g. texture,
35	clay mineralogy, native soil carbon content; Yang et al., 2022); plant inputs (Hammes
36	& Schmidt, 2009), and environmental factors such as moisture and temperature (Joseph
37	et al., 2021). If a low portion of biochar seems prone to microbial degradation (Palan-
38	sooriya et al., 2019), biochar can also be lost via lateral movement or downward migra-
39	tion in the soil as a consequence of coarse texture and/or bioturbation (Zhang et al., 2010).
40	Up to now, responses of biochar to degradation and downward migration remain poorly
41	documented in the field (Singh et al., 2015). Hence, long-term effect of biochar on soil
42	properties remains an open question. There are significant efforts to better understand
43	these processes, however better tools to characterize the presence and effect of biochar
44	in a soil are needed. That is, an accurate quantification of biochar in soils is required in
45	order to characterize their spatial and temporal dynamics at the field scale (Gao et al.,
46	2017; Mukherjee et al., 2021).
47	Charred particles counting (Rhodes, 1998; Lesven et al., 2022), chemical oxidation fol-
48	lowed by elemental analyses (Gustafsson et al., 1997; Knicker et al., 2008), Ultra-Violet
49	oxidation followed by solid-state $^{13}\mathrm{C}$ nuclear magnetic resonance (Skjemstad et al., 1996),
50	thermal degradation (Nakhli et al., 2019; Hardy et al., 2022), spectroscopy (Paetsch et
51	al., 2017) and acid digestion followed by the quantification of benzenepolycarboxylic acids

(Brodowski et al., 2005; Glaser et al., 1998) are among the main methods used to quan-

tify the biochar content in soil. All these methods require preliminary sampling, which
intrinsic limitations (e.g. quantity of analysed soil or accuracy in determining the investigated depth) can hamper our ability to detect a few but significant changes in biochar
degradation and/or migration. Because of their "invasive" nature, these methods are also
not suitable to provide a continuous monitoring of the spatial and temporal dynamics
of biochar in soil.

Geophysical methods are known to provide reliable information on soil properties at the 59 field scale (Robinson et al., 2008; Hermans et al., 2023), in a non-invasive way, partic-60 ularly electric and electromagnetic methods (see Friedman, 2005; Parsekian et al., 2015; 61 Binley & Slater, 2020), such as electrical resistivity tomography (ERT, Samouëlian et 62 al., 2005), electromagnetic induction (EMI, Benech et al., 2016; Tabbagh et al., 2021), 63 or induced polarization methods in the time or frequency domain (see Kemna et al., 2012). Among geo-electrical (induced polarization) methods, spectral induced polarization (SIP) 65 has been suggested as a valuable tool to detect and monitor biochar in soil (e.g., Haegel 66 et al., 2012; Gurin et al., 2014; Gao et al., 2017, 2019). SIP informs about the ability a 67 geo-material has to conduct electricity and polarize itself electrically. Gao et al. (2017) 68 mentions that biochar polarizes similarly to conductive or semi-conductive materials. Metal-69 lic and semi-conductors give rise to an electric polarization when an external electrical 70 field is imposed and the charge-carriers move according to the sinusoidal electrical field 71 (see Mao & Revil, 2016). 72

Following this, Gao et al. (2017, 2019) suggested that SIP derived parameters were
 related to the content of biochar in sand-biochar mixtures. Nonetheless, this pioneering

75	investigation was designed to evaluate the effect of the interaction between water sat-
76	uration and two biochar concentrations (1 and 2 $\%)$ on SIP signatures rather than evaluation of the second secon
77	uating SIP signatures as a mean to quantify biochar concentrations. Although these stud-
78	ies paved the way for the application of SIP to monitor the spatial and temporal dynam-
79	ics of biochar in soil, several intermediate steps are still required before a field applica-
80	tion. First, if sand presents numerous advantages from an experimental point of view,
81	it is not representative of the complex texture of natural soils with sand, silt and clay-
82	sized mineral particles, that have their own SIP response (Mendieta et al., 2021). Sec-
83	ond, the SIP response to the amendment of biochar was tested using two low concen-
84	trations (1 and 2 wt%, Gao et al., 2019) implying that additional experiments are re-
85	quired to assess the overall potential of SIP as a mean to quantify biochar content in soils.
86	In this study our aim is therefore to test the potential of SIP to quantify a wide range
87	in biochar content in a natural soil with simple and traditional geophysical parameters
88	with the use of three replicates.

