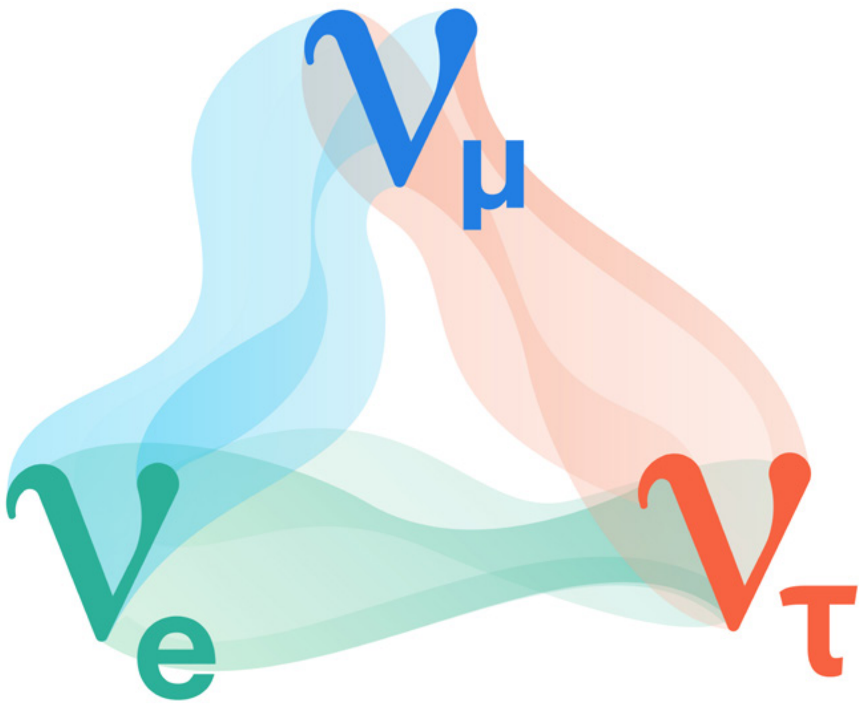
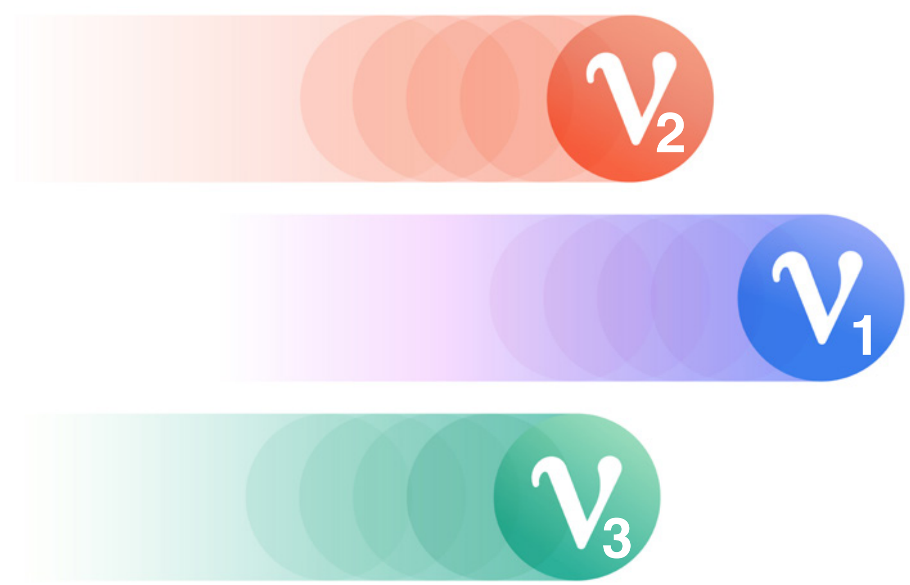
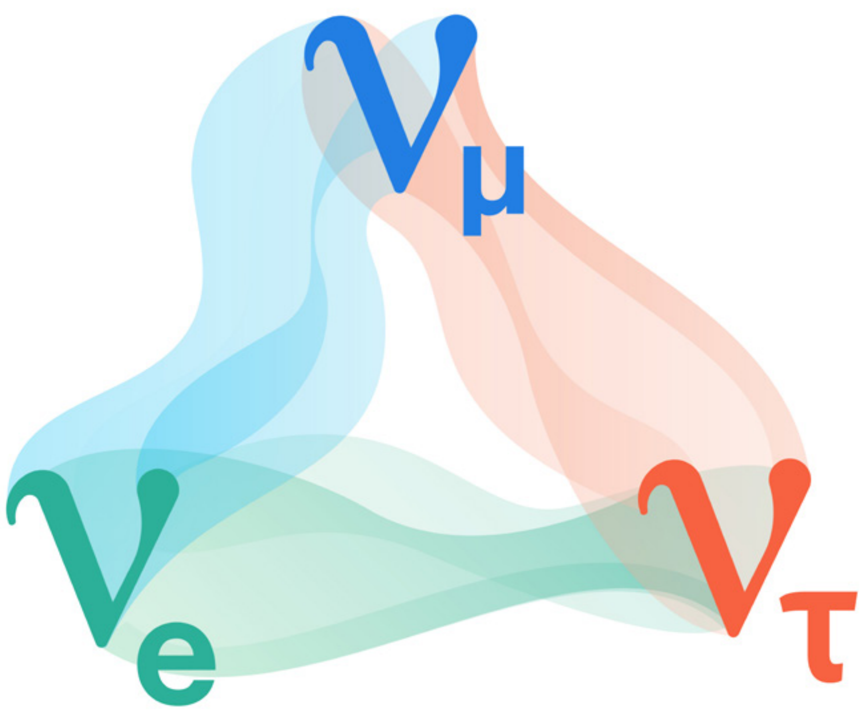
Theoretical Aspects of the Quantum Neutrino



