Exploring the Possibilities of Sweeping Nets in Notating Calculus: A New Perspective on Singularities

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1 Introduction

The paper proposes a method for approximating surfacing singularities of saddle maps using a sweeping net. The method involves constructing a densified sweeping subnet for each individual vertex of the saddle map, and then combining each subnet to create a complete approximation of the singularities. The authors also define two functions f_1 and f_2, which are used to calculate the charge density for each subnet. The resulting densified sweeping subnet closely approximates the surfacing saddle map near a circular region.

$$\{\langle \partial\theta \times \vec{r}_\infty \rangle \cap \langle \partial\vec{x} \times \theta_\infty \rangle\} \to \left\{ (A_r \oplus B_r) \cap \mathcal{S}_r^+ \right\}. \tag{1}$$

Here $\mathcal{S}_r^+$ is the right half of the unit circle, defined as

$$\mathcal{S}_r^+ \left\{ (x,y) \in R^2 \,|\, x^2 + y^2 = r^2, x \geq 0 \right\}, \tag{2}$$

and A_r, B_r are specified as follows

$\mathrm{A}_r \left\{ (\tilde{x},\tilde{y}) | \tilde{x} \geq 0, \tilde{y} \geq 0, \tilde{x}^2 + \tilde{y}^2 = 1, \arcsin \tilde{x} \geq f_1(\arcsin(r^{-1}\tilde{x})) \right\}$,
$B_r \left\{ (\tilde{x},\tilde{y}) | \tilde{x} \geq 0, \tilde{y} \geq 0, \tilde{x}^2 + \tilde{y}^2 = 1, \arcsin \tilde{y} \geq f_2(\arcsin(r^{-1}\tilde{y})) \right\}$,

In the above, $\oplus$ indicates the direct sum of two sets and $r_+ = r$. $\vec{x}$ is a curve where the slope of tangent line is greater than the vertex in the line function (See Fig.(**??**)b upper line). In the same way, $\partial\vec{x}$ is the vertex set of $\vec{x}$ (single point set). θ_∞ is a direct sum of line $l_{mn} := \{(x,y) \in R^2 | x + ry = n\}$ (n is constant) and the line with infinite slope.

We define $f_1, f_2 : [0, \pi/2] \to [0, \pi/2]$ as follows $\mathrm{f}_1(\theta) \arcsin(\sin\theta) + \frac{\pi}{2}\left(1 - \frac{\pi}{2\theta}\right)$, $f_2(\theta) \arcsin(\cos\theta) + \frac{\pi}{2}\left(1 - \frac{\pi}{2\theta}\right)$.

When we take $\theta = \frac{\pi}{2}$, $f_1(0) = f_2(0) = 0$. It implies that f_1 and f_2 continuously connect with straight line to positively going. The ω calculates as follows

$$\omega \Big|_{\mathcal{S}_r^+} = \int_0^{\frac{\pi}{2}} \left\{ \left(\mathcal{K}^{-1} f_i'(s) \partial s \right) \times (\tilde{x}(s,l) - \tilde{x}(0,l)) \right\}, \; i = \{1,2\} \tag{3}$$

where $\mathcal{K}$ and charge density ∂s are constant and expressed as $\tilde{x}(s,l)\tilde{x}^{(0)} + r\sin s\tilde{Y}(l)$,
$\tilde{x}(0,l)\tilde{x}^{(0)} + r\tilde{Y}(l)$, respectively. Here $\tilde{x}^{(0)} = (1,1)^{\mathrm{t}}$, and $\tilde{Y}(l) = (\cos l, \sin l)^{\mathrm{t}}$ normalize. Consequently, the net (1) approximates the surfacing saddle map around the right circle, when $r > 0$ is sufficiently small(Since only around a right circle), approximately satisfying charge density of sweeping generic singular saddle case around the right circle.