⁸⁹ 2 Materials and methods

2.1 Sample preparati

q

The soil used in the samples came from a test site located at the CEREMA (Centre for Studies and Expertise on Risks, the Environment, Mobility and Urban Planning) near the city of Rouen, France. The cationic exchange capacity (CEC) of the soil was measured by blue methylene tests which give an average value of 3.12 ± 0.36 g/100g eq. It means that the utilized soil has small reactive content. It was then determined, by granulometry that this soil contains, on average 19.9 ± 5.5 % clay, 69.8 ± 6.4 % silt, and 10.3

97 $\pm 3.7\%$ sand.

98	Firstly, the soil was dried at a temperature of 40 $^{\circ}\mathrm{C},$ during 10 days. Pyrolitic biochar
99	was used in this experiment. Afterwards, we crushed separately both biochar (74.08 $\%$
100	C, 0.72% N, and 3% H) and soil and sifted them to a grain size lower than 2 mm. Later,
101	the dry soil and biochar were mixed with an electric drill, obtaining mass proportions
102	of biochar content of: 0%, 0.1%, 1%, 5%, and 10%. Following, a mixing of de-ionized wa-
103	ter, biochar and soil was done with an electric drill. The mass water content of all sam-
104	ples was set at 27.5%. The choice of this water content value allows for a good plastic-
105	ity to fill the non-watertight sample holder (see Mendieta et al., 2021, 2023, for details
106	on the choice of sample holder). The decision on using de-ionized water was made be-
107	cause the soil samples had not been flushed, thus the original ions present in the soil re-
108	mained. After adding water and a period of 24 h, we placed the wet sample in the sam-
109	ple holder. Three separate batches (replicates) were made, this is not an usual practice
110	in SIP studies but it is in soil science studies.

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2.2 Spectral induced polarization

The SIP measurements were performed with the SIP-FUCHS III (Radic Research, www.radic-research.de).

For the SIP measurement and with the SIP-FUCHS III, a sinusoidal potential difference is imposed between a pair of electrical current injection electrodes. A resulting potential difference is measured in a different pair of electrical potential measuring elec-



Figure 1. a) Principle of the injection of an electrical current and its voltage response at a single frequency. b) The amplitude of the resistivity and phase at multiple frequencies. Schematic of a double Pelton model from the (c) amplitude of the resistivity and (d) the phase, with 2 single Pelton models and a double-Pelton model that is the sum of the previous single Pelton models. (e) Concept of the parameter α .

117	trodes. Before starting the measurements over the full frequency range (1 mHz to 20 kHz),
118	short tests are performed to adapt the imposed input electric potential difference (1 Hz $$
119	to 20 kHz). A very low potential difference yield poor quality data (low signal-to-noise
120	ratio), and a very high potential difference could be out of the validity of Ohm's law (lin-
121	earity). In this case, for the whole dataset the range of injected electrical current is from
122	0.1 to 0.05 mA, yielding measurement errors below 1%.
123	Following this, SIP yields information about the electrical resistivity (ρ or its inverse,
124	the electrical conductivity σ) and polarization through the amplitude of the measured
125	electrical potential difference and injected current and the phase-lag (in mrad) between
126	them (see figure 1a) at different frequency values (figure 1b).
127	Previous SIP biochar experiments (see Gao et al., 2017, 2019) have been carried out with
128	sand and biochar mixtures. The polarization of silica sand is thought to be mainly from
129	the Stern layer polarization and very small (i.e., negligible); hence the presence of biochar
130	could clearly and undoubtably be highlighted. Natural soils being a mixture of clays, sands,
131	water, among others make difficult to interpret their SIP response. Nevertheless accord-
132	ing to Revil et al. (2017) it is possible to model the SIP response of a polarizing back-
133	ground (silty soil for this study) and add the polarization of an inclusion (biochar in this
134	study).

