Solving Optimization Problems via Vortex Optimization Algorithm and Cognitive Development Optimization Algorithm

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Abstract

In the fields which require finding the most appropriate value, optimization became a vital approach to employ effective solutions. With the use of optimization techniques, many different fields in the modern life have found solutions to their real-world based problems. In this context, classical optimization techniques have had an important popularity. But after a while, more advanced optimization problems required the use of more effective techniques. At this point, Computer Science took an important role on providing software related techniques to improve the associated literature. Today, intelligent optimization techniques based on Artificial Intelligence are widely used for optimization problems. The objective of this paper is to provide a comparative study on the employment of classical optimization solutions and Artificial Intelligence solutions for enabling readers to have idea about the potential of intelligent optimization techniques. At this point, two recently developed intelligent optimization algorithms, Vortex Optimization Algorithm (VOA) and Cognitive Development Optimization Algorithm (CoDOA), have been used to solve some multidisciplinary optimization problems provided in the source book Thomas' Calculus 11th Edition and the obtained results have compared with classical optimization solutions.

Keywords: optimization, classical optimization, vortex optimization algorithm, cognitive development optimization algorithm, Artificial Intelligence.

1. Introduction

The concept of optimization is briefly defined as the choosing of the best set of alternatives in hand by also taking some rules into consideration.[1] When we examine optimization in the context of mathematics, we can say that a common optimization problem provides one or more functions to be optimized and besides the optimization process it requires meeting with some constraints while trying to reach optimum value(s). One cannot deny that the optimization has become an important solution approach especially for real world problems, ever since its first simple uses. Classical optimization techniques have provided a way of getting accurate enough results especially on complex problems of applied sciences. In time, many different fields in our life have been associated with the applications within optimization.

Like many other unstoppable changes in technologies (and even in the world), also the field of optimization has changed. Because more advanced optimization problems appeared with technological developments, classical optimization solutions became too weak to be employed. Especially, the requirements involved by advanced optimization problems enabled researchers / scientists to think about alternative solutions and thus, many different advanced optimization solutions have been introduced in time. If we look at the current associated literatures, it can be seen that Computer Science has an important role in improving the optimization field with software oriented techniques. In other words, Artificial Intelligence, which is a great sub-research field of

[1] INFORMS Computing Society, The nature of mathematical programming, Mathematical Programming Glossary. Online (Retrieved 10 June 2016):
http://glossary.computing.society.informs.org/ver2/mpgwiki/index.php?title=Extra:Mathematical_programming
MedLibrary. (2016). Mathematical optimization. Online (Retrieved 10 June 2016):
http://medlibrary.org/medwiki/Mathematical_optimization

Computer Science, currently occupies a remarkable place in optimization, with its intelligent approaches, methods, and techniques to solve advanced optimization problems.

The current literature of Artificial Intelligence about optimization is mostly associated with research works on Swarm Intelligence. With an increasing popularity especially in the last decade, the solutions ways introduced in Swarm Intelligence are generally characterized by a decentralized way of working mimicking behaviors / functions of swarms regarding to social insects, flocks of birds, or even schools of fish, and the most common advantages of these solution ways are robustness and flexibility against especially complex, advanced problems (Blum & Li, 2008). As a result of the successful results obtained with the techniques / algorithms in Swarm Intelligence, there has been a remarkable interest in this sub-field of Artificial Intelligence (in time, even different sub-research subjects on Swarm Intelligence have appeared). If we focus on the most recent literature, we can see that there are some techniques / algorithms, which are popular and widely used. These are: Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Artificial Bee Colony (ABC), Firefly Algorithm (FA), Bat Algorithm (BA), Cuckoo Search (CS). For more information about them, readers are referred to Blum & Li (2008), Bonabeau et al. (1999), Engelbrecht (2006), Kennedy et al. (2001), Kennedy (2011), Dorigo & Blum, (2005), Karaboga & Basturk (2008), Yang (2009), Yang & Deb (2009), Garnier et al. (2007), Rosenberg (2016), Foss (2016).

The objective of this paper is to provide a comparative study on employing classical optimization solutions and Artificial Intelligence based solutions for enabling readers to have an idea about the potential of intelligent optimization techniques. At this point, two recently developed intelligent optimization algorithms, Vortex Optimization Algorithm (VOA) and Cognitive Development Optimization Algorithm (CoDOA), have been used to solve some multidisciplinary optimization problems provided by the source book Thomas' Calculus 11th Edition, and the obtained results have been compared with classical optimization solution ways. By this, the authors aimed to show the effectiveness of two recent intelligent optimization techniques in optimization problems from a multidisciplinary perspective.

In the context of the objective of the paper, the organization of the remaining content is as follows: the next two sections are devoted to brief explanations of the employed techniques: Vortex Optimization Algorithm (VOA) and Cognitive Development Optimization Algorithm (CoDOA). Following that, the fourth section is related to applications on optimization problems in order to compare the potential of the related algorithms / techniques with classical optimization solution ways. Finally, the paper ends with conclusions and discussions about future works.

2. Vortex Optimization Algorithm

As developed by Kose and Arslan, Vortex Optimization Algorithm (VOA) is an intelligent optimization technique, which is inspired from vortex flows / behaviors in nature (Kose & Arslan, 2015). In this context, VOA tries to simulate some dynamics occurred in the context of vortex nature. Generally, the algorithm is a swarm intelligence based evolutional technique, which includes many methods of eliminating weak swarm members and improving the solution process by supporting the solution space via new swarm members having adaptive parameters.

The most recent algorithmic steps of the VOA can be explained briefly as follows (Kose & Arslan, 2015a; Kose et al., 2015):

- **Step 1:** Define initial parameters (*N* for number of particles; vorticity (*v*) values of each particle; max. and min. limits for vorticity value and other values related to function, problem…etc.; and *e* for elimination rate).
- **Step 2:** Locate the particles randomly within the solution space and calculate the fitness values for each of them. Update the v value of the particle with the best fitness value by using a random value. Mark this particle as a vortex and keep its values as the best so far.
- **Step 3:** Repeat the sub-steps below until the stopping criteria:

- **Step 3.1:** Mark each particle, whose fitness value is under the average fitness of all particles, as the vortex. The other – remaining particles are in the 'normal' particle status.
- **Step 3.2:** Update *v* value of each particle by using the Equation 1:
  *particlei_v_change = particlei_v + (random_value * (global_best_v / particlei_v))*
  particlei_v = particlei_v_change (1)
- **Step 3.3:** Update the *v* value of each vortex particle (except from the best particle so far) by using a random value.
- **Step 3.4:** Update the position of each particle (except from the best particle so far) by using the following equation (Equation 2):
  *particlei_position += (random_value * (particlei_v_change * (global_best_position – particlei_position)))* (2)
- **Step 3.5:** Calculate the fitness values according to the new positions of each particle. Mark the particle with the best value as a vortex (if it is not a vortex yet) and keep its values as the best so far.
- **Step 3.6:** If the number of non-vortex particles is under the value of *e*, remove all non-particles from the solution space and create new particles according to the number of removed particles. Locate these new particles randomly within the solution space. Perform in-system optimization in bigger problems.
- **Step 3.7:** Return to the Step 3.1. if the stopping criteria has not reached yet.

