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Selecting Project Delivery Systems Based on Simplified Neutrosophic Linguistic Preference Relations

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Abstract: Project delivery system selection is an essential part of project management. In the process of choosing appropriate transaction model, many factors should be under consideration, such as the capability and experience of proprietors, project implementation risk, and so on. How to make their comprehensive evaluations and select the optimal delivery system? This paper proposes a decision-making approach based on an extended linguistic preference structure: simplified neutrosophic linguistic preference relations (SNLPRs). The basic elements in SNLPRs are simplified neutrosophic linguistic numbers (SNLNs). First, several distance measures of SNLNs are introduced. A distance-based consistency index is provided to measure the consistency degree of a simplified neutrosophic linguistic preference relation (SNLPR). When the SNLPR is not acceptably consistent, a consistency-improving automatic iterative algorithm may be used. Afterwards, a decision-making method with SNLPRs is developed. The example of its application in project delivery systems' selection is offered, and a comparison analysis is given in the end as well.

Keywords: project delivery system selection; preference relations; simplified neutrosophic linguistic number; distance-based consistency index; improving consistency

1. Introduction

Construction is not only a carrier of fixed asset investment in a country, but also a channel to adjust products and industrial structures [1,2]. The choice of a project delivery system may be one of the most crucial elements of a project. There are multiple trading models that can be chosen. According to the complexity of projects and the relationships of owners with contractors, the delivery systems can be divided into four categories [3]. The general contract mode, which is the fixed price contract, mainly includes the Design Build (DB), Engineer Procure Construct Tumkey, and Design Build Operate (DBO). The management contract mode, namely the cost plus contract, principally contains the Construction-Management (CM) and Project-Management Contracting. The traditional trading model, called Design Bid Build (DBB), carries out the unit price contract. Others comprise Private Participating Infrastructure, Build Operate Transfer, Private Finance Initiative, and so on.

A lot of aspects, such as projects' characteristics, construction environment, owners' capacity, and market conditions, need to be decided in the system selection process [4]. Nevertheless, the choice of transaction modes is usually based on the subjective consciousness of the decision makers (DMs)

in engineering practice. Only a few scientific and rational decision-making methods related to the selection of project delivery systems have been established. For instance, a fuzzy approach to pick out appropriate transaction systems was presented by Mostafavi and Karamouz [5]. Wang et al. [6] constructed a project delivery system model based on fuzzy sets. After that, the analytical hierarchy process (AHP) for choosing the trade model was provided as well [7,8].

Neutrosophic sets (NSs), as a generalization of intuitionistic fuzzy sets, were originally proposed by Smarandache [9]. They can deal with consistent, hesitant, and inconsistent information at the same time. Figure 1 in Chen and Ye [10] shows the flow chart extended from fuzzy sets to neutrosophic sets (as well as simplified neutrosophic sets, single-valued neutrosophic sets, and interval neutrosophic sets). There are many extensions of NSs [11–15]. Numerous multi-criteria decision-making methods based on NSs and their extensions have been studied. For example, the ELECTRE approaches of NSs and interval neutrosophic sets (INSs) were presented by Peng et al. [16] and Zhang et al. [17], respectively. The extended TODIM [18] and MULTIMOORA [19] methods with NSs were also offered. Peng et al. [20] put forward a likelihood-based QUALIFLEX method of multi-valued neutrosophic numbers. Besides, the VIKOR method in line with INSs was developed by Bausys and Zavadskas [21]. Pouresmaeil et al. [22] proposed an extended TOPSIS and VIKOR approaches based on NSs. Furthermore, other approaches, such as the heronian mean operators [23], correlation coefficients [24,25], the WASPAS method [26–29], and the COPRAS [30,31] method were discussed and applied in diverse areas.

However, no matter which of the methods mentioned above is used, DMs are asked to give their evaluation values directly. It may be not easy for them in some cases. Sometimes, people may be accustomed to make a judgment through comparing each pair of delivery systems, especially when they cannot make direct evaluations for each single model [32]. Hence, the decision-making methods based on a judgment matrix (preference relations) with NSs or other extensions may be valuable and necessary.

In general, there are two main types of preference relations. One expresses quantitative data, such as reciprocal preference relations [33,34], interval fuzzy preference relations [35–37], intuitionistic fuzzy preference relations [38,39] triangular fuzzy preference relations [40,41], hesitant fuzzy preference relations [42,43], and some extensions [44–46]. Another contains qualitative information, like linguistic preference relations (LPRs) [47–51], hesitant fuzzy linguistic preference relations (HFLPRs) [52–54], intuitionistic linguistic preference relations (ILPRs) [55], and so on [56–58].

This paper introduces a new type of preference relations, simplified neutrosophic linguistic preference relations (SNLPRs). The basic element in SNLPRs is a simplified neutrosophic linguistic number (SNLN). The aims of the paper are as follows. On the one hand, a matrix with qualitative information may be more suitable for selecting project delivery systems as many qualitative factors are considered such as owners' ability, the technical difficulty of the project, the uncertainty of the external environment, and so on. On the other hand, the aforementioned preference relations cannot describe DMs' degrees of certainty, hesitation, and negation of their qualitative judgment simultaneously. There is a hypothesis that the membership degree of linguistic values is 1 in LPRs and HFLPRs. ILPRs only express the consistency and inconsistency of linguistic values.

On the basis of linguistic term sets and simplified neutrosophic numbers [59], simplified neutrosophic linguistic numbers (SNLNs) may be one of the most widely used extensions [60–62]. The former stands for the qualitative evaluation values of project systems [63–65], and the latter are the truth-membership, indeterminacy-membership, and false-membership of the qualitative assessment. Most importantly, these degrees are independent of each other. Consequently, considering the fuzziness of human thought and the complexity of reality, expressing preference in terms of SNLNs may be more suitable.

This paper studies a decision-making method with preference relations under simplified neutrosophic linguistic environment. The following are our innovations:

(1)　Propose the Hamming distance, Euclidean distance, and Hausdorff distance of two SNLNs. In addition, several relevant properties are discussed.

(2)　Present a new concept, SNLPRs. Subsequently, a distance-based consistency index is introduced to measure the consistency degree of SNLPRs.

(3)　Develop a consistency-improving algorithm and a ranking method based on aggregation operators. A decision-making approach based on SNLPRs is described as well.

(4)　Apply the proposed method to the project transaction model selection process. The practicability and effectiveness are demonstrated in a comparison analysis.

The remains of this paper are arranged as follows. Basic theories about SNLNs and LPRs are introduced in Section 2. Section 3 proposed some distance measures of SNLNs. In Section 4, the consistency-checking and consistency-improving issues of SNLPRs are discussed. Afterwards, there is an example and some analysis in Section 5. At last, some conclusions are drawn.

2. Preliminaries

In this section, some basic concepts and operations of linguistic term sets, SNLNs and LPRs, are reviewed.

A linguistic tem set is a collection of multiple linguistic values, like

$$\overline{S} = \{s_i | i = -u, \ldots, -1, 0, 1, \ldots, u\},$$

where s_i is a possible linguistic value, and the negation operator is $neg(s_i) = s_{-i}$. Furthermore, if and only if $i > j$, then $s_i > s_j$ [66].

Note that the linguistic term set above is discrete. In some cases, the aggregated results may be used, which are not contained in this set. Hence, Xu [66] further defined a continuous term set, like

$$S = \{s_i | i \in [-g, g]\}(g > u)$$

to extend the old one.

The following are some operations of two linguistic terms $s_i, s_j \in S$.

$$s_i \oplus_{Xu} s_j = s_{i+j} \tag{1}$$

$$s_i \oplus_{Xu} s_j = s_j \oplus_{Xu} s_i \tag{2}$$

$$\lambda s_i = s_{\lambda i}, 0 \leq \lambda \leq 1 \tag{3}$$

Definition 1. *In Reference [67], let $S = \{s_i | i \in [-g, g]\}$ be a linguistic term set. The subscript of any element s_i can be obtained by the function $N(s_i) = i$. The inverse function is $N^{-1}(i) = s_i$.*

Definition 2. *In Reference [68], suppose a crisp number $\vartheta_i \in [0, 1]$. If there is a mapping from s_i to ϑ_i, then the linguistic scale function f^* is denoted as $f^* : s_i \to \vartheta_i$ $(i = -g, -g + 1, \cdots, g - 1, g)$, where $0 \leq \vartheta_{-g} < \vartheta_{-g+1} < \cdots < \vartheta_{g-1} < \vartheta_g$. And f^{*-1} is the inverse function of f^*.*

The linguistic scale function $f^*(s_i) = \frac{1}{2} + \frac{i}{2g}(i \in [-g, g])$ is used in this paper.

Definition 3. *In References [69,70], let $S = \{s_i | i \in [-g, g]\}$ be a linguistic term set. $\eta = < h_\eta, (T_\eta, I_\eta, F_\eta) >$ is a SNLN, where $h_\eta(x) \in S$, the truth-membership degree $T_\eta(x) \in [0, 1]$, indeterminacy-membership degree $I_\eta(x) \in [0, 1]$, and falsity-membership degree $F_\eta(x) \in [0, 1]$, and $0 \leq T_\eta(x) + I_\eta(x) + F_\eta(x) \leq 3$.*

Definition 4. *In Reference [71], $A = < h_A, (T_A, I_A, F_A) >$ and $B = < h_B, (T_B, I_B, F_B) >$ are two arbitrary SNLNs, and their operations can be defined as follows:*

(1)　*If $S(a) > S(b)$, then $a > b$;*

(2)　*If $S(a) = S(b)$ and $A(a) > A(b)$, then $a > b$;*

(3)   If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) > C(b)$, then $a > b$;

(4)   If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) = C(b)$, then $a = b$.

Definition 5. *In Reference [71], assume $a_i = (s_{\theta(a_i)}, < T_i, I_i, F_i >)$ $(i = 1, 2, \ldots, n)$ are a sequence of SNLNs. Then the simplified neutrosophic linguistic arithmetic mean (SNLAM) operator is*

$$
\begin{aligned}
SNLAM(a_1, a_2, \ldots, a_n) &= \overset{n}{\underset{i=1}{\oplus}} \frac{1}{n} a_i \\
&= \left\langle f^{*-1}\left(\frac{1}{n} \sum_{i=1}^{n} \left(f^*(s_{\theta(a_i)}) \right) \right), \left(\frac{\sum_{i=1}^{n} \left(f^*(s_{\theta(a_i)}) T_i \right)}{\sum_{i=1}^{n} f^*(s_{\theta(a_i)})}, \frac{\sum_{i=1}^{n} \left(f^*(s_{\theta(a_i)}) I_i \right)}{\sum_{i=1}^{n} f^*(s_{\theta(a_i)})}, \frac{\sum_{i=1}^{n} \left(f^*(s_{\theta(a_i)}) F_i \right)}{\sum_{i=1}^{n} f^*(s_{\theta(a_i)})} \right) \right\rangle.
\end{aligned}
\tag{4}
$$

Definition 6. *In Reference [71], suppose $a_i = (s_{\theta(a_i)}, < T_i, I_i, F_i >)$ $(i = 1, 2, \ldots, n)$ are a sequence of SNLNs. Then the simplified neutrosophic linguistic geometric mean (SNLGM) operator is*

$$
SNLGM(a_1, a_2, \ldots, a_n) = \overset{n}{\underset{i=1}{\otimes}} a_i^{\frac{1}{n}} = \left\langle f^{*-1}\left(\prod_{i=1}^{n} \left(f^*(s_{\theta(a_i)}) \right)^{\frac{1}{n}} \right), \left(\prod_{i=1}^{n} T_i^{\frac{1}{n}}, 1 - \prod_{i=1}^{n} (1 - I_i)^{\frac{1}{n}}, 1 - \prod_{i=1}^{n} (1 - F_i)^{\frac{1}{n}} \right) \right\rangle.
\tag{5}
$$

Definition 7. *In Reference [71], let $a = < h_a, (T_a, I_a, F_a) >$ be a SNLN. Then the score function is $SF(a) = \frac{1}{3} f^*(h_a)(T_a + 1 - I_a + 1 - F_a)$, the accuracy function is $AF(a) = f^*(h_a)(T_a - F_a)$, and the certainty function is $CF(a) = f^*(h_a) T_a$.*

Definition 8. *In Reference [71], for two SNLNs $a = < h_a, (T_a, I_a, F_a) >$ and $b = < h_b, (T_b, I_b, F_b) >$, the comparison method is:*

(1)   If $S(a) > S(b)$, then $a > b$;

(2)   If $S(a) = S(b)$ and $A(a) > A(b)$, then $a > b$;

(3)   If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) > C(b)$, then $a > b$;

(4)   If $S(a) = S(b)$, $A(a) = A(b)$ and $C(a) = C(b)$, then $a = b$.

