

Classic Filters

There are 4 classic analogue filter types: Butterworth, Chebyshev, Elliptic and Bessel. There is no ideal filter; each filter is good in some areas but poor in others.

- **Butterworth:** Flattest pass-band but a poor roll-off rate.
- **Chebyshev:** Some pass-band ripple but a better (steeper) roll-off rate.
- **Elliptic:** Some pass- and stop-band ripple but with the steepest roll-off rate.
- **Bessel:** Worst roll-off rate of all four filters but the best phase response. Filters with a poor phase response will react poorly to a change in signal level.

Butterworth

The first, and probably best-known filter approximation is the **Butterworth** or **maximally-flat** response. It exhibits a nearly flat passband with no ripple. The rolloff is smooth and monotonic, with a low-pass or high-pass rolloff rate of 20 dB/decade (6 dB/octave) for every pole. Thus, a 5th-order Butterworth low-pass filter would have an attenuation rate of 100 dB for every factor of ten increase in frequency beyond the cutoff frequency. It has a reasonably good phase response.

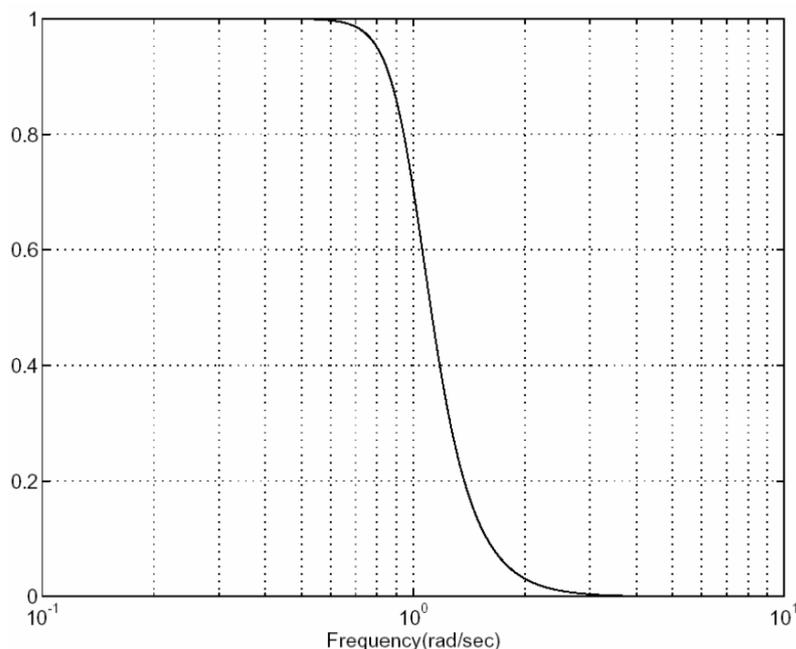


Figure 1 Butterworth Filter

Chebyshev

The Chebyshev response is a mathematical strategy for achieving a faster *roll-off* by allowing *ripple* in the frequency response. As the ripple increases (bad), the roll-off becomes sharper (good). The Chebyshev response is an optimal trade-off between these two parameters. Chebyshev filters where the ripple is only allowed in the *passband* are called **type 1** filters. Chebyshev filters that have ripple only in the *stopband* are called type 2 filters, but are seldom used. Chebyshev filters have a poor phase response.

It can be shown that for a passband flatness within 0.1dB and a stopband attenuation of 20dB an 8th order Chebyshev filter will be required against a 19th order Butterworth filter. This may be important if you are using a lower specification processor.

The following figure shows the frequency response of a lowpass Chebyshev filter.

