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A SIMPLE DEFINITION OF A DISTRIBUTED SLACK BUS FOR NETWORK ANALYSIS

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Abstract. The most important and oldest challenge in power system analysis is to solve the load flow problem. Simple examples are presented here to illustrate the old idea of distributed Slack-bus. The load flow equations are shown to include distributed bus slack. The Newton-Raphson equations for load flow have been rewritten in a matrix format as well. Distributed slack bass is demonstrated with a 5-bus system. Reducing power system losses is its most important advantage.

Keywords: Load flow, Economic dispatch, Slack bus, Distributed Slack bus.

چکیده: حل پخش بار سیستم قدرت مهم‌ترین و قدیمی‌ترین چالش در تحلیل سیستم‌های قدرت به شمار می‌آید. در این مقاله ایده قدیمی اسلک باس توزیع شده با مثالهایی ساده ارائه شده است. نشان داده شده است چگونه باید اسلک باس توزیع شده را در معادلات پخش بار اضافه کرد. همچنین معادلات پخش بار نیوتن رافسون به صورت ماتریسی دوباره بازنویسی شده‌اند. با یک سیستم ۵ باسه، مزایای اسلک باس توزیع شده نشان داده شده است. مهم‌ترین مزیت آن کاهش تلفات سیستم قدرت است.

کلمات کلیدی: پخش بار، پخش بار اقتصادی، اسلک باس، اسلک باس توزیع شده

1- Introduction

The slack bus-based power distribution to balance the system is distributed into a set of generating units, referred to here as economic dispatch (ED) units. Generation scheduling to compensate for the power imbalance can be done in a variety of ways. Suggested in this paper is to use participation factors-based economic dispatch [1,2]. It is shown that the net system power imbalance resulting from each transaction is a function of all transactions present in the system. Allocating the net power imbalance among different ED-generating units according to the participation factors can compensate for the power imbalance caused by a particular transaction.

The flows are decomposed into components associated with each particular transaction, and interaction components based on the superposition of all transactions. The interaction component accounts for the non-linearity of the power flow equations. The interaction component of all transactions on the system cannot be associated with a single transaction. Moreover, only a tiny percentage of a given transaction contributes to this interaction component. This decomposition forms an essential part of our framework for recovering fixed costs based on actual network power flows.

This paper is organized as follows: First, mathematical formulae for the decomposition of power flows are derived assuming a single slack bus. Next, the concept of a distributed slack bus is introduced to account for the fact that many generators are participating in ED generation when balancing the power on the system. The derived decomposition formulae are generalized to account for the ramifications of the distributed slack bus in our proposed approach. Using the derived decomposition formulae, the power imbalance at the ED generating units is calculated.

The latest references to the application of load flow to the determination of available transmission capability (ATC) are included at the end [3-15].

2- Notation

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- Bus enumeration: i=1,2,...,n nodes, node "1" is the slack bus, and node "2" for the distributed slack bus.
- $\hat{V}_i$: The complex-valued voltage at nodes "i", and $\hat{\mathbf{V}}_i = [\hat{V}_1 \cdots \hat{V}_n]^T$ is a vector of all node voltages in the system. Note that a boldface font is used for vector symbols. $\hat{V}_i = V_i e^{j\delta_i}$, i=1,2...,n. i=1,...,m generator and i=m+1,...,n are load buses.
- $\hat{I}_i$: Complex valued current at nodes i and $\hat{\mathbf{I}}_i = [\hat{I}_1 \cdots \hat{I}_n]^T$ is a vector of all node currents in the system.
- Q_{gi} : The reactive power generated at node i, i=1,...,n.
- Q_{li} : The reactive power consumed at node i, i=1,...,n.
- P_{gi} : The active power generated at node i, i=1,...,n.
- P_{li} : The active power consumed at node i, i=1,...,n.
- $Q_{\max i}$: The maximum reactive power generated at node i, i=2,...,m.
- $Q_{\min i}$: The minimum reactive power generated at node i, i=2,...,m.
- V_{0i} : The initial absolute voltage bus i, i=1,...,n.
- δ_{0i} ; The initial angle voltage bus i, i=1,...,n.
- V_{si} : The specified voltages for the control voltage bus i, i=1,...,m.
- $S_i = P_i + jQ_i$: Net complex valued power generated at node i, P_i is the real power, Q_i is the reactive power part and $S=[S_1 \ldots S_n]^T$ is a vector of all nodes complex valued power in the system.
- $y_s(i,j)$: Admittance of series branch connecting nodes i and j. Y_s is the series admittance matrix.
- $y_p(i,j)$: Admittance of parallel branch connecting nodes i and j. Y_p is the parallel admittance matrix.
- $ones(1,n) = [1 \cdots 1]_{1\times n}$
- $diag(\hat{V}) = diagonal(\hat{V}) = \begin{bmatrix} \hat{V}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}_n \end{bmatrix}$