- **Step 4:** The best values obtained within the loop is the optimum solution.

A flow chart regarding the VOA is presented under Figure 1.

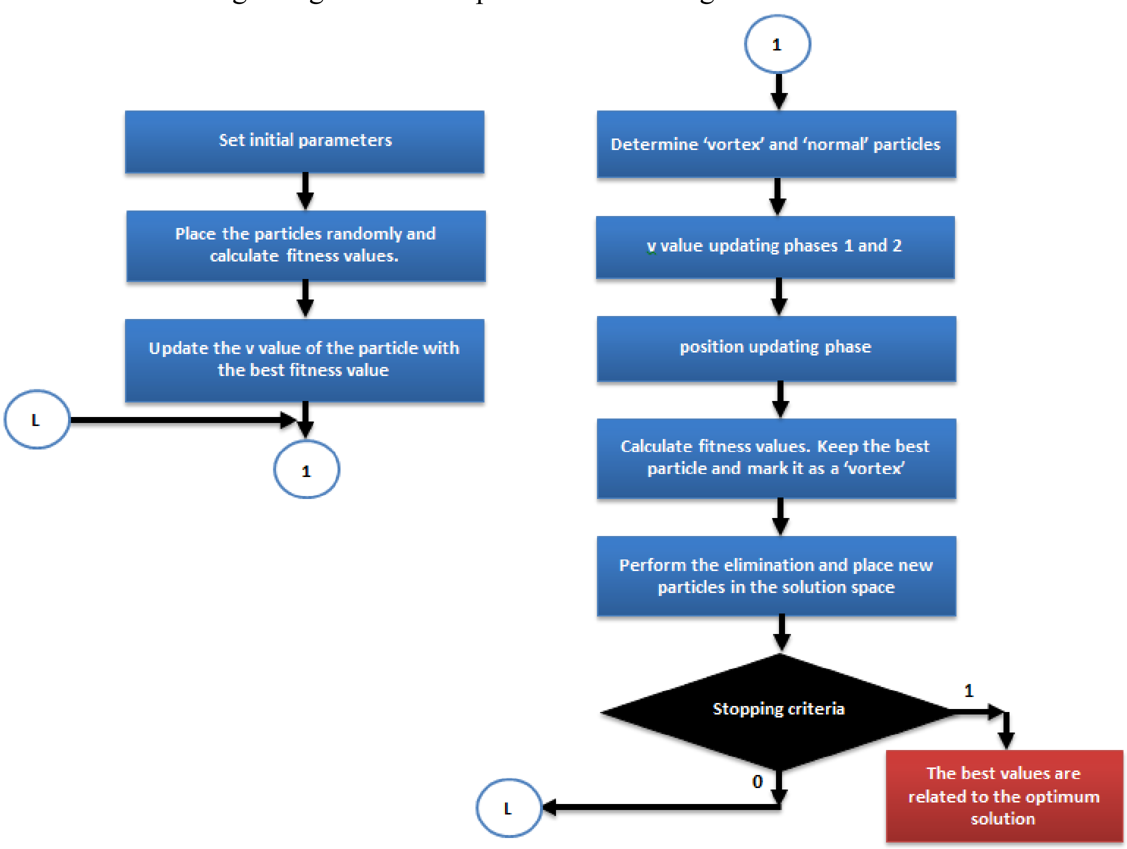


Figure 1. Flow chart of the VOA

3. Cognitive Development Optimization Algorithm

Cognitive Development Optimization Algorithm (CoDOA) is another intelligent optimization technique, which was introduced by Kose and Arslan (2015b). As similar to the VOA, CoDOA includes simple algorithmic steps and equations, which form a solution frame inspired from Piaget's Theory on Cognitive Development. Piaget's Theory explains that each individual goes through different stages of cognitive development like maturation, social interaction, balancing while learning new concepts and finally improving cognitive infrastructure (Kose & Arslan, 2015b; Piaget, 1964; Piaget, 1973; Singer & Revenson, 1997).

CoDOA has been built based on the following phases: Initialization Phase, Socialization Phase, Maturation Phase, Rationalizing Phase, and Balancing Phase. All of these phases are some kind of calculation steps, which have been formed as a result of inspirations from the stages of cognitive development. The phases are repeated until the stopping criterion is met (Kose & Arslan, 2015b).

Algorithmic steps of the CoDOAare as follows (Kose & Arslan, 2015b; Piaget, 1964; Piaget, 1973; Singer & Revenson, 1997; Kose & Arslan, 2016):

- **Step 1 (Initialization Phase):** Set initial parameters (*N*: number of particles; initial interactivity rate (*ir*) and experience (*ex*) values for each particle; max. and min. limits (min. limit is 0.0) for *ir* value (*max_ir* and *min_ir*); *ml* for the maturity limit; and *r* for the rationality rate.
  Also, set other values related to the function, problem…etc. (e.g. dimension, search domain…etc.).
- **Step 2:** Place the particles randomly in the solution space and calculate fitness values for each of them. Update the *ir* value of the particle with the best fitness value by using a random value (Equation 3).

$$best\_particle\_ir\_(new)=best\_particle\_ir\_(current)+(rand. * best\_particle\_ir\_(current)) \quad (3)$$

  Also, increase the *ex* value of this particle by 1.
- **Step 3:** Repeat the loop steps below until the stopping criterion (e.g. iteration number) is met:

  ○ **Step 3.1 (Socialization Phase):** Decrease (by 1) the *ex* value of each particle, whose fitness value is equal to or above the average fitness of all particles (if the problem is minimization).
  Also, increase (by 1) the *ex* value of each particle, whose fitness value is under the average fitness of all particles (if the problem is minimization). Finally, update the *ir* value of these particles by using a random value (Equation 4).

$$particle_j\_ir\_(new) = particle_j\_ir\_(current) + (rand. * particle_j\_ir\_(current)) \quad (4)$$

  ○ **Step 3.2:** Update the *ir* value of all particles by using the following equation (Equation 5):

$$particle_i\_ir\_(new) = rand. * particle_i\_ir\_(current) \quad (5)$$

  ○ **Step 3.3:** Update the position of each particle (except from the best particle so far) by using the Equation 6:

$$particle_i\_pos.\_(new) = particle_i\_pos.\_(current) + (rand. * (particle_i\_ir\_(current) * (global\_best\_pos. - particle_i\_pos.\_(current)))) \quad (6)$$

  ○ **Step 3.4:** Calculate fitness values according to the new position of each particle. Update the *ir* value of the particle with the better / best fitness value by using a random value (Equation 7).

$$best\_particle\_ir\_(new) = best\_particle\_ir\_(current) + (rand. * best\_particle\_ir\_(current)) \quad (7)$$

  Also, increase the *ex* value of this particle by 1.