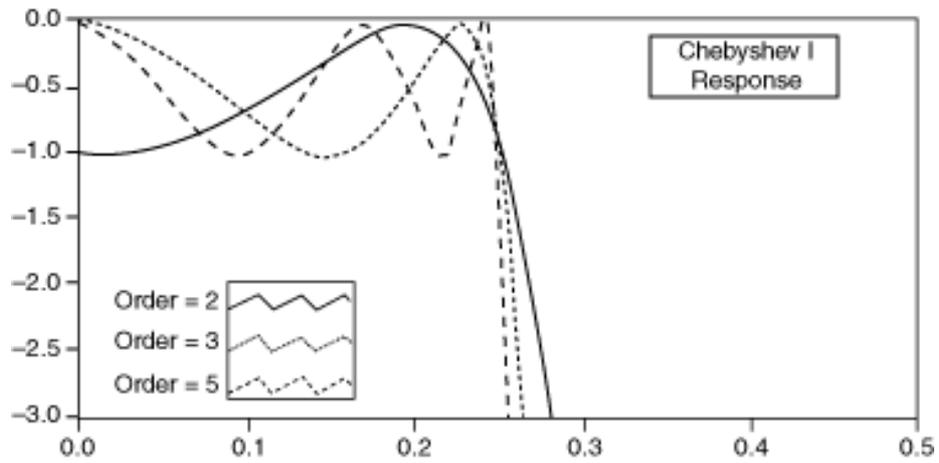


Figure 2

Compared to a Butterworth filter, a Chebyshev filter can achieve a sharper transition between the passband and the stopband with a lower order filter. The sharp transition between the passband and the stopband of a Chebyshev filter produces smaller absolute errors and faster execution speeds than a Butterworth filter.

The following figure shows the frequency response of a lowpass Chebyshev II filter.

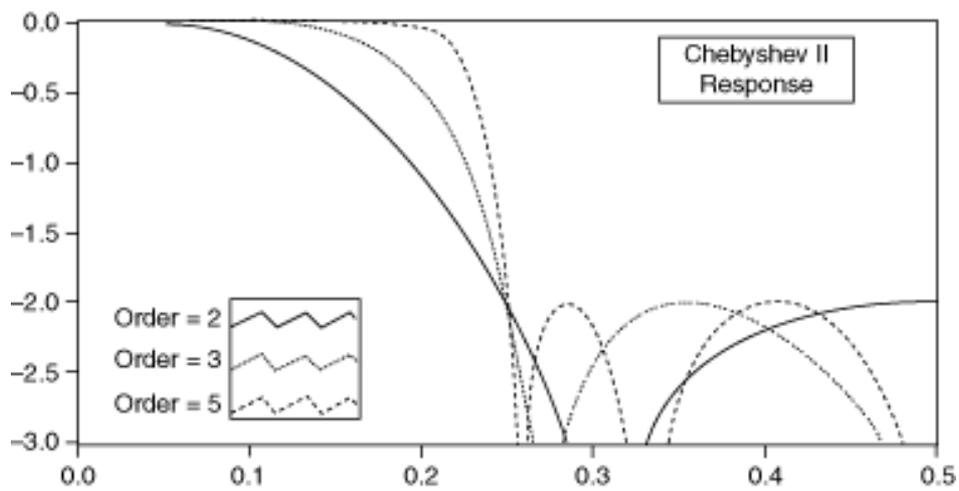


Figure 3

Chebyshev II filters have the same advantage over Butterworth filters that Chebyshev filters have—a sharper transition between the passband and the stopband with a lower order filter, resulting in a smaller absolute error and faster execution speed.

Elliptic

The cut-off slope of an **elliptic** filter is steeper than that of a Butterworth, Chebyshev, or Bessel, but the amplitude response has ripple in both the passband and the stopband, and the phase response is very non-linear. However, if the primary concern is to pass frequencies falling within a certain frequency band and reject frequencies outside that band, regardless of phase shifts or ringing, the elliptic response will perform that function with the lowest-order filter.