2- The Newton-Raphson method for the single slack bus

The elements of the bus admittance matrix, Y_{bus} are:

$$y_{bus}(i,j) = -y_s(i,j) \quad i \neq j , \qquad y_{bus}(i,i) = \sum_{j=1}^{n} \left(y_s(i,j) + y_p(i,j) \right) \tag{1}$$

Equation (1) is rewritten explicitly in terms of the matrix component of the form:

$$Y_{bus} = -Y_S + diag\left((Y_S + Y_P) \times ones(n,1) \right) \tag{2}$$

Then we are going to show the Jacobian matrix:

$$P_i = \sum_{j=1}^{n} V_i V_j \left| Y_{ij} \right| \cos(\delta_i - \delta_j - \gamma_{ij}) \triangleq f_{Pi}(V,\delta) \Rightarrow P_i = V_i^2 G_{ii} + \sum_{j=1 \neq i}^{n} V_i V_j \left| Y_{ij} \right| \cos(\delta_i - \delta_j - \gamma_{ij})$$

$$\Rightarrow P_i = V_i^2 G_{ii} + V_i \sum_{j=1\neq i}^{n} V_j (G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)) \tag{3-1}$$

$$Q_i = \sum_{j=1}^{n} V_i V_j |Y_{ij}| \sin(\delta_i - \delta_j - \gamma_{ij}) \triangleq f_{Qi}(V, \delta) \quad \Rightarrow Q_i = -V_i^2 B_{ii} + \sum_{j=1\neq i}^{n} V_i V_j |Y_{ij}| \sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\Rightarrow Q_i = -V_i^2 B_{ii} + V_i \sum_{j=1\neq i}^{n} V_j (G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)) \tag{3-2}$$

If the Jacobian matrix is shared to four submatrix J1, J2, J3 and J4, we have:

$$J_{1ij} = \frac{\partial f_{Pi}}{\partial \delta_j},\ J_{2ij} = \frac{\partial f_{Pi}}{\partial V_j},\ J_{3ij} = \frac{\partial f_{Qi}}{\partial \delta_j},\ J_{4ij} = \frac{\partial f_{Qi}}{\partial V_j} \Rightarrow \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \tag{4}$$

As a result of differentiation from (4), we have:

$$\begin{aligned}
J_{1ii} &= \frac{\partial f_{Pi}}{\partial \delta_i} = -V_i \sum_{k=1\neq i}^{n} V_k . |Y_{ik}| \sin(\delta_i - \delta_k - \gamma_{ik}) = -f_{Qi} - V_i^2 B_{ii} \simeq -Q_i - V_i^2 B_{ii} \\
J_{2ii} &= \frac{\partial f_{Pi}}{\partial V_i} = 2V_i |Y_{ii}| \cos(\gamma_{ii}) + \sum_{k=1\neq i}^{n} V_k |Y_{ik}| \cos(\delta_i - \delta_k - \gamma_{ik}) = \frac{f_{Pi}}{V_i} + V_i G_{ii} \simeq \frac{P_i}{V_i} + V_i G_{ii} \\
J_{3ii} &= \frac{\partial f_{Qi}}{\partial \delta_i} = V_i \sum_{k=1\neq i}^{n} V_k . |Y_{ik}| \cos(\delta_i - \delta_k - \gamma_{ik}) = f_{Pi} - V_i^2 G_{ii} \simeq P_i - V_i^2 G_{ii} \\
J_{4ii} &= \frac{\partial f_{Qi}}{\partial V_i} = -2V_i |Y_{ii}| \sin(\gamma_{ii}) + \sum_{k=1\neq i}^{n} V_k |Y_{ik}| \sin(\delta_i - \delta_k - \gamma_{ik}) = \frac{f_{Qi}}{V_i} - V_i B_{ii} \simeq \frac{Q_i}{V_i} - V_i B_{ii} \\
J_{1ik} &= \frac{\partial f_{Pi}}{\partial \delta_k} = V_i V_k . |Y_{ik}| \sin(\delta_i - \delta_k - \gamma_{ik}) = a_{ik} f_i - b_{ik} e_i \\
J_{2ik} &= \frac{\partial f_{Pi}}{\partial V_k} = V_i . |Y_{ik}| \cos(\delta_i - \delta_k - \gamma_{ik}) = \frac{a_{ik} e_i + b_{ik} f_i}{V_k} = \frac{a_{ik} e_i + b_{ik} f_i}{\sqrt{e_k^2 + f_k^2}} \\
J_{3ik} &= \frac{\partial f_{Qi}}{\partial \delta_k} = -V_i V_k . |Y_{ik}| \cos(\delta_i - \delta_k - \gamma_{ik}) = -a_{ik} e_i - b_{ik} f_i \\
J_{4ik} &= \frac{\partial f_{Qi}}{\partial V_k} = V_i . |Y_{ik}| \sin(\delta_i - \delta_k - \gamma_{ik}) = \frac{a_{ik} f_i - b_{ik} e_i}{V_k} = \frac{a_{ik} f_i - b_{ik} e_i}{\sqrt{e_k^2 + f_k^2}}
\end{aligned} \tag{5}$$

S is a vector of all nodes' complex valued power in the system, Y_{bus} is the admittance matrix and **I** is a vector of all node currents in the system. We have:

$$S = diag(\hat{V}).\hat{I}^*, \hat{I} = Y_{bus}.\hat{V} \Rightarrow S = diag(\hat{V}) Y_{bus}^* . \hat{V}^* = [P\,Q]^T \tag{6}$$

Now we define the Jacobian matrix:

$$J_1 = \frac{\partial f_P}{\partial \delta},\ J_2 = \frac{\partial f_P}{\partial V},\ J_3 = \frac{\partial f_Q}{\partial \delta},\ J_4 = \frac{\partial f_Q}{\partial V} \tag{7}$$

Equation (7) can be rewritten as:

$$J_1 = \text{Real}\frac{\partial S}{\partial \delta},\ J_2 = \text{Real}\frac{\partial S}{\partial V},\ J_3 = \text{Imag}\frac{\partial S}{\partial \delta},\ J_4 = \text{Imag}\frac{\partial S}{\partial V} \tag{8}$$

As a result of differentiation from (6), we have:

$$\frac{\partial S}{\partial \delta} = \frac{\partial}{\partial \delta}(diag(\hat{V})).diag(Y_{bus}^* . \hat{V}^*) + diag(\hat{V}).Y_{bus}^* . \frac{\partial}{\partial \delta}(\hat{V}^*) \tag{9}$$

$$\frac{\partial S}{\partial V} = \frac{\partial}{\partial V}(diag(\hat{V})).diag(Y_{bus}^{*}.\hat{V}^{*}) + diag(\hat{V}).Y_{bus}^{*}.\frac{\partial}{\partial V}(\hat{V}^{*}) \tag{10}$$