Definition 9. *In Reference [72], let $X = \{x_1, x_2, \ldots, x_n\}$ be a collection of n alternatives and $B = (b_{ij})_{n \times n} \subset X \times X$ be a judgment matrix. If for all $i, j = 1, 2, \ldots, n$, there are*

$$
b_{ij} \oplus b_{ji} = s_0 \text{ and } b_{ii} = s_0,
\tag{6}
$$

then $B = (b_{ij})_{n \times n}$ is a LPR, where b_{ij} is the preference degree of the alternative x_i over x_j. In particular, if $b_{ij} < s_0$, x_i is non-preferred to x_j; x_i is preferred to x_j if $b_{ij} > s_0$; if not, x_j is equivalent to x_i.

Definition 10. *In References [72,73], if $B = (b_{ij})_{n \times n} \subset X \times X$ is a LPR, and*

$$
b_{ij} = b_{ik} \oplus b_{kj}, (i, k, j = 1, 2, \ldots, n)
\tag{7}
$$

then B is a perfectly consistent LPR.

3. Distance Measures of SNLNs

Ye [70] defined the distance measure between two SNLNs, but this method has some drawbacks. Thus, some distance measures of SNLNs are redefined in this section.

Distance measure is a universal and effective way to calculate the difference between two elements. There are several common distance measures, such as Hamming distance, Euclidean distance, and Hausdorff distance.

Definition 11. *In Reference [70], suppose $\alpha = (T_\alpha, I_\alpha, F_\alpha)$ and $\beta = (T_\beta, I_\beta, F_\beta)$ are two optional SNLNs, and the subscript function is $N(s_i) = i$. $\lambda \geq 0$. The distance between α and β can be defined as below:*

$$d_Y(\alpha, \beta) = (|T_\alpha \cdot N(h_\alpha) - T_\beta \cdot N(h_\beta)|_\lambda + |I_\alpha \cdot N(h_\alpha) - I_\beta \cdot N(h_\beta)|_\lambda + |F_\alpha \cdot N(h_\alpha) - F_\beta \cdot N(h_\beta)|_\lambda)^{\frac{1}{\lambda}}. \quad (8)$$

Specially, when $\lambda = 1$, Equation (8) can be reduced to Hamming distance; when $\lambda = 2$, Equation (8) can be reduced to Euclidean distance.

The limitations of this definition are noticeable. Firstly, the calculation depends on linguistic subscripts directly, and different semantics cannot be distinguished. Secondly, this distance does not satisfy $0 \leq d_Y(\alpha, \beta) \leq 1$ and the property of triangle inequality. Thirdly, the truth-membership, indeterminacy-membership, and false-membership are put on an equal footing in the calculation process. This is intuitively irrational.

To overcome these shortcomings, the following distance measures between two SNLNs are defined.

Definition 12. *For two arbitrary SNLNs $a = <h_a, (T_a, I_a, F_a)>$ and $b = <h_b, (T_b, I_b, F_b)>$, assume the linguistic term set is $S = \{s_i | i \in [-g, g]\}$ and the subscript function is $N(s_i) = i$. Then the Hamming distance $d_H(a, b)$, Euclidean distance $d_E(a, b)$, and Hausdorff distance $d_{Ha}(a, b)$ can be defined as follows:*

$$d_H(a, b) = \frac{1}{6g}[N(|T_a \cdot h_a - T_b \cdot h_b|) + N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|) + N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|)], \quad (9)$$

$$d_E(a, b) = \sqrt{\frac{1}{12g^2}[N(|T_a \cdot h_a - T_b \cdot h_b|)^2 + N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|)^2 + N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|)^2]}, \quad (10)$$

$$d_{Ha}(a, b) = \frac{1}{2g}\max\{N(|T_a \cdot h_a - T_b \cdot h_b|), (|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|), N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|)\}. \quad (11)$$

Property 1. *Assume Ω is the set of all SNLNs, $a = <h_a, (T_a, I_a, F_a)>$, $b = <h_b, (T_b, I_b, F_b)>$ and $c = <h_c, (T_c, I_c, F_c)>$, $S = \{s_i | i \in [-g, g]\}$, and then the following properties are satisfied:*

(1) $0 \leq d_H(a, b) \leq 1, 0 \leq d_E(a, b) \leq 1$, and $0 \leq d_{Ha}(a, b) \leq 1$, for $\forall a, b \in \Omega$;

(2) $d_H(a, b) = d_H(b, a), d_E(a, b) = d_E(b, a)$, and $d_{Ha}(a, b) = d_{Ha}(b, a)$, for $\forall a, b \in \Omega$;

(3) If $a = b$, then $d_H(a, b) = 0, d_E(a, b) = 0$, and $d_{Ha}(a, b) = 0$, for $\forall a, b \in \Omega$;

(4) If $s_0 \leq h_a \leq h_b \leq h_c, T_a \leq T_b \leq T_c, I_a \geq I_b \geq I_c$ and $F_a \geq F_b \geq F_c$, then $d_H(a, b) \leq d_H(a, c)$, $d_H(b, c) \leq d_H(a, c), d_E(a, b) \leq d_E(a, c), d_E(b, c) \leq d_E(a, c), d_{Ha}(a, b) \leq d_{Ha}(a, c)$ and $d_{Ha}(b, c) \leq d_{Ha}(a, c)$.

Proof 1.

(1) Because $i \in [-g, g]$ and $0 \leq T_a, T_b \leq 1$, $N(|T_a \cdot h_a - T_b \cdot h_b|) \in [0, 2g]$; Similarly, $0 \leq (1 - I_a) \leq 1, 0 \leq (1 - I_b) \leq 1 \Rightarrow N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|) \in [0, 2g]$, and $0 \leq (1 - F_a) \leq 1, 0 \leq (1 - F_b) \leq 1 \Rightarrow N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|) \in [0, 2g]$; thus $0 \leq [N(|T_a \cdot h_a - T_b \cdot h_b|) + N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|) + N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|)]$ $\leq 6g \Rightarrow 0 \leq d_H(a, b) \leq 1$. Likewise, $0 \leq d_E(a, b) \leq 1$ and $0 \leq d_{Ha}(a, b) \leq 1$.

(2) $N(|T_a \cdot h_a - T_b \cdot h_b|) = N(|T_b \cdot h_b - T_a \cdot h_a|), N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|) = N(|(1 - I_b) \cdot h_b - (1 - I_a) \cdot h_a|)$, and $N(|(1 - F_a) \cdot h_a - (1 - F_b) \cdot h_b|) = N(|(1 - F_b) \cdot h_b - (1 - F_a) \cdot h_a|)$, therefore $d_H(a, b) = d_H(b, a)$. Likewise, $d_E(a, b) = d_E(b, a)$ and $d_{Ha}(a, b) = d_{Ha}(b, a)$.

(3) $a = b \Rightarrow N(|T_a \cdot h_a - T_b \cdot h_b|) = 0, N(|(1 - I_a) \cdot h_a - (1 - I_b) \cdot h_b|) = 0$ and $N(|(1 - F_b) \cdot h_b|) = 0$, therefore $d_H(a, b) = 0$. Similarly, $d_E(a, b) = 0$ and $d_{Ha}(a, b) = 0$.

(4) Because $s_0 \leq sa_{ij} \leq sb_{ij} \leq sc_{ij}, T_{ij}^a \leq T_{ij}^b \leq T_{ij}^c$, and $N(s_i) = i$ is a monotone increasing function, $T_{ij}^a \cdot (sa_{ij}) \leq T_{ij}^b \cdot (sb_{ij}) \leq T_{ij}^c \cdot (sc_{ij}) \Rightarrow |T_{ij}^a \cdot (sa_{ij}) - T_{ij}^b \cdot (sb_{ij})| \leq |T_{ij}^a \cdot (sa_{ij}) - T_{ij}^c \cdot (sc_{ij})|$ and $N(|T_{ij}^a \cdot (sa_{ij}) - T_{ij}^b \cdot (sb_{ij})|) \leq N(|T_{ij}^a \cdot (sa_{ij}) - T_{ij}^c \cdot (sc_{ij})|)$; Likewise, $N(|(1 - I_{ij}^a) \cdot (sa_{ij}) - (1 - I_{ij}^b) \cdot (sb_{ij})|) \leq N(|(1 - I_{ij}^a) \cdot (sa_{ij}) - (1 - I_{ij}^c) \cdot (sc_{ij})|)$ and $N(|(1 - F_{ij}^a) \cdot (sa_{ij}) - (1 - F_{ij}^b) \cdot (sb_{ij})|) \leq N(|(1 - F_{ij}^a) \cdot (sa_{ij}) - (1 - F_{ij}^c) \cdot (sc_{ij})|)$, so $d_H(a, b) \leq d_H(a, c)$. Similarly, $d_H(b, c) \leq d_H(a, c)$, $d_E(a, b) \leq d_E(a, c), d_E(b, c) \leq d_E(a, c), d_{Ha}(a, b) \leq d_{Ha}(a, c)$ and $d_{Ha}(b, c) \leq d_{Ha}(a, c)$.

Then, the proof is completed.   □

Example 1. *$a = < s_1, (0.2, 0.3, 0.6) >$ and $b = < s_2, (0.5, 0.1, 0.4) >$ are two SNLNs, and $g = 4$. Then $d_H(a,b) \approx 0.1125$, $d_E(a,b) \approx 0.1139$ and $d_{Ha}(a,b) \approx 0.1375$.*

4. Decision-Making Method Based on SNLPRs

In this section, the concept of SNLPRs is presented. A decision-making method is proposed after discussing the checking and improving of consistency.

4.1. The Concept of SNLPRs

Definition 13. *Given a group of n alternatives $X = \{x_1, x_2, \ldots, x_n\}$ and a matrix $K = (k_{ij})_{n \times n} \subset X \times X$. If all the elements are presented with SNLNs, $k_{ij} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)$, and satisfy these conditions in the following:*

$$s_{ij} \oplus_{Xu} s_{ji} = s_0 \tag{12}$$

$$T_{ij} = T_{ji}, \ I_{ij} = I_{ji}, \ F_{ij} = F_{ji} \tag{13}$$

$$(s_{ii}, < T_{ii}, I_{ii}, F_{ii} >) = (s_0, < 1, 0, 0 >), \ (i, j = 1, 2, \ldots, n), \tag{14}$$

then the matrix K on X can be regarded as a SNLPR, where s_{ij} is the degree of x_i preferred to x_j, and T_{ij}, I_{ij} and F_{ij} represent the truth-membership degree, the indeterminacy-membership degree, and the falsity-membership degree of s_{ij}, respectively.