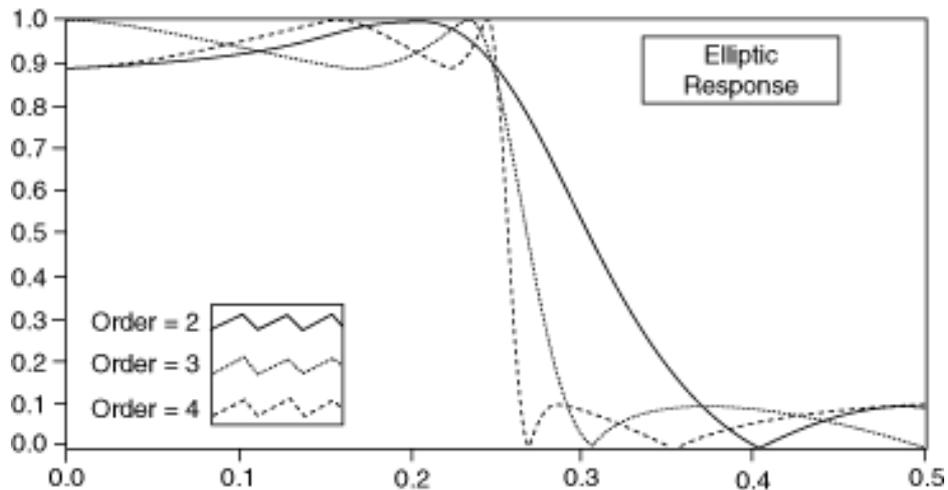


Figure 4

Compared with the same order Butterworth or Chebyshev filters, the elliptic filters provide the sharpest transition between the passband and the stopband, which accounts for their widespread use.

Bessel

- Maximally flat response in both magnitude and phase
- Nearly linear-phase response in the passband

You can use Bessel filters to reduce nonlinear-phase distortion inherent in all IIR filters. High-order IIR filters and IIR filters with a steep roll-off have a pronounced nonlinear-phase distortion, especially in the transition regions of the filters. You also can obtain linear-phase response with FIR filters.