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2.3 Data pre-processing and Pelton model adjustment

First, the correction proposed by Florsch et al. (2014) was applied to the SIP measured data (see figure 2 c and d) to remove the high frequency noise. Second, we performed an optimization procedure (following Maineult, 2016) that allows us to obtain the Pelton parameters.

Pelton-type models (see figure 1c and d) are empirical and allow to describe a SIP 140 curve with few parameters (Ghorbani et al., 2009). Such models consist of at least the 141 four following parameters: the direct current resistivity ρ_0 (in Ω m, or its inverse σ_0 in 142 S m⁻¹), the chargeability m (mV/V), the Cole-Cole exponent c (no units), and the re-143 laxation time τ (in s). ρ_0 informs about the resistance of a material to an electrical cur-144 rent, m about the capacity of a material to store charges reversely, τ determines the char-145 acteristic time at which a material electrically discharges itself, and c is a constant that 146 informs about the distribution of relaxation times of a material (for more information 147 see Tarasov & Titov, 2013; Revil et al., 2012). A single Pelton model describes a single 148 polarization peak of a material (that is a singular cause for polarization). In this work 149 we use a double Pelton model (i.e., two polarization peaks), one peak describing the po-150 larization of biochar at lower frequencies and one describing the high frequency behaviour 151 (see figure 1 c and d). The equation that describes the double-Pelton model is: 152

$$\rho^*(\omega) = \rho_0 \left[1 - m_1 \left(1 - \frac{1}{1 + (i\omega\tau_1)^{c_1}} \right) - m_2 \left(1 - \frac{1}{1 + (i\omega\tau_2)^{c_2}} \right) \right],\tag{1}$$

where $\rho^*(\omega)$ is the complex and frequency dependent electrical resistivity (in Ω m). Note that the subscripts 1 and 2 correspond to the Pelton for the the low and high frequency polarizations, respectively. The high frequency peak contains both the polarization of the material and a high frequency capacitive effect which may have a component linked ¹⁵⁸ For a four-electrode system this effect is normal (see Kemna et al., 2012).

¹⁵⁹ 2.4 Curve parametrization from the polarization spectra

160	In addition to Pelton derived parameters, we defined simple characterization pa-
161	rameters, such as: the phase at 11.7 Hz, the resistivity at 11.7 Hz, and the average phase
162	at a mid-frequency range (1.46 - 46.88 Hz). The frequency of 11.7 Hz was chosen because
163	the maximum of the phase curve is at 11.7 Hz.
164	As an attempt to characterize the effect of biochar on SIP measurements we introduce
165	the shape parameter α . We define it as the height of a triangle located in a the phase
166	spectrum. The height of the triangle can be measured graphically or numerically, we pro-
167	pose an analytical solution in Appendix 1. The triangle is constructed from three points
168	in the curve, given by its inflection points. That is, one point at a low, medium and high
169	frequencies (lm, mf, and hf, respectively), with the medium point at a maximum or min-
170	imum of the curve (see figure 1e). A positive value of α denotes a downward concavity
171	and on the contrary a negative value represents an upward concavity. An α value of zero
172	represents a straight line, that is the lack of a peak in the chosen frequency range of the
173	phase plot.

¹⁷⁴ 3 Results and discussion

Up until now SIP has been tested on mixtures of sand and biochar (Gao et al., 2017, 2019). However, natural soils have a complex texture, including sand, silt, and also clay

which also has its own significant SIP signature (see for instance, Revil et al., 2012; Mendi-177 eta et al., 2021, among others). 178