Stephen Parke
Theoretical Physicist
Chicago, USA



Theoretical Aspects of the Quantum Neutrino



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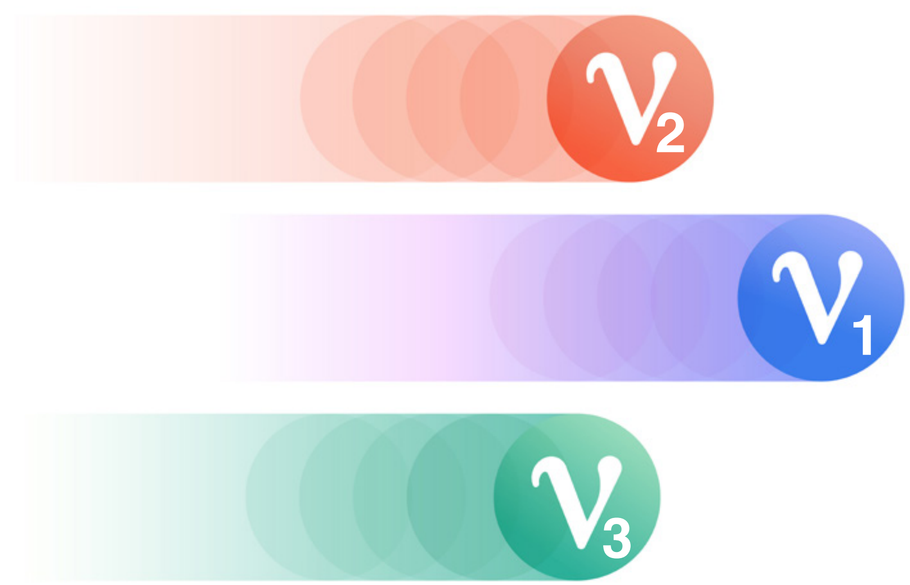