Specifically, when $T_{ij} = 1$ and $I_{ij} = F_{ij} = 0$ for all $i, j = 1, 2, \ldots, n$, then the SNLPR is reduced to a LPR. Compared to LPRs, SNLPRs contain not only the linguistic values, but also the degrees of accuracy, hesitation, and mistake. The discrete linguistic term set can be extended to be a continuous one and DMs can express their qualitative preference information more flexibly.

From Definition 13, it can be seen that $k_{ij} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)$ is the preferred value of the scheme x_i to x_j, and it could be the same as $k_{ij} = (< s_{ij}, T_{ij} >, < s_{ij}, I_{ij} >, < s_{ij}, F_{ij} >)$, where $< s_{ij}, T_{ij} >$ shows x_i is s_{ij} to x_j with the true possibility T_{ij}; $< s_{ij}, I_{ij} >$ shows x_i is s_{ij} to x_j with the hesitant possibility I_{ij}; $< s_{ij}, F_{ij} >$ shows x_i is s_{ij} to x_j with the false possibility F_{ij}.

As well as LPR, the preference degree of x_j to x_i can be denoted as $< s_{-ij}, T_{ij} >$, $< s_{-ij}, I_{ij} >$ and $< s_{-ij}, F_{ij} >$, individually. That is to say, $k_{ji} = (< s_{-ij}, T_{ij} >, < s_{-ij}, I_{ij} >, < s_{-ij}, F_{ij} >)$ or $k_{ji} = (s_{-ij}, < T_{ij}, I_{ij}, F_{ij} >)$.

Example 2. *There are three alternatives $X = \{x_1, x_2, x_3\}$, and the linguistic term set $S = \{s_i | i \in [-4, 4]\}$ is used, where $s = \{s_{-4} = tremendously \ poorer, \ s_{-3} = much \ poorer, \ s_{-2} = poorer, \ s_{-1} = a \ little \ poorer, \ s_0 = fair, \ s_1 = a \ little \ better, \ s_2 = better, \ s_3 = much \ better, \ s_4 = tremendously \ better\}$. If a decision maker believes the degree of x_1 preferred to x_2 is s_2, but he is not sure that he is absolutely right. According to his professional knowledge and experience in the past, he deems that he is correct with a probability of 40%, but the probability of error is 50%, and the uncertainty is 10%. In that case, his preference can be described using a SNLN, that is, $(s_2, < 0.4, 0.1, 0.5 >)$. In this way, a SNLPR can be obtained as all the alternatives above are compared with each other in a proper sequence. An example is given as follows:*

$$K_1 = \begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_{-1}, < 0.4, 0.2, 0.1 >) & (s_2, < 0.3, 0.1, 0.2 >) \\ (s_1, < 0.4, 0.2, 0.1 >) & (s_0, < 1, 0, 0 >) & (s_{-3}, < 0.2, 0.5, 0.3 >) \\ (s_{-2}, < 0.3, 0.1, 0.2 >) & (s_3, < 0.2, 0.5, 0.3 >) & (s_0, < 1, 0, 0 >) \end{bmatrix}.$$

Definition 14. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a cluster of n alternatives and a SNLPR be $K = (k_{ij})_{n \times n} \subset X \times X$, where $k_{ij} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)$. Then the matrix $T = (< s_{ij}, T_{ij} >)_{n \times n} \subset X \times X$ is regarded as the true*

linguistic judgment matrix of K, $I = (< s_{ij}, I_{ij} >)_{n \times n} \subset X \times X$ is the hesitant linguistic judgment matrix of K, and $F = (< s_{ij}, F_{ij} >)_{n \times n} \subset X \times X$ is the false linguistic judgment matrix of K, respectively.

From Definition 14, it can be known that for an arbitrary SNLPR, it is easy to derive its corresponding true linguistic judgment matrix, hesitant linguistic judgment matrix, and false linguistic judgment matrix. Furthermore, these linguistic judgment matrices are all defined based on the continuous linguistic terms.

Example 3. *Suppose a SNLPR is the same as in Example 2. Then, according to Definition 14, its corresponding true linguistic judgment matrix, hesitant linguistic judgment matrix, and false linguistic judgment matrix are* $T_1 = \begin{bmatrix} (< s_0, 1 >) & (< s_{-1}, 0.4 >) & (< s_2, 0.3 >) \\ (< s_1, 0.4 >) & (< s_0, 1 >) & (< s_{-3}, 0.2 >) \\ (< s_{-2}, 0.3 >) & (< s_3, 0.2 >) & (< s_0, 1 >) \end{bmatrix}$,

$I_1 = \begin{bmatrix} (< s_0, 0 >) & (< s_{-1}, 0.2 >) & (< s_2, 0.1 >) \\ (< s_1, 0.2 >) & (< s_0, 0 >) & (< s_{-3}, 0.5 >) \\ (< s_{-2}, 0.1 >) & (< s_3, 0.5 >) & (< s_0, 0 >) \end{bmatrix}$ *and* $F_1 =$

$\begin{bmatrix} (< s_0, 0 >) & (< s_{-1}, 0.1 >) & (< s_2, 0.2 >) \\ (< s_1, 0.1 >) & (< s_0, 0 >) & (< s_{-3}, 0.3 >) \\ (< s_{-2}, 0.2 >) & (< s_3, 0.3 >) & (< s_0, 0 >) \end{bmatrix}$.

4.2. Consistency Checking of SNLPRs

The deviation between two SNLPRs is calculated in this subsection, and then a distance-based consistency index is presented as well.

Definition 15. *Assume there are several alternatives $X = \{x_1, x_2, \ldots, x_n\}$. For an arbitrary SLNLPR, if for all $i, j, k = 1, 2, \ldots, n$, there is $(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj}) = (T_{ij} \cdot s_{ij})$, then it can be regarded that $T = (T_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ is consistent; if for all $i, j, k = 1, 2, \ldots, n$, $((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj}) = ((1 - I_{ij}) \cdot s_{ij})$, $I = (I_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ is consistent; Similarly, if for all $i, j, k = 1, 2, \ldots, n$, $((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj}) = ((1 - F_{ij}) \cdot s_{ij})$, $F = (< F_{ij} \cdot s_{ij} >)_{n \times n} \subset X \times X$ is consistent.*

Definition 16. *Let $K = (k_{ij})_{n \times n} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)_{n \times n} \subset X \times X$ be a SNLPR. If the following equations are true for all $i, j, k = 1, 2, \ldots, n$:*

$$(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj}) = (T_{ij} \cdot s_{ij}) \tag{15}$$

$$((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj}) = ((1 - I_{ij}) \cdot s_{ij}) \tag{16}$$

$$((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj}) = ((1 - F_{ij}) \cdot s_{ij}), \tag{17}$$

then K is regarded as a consistent SNLPR.

Example 4. *Suppose a SNLPR is the same one as Example 2. Because $(T_{12} \cdot s_{12}) \oplus (T_{23} \cdot s_{23}) = 0.4s_{-1} \oplus 0.2s_{-3} = s_{-1} \neq T_{13} \cdot s_{13}$, based on Definition 16, K_1 is not considered a consistent SNLPR.*

Theorem 1. *Given some alternatives $X = \{x_1, x_2, \ldots, x_n\}$, and the related SNLPR is $K = (k_{ij})_{n \times n} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)_{n \times n} \subset X \times X$. If $T = (T_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$, $I = (I_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ and $F = (F_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ all have perfect consistency, then the SNLPR K is consistent, too. On the contrary, when a SNLPR K has complete consistency, then $T = (T_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$, $I = (I_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ and $F = (F_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ are all absolutely consistent.*

Proof 2.

(1) Because $T = (T_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ is consistent, for all $i, j, k = 1, 2, \ldots, n$, there is $(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj}) = (T_{ij} \cdot s_{ij})$ based on Definition 15. In the same way, $((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj}) = ((1 - I_{ij}) \cdot s_{ij})$ and $((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj}) = ((1 - F_{ij}) \cdot s_{ij})$, as $I = (I_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ and $F = (F_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ are consistent. That is to say, $(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj}) = (T_{ij} \cdot s_{ij})$, $((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj}) = ((1 - I_{ij}) \cdot s_{ij})$ and $((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj}) = ((1 - F_{ij}) \cdot s_{ij})$ for all $i, j, k = 1, 2, \ldots, n$. On the basis of Definition 16, it can be seen that K is consistent.

(2) Since K has complete consistency, then these equations hold based on Definition 16: $(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj}) = (T_{ij} \cdot s_{ij})$, $((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj}) = ((1 - I_{ij}) \cdot s_{ij})$ and $((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj}) = ((1 - F_{ij}) \cdot s_{ij})$. In the light of Definition 15, $T = (T_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$, $I = (I_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ and $F = (F_{ij} \cdot s_{ij})_{n \times n} \subset X \times X$ are all consistent as well.

The proof is done now. $\square$

Theorem 2. *Given an arbitrary SNLPR* $K = (k_{ij})_{n \times n} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)_{n \times n} \subset X \times X$, $i, j, k = 1, 2, \ldots n$, *if*

$$T^*_{ij} \cdot s^*_{ij} = \frac{1}{n} \{ \oplus^n_{k=1} [(T_{ik} \cdot s_{ik}) \oplus (T_{kj} \cdot s_{kj})] \} \tag{18}$$

$$(1 - I^*_{ij}) \cdot s^*_{ij} = \frac{1}{n} \{ \oplus^n_{k=1} [((1 - I_{ik}) \cdot s_{ik}) \oplus ((1 - I_{kj}) \cdot s_{kj})] \} \tag{19}$$

$$(1 - F^*_{ij}) \cdot s^*_{ij} = \frac{1}{n} \{ \oplus^n_{k=1} [((1 - F_{ik}) \cdot s_{ik}) \oplus ((1 - F_{kj}) \cdot s_{kj})] \}, \tag{20}$$

then a consistent SNLPR $K^* = (k^*_{ij})_{n \times n} = (s^*_{ij}, < T^*_{ij}, I^*_{ij}, F^*_{ij} >)_{n \times n}$ *is obtained.*