  ○ **Step 3.5 (Maturation Phase):** Update the *ir* value of each particle, whose *ex* value is equal to or under the *ml* value by using the Equation 8:

$$particle_j\_ir\_(new) = particle_j\_ir\_(current) + (rand. * particle_j\_ir\_(current)) \quad (8)$$

  Calculate the fitness values according to the new position of each particle. Update the *ir* value of the particle with the better / best fitness value by using a random value (Equation 9).

*best_particle_ir_(new) = best_particle_ir_(current) + (rand. * best_particle_ir_(current))*(9)
Also, increase ex value of this particle by 1.

○**Step 3.6 (Rationalizing Phase):** Update *ir* and positions of each particle, whose *ex* value is under 0, by using the following equations:

$particle_j\_ir\_(new) = particle_j\_ir\_(current) + (rand. * (best\_particle\_ir\_(current) / particle_j\_ir\_(current)))$ (10)

$particle_i\_pos.\_(new) = particle_i\_pos.\_(current) + (rand. * (particle_i\_ir\_(current) * (global\_best\_pos. - particle_i\_pos.\_(current))))$ (11)

Update *ir* of each particle, whose *ex* value is equal to or above 0, and repeat this *r* times; by using the Equation 12:

$particle_j\_ir\_(new) = particle_j\_ir\_(current) + (rand. * (best\_particle\_ir\_(current) / particle_j\_ir\_(current)))$ (12)

○**Step 3.7 (Balancing Phase):** Update the *ir* value of all particles by using the Equation 13:

$particle_i\_ir\_(new) = rand. * particle_i\_ir\_(current)$ (13)

Calculate fitness values according to the new position of each particle. Update the *ir* value of the particle with the better / best fitness value by using a random value (Equation 14).

*best_particle_ir_(new) = best_particle_ir_(current) + (rand. * best_particle_ir_(current))*(14)

Also, increase the *ex* value of this particle by 1. For big problems, perform in-system optimization. Return to the Step 3.1. if the stopping criteria is not achieved yet.

▪ **Step 4:** The best values obtained within the loop are related to the optimum solution.

Figure 2 shows a flow chart regarding the CoDOA (Kose, & Arslan, 2015b).

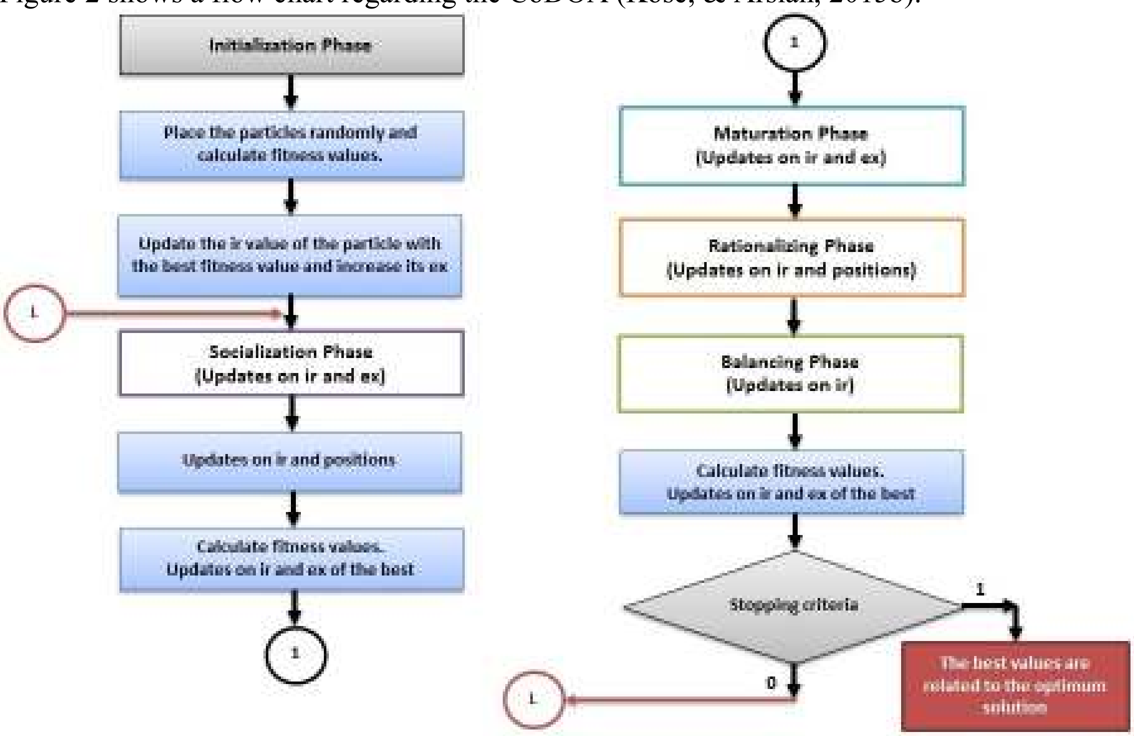


Figure 2.Flow chart of the CoDOA (Kose & Arslan, 2015b).

4. Solving Optimization Problems via VOA and CoDOA

The related problems solved here are from Thomas' Calculus 11th Edition (Thomas et al., 2005a). In order to see the success of the algorithms: VOA and CoDOA, the solutions offered by these algorithms have been compared with the solutions provided in the solution manual of Thomas Calculus book (Thomas et al., 2005b). In this context, a total of 10 problems have been taken into consideration. In order not to make the readability complex, the problems and their solutions with classical approaches are provided first, and then a comparison with the results provided by VOA and CoDOA are shown under a general table.

In order to understand more about the background of classical approaches, The First Derivative Theorem for Local Extreme Values and also some solution rules in this manner are expressed briefly in the following paragraphs as follows (Thomas et al., 2005a):

4.1. The First Derivative Theorem for Local Extreme Values

If f has a local maximum or minimum value at an interior point c of its domain, and if it is defined at c, then $f'(c)=0$.

4.1.1. The trapezoidal rule

In order to approximate $\int_a^b f(x)\,dx$, use $T=\frac{\Delta x}{2}\left(y_0+2y_1+2y_2+...+2y_{n-1}+y_n\right)$. The y's are the values of f at the partition points $x_0=a$, $x_1=a+\Delta x$, $x_2=a+2\Delta x$, …, $x_{n-1}=a+(n-1)\Delta x$, $x_n=b$, where $\Delta x=(b-a)/n$.