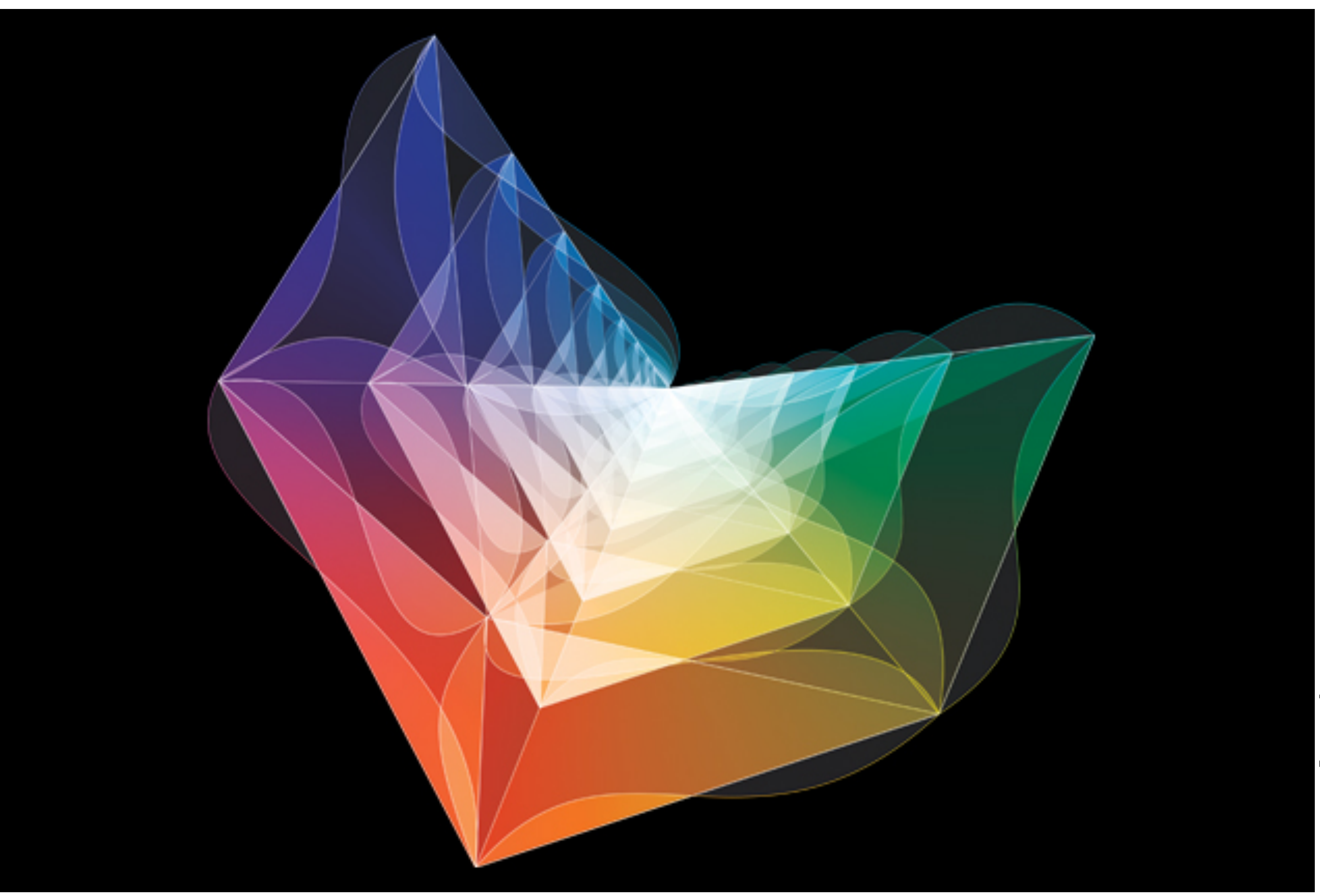
DUALITY AND MULTI-GLUON SCATTERING

Michelangelo MANGANO, Stephen PARKE and Zhan XU[1]

Fermi National Accelerator Laboratory[2], P.O. Box 500, Batavia, IL 60510, USA

Received 13 July 1987

For the *six*-gluon scattering process we give explicit and simple expressions for the amplitude and its square. To achieve this we use an analogy with string theories to identify a unique procedure for writing the multi-gluon scattering amplitudes in terms of a sum of gauge invariant dual sub-amplitudes multiplied by an appropriate color (Chan-Paton) factor. The sub-amplitudes defined in this way are invariant under cyclic permutations, satisfy powerful identities which relate different non-cyclic permutations and factorize in the soft gluon limit, the two-gluon collinear limit and on multi-gluon poles. Also, to leading order in the number of colors these sub-amplitudes sum *incoherently* in the square of the full matrix element. The results contained



Xu Zhan, Tsinghua

AMPLITUHEDRON

HEP 4 records found Search took 0.31 seconds.