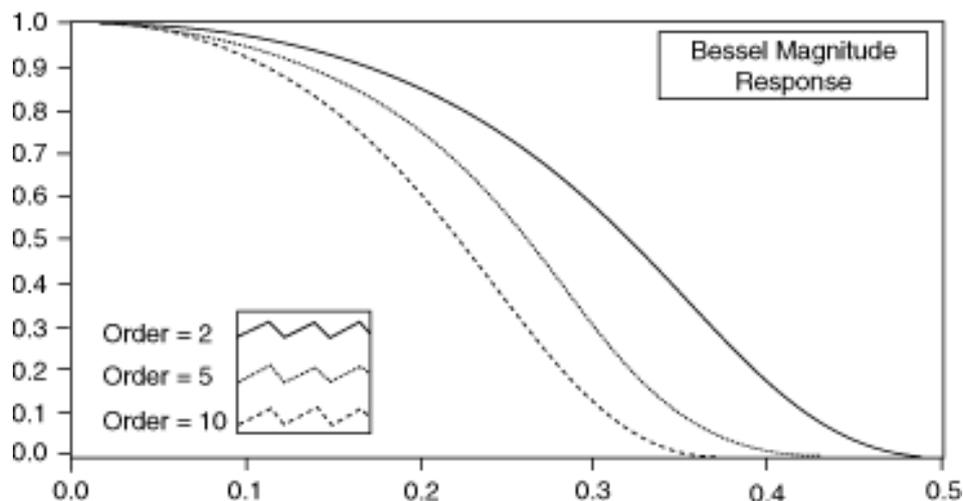


Figure 5

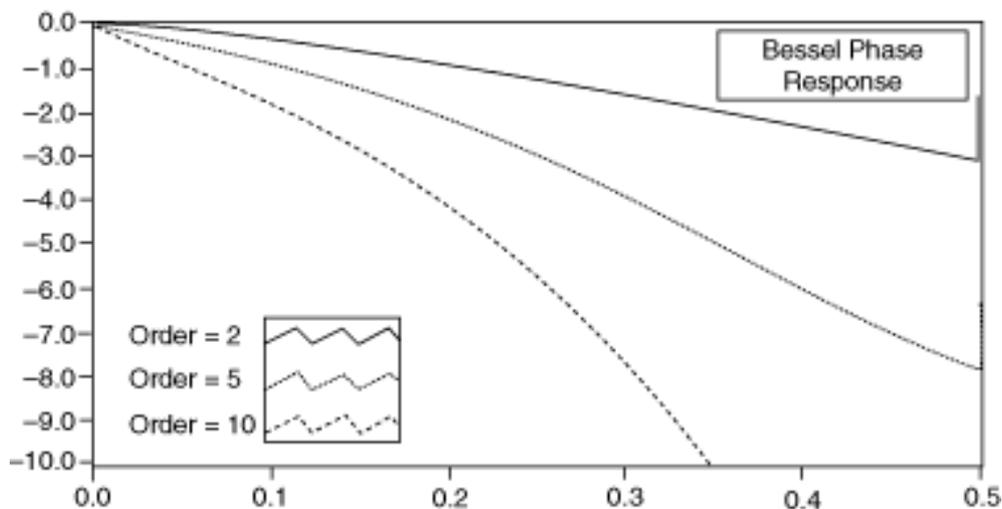


Figure 6

You can use Bessel filters to reduce nonlinear-phase distortion inherent in all IIR filters. High-order IIR filters and IIR filters with a steep roll-off have a pronounced nonlinear-phase distortion, especially in the transition regions of the filters. You also can obtain linear-phase response with FIR filters.

All the filters described above may be analogue or digital. However there is a lot of recorded data about the analogue varieties, so it is often the case that designers use the analogue equations and parameters used and convert them to their digital equivalents. There are two main methods for this, namely the **Impulse Invariant** method and the **Bilinear Transform** method.

Bilinear Transform

Analogue filters are designed using the Laplace transform (s domain) which is the analogue equivalent of the Z transform for digital filters.

Filters designed in the s domain have a transfer function like:

$$T(s) = \frac{1}{1 + \frac{s}{10}}$$

If we have a filter where 10 rads/sec = ω_c . Then multiply top and bottom by 10

$$T(s) = \frac{10}{s + 10}$$

To apply the Bilinear transform we just need to replace the s by:

$$s = \frac{2(z - 1)}{T(z + 1)}$$

Where T is the sampling period. So for a sampling frequency of 16Hz (T= 0.065 s)

$$t(z) = \frac{10}{\frac{2(z - 1)}{0.0625(z + 1)} + 10}$$

And then just work it out!

Near zero frequency, the relation between the analogue and digital frequency response is essentially linear. However as we near the Nyquist frequency it tends to become non-linear. This nonlinear compression is called *frequency warping*.

In the design of a digital filter, the effects of the frequency warping must be taken into account. The prototype filter frequency scale must be prewarped so that after the bilinear transform, the critical frequencies are in the correct places.

Impulse Invariant method

The approach here is to produce a digital filter that has the same impulse response as the analogue filter. It requires the following steps:

1. Compute the Inverse Laplace transform to get impulse response of the analogue filter
2. Sample the impulse response
3. Compute z-transform of resulting sequence

Sampling the impulse response has the advantage of preserving resonant frequencies but its big disadvantage is *aliasing* of the frequency response. Before a continuous impulse response is sampled, a *lowpass filter* should be used to eliminate all frequency components at half the sampling rate and above.

Using the low pass filter transfer function from the previous example:

$$T(s) = \frac{10}{s+10}$$

Now find the inverse Laplace transform from the Laplace transform tables, gives is:

$$y(t) = 10e^{-10t}$$

The final step is to find the z transform, $Y(z)$ of this time variation. Once again from the Laplace/z transform tables, e^{at} has a z transformation of $z/(z - z^{-aT})$. With a sampling frequency of 16Hz:

$$Y(z) = \frac{10z}{z - e^{-0.625}} = \frac{10z}{z - 0.535}$$

As $Y(z) = T(z) \times 1$ for an impulse then:

$$T(z) = \frac{10z}{z - 0.535}$$