4.1.2. Simpson's rule

In order to approximate $\int_a^b f(x)\,dx$, use $S=\frac{\Delta x}{3}\left(y_0+4y_1+2y_2+4y_3+...+2y_{n-2}+4y_{n-1}+y_n\right)$. The y's are the values of f at the partition points $x_0=a$, $x_1=a+\Delta x$, $x_2=a+2\Delta x$, …, $x_{n-1}=a+(n-1)\Delta x$, $x_n=b$, The number n is even, and $\Delta x=(b-a)/n$.

4.2. Optimization Problems from Thomas' Calculus

The chosen problems from Thomas' Calculus 11th Edition and their solutions with classical approaches are as follows (problem definitions and necessary explanations regarding the solutions are taken directly from the source books in order to enable readers to get exact information from the original source) (Thomas et al., 2005a; Thomas et al., 2005b):

Problem – 1: "Piping oil from a drilling rig to a refinery":

A drilling rig 12 mi offshore is to be connected by pipe to a refinery onshore, 20 mi straight down the coast from the rig. If a underwater pipe costs $500,000 per mile and a land-based pipe costs $300,000 per mile, what combination of the two will render the least expensive connection?

Solution – 1:

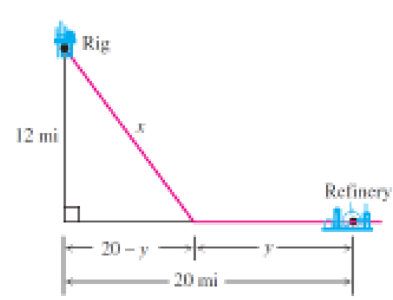


Figure 3.Problem – 1: "Piping oil from a drilling rig to a refinery" (Thomas et al., 2005a; Thomas et al., 2005b).

Now we introduce the length x of the underwater pipe and the length y of the land-based pipe as variables. The right angle opposite the rig is the key to expressing the relationship between x and y, for the Pythagorean Theorem gives:

$$x^2 = 12^2 + (20-y)^2$$
$$x = \sqrt{144+(20-y)^2}$$

Only the positive root has meaning in this model. The dollar cost of the pipeline is:
$c = 500,000x + 300,000y$.

To express c as a function of a single variable, we can substitute for x, using the following equation:
$$c(y) = 500,000\sqrt{144+(20-y)^2} + 300,000y$$

Our goal now is to find the minimum value of c(y) on the interval 0 ≤ y ≤ 20. The first derivative of c(y) with respect to y according to the Chain Rule is:
$$c'(y) = 500,000\frac{1}{2}\frac{2(20-y)(-1)}{\sqrt{144+(20-y)^2}} + 300,000$$
$$= -500,000\frac{(20-y)}{\sqrt{144+(20-y)^2}} + 300,000$$

Setting c’ equal to zero gives:
$$500,000(20-y) = 300,000\sqrt{144+(20-y)^2}$$
$$\Rightarrow \frac{5}{3}(20-y) = \sqrt{144+(20-y)^2}$$
$$\Rightarrow \frac{25}{9}(20-y)^2 = 144+(20-y)^2$$
$$\Rightarrow \frac{16}{9}(20-y)^2 = 144$$
$$\Rightarrow (20-y) = \pm\frac{3}{4}12 = \pm 9$$
$$\Rightarrow y = 20 \pm 9$$
$$\Rightarrow y = 11 \, or \, y = 29$$

Only y = 11 lies in the interval of interest. The values of c at this critical point and at the endpoints are:
$$c(11) = 10,800,000$$
$$c(0) = 11,661,900$$
$$c(20) = 12,000,000$$

The least expensive connection costs $10,800,000, and we achieve it by running the line underwater to the point on shore 11 mi from the refinery.

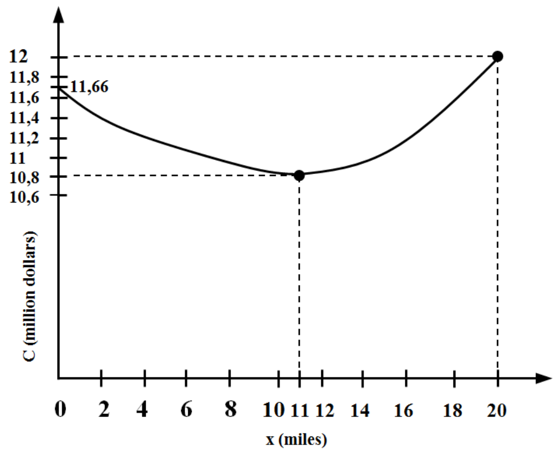


Figure 4.Problem – 1: "Piping oil from a drilling rig to a refinery" – solution (as drawn by the authors).

Problem – 2: "Constructing a pipeline":

Supertankers off-load oil at a dockingfacility 4 mi offshore. The nearest refinery is 9 mi east of theshore point nearest the docking facility. A pipeline must be constructedconnecting the docking facility with the refinery. The pipeline costs $300,000 per mile if constructed underwater and $200,000 per mile if constructed overland.

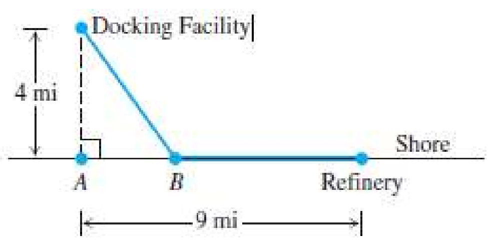


Figure 5.Problem – 2: "Constructing a pipeline" (Thomas et al., 2005a; Thomas et al., 2005b).

a. Locate Point *B* to minimize the cost of the construction.
b. The cost of underwater construction is expected to increase, whereas the cost of overland construction is expected to stay constant. At what cost does it become optimal to construct the pipeline directly to Point *A*?

Solution – 2:

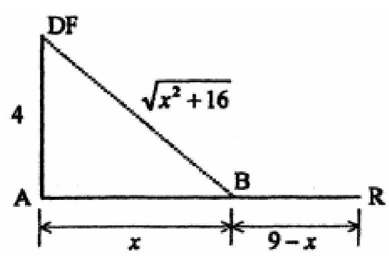


Figure 6.Problem – 2: "Constructing a pipeline" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

a. The construction cost is $C(x)=0.3\sqrt{16+x^2}+0.2(9-x)$ million dollars, where $0 \le x \le 9$ miles. The following is a graph of $C(x)$.