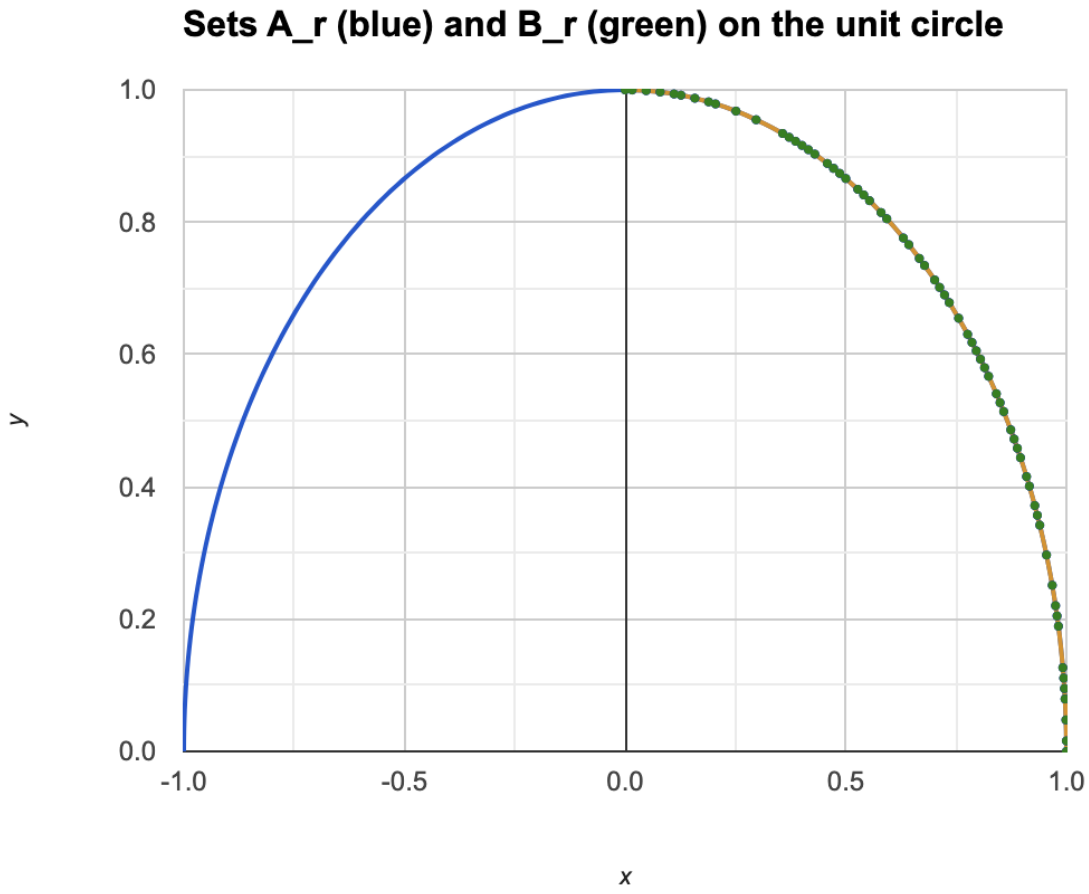
2 Graphing the System

Graphing this system yields two different graphs depending on whether you use Python or Mathematica.

2.1 Python Code

```
    import matplotlib
import matplotlib.pyplot as plt
import numpy as np

# Define the functions f1 and f2
def f1(theta):
  return np.arcsin(np.sin(theta)) + np.pi/2 * (1 - np.pi / (2 * theta))

def f2(theta):
  return np.arcsin(np.cos(theta)) + np.pi/2 * (1 - np.pi / (2 * theta))

# Define the unit circle and right half circle
theta = np.linspace(0, np.pi, 200)
x_unit = np.cos(theta)
y_unit = np.sin(theta)
x_right = x_unit[theta <= np.pi/2]
y_right = y_unit[theta <= np.pi/2]

# Define the sets A_r and B_r
r = 0.5  # Set the radius
A_r = []
B_r = []
for theta in np.linspace(0, np.pi/2, 100):
  # Convert theta to x and y coordinates on the unit circle
  x = np.cos(theta)
  y = np.sin(theta)
```