Proof 3. Since $(T^*_{ik} \cdot s^*_{ik}) \oplus (T^*_{kj} \cdot s^*_{kj}) = \frac{1}{n} \{ \oplus^n_{e=1} [(T_{ie} \cdot s_{ie}) \oplus (T_{ek} \cdot s_{ek})] \} \oplus \frac{1}{n} \{ \oplus^n_{e=1} [(T_{ke} \cdot s_{ke}) \oplus (T_{ej} \cdot s_{ej})] \}$ $= \frac{1}{n} \{ \oplus^n_{e=1} [(T_{ie} \cdot s_{ie}) \oplus (T_{ek} \cdot s_{ek}) \oplus (T_{ke} \cdot s_{ke}) \oplus (T_{ej} \cdot s_{ej})] \} = \frac{1}{n} \{ \oplus^n_{e=1} [(T_{ie} \cdot s_{ie}) \oplus (T_{ej} \cdot s_{ej}) \oplus (T_{ek} \cdot s_0)] \}$ $= \frac{1}{n} \{ \oplus^n_{e=1} [(T_{ie} \cdot s_{ie}) \oplus (T_{ej} \cdot s_{ej})] \} = T^*_{ij} \cdot s^*_{ij}$; then $(T^*_{ik} \cdot s^*_{ik}) \oplus (T^*_{kj} \cdot s^*_{kj}) = T^*_{ij} \cdot s^*_{ij}$; Similarly, $((1 - I^*_{ik}) \cdot s^*_{ik}) \oplus ((1 - I^*_{kj}) \cdot s^*_{kj}) = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - I_{ie}) \cdot s_{ie}) \oplus ((1 - I_{ek}) \cdot s_{ek})] \} \oplus \frac{1}{n} \{ \oplus^n_{e=1} [((1 - I_{ke}) \cdot s_{ke}) \oplus ((1 - I_{ej}) \cdot s_{ej})] \}$ $= \frac{1}{n} \{ \oplus^n_{e=1} [((1 - I_{ie}) \cdot s_{ie}) \oplus ((1 - I_{ek}) \cdot s_{ek}) \oplus ((1 - I_{ke}) \cdot s_{ke}) \oplus ((1 - I_{ej}) \cdot s_{ej})] \} = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - I_{ie}) \cdot s_{ie})$ $\oplus ((1 - I_{ej}) \cdot s_{ej}) \oplus ((1 - I_{ek}) \cdot s_0)] \} = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - I_{ie}) \cdot s_{ie}) \oplus ((1 - I_{ej}) \cdot s_{ej})] \}$ $= (1 - I^*_{ij}) \cdot s^*_{ij}$ and $((1 - F^*_{ik}) \cdot s^*_{ik}) \oplus ((1 - F^*_{kj}) \cdot s^*_{kj}) = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - F_{ie}) \cdot s_{ie}) \oplus ((1 - F_{ek}) \cdot s_{ek})] \} \oplus$ $\frac{1}{n} \{ \oplus^n_{e=1} [((1 - F_{ke}) \cdot s_{ke}) \oplus ((1 - F_{ej}) \cdot s_{ej})] \} = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - F_{ie}) \cdot s_{ie}) \oplus ((1 - F_{ek}) \cdot s_{ek}) \oplus ((1 - F_{ke}) \cdot s_{ke})$ $\oplus ((1 - F_{ej}) \cdot s_{ej})] \} = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - F_{ie}) \cdot s_{ie}) \oplus ((1 - F_{ej}) \cdot s_{ej}) \oplus ((1 - F_{ek}) \cdot s_0)] \} = \frac{1}{n} \{ \oplus^n_{e=1} [((1 - F_{ie}) \cdot s_{ie}) \oplus ((1 - F_{ej}) \cdot s_{ej})] \} = (1 - F^*_{ij}) \cdot s^*_{ij}$.

According to Equations (15)–(17), it can be seen that $K^* = (k^*_{ij})_{n \times n} = (s^*_{ij}, < T^*_{ij}, I^*_{ij}, F^*_{ij} >)_{n \times n}$ is a consistent SNLPR.

This is the end of Proof 3. $\square$

Note that there are only three equations above, but four variables s^*_{ij}, T^*_{ij}, I^*_{ij} and F^*_{ij} are contained. Thus, there may be many possible answers. In order to get a unique solution, the following method is used:

(1) In a general way, assume $s^*_{ij} \geq s_o$, and then $s_o \leq T^*_{ij} \cdot s^*_{ij} \leq s^*_{ij}$, $s_o \leq (1 - I^*_{ij}) \cdot s^*_{ij} \leq s^*_{ij}$, and $s_o \leq (1 - F^*_{ij}) \cdot s^*_{ij} \leq s^*_{ij}$; suppose $\max\{N(T^*_{ij} \cdot s^*_{ij}), N((1 - I^*_{ij}) \cdot s^*_{ij}), N((1 - F^*_{ij}) \cdot s^*_{ij})\} \in [a - 1, a]$, $a \leq N(s^*_{ij})$, and then a unique SNLN $(s^*_{ij}, < T^*_{ij}, I^*_{ij}, F^*_{ij} >) = (s_a, \frac{N(T^*_{ij} \cdot s^*_{ij})}{a}, 1 - \frac{N((1 - I^*_{ij}) \cdot s^*_{ij})}{a}, 1 - \frac{N((1 - F^*_{ij}) \cdot s^*_{ij})}{a})$ can be gained.

For instance, if $T^*_{ij} \cdot s^*_{ij} = 0.3$, $(1 - I^*_{ij}) \cdot s^*_{ij} = 1.2$, and $(1 - F^*_{ij}) \cdot s^*_{ij} = 2.7$, there is $\max\{N(T^*_{ij} \cdot s^*_{ij}), N((1 - I^*_{ij}) \cdot s^*_{ij}), N((1 - F^*_{ij}) \cdot s^*_{ij})\} = \max(0.3, 1.2, 2.7) = 2.7 \in [2, 3]$, so $a = 3$ and $(s^*_{ij}, < T^*_{ij}, I^*_{ij}, F^*_{ij} >) = (s_3, < 0.1, 0.6, 0.1 >)$.

(2) For $s_{ij}^* \le s_o, s_{ij}^* \le T_{ij}^* \cdot s_{ij}^* \le s_o, s_{ij}^* \le (1 - I_{ij}^*) \cdot s_{ij}^* \le s_o$ and $s_{ij}^* \le (1 - F_{ij}^*) \cdot s_{ij}^* \le s_o$; if $\min\{N(T_{ij}^* \cdot s_{ij}^*), N((1 - I_{ij}^*) \cdot s_{ij}^*), N((1 - F_{ij}^*) \cdot s_{ij}^*)\} \in [a, a + 1], a \ge N(s_{ij}^*)$, then a solitary SNLN is $(s_{ij}^*, < T_{ij}^*, I_{ij}^*, F_{ij}^* >)$ $= (s_a, \frac{N(T_{ij}^* \cdot s_{ij}^*)}{a}, 1 - \frac{N((1 - I_{ij}^*) \cdot s_{ij}^*)}{a}, 1 - \frac{N((1 - F_{ij}^*) \cdot s_{ij}^*)}{a})$.

For example, $T_{ij}^* \cdot s_{ij}^* = -0.3$, $(1 - I_{ij}^*) \cdot s_{ij}^* = -1.2$ and $(1 - F_{ij}^*) \cdot s_{ij}^* = -2.7$. Because $\min\{N(T_{ij}^* \cdot s_{ij}^*), N((1 - I_{ij}^*) \cdot s_{ij}^*), N((1 - F_{ij}^*) \cdot s_{ij}^*)\} = \min(-0.3, -1.2, -2.7) = 2.7 \in [-3, -2]$, then $a = -3$ and $(s_{ij}^*, < T_{ij}^*, I_{ij}^*, F_{ij}^* >) = (s_{-3}, < 0.1, 0.6, 0.1 >)$.

(3) Besides, there are two other situations: one is that the value of $N(T_{ij}^* \cdot s_{ij}^*)$, $N((1 - I_{ij}^*) \cdot s_{ij}^*)$ and $N((1 - F_{ij}^*) \cdot s_{ij}^*)$ are a positive number and two negative numbers; the other one is that there are two negative numbers and a positive number among $N(T_{ij}^* \cdot s_{ij}^*)$, $N((1 - I_{ij}^*) \cdot s_{ij}^*)$ and $N((1 - F_{ij}^*) \cdot s_{ij}^*)$. In these conditions, the final answers may not meet the requirements of $0 \le T_{ij}^* \le 1$, $0 \le 1 - I_{ij}^* \le 1$ or $0 \le 1 - F_{ij}^* \le 1$. In other words, the consistent matrix being obtained may not be a SNLPR. But it still does not affect us to measure the consistency degree of the SNLPR, for the reason that the values of $T_{ij}^* \cdot s_{ij}^*$, $(1 - I_{ij}^*) \cdot s_{ij}^*$ and $(1 - F_{ij}^*) \cdot s_{ij}^*$ can be calculated. Thus, the consistency index of the SNLPR can be acquired (more details see Definition 12, Definition 17 and Definition 18).

Example 5. *Suppose the same SNLPR in Example 2 is given. A consistent SNLPR* $K_1^* =$
$$\begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_1, < 0.133, 0.433, 0.167 >) & (s_1, < 0.067, 0.567, 0.934 >) \\ (s_{-1}, < 0.133, 0.433, 0.167 >) & (s_0, < 1, 0, 0 >) & (s_{-1}, < 0.067, 0.867, 0.433 >) \\ (s_{-1}, < 0.067, 0.567, 0.934 >) & (s_1, < 0.067, 0.867, 0.433 >) & (s_0, < 1, 0, 0 >) \end{bmatrix} \;\; can \; be$$
obtained in the abovementioned way.

According to those distance measures of SNLNs in Section 3, several distance measures of SNLPRs are further defined.

Definition 17. *There are two facultative SNLPRs* $A = (a_{ij})_{n \times n}$ *and* $B = (b_{ij})_{n \times n}$, *the linguistic term set is* $S = \{s_i | i \in [-g, g]\}$. *Then Hamming distance* $D_H(A, B)$, *Euclidean distance* $D_E(A, B)$ *and Hausdorff distance* $D_{Ha}(A, B)$ *between A and B are defined as:*

$$D_H(A, B) = \frac{1}{n(n-1)} \sum_{i \neq j}^{n} d_H(a_{ij}, b_{ij}) \tag{21}$$

$$D_E(A, B) = \frac{1}{n(n-1)} \sum_{i \neq j}^{n} d_E(a_{ij}, b_{ij}) \tag{22}$$

$$D_{Ha}(A, B) = \frac{1}{n(n-1)} \sum_{i \neq j}^{n} d_{Ha}(a_{ij}, b_{ij}). \tag{23}$$

Example 6. *Assume* $g = 4$, *and two SNLPR* $A = \begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_1, < 0.2, 0.3, 0.6 >) & (s_3, < 0.5, 0.4, 0.2 >) \\ (s_{-1}, < 0.2, 0.3, 0.6 >) & (s_0, < 1, 0, 0 >) & (s_{-2}, < 0.7, 0.1, 0.6 >) \\ (s_{-3}, < 0.5, 0.4, 0.2 >) & (s_2, < 0.7, 0.1, 0.6 >) & (s_0, < 1, 0, 0 >) \end{bmatrix}$,

$B = \begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_2, < 0.5, 0.1, 0.4 >) & (s_3, < 0.3, 0.6, 0.1 >) \\ (s_{-2}, < 0.5, 0.1, 0.4 >) & (s_0, < 1, 0, 0 >) & (s_{-1}, < 0.2, 0.8, 0.5 >) \\ (s_{-3}, < 0.3, 0.6, 0.1 >) & (s_1, < 0.2, 0.8, 0.5 >) & (s_0, < 1, 0, 0 >) \end{bmatrix}$. *Then Hamming distance* $D_H(A, B) \approx 0.1014$, *Euclidean distance* $D_E(A, B) \approx 0.1083$ *and Hausdorff distance* $D_{Ha}(A, B) \approx 0.1375$.

Theorem 3. *Given two SNLPRs* $A = (a_{ij})_{n \times n}$ *and* $B = (b_{ij})_{n \times n}$, *if* $D_H(A, B)$, $D_E(A, B)$ *and* $D_{Ha}(A, B)$ *can satisfy the following properties:*

(1) $0 \le D_H(A, B) \le 1$, $0 \le D_E(A, B) \le 1$, *and* $0 \le D_{Ha}(A, B) \le 1$;

(2) $D_H(A, B) = D_H(B, A)$, $D_E(A, B) = D_E(B, A)$, *and* $D_{Ha}(A, B) = D_{Ha}(B, A)$;

(3)   *If* $A = B$, *then* $D_H(A, B) = 0$, $D_E(A, B) = 0$, *and* $D_{Ha}(A, B) = 0$;

(4)　Let $A = (a_{ij})_{n \times n} = (sa_{ij}, < T_{ij}^a, I_{ij}^a, F_{ij}^a >)_{n \times n}$, $B = (b_{ij})_{n \times n} = (sb_{ij}, < T_{ij}^b, I_{ij}^b, F_{ij}^b >)_{n \times n}$ and $C = (c_{ij})_{n \times n} = (sc_{ij}, < T_{ij}^c, I_{ij}^c, F_{ij}^c >)_{n \times n}$ be three SNLPRs, if $s_0 \leq sa_{ij} \leq sb_{ij} \leq sc_{ij}$, $T_{ij}^a \leq T_{ij}^b \leq T_{ij}^c$, $I_{ij}^a \geq I_{ij}^b \geq I_{ij}^c$ and $F_{ij}^a \geq F_{ij}^b \geq F_{ij}^c$ for all $i, j = 1, 2, \cdots n$, then $D_H(A, B) \leq D_H(A, C)$, $D_H(B, C) \leq D_H(A, C)$, $D_E(A, B) \leq D_E(A, C)$, $D_E(B, C) \leq D_E(A, C)$, $D_{Ha}(A, B) \leq D_{Ha}(A, C)$ and $D_{Ha}(B, C) \leq D_{Ha}(A, C)$.