Then:

$$\frac{\partial}{\partial \delta}(diag(\hat{V})) = j\, diag(\hat{V}), \quad \frac{\partial}{\partial V}(diag(\hat{V})) = diag(e^{j\delta}) \tag{11}$$

$$\frac{\partial}{\partial \delta}(diag(\hat{V}^{*})) = -j\, diag(\hat{V}^{*}), \quad \frac{\partial}{\partial V}(diag(\hat{V}^{*})) = diag(e^{-j\delta}) \tag{12}$$

Or:

$$\frac{\partial S}{\partial \delta} = j\, diag(\hat{V}).\left(diag(Y_{bus}^{*}.\hat{V}^{*}) - Y_{bus}^{*}.diag(\hat{V}^{*})\right) \tag{13}$$

$$\frac{\partial S}{\partial V} = diag(e^{j\delta}).diag(Y_{bus}^{*}.\hat{V}^{*}) + diag(\hat{V}).Y_{bus}^{*}.diag(e^{-j\delta}) \tag{14}$$

Equations (13) and (14) show the Jacobian matrix. For the load flow, we have to calculate the AX=B equation such that A, B, and X can be shown as:

$$A = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}, \; X = \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}, \; B = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{15}$$

Firstly, we have to calculate ΔP and ΔQ. We can obtain these values from this formula:

$$\Delta P = P_g - P_L - \mathrm{Real}\left(diag(\hat{V}).Y_{bus}^{*}.\hat{V}^{*}\right) \tag{16}$$

$$\Delta Q = Q_g - Q_L - \mathrm{Imag}\left(diag(\hat{V}).Y_{bus}^{*}.\hat{V}^{*}\right) \tag{17}$$

For the load flow, we can use:

X=A\B (18)

3- The distributed slack bus

Electrical power systems do not operate as slack buses, which are mathematical artifacts. Instead, there exists a relatively small number of generating units designated as load-following units. These units participate in load frequency control (LFC) and automatic generation control (AGC) with the purpose of balancing the interconnected systems in response to demand uncertainties. Effectively they manage to maintain the system frequency at 60 HZ. These units are distributed at various geographical locations in the system. They are necessary in order to make a transaction feasible without degrading the quality of supply and reliability of the system. The amount of real power imbalance in the system is distributed among these units based on participation factors, Ki. These are determined based on combined cost and reliability criteria. They all add up to unity. $\sum ki = 1.00$

There may be a difference between the units used for steady-state loss compensation and the units participating in AGC. Distributed slack buses can be modeled similarly to how participation factors are used for AGC. Following are the equations that modify the Jacobian of the system.

$$\begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_n \\ -- \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \Delta\delta_n \\ -- \\ \Delta V_3 \\ \vdots \\ \Delta V_n \end{bmatrix}, \quad \begin{matrix} \Delta P_2 = \dfrac{K_1}{K_2}\Delta P_2 \\ \Delta V_1 = \Delta V_2 = \Delta\delta_1 = 0 \end{matrix} \tag{19}$$

4- Numerical example

In this section, we present numerical results that illustrate our theoretical analysis. First, we study a simple five-bus system with a single Slack bus. As a second example, we study the same system with a generator that acts as a second Slack bus. We show the major flows and interaction (minor) components in the two cases.

4-1- Single slack bus

In this section, a simple five-bus system given in Figure (1) is used as a numerical example of the above mathematical formulation using a single slack bus. This example helps in understanding different complex issues pertaining to network interaction. MATLAB was used to achieve the following results. Data for the five bus systems is given in Table (1).