179	When a mineral is in contact with an electrolyte, an electrical charge builds up in
180	its surface, creating an electrical double layer (EDL). When subjecting these EDL con-
181	taining systems (e.g., soils) to an external electrical field, the EDL polarizes. EDL po-
182	larization happens at around the Hz frequency (Loewer et al., 2017) which is in the range
183	expected for biochar (Gao et al., 2017). It is then important to test if the presence of
184	clay interferes with the SIP signature of biochar. In the present study, we show that the
185	presence of biochar can indeed be detected by SIP despite the presence of clay. This was
186	observed (1) according to its correlation between the amplitude of the resistivity and the
187	phase with biochar content; and (2) following the evolution of the shape of the phase curves
188	with biochar content.
189	As mentioned by Gao et al. (2017), biochar acts as conductors or semi-conductors, it was
190	then expected that with an increase of its content the electrical resistivity of the mea-
191	sured media decreases. Similarly, for the measured phase, an increase of the phase value
192	is expected with an increase of the weight fraction of biochar (see Revil et al., 2017).
193	The sample with the highest biochar content (10% wt.) has the lowest resistivity
194	(41.43 Ωm at 11.7 Hz) and conversely the sample without biochar presents the highest
195	resistivity values (69.12 Ωm at 11.7 Hz, figures 2a and c). In addition to changes of the
196	SIP signature at low and medium frequencies, an increase of the phase at the highest fre-
197	quencies (above 1 kHz) was observed (figure 2b). Figures 2 c and d describe the evolu-
198	tion of the amplitude of the resistivity and the phase as a function of frequency of soil

199	biochar mixtures, with the high frequency correction. We observe a peak in the phase
200	near 10 Hz for the soils in which, 5 and 10% of biochar were incorporated (figures 2b and
201	d). The amplitude of this peak decreases with biochar content. It is worth mentioning
202	that the phase at high frequencies comes from both polarization of the sample and a ca-
203	pacitive effect linked to the electromagnetic coupling (see Zimmermann et al., 2008).
204	The traditional way to interpret SIP curves is through physical models or phenomeno-
205	logical (notably Pelton type models, see for instance, Tarasov & Titov, 2013; Leroy et
206	al., 2017; Weller & Slater, 2022), here applied to SIP curves corrected from high frequency
207	noise (figures 2 c and d).
208	A double-Pelton model is able to represent a double-peaked SIP dataset with seven pa-
209	rameters (ρ_0 , m_1 , τ_1 , c_1 , m_2 , τ_2 , and c_2 , see eqn.1). The high frequency peak is deter-
210	mined by the Pelton parameters presenting the subscript 1, while the peak near 10 Hz $$
211	is defined according to the parameters with the subscript two (following figure 1 c and
212	d). Table 1 presents the fitted Pelton parameters for both peaks and their correlation
213	coefficient with biochar. In the following, we will focus on peak 2 (near 10 Hz) where Gao $$
214	et al. (2017) already observed an effect of biochar. Overall, we verify that the resistiv-
215	ity (ρ_0) decreases with an increase of biochar content (70.2 to 43.6 Ωm). Additionally,
216	the chargeability (m_2) increases with biochar content (0.066 to 0.114 mV/V), as the re-
217	laxation time (τ_2 , 3.80×10 ⁻⁴ to 7.01×10 ⁻³ s), and the Cole-Cole exponent (c_2 , 0.334 to
218	0.436).
219	In addition to double-Pelton parameters, we introduce three other parameters, namely

the mean phase in the medium frequency range ($\bar{\varphi}_{MF}$), the phase at 11.7 Hz and the

221	parameter α , all determined on the uncorrected and corrected data (see figure 1e). The
222	choice of focusing in the mid-frequency region (1.46 to 46.88 Hz) was made because it is
223	both where the polarization peak presents itself, and because this frequency range is within
224	a typical frequency range that could be used in a field campaign (see Martin et al., 2021).
225	A high correlation coefficient (0.99-0.97) was determined between the biochar content
226	and these three parameters (see figure 3). Interestingly, the phase measured at 11.7 Hz
227	on an uncorrected spectra constitutes the best descriptor of biochar content ranging be-
228	tween 0 and 10 $\%$ (wt.%). These results show that the double-Pelton model and the re-
229	moval of the high frequency noise can be avoided to determine biochar content as a func-
230	tion of the SIP signature. Hence, our simplistic but reliable approach (phase at 11.7 Hz)
231	opens the possibility to track biochar content with SIP through a new routine that can
232	be useful in soil science.