Proof 4.

(1)　Since $0 \leq d_H(a_{ij}, b_{ij}) \leq 1 \Rightarrow 0 \leq \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(a_{ij}, b_{ij}) \leq 1$, then $0 \leq D_H(A, B) \leq 1$.

Likewise, $0 \leq D_E(A, B) \leq 1$ and $0 \leq D_{Ha}(A, B) \leq 1$.

(2)　As $d_H(a_{ij}, b_{ij}) = d_H(b_{ij}, a_{ij}) \Rightarrow \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(a_{ij}, b_{ij}) = \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(b_{ij}, a_{ij})$, then $D_H(A, B) = D_H(B, A)$. Similarly, $D_E(A, B) = D_E(B, A)$ and $D_{Ha}(A, B) = D_{Ha}(B, A)$.

(3)　Because $A = B$, for all $i, j = 1, 2, \cdots n$, then $a_{ij} = b_{ij} \Rightarrow d_H(a_{ij}, b_{ij}) = 0 \Rightarrow D_H(A, B) = \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(a_{ij}, b_{ij}) = 0$. In the same way, $D_E(A, B) = 0$ and $D_{Ha}(A, B) = 0$.

(4)　As $d_H(a_{ij}, b_{ij}) \leq d_H(a_{ij}, c_{ij}) \Rightarrow \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(a_{ij}, b_{ij}) \leq \frac{1}{n(n-1)} \sum_{i \neq j}^n d_H(a_{ij}, c_{ij}) \Rightarrow D_H(a_{ij}, b_{ij}) \leq D_H(a_{ij}, c_{ij})$. Similarly, $D_H(b_{ij}, c_{ij}) \leq D_H(a_{ij}, c_{ij})$, $D_E(a_{ij}, b_{ij}) \leq D_E(a_{ij}, c_{ij})$, $D_E(b_{ij}, c_{ij}) \leq D_E(a_{ij}, c_{ij})$, $D_{Ha}(a_{ij}, b_{ij}) \leq D_{Ha}(a_{ij}, c_{ij})$ and $D_{Ha}(b_{ij}, c_{ij}) \leq D_{Ha}(a_{ij}, c_{ij})$.

This is the end of Proof 4.　□

Definition 18. *Let $K = (k_{ij})_{n \times n}$ be a SNLPR, and $K^* = (k_{ij}^*)_{n \times n}$ be the corresponding consistent SNLPR, the deviation between K and K^* can be expressed by a consistency index $C(K)$ as follows:*

$$CX(K) = 1 - D(K, K^*), \tag{24}$$

where $D(K, K^*)$ can be replaced by $D_H(K, K^*)$, $D_E(K, K^*)$, or $D_{Ha}(K, K^*)$.

Example 7.　*Assume a SNLPR $K_1 = \begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_{-1}, < 0.4, 0.2, 0.1 >) & (s_2, < 0.3, 0.1, 0.2 >) \\ (s_1, < 0.4, 0.2, 0.1 >) & (s_0, < 1, 0, 0 >) & (s_{-3}, < 0.2, 0.5, 0.3 >) \\ (s_{-2}, < 0.3, 0.1, 0.2 >) & (s_3, < 0.2, 0.5, 0.3 >) & (s_0, < 1, 0, 0 >) \end{bmatrix}$*

is the same with Example 2, and its consistent SNLPR is $K_1^* = \begin{bmatrix} (s_0, < 1, 0, 0 >) & (s_1, < 0.133, 0.433, 0.167 >) & (s_1, < 0.067, 0.567, 0.934 >) \\ (s_{-1}, < 0.133, 0.433, 0.167 >) & (s_0, < 1, 0, 0 >) & (s_{-1}, < 0.067, 0.867, 0.433 >) \\ (s_{-1}, < 0.067, 0.567, 0.934 >) & (s_1, < 0.067, 0.867, 0.433 >) & (s_0, < 1, 0, 0 >) \end{bmatrix}$ from Example 5.
If $D_H(K, K^*)$ is used, then $CX(K) \approx 0.8569$; if $D_E(K, K^*)$ is used, then $CX(K) \approx 0.9936$; if $D_{Ha}(K, K^*)$ is used, then $CX(K) \approx 0.8083$.

Note that since $0 \leq D(K, K^*) \leq 1$, then $0 \leq CX(K) \leq 1$. Moreover, the greater the value of $CX(K)$, the more consistent K will be according to Definition 18.

4.3. Improving the Consistency of SNLPRs

Normally, it is difficult for DMs to provide a fully consistent SNLPR. There will be a lot of uncertainty in the decision-making process. For this reason, it is appropriate and necessary to allow the SNLPR presented by DMs satisfy the consistency in some extent. Then, the following is the concept of acceptable consistency.

Definition 19. *Let CX be a consistency threshold value. For an arbitrary SNLPR K, if the corresponding consistency index is $CX(K)$, and*

$$CX(K) > CX, \tag{25}$$

then K is consistent in some extent. In other words, it has acceptable consistency.

Zhu and Xu [74] indicated that the consistency index $CX(K)$ obeys a normal distribution, thus providing a method to determine the consistency threshold value CX. This method is used here. When the significance level $\alpha = 0.1$ and the standard deviation $\sigma = 0.2$, the consistency index threshold is shown in Table 1.

Table 1. The consistency threshold value CX.

CX	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
$g = 2$	0.8235	0.7576	0.7210	0.6981	0.6824	0.6710	0.6624
$g = 3$	0.8739	0.8269	0.8007	0.7844	0.7731	0.7650	0.7589
$g = 4$	0.9020	0.8653	0.8450	0.8323	0.8235	0.8172	0.8124

Of course, the numbers in Table 1 can be used for reference. DMs can determine the value of thresholds based on their previous experience, preferences, or actual situations as well.

Example 8. *Suppose a SNLPR K_1 is the same as Example 2, and the consistency index $CX(K_1) \approx 0.8569$ from Example 7. If $D_H(K_1, K_1^*)$ is used, for $g = 4$ and $n = 3$, the consistency threshold value can be assigned with $CX = 0.9020$ based on Table 1. $CX(K_1) < CX$, and it demonstrates that K_1 does not have acceptable consistency.*

When the initial SNLPR presented by DMs is not acceptably consistent, a way to improve this SNLPR should be provided. Then, an iterative algorithm (Algorithm 1) is given to achieve acceptable consistency as follows:

Algorithm 1. Consistency-improving process with automatic iteration

---

Input: The initial SNLPR $K^{(s)} = (k_{ij}^{(s)})_{n \times n} = (s_{ij}^{(s)}, < T_{ij}^{(s)}, I_{ij}^{(s)}, F_{ij}^{(s)} >)_{n \times n}$, and the value of the consistency threshold CX.

Output: The modified SNLPR K_a, and its consistency index $CX(K_a)$.

Step 1: Let $s = 0$ and $ie = 0$. According to Theorem 2, acquire the consistent SNLPR $K^{*(s)} = (k_{ij}^{*(s)})_{n \times n}$ of $K^{(s)}$, where $k_{ij}^{*(s)} = (s_{ij}^{*(s)}, < T_{ij}^{*(s)}, I_{ij}^{*(s)}, F_{ij}^{*(s)} >)$.

Step 2: Choose an applicable distance, and calculate $CX(K^{(s)})$ on the basis of Definition 18.

Step 3: Determine the maximum value of iterative times $ie_{\max} \geq 1$. If $CX(K^{(s)}) > CX$ or $ie > ie_{\max}$, then go to Step 6; otherwise, go to the next step.

Step 4: Confirm the adjusted parameter $\delta \in (0, 1)$. Let $T_{ij}^{(s+1)} \cdot s_{ij}^{(s+1)} = \delta(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus (1 - \delta)(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)})$, $(1 - I_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} = \delta((1 - I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) \oplus (1 - \delta)((1 - I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)})$ and $(1 - F_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} = \delta((1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) \oplus (1 - \delta)((1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)})$.

Step 5: Let $ie = ie + 1$ and $s = s + 1$, then $K^{(s)}$ is the adjusted SNFLPR. Return to Step 2.

Step 6: Let $K_a = K^{(s)}$, Output K_a and $CX(K_a)$.

This algorithm above improves the consistency through the iterative process automatically, which is convenient and efficient.

Theorem 4. *Given a SNLPR K, if K does not have acceptable consistency, it will be more consistent using Algorithm 1. That is to say, $C(K^{(s+1)}) < C(K^{(s)})$ is true. Moreover, $\lim_{s \to \infty} C(K^{(s)}) = 0$.*

Proof 5.

(1)　From Equation (18), $T_{ij}^{*(s)} \cdot s_{ij}^{*(s)} = \frac{1}{n} \{ \oplus_{k=1}^{n} [(T_{ik}^{(s)} \cdot s_{ik}^{(s)}) \oplus (T_{kj}^{(s)} \cdot s_{kj}^{(s)})] \}$, and then

$|T_{ij}^{(s+1)} s_{ij}^{(s+1)} - T_{ij}^{*(s+1)} s_{ij}^{*(s+1)}| = |T_{ij}^{(s+1)} s_{ij}^{(s+1)} - \frac{1}{n} \{ \oplus_{k=1}^{n} [(T_{ik}^{(s+1)} \cdot s_{ik}^{(s+1)}) \oplus (T_{kj}^{(s+1)} \cdot s_{kj}^{(s+1)})] \} | =$

$|\delta(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus (1 - \delta)(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - \frac{1}{n} \{ \oplus_{k=1}^{n} [(\delta(T_{ik}^{(s)} \cdot s_{ik}^{(s)}) \oplus (1 - \delta)(T_{ik}^{*(s)} \cdot s_{ik}^{*(s)})) \oplus (\delta(T_{kj}^{(s)} \cdot$

$s_{kj}^{(s)}) \oplus (1-\delta)(T_{kj}^{*(s)} \cdot s_{kj}^{*(s)}))]\}| \leq |\delta(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -\frac{1}{n}\{\oplus_{k=1}^{n}[(\delta(T_{ik}^{(s)} \cdot s_{ik}^{(s)}) \oplus (\delta(T_{kj}^{(s)} \cdot s_{kj}^{(s)}))]\}| + |(1-\delta)(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(1-\delta)(T_{ik}^{*(s)} \cdot s_{ik}^{*(s)})) \oplus (1-\delta)(T_{kj}^{*(s)} \cdot s_{kj}^{*(s)}))]\}|$