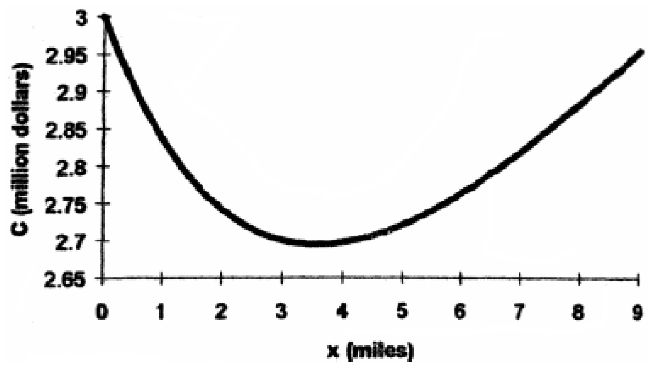


Figure 7.Problem – 2: "Constructing a pipeline" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Solving $C'(x)=\frac{0.3}{\sqrt{16+x^2}}-0.2=0$ gives $x=\pm\frac{8\sqrt{5}}{5}\approx\pm3.58$ miles, but only $x=3.58$ miles is a critical point is the specified domain. Evaluating the costs at the critical and endpoints gives $C(0)=\$3$ million, $C\left(\frac{8\sqrt{5}}{5}\right)\approx\$2.694$ million, and $C(9)\approx\$2.955$ million. Therefore, to minimize the cost of construction, the pipeline should be placed from the docking facility to point B, 3.58 miles along the shore from point A, and then along the shore from B to the refinery.

b. If the per mile cost of underwater construction is p, then $C(x)=p\sqrt{16+x^2}+0.2(9-x)$ and $C'(x)=\frac{px}{\sqrt{16+x^2}}-0.2=0$ gives $x_c=\frac{0.8}{\sqrt{p^2-0.04}}$, which minimizes the construction cost provided $x_c \le 9$. The value of p that gives $x_c=9$ miles is 0.218864. Consequently, if the underwater construction costs \$0.218864 per mile or less, then running the pipeline along a straight line directly from the docking facility to the refinery will minimize the cost of construction.
In theory, p would have to be infinite to justify running the pipe directly from the docking facility to point A (i.e., for x_c to be zero). For all values of $p > 0.218864$ there is always an $x_c \in (0,9)$ that will give a minimum value for C. This is proved by looking at $C''(x_c)=\frac{16p}{\left(16+x_c^2\right)^{3/2}}$ which is always positive for $p > 0$.

Problem – 3: "Upgrading a highway":

A highway must be constructed to connect Village *A* with Village *B*. There is a rudimentary roadway that can be upgraded 50 mi south of the line connecting the two villages. The cost of upgrading the existing roadway is of \$300,000 per mile, whereas the cost of constructing a new highway is of \$500,000 per mile. Find the combination between upgrading the existing construction and a new construction, that minimizes the cost of connecting the two villages. Clearly define the location of the proposed highway.

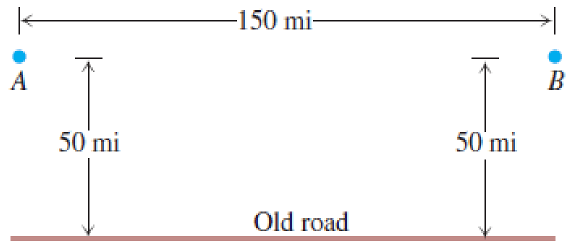
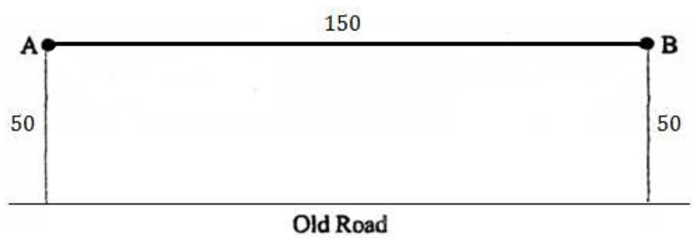


Figure 8.Problem – 3: "Upgrading a highway" (Thomas et al., 2005a; Thomas et al., 2005b).

Solution – 3:

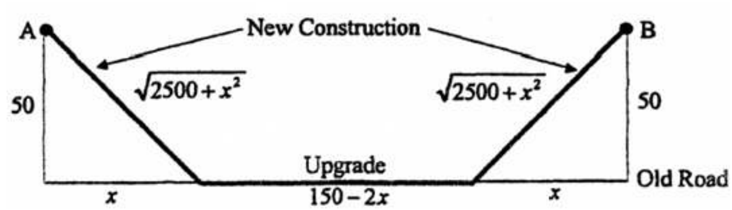
There are two options to consider. The first one is to build a new road straight from Village A to Village B. The second one is to build a new highway segment from Village A to the Old Road, reconstruct a segment of the Old Road, and build a new highway segment from the Old Road to Village B.



C_1

Figure 9. Problem – 3: "Upgrading a highway" – solution (as drawn by the authors).

The cost of the first option is $C_1 = 0.5(150)$ million dollars =75 million dollars.



C_2

Figure 10.Problem – 3: "Upgrading a highway" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

The construction cost for the second option is $C_2(x) = 0.5\left(2\sqrt{2500+x^2}\right) + 0.3(150-2x)$ million dollars for $0 \le x \le 75$ miles. A graph of $C_2(x)$ is shown in Figure 11.

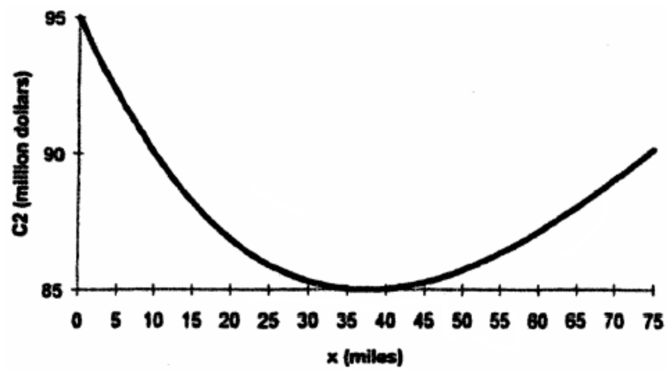


Figure 11.Problem – 3: "Upgrading a highway" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Solving $C_2'(x)=\dfrac{x}{\sqrt{2500+x^2}}-0.6=0$ give $x=\pm 37.5$ miles, but only $x=37.5$ miles is in the specified domain. In summary, $C_1=\$75$ million, $C_2(0)=\$95$ million, $C_2(37.5)=\$85$ million, and $C_2(75)=\$90.139$ million. Consequently, a new road straight from village A to village B is the least expensive option.

Problem – 4: "Locating a pumping station":

Two towns lie on the south side of a river. A pumping station is to be located to serve the two towns. A pipeline will be constructed from the pumping station to each of the towns along the line connecting the town and the pumping station. Locate the pumping station to minimize the amount of pipeline that must be constructed.

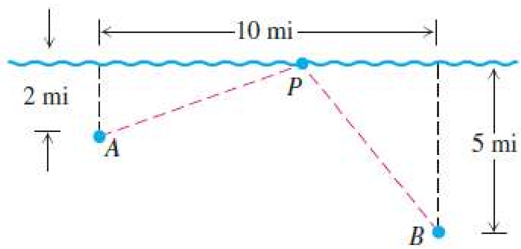


Figure 12.Problem – 4: "Locating a pumping station" (Thomas et al., 2005a; Thomas et al., 2005b).