Figure 2. a) Average (n=3) amplitude of the resistivity from the soil-biochar mixtures. b) Average (n=3) phase from the soil and biochar samples. c) Average (n=3) corrected amplitude of the resistivity from the soil and biochar samples. d) Average (n=3) corrected phase from the soil-biochar mixtures.

Table 1. Fit	ted Pelton paran	neters, presented in	ı average and its standard dev	iation. The last ro	w presents Pearsor	ı's correlation (between bioch	ar content and its
corresponding]	parameter).						
biochar $(\%)$	$ ho_0 (\Omega m)$	$m_1 \; ({ m mV/V})$	$ au_1$ (s)	c1 (-)	$m_2 \ ({ m mV/V})$	$ au_2$ (s)	c2 (-)
0	70.2 ± 1.7	0.042 ± 0.006	$2.37 \times 10^{-5} \pm 2.04 \times 10^{-6}$	0.752 ± 0.037	0.066 ± 0.013	$3.80{ imes}10^{-4}{\pm}1.30{ imes}10^{-4}$	0.334 ± 0.004
0.1	69.2 ± 1.9	0.043 ± 0.003	$2.48 \times 10^{-5} \pm 1.67 \times 10^{-6}$	0.747 ± 0.007	0.078 ± 0.003	$5.07 imes 10^{-4} \pm 2.24 imes 10^{-4}$	0.336 ± 0.010
1	64.4 ± 5.8	0.055 ± 0.013	$2.14{ imes}10^{-5}{\pm}1.27{ imes}10^{-6}$	0.681 ± 0.046	0.072 ± 0.007	$1.50{ imes}10^{-3}{\pm}~9.59{ imes}10^{-4}$	0.357 ± 0.008
a	46.8 ± 1.2	0.065 ± 0.013	$2.55 \times 10^{-5} \pm 4.06 \times 10^{-6}$	0.618 ± 0.057	0.084 ± 0.003	$5.30 imes 10^{-3} \pm 1.67 imes 10^{-3}$	0.427 ± 0.010
10	43.6 ± 3.2	0.075 ± 0.010	$2.72 \times 10^{-5} \pm 3.47 \times 10^{-6}$	0.583 ± 0.046	0.114 ± 0.010	$7.01 \times 10^{-3} \pm 2.28 \times 10^{-3}$	0.436 ± 0.021
r	-0.92	0.81	0.50	-0.83	0.89	0.90	0.91

233	Beyond the correlation between the SIP derived parameters and the biochar con-
234	tent, the parameter α can identify the presence of biochar above 5% (0.33 \pm 0.09) in con-
235	tent within a natural soil sample (-0.48 \pm 0.10). Between 0 and 1% the value of α is neg-
236	ative, while above 5% it is positive. Thus, we could infer that the approach presented
237	in this paper could be an useful tool to find potential biochar hot-spots in the field with
238	simple to obtain parameters (e.g.: positive α values).
239	It is important to emphasize that the source of the SIP signature arises from the
240	whole mixture, that is: soil, water and biochar. Indeed, variable water content, soil tex-
241	ture, feedstocks, carbonization degree (depending on temperature and residence time),
242	biochar particle size, additional carbon inputs into soils (from plant and microbial biomass)
243	and biochar aging can conspicuously modify the SIP signature of biochar; according sev-
244	eral conduction paths such as the pore structure (and pore water).
245	In this study, we investigated the quantification of the biochar content by SIP un-
246	der controlled laboratory conditions. The diversity in soil properties, water content as
247	well as the physical and chemical structure of biochar are not considered here although
248	it can play a key role in the field on the response of SIP signatures. Soil can contain or-
249	ganic matter (e.g., litter, plant roots, bacteria, fungi,). Several studies also entail the
250	impact of organic matter on SIP signatures (e.g.: Schwartz et al., 2014; Mellage et al.,
251	2022; Strobel et al., 2023). In addition, activity of below ground life can also alter the
252	SIP response (see more in Atekwana & Slater, 2009; Kessouri et al., 2019). One of the
253	main electrical conduction and polarization paths is through the pore water and its in-
254	terface with minerals. Therefore, water content can highly impact the SIP response in



Figure 3. (a) The phase at 11.7 Hz, (b) the resistivity at 11.7 Hz, (c) the average phase at a medium frequency range, and (d) the parameter α for the non-corrected datasets. The limits of the shaded areas represent the minimum and maximum value of each parameter for the 0% biochar content.