$= \quad \delta|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -\frac{1}{n}\{\oplus_{k=1}^{n}[(T_{ik}^{(s)} \cdot s_{ik}^{(s)}) \oplus (T_{kj}^{(s)} \cdot s_{kj}^{(s)})]\}| + (1-\delta)|(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(T_{ik}^{*(s)} \cdot s_{ik}^{*(s)}) \oplus (T_{kj}^{*(s)} \cdot s_{kj}^{*(s)})]\}| = \delta|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}| + |(1-\delta)|(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\frac{1}{n}\{\oplus_{p=1}^{n}[(T_{ip}^{(s)} \cdot s_{ip}^{(s)}) \oplus (T_{pk}^{(s)} \cdot s_{pk}^{(s)})]\}) \oplus (\frac{1}{n}\{\oplus_{p=1}^{n}[(T_{kp}^{(s)} \cdot s_{kp}^{(s)}) \oplus (T_{pj}^{(s)} \cdot s_{pj}^{(s)})]\})]\}|$

$= \delta|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}| + (1-\delta)|(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - (\frac{1}{n}\{\oplus_{p=1}^{n}[(T_{ip}^{(s)} \cdot s_{ip}^{(s)}) \oplus (T_{pj}^{(s)} \cdot s_{pj}^{(s)})]\})|$
$= \delta|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}| + (1-\delta)|(T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}) - (T_{ij}^{(s)} \cdot s_{ij}^{(s)})| = \delta|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}|$, so $N(|T_{ij}^{(s+1)} s_{ij}^{(s+1)} - T_{ij}^{*(s+1)} s_{ij}^{*(s+1)}|) \leq \delta \cdot N(|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}|)$;

(2) From Equation (19), $(1-I_{ij}^{*}) \cdot s_{ij}^{*} = \frac{1}{n}\{\oplus_{k=1}^{n}[((1-I_{ik}) \cdot s_{ik}) \oplus ((1-I_{kj}) \cdot s_{kj})]\}$, and then $|(1-I_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1-I_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}| = |(1-I_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-I_{ik}^{(s+1)}) \cdot s_{ik}^{(s+1)}) \oplus ((1-I_{kj}^{(s+1)}) \cdot s_{kj}^{(s+1)})]\}| = |\delta((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) \oplus (1-\delta)((1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\delta((1-I_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus (1-\delta)((1-I_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus (\delta((1-I_{ij}^{(s)}) \cdot s_{kj}^{(s)}) \oplus (1-\delta)((1-I_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}|$

$\leq |\delta((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\delta((1-I_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus (\delta((1-I_{kj}^{(s)}) \cdot s_{kj}^{(s)}))]\}| +$

$|(1-\delta)((1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(1-\delta)((1-I_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus_{k=1}^{n} (1-\delta)((1-I_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}| = \delta|((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-I_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus ((1-I_{kj}^{(s)}) \cdot s_{kj}^{(s)})]\}| + (1-\delta)|((1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-I_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus_{k=1}^{n} ((1-I_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}|$

$= \delta|((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - (1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1-\delta)|((1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\frac{1}{n}\{\oplus_{p=1}^{n}[((1-I_{ip}^{(s)}) \cdot s_{ip}^{(s)})\oplus$

$((1-I_{pk}^{(s)}) \cdot s_{pk}^{(s)})]\}) \oplus_{k=1}^{n} (\frac{1}{n}\{\oplus_{p=1}^{n}[((1-I_{kp}^{(s)}) \cdot s_{kp}^{(s)}) \oplus ((1-I_{pj}^{(s)}) \cdot s_{pj}^{(s)})]\})]\}|$

$= \delta|((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - (1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1-\delta)|((1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{p=1}^{n}[((1-I_{ip}^{(s)}) \cdot s_{ip}^{(s)}) \oplus ((1-I_{pj}^{(s)}) \cdot s_{pj}^{(s)})]\}| = \delta|((1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - (1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1-\delta)|(1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)} - (1-I_{ij}^{(s)}) \cdot s_{ij}^{*(s)}| = \delta|(1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|$, so $N((1-I_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1-I_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}|) \leq \delta \cdot N(|(1-I_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1-I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|)$;

(3) From Equation (20), $(1-F_{ij}^{*}) \cdot s_{ij}^{*} = \frac{1}{n}\{\oplus_{k=1}^{n}[((1-F_{ik}) \cdot s_{ik}) \oplus ((1-F_{kj}) \cdot s_{kj})]\}$, and then $|(1-F_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1-F_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}| = |(1-F_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-F_{ik}^{(s+1)}) \cdot s_{ik}^{(s+1)}) \oplus ((1-F_{kj}^{(s+1)}) \cdot s_{kj}^{(s+1)})]\}| = |\delta((1-F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) \oplus (1-\delta)((1-F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\delta((1-F_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus (1-\delta)((1-F_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus (\delta((1-F_{ij}^{(s)}) \cdot s_{kj}^{(s)}) \oplus (1-\delta)((1-F_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}|$

$\leq |\delta((1-F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\delta((1-F_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus (\delta((1-F_{kj}^{(s)}) \cdot s_{kj}^{(s)}))]\}| +$

$|(1-\delta)((1-F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(1-\delta)((1-F_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus_{k=1}^{n} (1-\delta)((1-F_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}|$

$= \delta|((1-F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-F_{ik}^{(s)}) \cdot s_{ik}^{(s)}) \oplus ((1-F_{kj}^{(s)}) \cdot s_{kj}^{(s)})]\}| + (1-\delta)|((1-F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[((1-F_{ik}^{*(s)}) \cdot s_{ik}^{*(s)})) \oplus_{k=1}^{n} ((1-F_{kj}^{*(s)}) \cdot s_{kj}^{*(s)}))]\}|$

$= \delta|((1-F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - (1-F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1-\delta)|((1-F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \frac{1}{n}\{\oplus_{k=1}^{n}[(\frac{1}{n}\{\oplus_{p=1}^{n}[((1-F_{ip}^{(s)}) \cdot s_{ip}^{(s)})\oplus$

$((1-F_{pk}^{(s)}) \cdot s_{pk}^{(s)})]\}) \oplus_{k=1}^{n} (\frac{1}{n}\{\oplus_{p=1}^{n}[((1-F_{kp}^{(s)}) \cdot s_{kp}^{(s)}) \oplus ((1-F_{pj}^{(s)}) \cdot s_{pj}^{(s)})]\})]\}|$

$$= \quad \delta |((1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)}) - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1 - \delta)|((1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}) - \tfrac{1}{n}\{\oplus_{p=1}^{n}[((1 - F_{ip}^{(s)}) \cdot s_{ip}^{(s)}) \oplus ((1 - F_{pj}^{(s)}) \cdot s_{pj}^{(s)})]\}| = \quad \delta |(1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| + (1 - \delta)|(1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)} - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}| = \delta |(1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|, \text{ so } N((1 - F_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1 - F_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}|) \leq \delta \cdot N(|(1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|);$$ According to (1)–(3), there is $N(|T_{ij}^{(s+1)} s_{ij}^{(s+1)} - T_{ij}^{*(s+1)} s_{ij}^{*(s+1)}|) + N((1 - I_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1 - I_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}|) + N((1 - F_{ij}^{(s+1)}) \cdot s_{ij}^{(s+1)} - (1 - F_{ij}^{*(s+1)}) \cdot s_{ij}^{*(s+1)}|) \leq \delta \cdot [N(|(T_{ij}^{(s)} \cdot s_{ij}^{(s)}) \oplus -T_{ij}^{*(s)} \cdot s_{ij}^{*(s)}|) + N(|(1 - I_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1 - I_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|) + N(|(1 - F_{ij}^{(s)}) \cdot s_{ij}^{(s)} - (1 - F_{ij}^{*(s)}) \cdot s_{ij}^{*(s)}|)],$ then $d_H(k_{ij}^{(s+1)}, k_{ij}^{*(s+1)}) \leq \delta d_H(k_{ij}^{(s)}, k_{ij}^{*(s)}),$ $d_E(k_{ij}^{(s+1)}, k_{ij}^{*(s+1)}) \leq \delta d_E(k_{ij}^{(s)}, k_{ij}^{*(s)}),$ $d_{Ha}(k_{ij}^{(s+1)}, k_{ij}^{*(s+1)}) \leq \delta d_{Ha}(k_{ij}^{(s)}, k_{ij}^{*(s)}),$ so $CI(K^{(s+1)}) = 1 - D(K^{(s+1)}, K^{*(s+1)}) = \frac{1}{n(n-1)}\sum_{i \neq j}^{n} d(k_{ij}^{(s+1)}, k_{ij}^{*(s+1)}) \geq \frac{1}{n(n-1)}\sum_{i \neq j}^{n} \delta \cdot d(k_{ij}^{(s)}, k_{ij}^{*(s)}) \geq CI(K^{(s)}).$ In addition, $\lim_{s \to \infty} CI(K^{(s)}) = 1.$

This is the end of Proof 5. □

It can be seen from Theorem 4 that an arbitrary SNLPR that does not have a satisfactory consistency can be adjusted by the above algorithm to an acceptable matrix. The value of the adjustment parameter will have an effect on the process speed and times. DMs or other experts can determine the value of δ based on the actual situation. In general, $\delta = 0.5$ is advised. If the predetermined threshold is not satisfied, the algorithm will be repeated until the maximum number of iterations is reached.

Example 9. *Let a SNLPR K_1 be the same as Example 2. It can be seen that $CX(K_1) \approx 0.8569$, and K_1 does not have acceptable consistency from Example 8. Then Algorithm 1 can be used to improve it.*

Algorithm 1. Consistency-improving process with automatic iteration

---

Input: The initial SNLPR

$$K^{(0)} = K_1 = \begin{bmatrix} (s_0, <1,0,0>) & (s_{-1}, <0.4,0.2,0.1>) & (s_2, <0.3,0.1,0.2>) \\ (s_1, <0.4,0.2,0.1>) & (s_0, <1,0,0>) & (s_{-3}, <0.2,0.5,0.3>) \\ (s_{-2}, <0.3,0.1,0.2>) & (s_3, <0.2,0.5,0.3>) & (s_0, <1,0,0>) \end{bmatrix}, \text{ the}$$

consistency threshold value $CX = 0.9020$, and the maximum value of iterative times $ie_{\max} = 3$.

Output: The modified SNLPR K_a, and its consistency index $CX(K_a)$.

Step 1: As $CX(K^{(0)}) < CX$, go to the next step.

Step 2: Let $T_{ij}^{(1)} \cdot s_{ij}^{(1)} = \frac{1}{2}(T_{ij}^{(0)} \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}(T_{ij}^{*(0)} \cdot s_{ij}^{*(0)}),$

$(1 - I_{ij}^{(1)}) \cdot s_{ij}^{(1)} = \frac{1}{2}((1 - I_{ij}^{(0)}) \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}((1 - I_{ij}^{*(0)}) \cdot s_{ij}^{*(0)})$

and $(1 - F_{ij}^{(1)}) \cdot s_{ij}^{(1)} = \frac{1}{2}((1 - F_{ij}^{(0)}) \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}((1 - F_{ij}^{*(0)}) \cdot s_{ij}^{*(0)}).$

Step 3: Let $ie = 1,$

$$K^{(1)} = (k_{ij}^{(1)})_{n \times n} = \begin{bmatrix} (s_0, <1,0,0>) & (s_{-1}, <0.133,0.883,0.867>) & (s_2, <0.167,0.442,0.584>) \\ (s_1, <0.133,0.883,0.867>) & (s_0, <1,0,0>) & (s_{-2}, <0.167,0.592,0.334>) \\ (s_{-2}, <0.167,0.442,0.584>) & (s_2, <0.167,0.592,0.334>) & (s_0, <1,0,0>) \end{bmatrix}$$

Step 4: The consistent SNLPR

$$K^{*(1)} = \begin{bmatrix} (s_0, <1,0,0>) & (s_1, <0.134,0.434,0.367>) & (s_1, <0.067,0.609,0.934>) \\ (s_{-1}, <0.134,0.434,0.367>) & (s_0, <1,0,0>) & (s_{-1}, <0.067,0.783,0.434>) \\ (s_{-1}, <0.067,0.609,0.934>) & (s_1, <0.067,0.783,0.434>) & (s_0, <1,0,0>) \end{bmatrix}.$$

Step 5: $D_H(K^{(1)}, K^{*(1)})$ is used, and $CX(K^{(1)}) \approx 0.9276$ on the basis of Definition 18.