Table 1. Transmission lines parameters

From	To	R	X	From	To	R	X
5	1	0.01	0.10	3	1	0.01	0.10
5	3	0.01	0.10	1	2	0.20	2.00
3	4	0.20	2.00	4	2	0.01	0.10

Load flow system variables can be found in Table (2). All values are expressed as per-unit values. Bus #1 is the Slack bus. There are two scheduled transactions in the system. The first is from generator bus #3 to load bus #4, the contract is for 100 MW, 1.0 per unit. The second is from generator bus #2 to load bus #5, the contract is for 120 MW and 50 MVAr, and 1.20 and 0.5 per unit. Table 2 shows the system variables for all scheduled transactions. In this case, the solution is achieved using a simple load flow since there is only one ED unit, i.e., the Slack bus. Note that the active power imbalance at the Slack bus reflects only the losses since there is no mismatch between load and generation in both transactions.

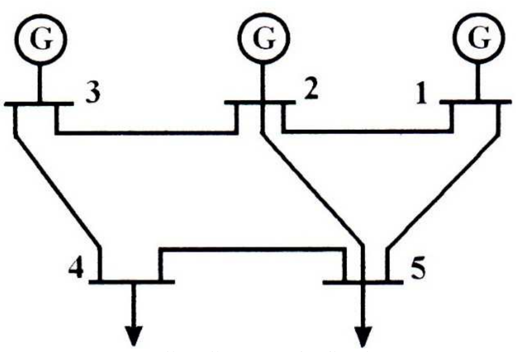

Fig. 1. One line diagram of a five bus system

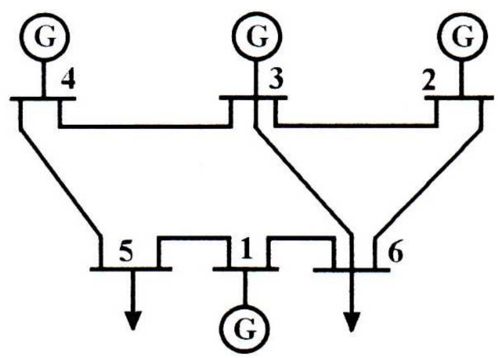

Fig. 2. One line diagram of a six bus system

Table 2. Vectors of complex valued power, current and voltage for all transactions.

Bus	S	I	V
1	0.02+0.05j	0.02+0.05j	1.0000
2	1.2+0.57j	1.2+0.52j	1.0193+0.0376j
3	1.00+0.10j	1.00+0.06j	0.9993+0.0387j
4	-1.00+0.00j	-1.01-0.06j	0.9825-0.0610j
5	-1.20-0.50j	-1.21-0.56j	0.9757-0.0393j

4-2- Distributed slack bus

The same transactions and transmission system parameters are used here as in the previous example. However, we have added another generator between buses 4 and 5, as in Figure 2. The new bus is considered an ED generator with no scheduled generation.

Table 3. Vectors of complex valued power, current and voltage for all transactions.

Bus	S	I	V
1	0.0054+0.0358j	0.0054+0.0358j	1.0000
2	0.0162+0.0382j	0.0180+0.0374j	0.9988+0.0485j
3	1.20+0.5643j	1.2195+0.4507j	1.0163+0.0872j
4	1.00+0.0774j	1.0032-0.0131j	0.9959+0.0902j
5	-1.00+0.0006j	-1.0141-0.0098j	0.9862-0.010lj
6	-1.20-0.4999j	-1.2319-0.5010j	0.9775+0.0083j

Table 3 shows the load flow solution for the system using participation factors. Generator #2 assigned a participation factor of 75% while Generator #1 assigned a participation factor of 25%. The real power losses of the system are lower than those of a single Slack bus, as expected. This means less operating costs and higher efficiency but it does not affect fixed cost recovery.

5- Conclusions

The paper's approach is further generalized to resemble the actual operation of power systems with distributed slack buses or many generation units participating in the ED. Second, the net power imbalance caused by each transaction can also be identified as a function of all transactions on the system. However, there exists a degree of freedom in dividing this imbalance among different generation units participating in the ED. Using the derivations introduced in this paper, one can calculate the power flows and imbalances on the network subject to each given economic transaction.

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