Solution – 4

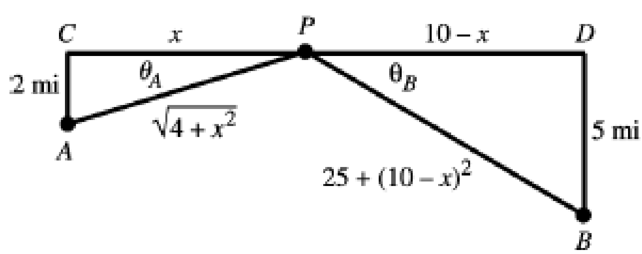


Figure 13.Problem – 4: "Locating a pumping station" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

The length of the pipeline is $L(x)=\sqrt{4+x^2}+\sqrt{25+(10-x)^2}$ for $0\le x\le 10$. A graph of $L(x)$ is shown in Figure 14.

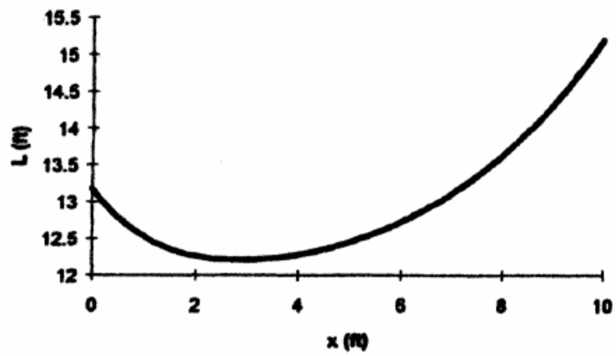


Figure 14.Problem – 4: "Locating a pumping station" – solution [22, 23].

Setting the derivative of $L(x)$ equal to zero gives $L'(x) = \frac{x}{\sqrt{4+x^2}} - \frac{(10-x)}{\sqrt{25+(10-x)^2}} = 0$. Note that $\frac{x}{\sqrt{4+x^2}} = \cos\theta_A$ and $\frac{(10-x)}{\sqrt{25+(10-x)^2}} = \cos\theta_B$ .Therefore, $L'(x) = 0$ when $\cos\theta_A = \cos\theta_B$, or $\theta_A = \theta_B$ and ΔACP is similar to ΔBDP. Use simple proportions to determine x as follows: $\frac{x}{2} = \frac{10-x}{5} \Rightarrow x = \frac{20}{7} \approx 2.857$ miles along the coast from town A to town B. If the two towns were on opposite sides of the river, the obvious solution would be to place the pump station on a straight line (the shortest distance) between two towns, again forcing $\theta_A = \theta_B$. The shortest length of pipe is the same regardless of whether the towns are on the same or opposite sides of the river.

Problem – 5: "The length of a guy wire":

One tower is 50 ft high and another tower is 30 ft high. The towers are 150 ft apart. A guy wire is to run from Point A to the top of each tower.

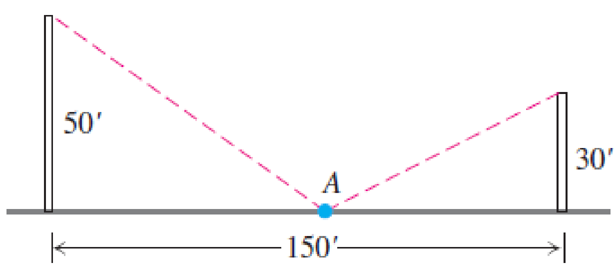


Figure 15. Problem – 5: "Length of a guy wire" (Thomas et al., 2005a; Thomas et al., 2005b).

a. Locate Point *A* so that the total length of guy wire is minimal.
b. Show in general that regardless of the height of the towers, the length of guy wire is minimized if the angles at *A* are equal.

Solution – 5:

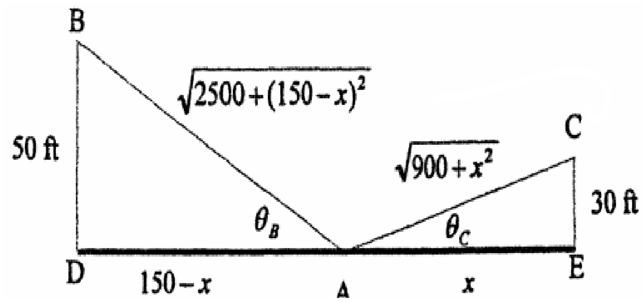


Figure 16. Problem – 5: "Length of a guy wire" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

a. The length of guy wire is $L(x)=\sqrt{900+x^2}+\sqrt{2500+(150-x)^2}$ for $0\le x\le 150$. A graph of $L(x)$ is shown in Figure 17.

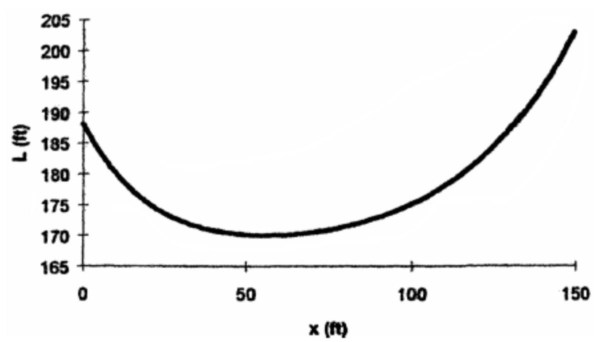


Figure 17. Problem – 5: "Length of a guy wire" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Setting $L'(x)$ equal to zero gives $L'(x)=\frac{x}{\sqrt{900+x^2}}-\frac{(150-x)}{\sqrt{2500+(150-x)^2}}=0$.

Note that $\frac{x}{\sqrt{900+x^2}}=\cos\theta_C$ and $\frac{(150-x)}{\sqrt{2500+(150-x)^2}}=\cos\theta_B$. Therefore, $L'(x)=0$ when $\cos\theta_C=\cos\theta_B$, or $\theta_C=\theta_B$ and ΔACE is similar to ΔABD. Use simple proportions to determine x as follows: $\frac{x}{30}=\frac{150-x}{50}\Rightarrow x=\frac{225}{4}=56.25$ feet.

b.

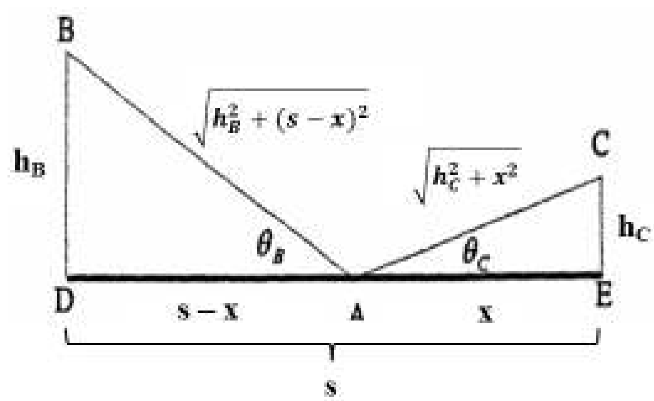


Figure 18. Problem – 5: "Length of a guy wire" – solution(as re-drawn by the authors).