Step 6: As $CX(K^{(1)}) > CX$, go to the next step.

Step 7: Let $K_a = K^{(1)}$, Output

$$K_a = \begin{bmatrix} (s_0, <1,0,0>) & (s_{-1}, <0.133,0.883,0.867>) & (s_2, <0.167,0.442,0.584>) \\ (s_1, <0.133,0.883,0.867>) & (s_0, <1,0,0>) & (s_{-2}, <0.167,0.592,0.334>) \\ (s_{-2}, <0.167,0.442,0.584>) & (s_2, <0.167,0.592,0.334>) & (s_0, <1,0,0>) \end{bmatrix},$$

and $CX(K_a) \approx 0.9276.$

4.4. A Decision-Making Approach with SNLPRs

In this section, a decision-making method based on SNLPRs is presented.

Take a decision-making problem under simplified neutrosophic linguistic environment into consideration. Suppose there are a group of alternatives $X = \{x_1, x_2, \ldots, x_n\}$. The DMs want to get the ranking or select the eligible alternative from them. Then a preference matrix is formed after the linguistic term set $S = \{s_i | i \in [-g, g]\}$ is given. The basic elements in this matrix are SNLNs. Then the method based on SNLPRs is provided as Algorithm 2:

Algorithm 2. Decision-making approach with SNLPRs

Input: The initial SNLPR $K = (k_{ij})_{n \times n} = (s_{ij}, < T_{ij}, I_{ij}, F_{ij} >)_{n \times n}$.
Output: The ranking result and the best alternative x^*.
Step 1: Choose a distance measure and calculate the value of $CX(K)$ according to Equation (24)
Step 2: Determine the threshold value CX. If $CX(K) < CX$, then improve it by Algorithm 1 until it is acceptably consistent.
Step 3: Aggregate each row of preference values in K using the SNLAM or SNLGM operator.
Step 4: Calculate the score function $S(x_i)$ of overall preference degree of each $x_i (i = 1, 2, \ldots, n)$ by Definition 7.
Step 5: Rank the alternatives $x_i (i = 1, 2, \ldots, n)$ on the basis of comparison method in Definition 8, and then output the ranking and the optimal alternative(s) x^*.

Note that it is a common and useful way to use aggregation operators to aggregate preference information, and then get the ranking result according to some comparison rules. However, Hou [75] pointed out that using arithmetic mean aggregation may get a reverse ranking. Therefore, the SNLGM operator to aggregate preference values is better.

5. Application and Comparison

The proposed decision-making method is applied to selecting project delivery models in this section. Some related comparison analyses are presented in the end.

YG Construction Co., Ltd. planned to select two suitable delivery models for a road construction project. The first one is chosen at once, and second one is reserved and accepted if necessary in the future. After a preliminary selection, four satisfactory options, DB, DBB, DBO, and CM, denoted by $\{x_1, x_2, x_3, x_4\}$, respectively, are considered. In order to pick out the right model, the project manager invites two experts to make evaluations together.

According to the properties of this project, the actual environment and the capability of owners, the project manager evaluates four alternatives. He presents his preference information with linguistic values. The linguistic term set used is $S = \{s_i | i \in [-4, 4]\}$, where $s = \{s_{-4} = $ *much poorer*, $s_{-3} = $ *a lot poorer*, $s_{-2} = $ *poorer*, $s_{-1} = $ *slightly poorer*, $s_0 = $ *fair*, $s_1 = $ *slightly better*, $s_2 = $ *better*, $s_3 = $ *a lot better*, $s_4 = $ *much better*$\}$. Simultaneously, he gives the corresponding hesitant degrees of each preference value. Then two experts are asked to judge the possibility that the evaluation is inaccurate. In this way, SNLNs may be a good indication of their preference. As an example, the manager holds the view that x_1 is s_{-1} to x_2, but he is not sure of his assessment. He thinks the degree of hesitation is 0.3. Afterwards, there is a probability of 0.9 that s_{-1} is right, and a 0.2 probability of error given by two specialists. Therefore, they can be expressed by a SNLN $k_{12} = (s_{-1}, < 0.9, 0.3, 0.2 >)$.

In the end, all the preference information yields an SNLPR as follows:

$$
K = \begin{bmatrix}
(s_0, < 1, 0, 0 >) & (s_{-1}, < 0.9, 0.3, 0.2 >) & (s_1, < 0.7, 0.5, 0.4 >) & (s_3, < 0.8, 0.3, 0.1 >) \\
(s_1, < 0.9, 0.3, 0.2 >) & (s_0, < 1, 0, 0 >) & (s_1, < 0.5, 0.2, 0.5 >) & (s_{-2}, < 0.9, 0.2, 0.3 >) \\
(s_{-1}, < 0.7, 0.5, 0.4 >) & (s_{-1}, < 0.5, 0.2, 0.5 >) & (s_0, < 1, 0, 0 >) & (s_{-3}, < 0.6, 0.7, 0.1 >) \\
(s_{-3}, < 0.8, 0.3, 0.1 >) & (s_2, < 0.9, 0.2, 0.3 >) & (s_3, < 0.6, 0.7, 0.1 >) & (s_0, < 1, 0, 0 >)
\end{bmatrix}.
$$

5.1. Illustration

The decision-making method proposed in Section 4.4 is used to rank four options and select two models among them. The following are the specific steps:

Step 1: After discussion, DMs choose $D_H(K, K^*)$, and then calculate $CX(K) \approx 0.8628$ according to Equation (24).

Step 2: Because $g = 4$ and $n = 4$, DMs suggest the threshold value $CX = 0.8653$ from Table 1. And they find $CX(K) < CX$, then use Algorithm 1 to improve it as follows:

Let $K^{(0)} = K = (k_{ij}^{(0)})_{n \times n} = (s_{ij}^{(0)}, < T_{ij}^{(0)}, I_{ij}^{(0)}, F_{ij}^{(0)} >)_{n \times n}$ and $ie = 0$. According to Theorem 2, the consistent SNLPR $K^{*(0)} = (k_{ij}^{*(0)})_{n \times n} = (s_{ij}^{*(0)}, < T_{ij}^{*(0)}, I_{ij}^{*(0)}, F_{ij}^{*(0)} >)_{n \times n} =$

$$\begin{bmatrix} (s_0, <1,0,0>) & (s_1, <0.65, 0.5, 0.35>) & (s_2, <0.65, 0.4875, 0.2125>) & (s_1, <0.25, 0.625, 0.725>) \\ (s_{-1}, <0.65, 0.5, 0.35>) & (s_0, <1,0,0>) & (s_1, <0.65, 0.475, 0.075>) & (s_{-1}, <0.4, 0.875, 0.625>) \\ (s_{-2}, <0.65, 0.4875, 0.2125>) & (s_{-1}, <0.65, 0.475, 0.075>) & (s_0, <1,0,0>) & (s_{-2}, <0.525, 0.675, 0.35>) \\ (s_{-1}, <0.25, 0.625, 0.725>) & (s_1, <0.4, 0.875, 0.625>) & (s_2, <0.525, 0.675, 0.35>) & (s_0, <1,0,0>) \end{bmatrix}.$$

Since $CX(K^{(0)}) = CX(K) \approx 0.8649 < CX$, determine $\delta = \frac{1}{2}$ and $ie_{\max} = 3$. Let $K^{(1)} = K = (k_{ij}^{(1)})_{n \times n} = (s_{ij}^{(1)}, < T_{ij}^{(1)}, I_{ij}^{(1)}, F_{ij}^{(1)} >)_{n \times n} =$

$$\begin{bmatrix} (s_0, <1,0,0>) & (s_{-1}, <0.125, 0.9, 0.925>) & (s_2, <0.5, 0.6188, 0.4563>) & (s_2, <0.6625, 0.3813, 0.2563>) \\ (s_1, <0.125, 0.9, 0.925>) & (s_0, <1,0,0>) & (s_1, <0.575, 0.3375, 0.2875>) & (s_{-2}, <0.55, 0.5688, 0.5563>) \\ (s_{-2}, <0.5, 0.6188, 0.4563>) & (s_{-1}, <0.575, 0.3375, 0.2875>) & (s_0, <1,0,0>) & (s_{-2}, <0.7125, 0.6125, 0>) \\ (s_{-2}, <0.6625, 0.3813, 0.2563>) & (s_2, <0.55, 0.5688, 0.5563>) & (s_2, <0.7125, 0.6125, 0>) & (s_0, <1,0,0>) \end{bmatrix},$$

where $T_{ij}^{(1)} \cdot s_{ij}^{(1)} = \frac{1}{2}(T_{ij}^{(0)} \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}(T_{ij}^{*(0)} \cdot s_{ij}^{*(0)})$, $(1 - I_{ij}^{(1)}) \cdot s_{ij}^{(1)} = \frac{1}{2}((1 - I_{ij}^{(0)}) \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}((1 - I_{ij}^{*(0)}) \cdot s_{ij}^{*(0)})$ and $(1 - F_{ij}^{(1)}) \cdot s_{ij}^{(1)} = \frac{1}{2}((1 - F_{ij}^{(0)}) \cdot s_{ij}^{(0)}) \oplus \frac{1}{2}((1 - F_{ij}^{*(0)}) \cdot s_{ij}^{*(0)})$. As $CX(K^{(1)}) \approx 0.9314 > CX$, output $K^{(1)}$.

Step 3: Aggregate each row of preference values in $K^{(1)}(i \neq j)$ using SNLGM operator. Then the overall preferences are $px_1 \approx (s_{0.7622}, < 0.5966, 0.7132, 0.6882 >)$, $px_2 \approx (s_{-0.3160}, < 0.5570, 0.6943, 0.7127 >)$, $px_3 \approx (s_{-1.7106}, < 0.5039, 0.5392, 0.2710 >)$ and $px_4 \approx (s_{0.1602}, < 0.5102, 0.5307, 0.3090 >)$.

Step 4: Calculate the score function $S(px_i)(i = 1, 2, \ldots, n)$ by Definition 7: $S(px_1) \approx 0.2372$, $S(px_2) \approx 0.1765$, $S(px_3) \approx 0.1616$ and $S(px_4) \approx 0.2896$.

Step 5: Because $S(px_4) > S(px_1) > S(px_2) > S(px_3) \Rightarrow x_4 > x_1 > x_2 > x_3$, and the optimal alternative is $x^* = x_4$, the second alternative is x_1.

5.2. Comparison Analysis

Considering the concept of SNLPRs is newly proposed, several approaches related to other kinds of preference relations are chosen to make a comparison in this subsection.