255	the presence of biochar as revealed by Gao et al. (2019). In a field experimental design,
256	the above-mentioned factors can be easily constrained notably through the investigation
257	of the initial conditions. The physicochemical properties of biochar itself should also have
258	a deep impact on soil polarization. Indeed, biochar can display a wide diversity in their
259	initial physicochemical properties depending on feedstocks carbonization degree (depend-
260	ing on temperature and residence time) and particle size (Tomczyk et al., 2020). These
261	factors are known to modify the the specific surface area of biochar (Leng et al., 2021)
262	and therefore its surface conductivity. surface conductivity of biochar and/or the specific $% \mathcal{A}$
263	surface area (Leroy et al., 2017; Leng et al., 2021). Again, the initial properties of biochar
264	can be constrained in a field experimental design in which the initial conditions are known.
265	However, with time biochar is subjected to aging, which implies fragmentation as well
266	as surface oxidation (Zeba et al., 2022). Aging therefore usually leads to a significant in-
267	crease in the CEC while its effect on the specific surface area of biochar is not straight-
268	forward (L. Wang et al., 2020). As surface conduction is highly dependent on the CEC
269	and specific surface area (see for instance: Leroy et al., 2008; Lévy et al., 2019; Mendi-
270	eta et al., 2021), biochar aging should cause changes in the surface conduction. One could
271	also expect that surface oxidation of biochar - implying a relative enrichment in carboxylic
272	and phenolic groups (Wiedner et al., 2015) also affect surface conduction. Characteriz-
273	ing mineral wastes, Placencia-Gómez et al. (2013) observe a decrease in the phase shift
274	with an increase in oxidation. Up to now, the effect of biochar aging on SIP signatures
275	remains undocumented although biochar aging may drive the modifications of these sig-
276	natures in the field. Following this, we suggest that SIP signatures measured in the field

following a time-lapse approach may be a suitable way to track processes involved in biochar aging.

279 4 Conclusion

280	We present a SIP dataset from natural soil and varying biochar concentration. Our
281	data shows a clear relation between the Pelton parameters $(\rho_0, m, c, \text{ and } \tau)$ and biochar
282	content. Pelton parameters are traditionally used in geophysics and are a result of data
283	processing. We propose simple parameters that directly relate to biochar content in a
284	soil sample, such as: the mean phase from a middle frequency range (1.46 - 46.88 Hz),
285	the phase at 11.7 Hz, and the shape parameter α . These parameters are simple to cal-
286	culate and effective even with no high-frequency correction. Thus they could be a help-
287	ful tool in the field in order to possibly identify biochar hot-spots.

²⁸⁸ 5 Funding declaration

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²⁹⁰ 6 Acknowledgements

201 We would like to thank Fayçal Rejiba for providing the soil samples used in these 202 experiments, with their respective characterization.

²⁹³ 7 Data availability

294	The dataset presented here is stored in a cloded access zenodo repository under the
295	doi: 10.5281/zenodo.8090793. After acceptance of the paper it will be open access.

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522 8 Appendix 1

We define three frequencies namely the lowest frequency of interest (LF),intermediate 523 frequency of interest (IF) and highest frequency of interest (HF). the only need in our 524 case is that LF<IF<HF. This triad of frequencies defines a triad of values on the curve 525 defining a triangle. The parameter α is determined as the scalar product between the 526 unitary vector defined by the couple (LF, HF) rotated by $\frac{\pi}{2}$ and the vector correspond-527 ing to the height of the triangle associated with the side (LF,HF) oriented from the side 528 to the point corresponding to MF. If this product is positive, then the curves exhibit a 529 concave shape on the frequency domain chosen. on the other hand, if α is negative then 530 the curve exhibit a convex shape on the frequency domain chosen. In addition, the am-531 plitude gives a hint on the strength of the effect. 532