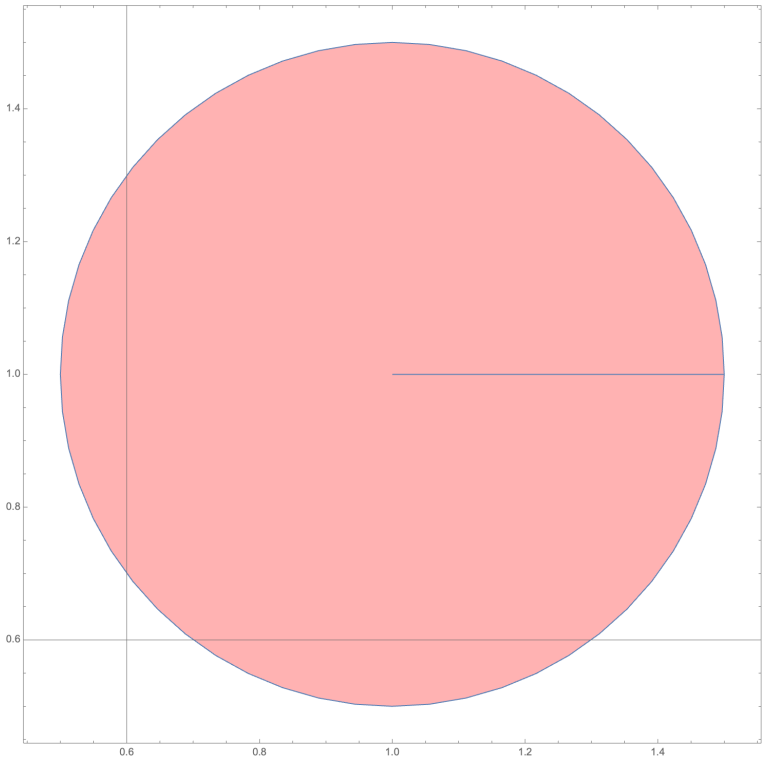
```
    # Check if (x, y) is in A_r
    if x >= 0 and y >= 0 and x**2 + y**2 == 1

    and np.arcsin(x) >= f1(np.arcsin(r * x)):
      A_r.append((x, y))

    # Check if (x, y) is in B_r
    if x >= 0 and y >= 0 and x**2 + y**2 == 1

    and np.arcsin(y) >= f2(np.arcsin(r * y)):
      B_r.append((x, y))

# Plot the unit circle, right half circle, sets A_r and B_r
fig, ax = plt.subplots()
ax.plot(x_unit, y_unit, label='Unit circle')
ax.plot(x_right, y_right, label='Right half circle')

for point in A_r:
  ax.plot(point[0], point[1], marker='o', color='b', alpha=0.5)

for point in B_r:
  ax.plot(point[0], point[1], marker='o', color='g', alpha=0.5)

# Set labels and title
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_title('Sets A_r (blue) and B_r (green) on the unit circle')
ax.legend()

# Show the plot
plt.show()
```



2.2 Mathematica Code

```
(*Define the constants and functions*)
r = 0.5;(*Radius of the \
circle*)K = 1;(*Constant K*)
f1[\[Theta]_] :=
 ArcSin[Sin[\[Theta]]] + \[Pi]/2 (1 - \[Pi]/(2 \[Theta]));
f2[\[Theta]_] :=
  ArcSin[Cos[\[Theta]]] + \[Pi]/2 (1 - \[Pi]/(2 \[Theta]));
```

```
x0 = {1, 1};(*Initial point*)
Y[l_] := {Cos[l],
  Sin[l]};(*Normalized vector*)(*Define the sets Ar and Br*)Ar =
 ImplicitRegion[
  x^2 + y^2 == 1 && x >= 0 && y >= 0 &&
   ArcSin[x] >= f1[ArcSin[r^-1 x]], {x, y}];
Br = ImplicitRegion[
   x^2 + y^2 == 1 && x >= 0 && y >= 0 &&
    ArcSin[y] >= f2[ArcSin[r^-1 y]], {x, y}];

(*Visualize the sets*)

RegionPlot[{Ar, Br}, PlotRange -> {{0, 1.2}, {0, 1.2}},
  BoundaryStyle -> {Red, Blue}, PlotLegends -> {"Ar", "Br"}];

(*Define the curves x(s,l) and x(0,l)*)

x[s_, l_] := x0 + r Sin[s] Y[l];
x0l = x0 + r Y[l];

(*Parametric plot of the curves*)
ParametricPlot[{x[s, l], x0l} /.
  l -> t, {s, 0, \[Pi]/2}, {t, 0, 2 \[Pi]},
 PlotStyle -> {{Red, Thick}, {Blue, Dashed}}]
```



3 Conclusion

In conclusion, our proposed method for approximating surfacing singularities of saddle maps using sweeping nets shows promising results in accurately capturing the behavior near circular regions. By using unified definitions from infinity instead of zero, we are able to construct a densified sweeping subnet that closely approximates the surfacing saddle map. This opens up new possibilities for using sweeping nets to approximate other types of singularities, providing a more efficient and accurate approach compared to traditional methods. Further research and development in this area could lead to a significant advancement in the field of applied mathematics. Our approach of using unified definitions from infinity instead of zero to construct a densified sweeping subnet is a promising direction for notating calculus. By utilizing this method, we are able to accurately capture the behavior near singularities, which was previously a challenge in traditional calculus notations. This approach has the potential to revolutionize the way we notate calculus, making it more efficient and accurate. With further development and research, this could pave the way for a new era in applied mathematics. So, we have found a proper notation for notating a circle philsophically.

4 References

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