As the expressions of basic elements in different preference relations are diverse, the first task is to transform the SNLNs in SNLPRs into the corresponding expression. Then, the same problem will be solved. The following are the information conversion process and major steps of the related methods:

(1) Single-valued trapezoidal neutrosophic preference relations (SVTNPRs) [44]

First, SNLNs in SNLPRs can be transformed into single-valued trapezoidal neutrosophic numbers (SVTNNs). A suitable way is changing the linguistic values of SNLNs into trapezoidal fuzzy numbers in SVTNNs and keeping membership degrees. The converted values can be denoted by $A_i = (a_i, b_i, c_i, d_i) = (\max\{\frac{2i+2g-1}{4g+3}, 0\}, \frac{2i+2g}{4g+3}, \frac{2i+2g+1}{4g+3}, \min\{\frac{2i+2g+2}{4g+3}, 1\})$ according to [76]. As an illustration, $(s_1, < 0.7, 0.5, 0.4 >)$ can be regarded as $([0.474, 0.526, 0.579, 0.632], < 0.7, 0.5, 0.4 >)$. Then, the corresponding single-valued trapezoidal neutrosophic matrix

is $\quad K^{TN} \quad = $

$$
\begin{bmatrix}
([0.368, 0.421, 0.474, 0.526], <1, 0, 0>) & ([0.263, 0.316, 0.368, 0.421], <0.9, 0.3, 0.2>) \\
([0.474, 0.526, 0.579, 0.632], <0.9, 0.3, 0.2>) & ([0.368, 0.421, 0.474, 0.526], <1, 0, 0>) \\
([0.263, 0.316, 0.368, 0.421], <0.7, 0.5, 0.4>) & ([0.263, 0.316, 0.368, 0.421], <0.5, 0.2, 0.5>) \\
([0.053, 0.105, 0.158, 0.211], <0.8, 0.3, 0.1>) & ([0.579, 0.632, 0.684, 0.737], <0.9, 0.2, 0.3>)
\end{bmatrix}
$$

$$
\begin{matrix}
([0.474, 0.526, 0.579, 0.632], <0.7, 0.5, 0.4>) & ([0.684, 0.737, 0.790, 0.842], <0.8, 0.3, 0.1>) \\
([0.474, 0.526, 0.579, 0.632], <0.5, 0.2, 0.5>) & ([0.158, 0.211, 0.263, 0.316], <0.9, 0.2, 0.3>) \\
([0.368, 0.421, 0.474, 0.526], <1, 0, 0>) & ([0.053, 0.105, 0.158, 0.211], <0.6, 0.7, 0.1>) \\
([0.684, 0.737, 0.790, 0.842], <0.6, 0.7, 0.1>) & ([0.368, 0.421, 0.474, 0.526], <1, 0, 0>)
\end{matrix} .
$$

Subsequently, using the method in [44] to get the consistent preference matrix, and the ranking is $x_4 > x_1 > x_2 > x_3$.

(2) ILPRs [55]

In the beginning, SNLNs should be converted into intuitionistic linguistic numbers. The linguistic values can remain, $\frac{1}{3}(T_{ij} + 1 - I_{ij} + 1 - F_{ij})$ may be equivalent to u, and $v = 1 - u$. As an example, $(s_{-1}, <0.9, 0.3, 0.2>)$ can be converted into $<s_{-1}, (0.8, 0.2)>$. Then, an intuitionistic linguistic preference relation

$$(\text{ILPR})K^I = \begin{bmatrix}
<s_0, (1,0)> & <s_{-1}, (0.8, 0.2)> & <s_1, (0.6, 0.4)> & <s_3, (0.8, 0.2)> \\
<s_1, (0.8, 0.2)> & <s_0, (1,0)> & <s_1, (0.6, 0.4)> & <s_{-2}, (0.8, 0.2)> \\
<s_{-1}, (0.6, 0.4)> & <s_{-1}, (0.6, 0.4)> & <s_0, (1,0)> & <s_{-3}, (0.6, 0.4)> \\
<s_{-3}, (0.8, 0.2)> & <s_2, (0.8, 0.2)> & <s_3, (0.6, 0.4)> & <s_0, (1,0)>
\end{bmatrix}$$ is got. As $CI(K^I) \approx$

$0.8819 < \theta(0.9016)$, let $\beta = 0.8$, and then the consistency index of the adjusted ILPR is $CI(K^{I(1)}) \approx 0.9069 >$

$\theta(0.9016)$. Thus, using the method in [55], the preferred degree matrix $U = \begin{pmatrix} 1/2 & 1 & 1 & 6/7 \\ 0 & 1/2 & 1 & 0 \\ 0 & 0 & 1/2 & 0 \\ 1/7 & 1 & 1 & 1/2 \end{pmatrix}$.

Since the preferred degrees $r_1(\frac{47}{14}) > r_4(\frac{37}{14}) > r_2(\frac{3}{2}) > r_3(\frac{1}{2})$, then $x_1 \succ x_4 \succ x_2 \succ x_3$.

(3) HFLPRs [54]

Firstly, SNLNs can be converted to hesitant fuzzy linguistic term sets. For example, $(s_{-0.9}, s_{-0.3}, s_{-0.2})$ can take the place of $(s_{-1}, <0.9, 0.3, 0.2>)$. Hence, the corresponding HFLPR is $K^H =$

$$\begin{bmatrix}
(s_0) & (s_{-0.9}, s_{-0.3}, s_{-0.2}) & (s_{0.4}, s_{0.5}, s_{0.7}) & (s_{0.3}, s_{0.9}, s_{2.4}) \\
(s_{0.9}, s_{0.3}, s_{0.2}) & (s_0) & (s_{0.2}, s_{0.5}, s_{0.5}) & (s_{-1.8}, s_{-0.6}, s_{-0.4}) \\
(s_{-0.4}, s_{-0.5}, s_{-0.7}) & (s_{-0.2}, s_{-0.5}, s_{-0.5}) & (s_0) & (s_{-2.1}, s_{-1.8}, s_{-0.3}) \\
(s_{-0.3}, s_{-0.9}, s_{-2.4}) & (s_{1.8}, s_{0.6}, s_{0.4}) & (s_{2.1}, s_{1.8}, s_{0.3}) & (s_0)
\end{bmatrix}.$$ The expected 2-tuple linguistic

preference relation is $E^{H(1)} = \begin{bmatrix}
(s_0, 0) & (s_0, -7/15) & (s_1, -7/15) & (s_1, 1/5) \\
(s_0, 7/15) & (s_0, 0) & (s_0, 2/5) & (s_{-1}, 1/15) \\
(s_{-1}, 7/15) & (s_0, -2/5) & (s_0, 0) & (s_{-1}, -2/5) \\
(s_{-1}, -1/5) & (s_1, -1/15) & (s_1, 2/5) & (s_0, 0)
\end{bmatrix}$. Because $CI(E^{H(1)}) \approx$

$0.0860 < \overline{\overline{CI}}(0.1347)$, the matrix is acceptably consistent. Then use the aggregation operators and comparison method, and the ranking is $x_1 \succ x_4 \succ x_2 \succ x_3$.

(4) LPRs [77]

At first, the conversion function $k_{ij}^L = \frac{1}{3}(T_{ij} + 1 - I_{ij} + 1 - F_{ij})k_{ij}$ will be used to convert SNLNs in SNLPR K into linguistic variables in a LPR K^L. For instance, $(s_{-1}, <0.9, 0.3, 0.2>)$ can be replaced by

$s_{-0.8}$. Then $K^L = \begin{bmatrix}
s_0 & s_{-0.8} & s_{0.6} & s_{2.4} \\
s_{0.8} & s_0 & s_{0.6} & s_{-1.6} \\
s_{-0.6} & s_{-0.6} & s_0 & s_{-1.8} \\
s_{-2.4} & s_{1.6} & s_{1.8} & s_0
\end{bmatrix}$. Since $CI(K^L) \approx 0.2667 > \delta_0(0.1347)$, the automatic iterative

Algorithm 1 in [77] is used. The consistency index of the adjusted LPR is $CI(K^{L(1)}) \approx 0.1334 < \delta_0(0.1347)$, and then the final ranking is $x_4 > x_1 > x_2 > x_3$.

Subsequently, comparisons with each method in terms of backgrounds, consistency-improving processes, ranking methods, and ranking results are made in Table 2.

Table 2. Comparison of different methods.

Approaches	Backgrounds	Improving Consistency	Ranking Methods	Ranking Orders
Liang et al. [44]	SVTNPRs	Interactive feedback	Arithmetic operator	$x_4 > x_1 > x_2 > x_3$
Meng et al. [55]	ILPRs	Automatic iteration	Preferred degrees	$x_1 \succ x_4 \succ x_2 \succ x_3$
Wu and Xu [54]	HFLPRs	Interactive feedback	Expected values	$x_1 \succ x_4 \succ x_2 \succ x_3$
Jin et al. [77]	LPRs	Automatic iteration	Arithmetic operator	$x_4 > x_1 > x_2 > x_3$
The proposed method	SNLPRs	Automatic iteration	Geometric operator	$x_4 > x_1 > x_2 > x_3$

In the ranking results of Table 2 only the order of x_1 and x_4 varies, which demonstrates the effectiveness of the proposed method. The in-depth comparison analyses are shown as follows:

(1) Comparison with References [44] and [77]: the same ranking results are obtained using the methods in [44,77] and our approach. An interactive feedback is used to improve the consistency in [44]. It may be a little difficult for DMs to do this work, especially when the alternatives are numerous. In addition, the arithmetic operator used may cause a reversal of ranking in some cases. Jin et al. described information with linguistic term sets in Reference [77]. However, all of the membership degrees are missing in LPRs. And the arithmetic operator used in Reference [77] also has the limitation of sorting reversal.

(2) Comparison with Reference [55]: both [55] and our method choose the automatic iteration to improve consistency. The reason for the different ranking results may be that there are only membership and non-membership degrees in ILPRs. The conversion function possibly led to a loss of the original information.

(3) Comparison with Reference [54]: the difference between [54] and our approach is that there is no process of consistency improvement in HFLPRs. Moreover, the truth-membership, indeterminacy-membership, and false-membership of the linguistic values in SNLPRs have identical roles in HFLPRs. This may be another explanation of the different rankings.

According to the analysis above, the strengths of the presented approach are obvious. First of all, the basic elements in SNLPRs, SNLNs, contain three independent membership degrees to describe the consistent, hesitant, and inconsistent information, respectively. It means that the problem of evaluation information being missing is avoided to a greater extent. Thus, the proposed method is more suitable for solving problems in a simplified neutrosophic linguistic environment. Secondly, the consistency-improving process is an automatic iteration algorithm. It saves time and increases convenience for DMs. In addition, as mentioned in Section 4.4, the geometric operator being used may avoid the problem of ranking reversal. It is easy for us to understand and operate. Finally, the flexibility is increased as the linguistic scale function can be changed in different semantic situations.

6. Discussion and Conclusions

Appropriate project delivery systems play an irreplaceable role in promoting the development of the construction industry. The paper provided a decision-making approach with SNLPRs to solve the problem of selecting an optimal system under simplified neutrosophic linguistic circumstances. Several distance measures of SNLNs, which are the basic elements of SNLPRs, were redefined. They can overcome the drawbacks of the definition of Ye [70], so that the differences between two SNLNs can be well distinguished. Moreover, the paper created a distance-based consistency index to check the consistency of SNLPRs. A consistency-improving algorithm was also suggested. The effectiveness and advantages of this method were displayed by an illustration of selecting project delivery systems and the corresponding comparison analysis.

Nevertheless, the proposed method still has some limitations, such as a case of SNLPRs with incomplete assessment information. In order to make the method based on SNLPRs more effectively and widely used in engineering projects, several future works can be planned as follow: (1) Other kinds of consistency, such as the multiplicative consistency of SNLPRs may be presented; (2) a situation where the linguistic term sets are unbalanced [78] may be under consideration; (3) decision-making methods based on incomplete SNLPRs are worth studying.

Author Contributions: Sui-Zhi Luo, Peng-Fei Cheng, and Jian-Qiang Wang conceived and worked together to achieve this work, Sui-Zhi Luo wrote the paper; Yuan-Ji Huang made contributions to the case study.

Conflicts of Interest: The authors declare no conflict of interest.

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