If the heights of the towers are hB h_B and h_C, and the horizontal distance between them is s, then $L(x)=\sqrt{h_C^2+x^2}+\sqrt{h_B^2+(s-x)^2}$ and $L'(x)=\frac{x}{\sqrt{h_C^2+x^2}}-\frac{(s-x)}{\sqrt{h_B^2+(s-x)^2}}$.

However, $\frac{x}{\sqrt{h_C^2+x^2}}=\cos\theta_C$ and $\frac{(s-x)}{\sqrt{h_B^2+(s-x)^2}}=\cos\theta_B$. Therefore, $L'(x)=0$ when $\cos\theta_C=\cos\theta_B$, or $\theta_C=\theta_B$ and ΔACE is similar to ΔABD. Simple proportions can again be used to determine the optimum x: $\frac{x}{h_C}=\frac{s-x}{h_B}\Rightarrow x=\left(\frac{h_C}{h_B+h_C}\right)s$.

Problem – 6:"The area of an athletic field":

An athletic field is to be built in the shape of a rectangle *x* units long capped by semicircular regions of radius *r* at the two ends. The field is to be bounded by a 400-m racetrack.

a. Express the area of the rectangular portion of the field as a function of *x* alone or *r* alone (your choice).

b. What values of *x* and *r* give the rectangular portion the largest possible area?

Solution – 6:

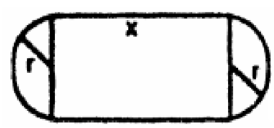


Figure 19.Problem – 6: "Area of an athletic field" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

a. From the diagram, the perimeter $P=2x+2\pi r=400\Rightarrow x=200-\pi r$. The area A is $2rx\Rightarrow A(r)=400r-2\pi r^2$ where $0\le r\le\frac{200}{\pi}$.

b. $A'(r)=400-4\pi r$ so the only critical point is $r=\frac{100}{\pi}$. Since $A(r)=0$ if $r=0$ and $x=200-\pi r=0$, the values $r=\frac{100}{\pi}\approx 31.83$ m and $x=100$ m maximize the area over the interval $0\le r\le\frac{200}{\pi}$.

Problem – 7:"The alternating current peak value":

Suppose that at any given time *t* (in seconds) the current *i* (in amperes) in an alternating current circuit is $i=2\cos t+2\sin t$. What is the peak value of current for this circuit (the largest magnitude)?

Solution – 7:

$\frac{dI}{dt}=-2\sin t+2\cos t$, solving $\frac{dI}{dt}=0\Rightarrow\tan t=1\Rightarrow t=\frac{\pi}{4}+n\pi$ where n is a nonnegative integer (in this exercise t is never negative) $\Rightarrow$ the peak value of current is $2\sqrt{2}$ amps.

Problem – 8:"Draining a swamp":

A town wants to drain and fill a small polluted swamp (Figure 20). The swamp has an average depth of 5 ft. About how many cubic yards of dirt will it take to fill the area after the swamp is drained?

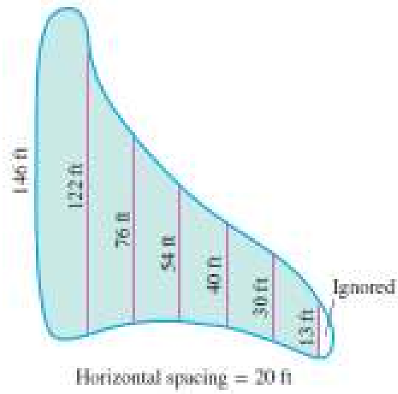


Figure 20.Problem – 8: "Draining a swamp" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Solution – 8:

To calculate the volume of the swamp, we estimate the surface area and multiply by 5. To estimate the area, we use Simpson's Rule with $\Delta x = 20$ ft and the y's equal to the distances measured across the swamp, as shown in Figure 20.

$$S = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right)$$

$$= \frac{20}{3}\left(146 + 488 + 152 + 216 + 80 + 120 + 13\right)$$

The volume is about $(8100)(5) = 40,500\ ft^3$ or $1500\ yd^3$

Problem – 9:"Stocking a fish pond":

As the fish and game warden of your township, you are responsible for stocking the town pond with fish before the fishing season. The average depth of the pond is 20 ft. Using a scaled map, the distances across the pond at 200-ft intervals are measured, as shown in the accompanying diagram.

a. Use the Trapezoidal Rule to estimate the volume of the pond.

b. You plan to start the season with one fish per 1000 cubic feet. You intend to have at least 25% of the opening day's fish population left at the end of the season. What is the maximum number of licenses the town can sell if the average seasonal catch is of 20 fish per license?

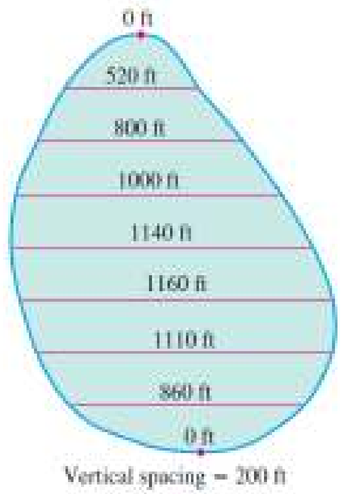


Figure 21.Problem – 9: "Stocking a fish pond" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Solution – 9:

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	200	520	2	1040
x_2	400	800	2	1600
x_3	600	1000	2	2000
x_4	800	1140	2	2280
x_5	1000	1160	2	2320
x_6	1200	1110	2	2220
x_7	1400	860	2	1720
x_8	1600	0	1	0

Figure 22.Problem – 9: "Stocking a fish pond" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

a. Using Trapezoid Rule, $\Delta x = 200 \Rightarrow \frac{\Delta x}{2} = \frac{200}{2} = 100$;

$\sum mf\left(x_i\right) = 13{,}180 \Rightarrow Area \approx 100\left(13{,}180\right) = 1{,}318{,}000\ ft^2$ Since the average depth = 20 ft we obtain Volume $\approx$20(Area) $\approx$26,360,000 ft^3.

b. Now, Number of fish $= \frac{Volume}{1000} = 26{,}360$ (to the nearest fish) $\Rightarrow$ Maximum to be caught =%75 of 26,360 = 19,770 $\Rightarrow$ Number of licenses $\frac{19{,}770}{20} = 988$.