Four points are necessary to compute the α parameter :

$$A = \begin{pmatrix} x_{LF} \\ y_{LF} \end{pmatrix}, B = \begin{pmatrix} x_{IF} \\ y_{IF} \end{pmatrix}, C = \begin{pmatrix} x_{HF} \\ y_{HF} \end{pmatrix}, O = \begin{pmatrix} x \\ y \end{pmatrix}$$
(2)

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The O point is as the intercept between the line (AC) and a perpendicular passing by B. Hence we can define O as the point which coordinate verify :

$$\begin{cases} y = ax + b \\ y = -\frac{x}{a} + \beta \end{cases}$$
(3)

with :

$$a = \frac{y_{HF} - y_{LF}}{x_{HF} - x_{LF}} \tag{4}$$

$$b = \frac{y_{LF}x_{HF} - y_{HF}x_{LF}}{x_{HF} - x_{LF}} \tag{5}$$

$$\beta = y_{IF} + \frac{x_{IF}}{a} \tag{6}$$

Given the x and y values obtained the α parameter is computed as :

$$\alpha = \vec{OB} \cdot \frac{\vec{AC_{\perp}}}{\|\vec{AC}\|} \tag{7}$$

536 where

$$\vec{OB} = \begin{pmatrix} x_{IF} - x \\ y_{IF} - y \end{pmatrix} \tag{8}$$

$$\vec{AC} = \begin{pmatrix} x_{HF} - x_{LF} \\ y_{HF} - y_{LF} \end{pmatrix}$$
(9)

$$\vec{AC}_{\perp} = \begin{pmatrix} y_{LF} - y_{HF} \\ x_{HF} - x_{LF} \end{pmatrix}$$
(10)

537 9 Appendix 2

A correction for the high frequency noise was applied to the SIP data. The applied

procedure is the following:

$$\frac{\sigma^*(\omega) - \sigma_\infty}{\sigma_0 - \sigma_\infty} = \sum_{k=1}^N \frac{m_k}{1 + i\omega\tau_k},\tag{11}$$

where ω is the angular frequency (in mrad), equals to $2\pi f$, with frequency f, σ_{∞} is the amplitude of the conductivity at an infinite frequency, and σ_0 is the amplitude of the conductivity at a null frequency, and with $\tau_k = 1/(2\pi f_k)$, with f_k , k = 1, ..., N a sampling of the used frequency range.

546 Therefore:

$$\sigma^*(\omega) = \sigma_\infty + (\sigma_0 - \sigma_\infty) \sum_{k=1}^N \frac{m_k}{1 + i\omega\tau_k}.$$
(12)

This formulation does not take into account the divergence of the signature that can be observed at high frequencies (>100-1000 Hz), linked to a natural dielectric component or to electromagnetic coupling effects (e.g., Florsch et al., 2014). It is therefore necessary to add a term to equation 12 to account for this response, namely $i\omega C$, where C is a capacitance.

⁵⁵² On the other hand, it is impossible to determine independently $(\sigma_0 - \sigma_\infty)$ and m_k . ⁵⁵³ We propose then to replace equation 12 with:

$$\sigma^*(\omega) = \sigma_{\infty} + M \sum_{k=1}^{N} \frac{m_k}{1 + i\omega\tau_k} + i\omega C,$$
(13)

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where M is a positive constant.

The values of σ_{∞} , M, m_k for k = 1, ..., N and C are searched by optimization, using the simulated annealing algorithm (see Aarts, E.H.L., Van Laarhoven, 1985; Maineult, 2016, for its implementation), so as to minimize the fit coefficient between the spectrum predicted by equation 13 (calculated at frequencies $f_k, k = 1, ..., N$) and the whole set of measurements $[f_k, \sigma_k^*], k = 1, ..., N$.

- Once these parameters are determined, the measured spectrum corrected for high
- frequency disturbances is expressed by:

$$\tilde{\sigma_k^*} = \sigma_k^* - i\omega C \tag{14}$$

for k = 1, ..., N.