Problem – 10:"Wing design":

The design of a new airplane requires a gasoline tank of constant cross-sectional area in each wing. A scale drawing of a cross-section is shown here. The tank must hold 5000 lb of gasoline, which has a density of 42 lb / ft^3. Estimate the length of the tank.

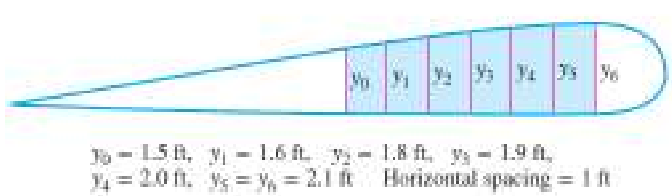


Figure 23.Problem – 10: "Wing design" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Solution – 10:

	x_i	y_i	m	my_i
x_0	0	1.5	1	1.5
x_1	1	1.6	4	6.4
x_2	2	1.8	2	3.6
x_3	3	1.9	4	7.6
x_4	4	2.0	2	4.0
x_5	5	2.1	4	8.4
x_6	6	2.1	1	2.1

Figure 24.Problem – 10: "Wing design" – solution (Thomas et al., 2005a; Thomas et al., 2005b).

Using Simpson's Rule, $\Delta x = 1 \Rightarrow \frac{\Delta x}{3} = \frac{1}{3}$; $\sum my_i = 33.6 \Rightarrow$ Cross Section Area $\approx \frac{1}{3}(33.6) = 11.2\ ft^2$. Let x be the length of the tank. Then the Volume V= (Cross Sectional Area) x=11.2x. Now 5000 lb of gasoline at 42 $lb/ft^3 \Rightarrow V = \frac{5000}{42} = 119.05\ ft^3 \Rightarrow 119.05 = 11.2x \Rightarrow x \approx 10.63\ ft^3$.

4.3. A comparison of Optimization Results

In addition to the solutions with classical approaches, the related algorithms (VOA and CoDOA) have been applied in each optimization problem.
Defined values for the parameters of VOA and CoDOAalong the solution processes are shown in Table 1.

Table 1.Defined values for the parameters of VOA and CoDOA.

VOA	**CoDOA**
Total number of particles (*N*): 50	**Total number of particles (*N*):** 50
Total iteration (the stopping criteria): 1500	**Total iteration (the stopping criteria):** 1500
Initial vorticity value: 0.50	**Initial interactivity rate:** 0.50
Max. vorticity value: 7.0	**Max. interactivity rate:** 10.0
Min. vorticity value: -7.0	**Maturity limit (*ml*):** 3
Elimination rate (*e*): 50	**Rationality rate (*r*):** 2

The obtained optimum values via different solution ways are provided briefly in Table 2.

Table 2.Comparison of results with classical optimization and intelligent optimization by VOA and CoDOA.

No.	Problem	Evaluated optimum valueswith;		
		Classical Optimization	**VOA**	**CoDOA**
1	**“Piping oil from a drilling rig to a refinery”**	11	11.00000023	11.00000025
2	**“Constructing a pipeline”**	**a.** 3.58 **b.** 0.218864	**a.** 3.57770878 **b.** 0.21886419	**a.** 3.57794999 **b.** 0.21886437
3	**“Upgrading a highway”**	C_1=150 C_2=37.5	C_1=150 C_2=37.50000060	C_1=150 C_2=37.50000058
4	**“Locating a pumping station”**	≈2.857	2.85714284	2.85714011
5	**“Length of a guy wire”**	**a.** 56.25 **b.** proved under the solution.	**a.** 56.25000174 **b.** keeping the angles equal, the function was always at minimum.	**a.** 56.25001853 **b.** keeping the angles equal, the function was always at minimum.
6	**“Area of an athletic field”**	**a.** out of evaluation concept. **b.** ≈31.83; 100	**a.** out of evaluation concept. **b.** 31.83010059; 100.00001703	**a.** out of evaluation concept. **b.** 31.83010968; 100.00001599
7	**“Peak alternating current”**	2.83	2.82986004	2.82979951
8	**“Draining a swamp”**	(distances across the swamp) 146; 122; 76; 54; 40; 30; 13	(distances across the swamp) 145.98660207; 122.00032280; 75.75901488; 53.99980502; 40.00000118; 30.11089200; 13.00651000	(distances across the swamp) 145.96803305; 122.00001999; 76.00739888; 53.99000855; 40.00099502; 29.87605209; 13.00788033
9	**“Stocking a fish pond”**	**a.** (distances across the pond) 0; 520; 800; 1000; 1140; 1160; 1110; 860; 0 **b.** max. lic.= 988	**a.** (distances across the pond) 0.00000000; 520.00003122; 800.00010055; 1000.01000180; 1139.99866225; 1160.01330288; 1109.78908801; 860.00012000; 0.00000000	**a.** (distances across the pond) 0.00000000; 520.00003122; 800.00010055; 1000.01000180; 1139.99866225; 1160.01330288; 1109.78908801; 860.00012000; 0.00000000

			b. max. lic.= 988	**b.** max. lic.= 988
10	**"Wing design"**	(distances across the wing / tank) 1.5; 1.6; 1.8; 1.9; 2.0; 2.1; 2.1	(distances across the wing / tank) 1.50002333; 1.59999850; 1.80011019; 1.90000111; 1.99968005; 2.10003689; 2.10003444	(distances across the wing / tank) 1.50111088; 1.60000199; 1.79983001; 1.90044608; 1.97688903; 2.09999809; 2.10099985

5. Conclusions and future work

This paper has provided a comparative study on classical optimization solutions and Artificial Intelligence based intelligent solutions for some optimization problems in a multidisciplinary manner. In order to achieve that, the authors have used two recently introduced intelligent optimization algorithm / techniques, Vortex Optimization Algorithm (VOA) and Cognitive Development Optimization Algorithm (CoDOA), on some optimization problems from the source book, Thomas' Calculus 11th Edition, and compared the obtained results with the ones rendered by the classical optimization. The obtained results show that the Artificial Intelligence and the techniques introduced under this important field have an effective role on providing desired optimization results. In detail, the employment of Artificial Intelligence (and so intelligent solutions) is also able to reduce calculation times and efforts spent to solve more advanced optimization problems, thanks to the power of computers and mathematically – logically improved solution steps. The authors believe that this paper is also a reference for enabling readers to have an idea about the potential of intelligent optimization techniques and their role on multidisciplinary cases.

In addition to the applications reported here, there are also some future works especially on the employment of VOA and CoDOA on different optimization problems. In detail, the authors plan to use the related techniques on solving more advanced optimization problems in mathematics and comparing the obtained results with some other strong solution ways and even alternative Artificial Intelligence techniques. On the other hand, there are also some other works to use VOA and CoDOA for designing hybrid models under a multidisciplinary perspective, including many different fields needing more effective and efficient solutions.

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