1. **Helicity Amplitudes for Multiple Bremsstrahlung in Massless Nonabelian Gauge Theories**
   Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Jul 1986. 37 pp.
   Published in **Nucl.Phys. B291 (1987) 392-428**
   TUTP-86/9a
   DOI: 10.1016/0550-3213(87)90479-2
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 445 records 250+

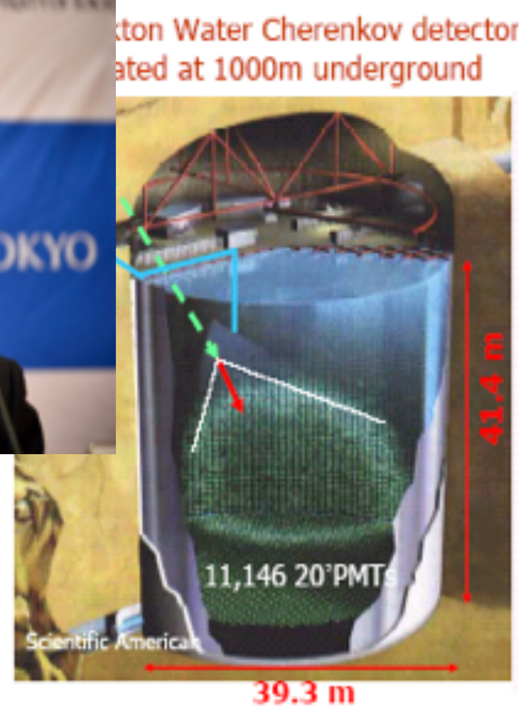
2. **Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 2. Decompositi Invariant Subsets**
   Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Mar 1985. 32 pp.
   TUTP 84/4
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record

3. **Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 3. Amplitudes O Processes Involving Gluon Selfcoupling Vertices**
   Da-hua Zhang, Zhan Xu, Lee Chang (Tsinghua U., Beijing). Jan 1985. 29 pp.
   TUTP-84/5a
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record

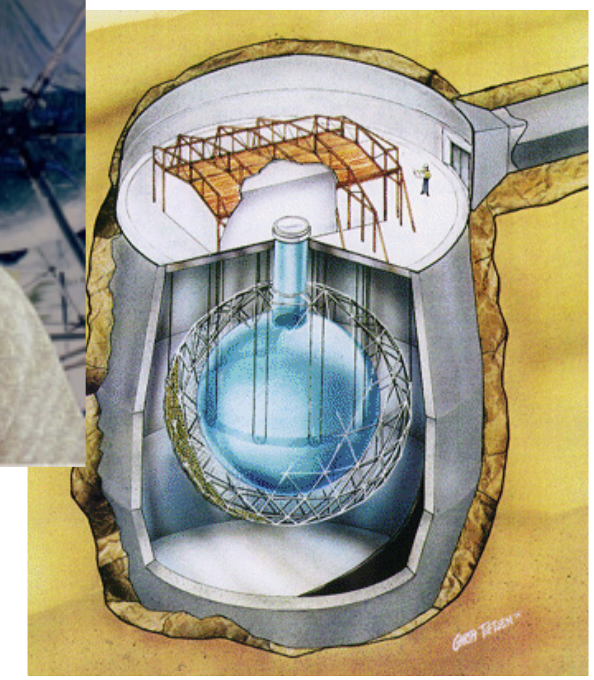
4. **Helicity Amplitudes For Multiple Bremsstrahlung In Massless Nonabelian Gauge Theory. 1. New Definitio Formulation Of Amplitudes In Grassmann Algebra**
   Zhan Xu, Da-Hua Zhang, Lee Chang (Tsinghua U., Beijing). Dec 1984. 20 pp.
   TUTP-84/3-TSINGHUA
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   Detailed record - Cited by 4 records

NOBEL 2015

"for the discovery of ***neutrino oscillations****,*
which shows that neutrinos have mass"



Takaaki Kajita
SuperKamiokaNDE



Art McDonald
SNO

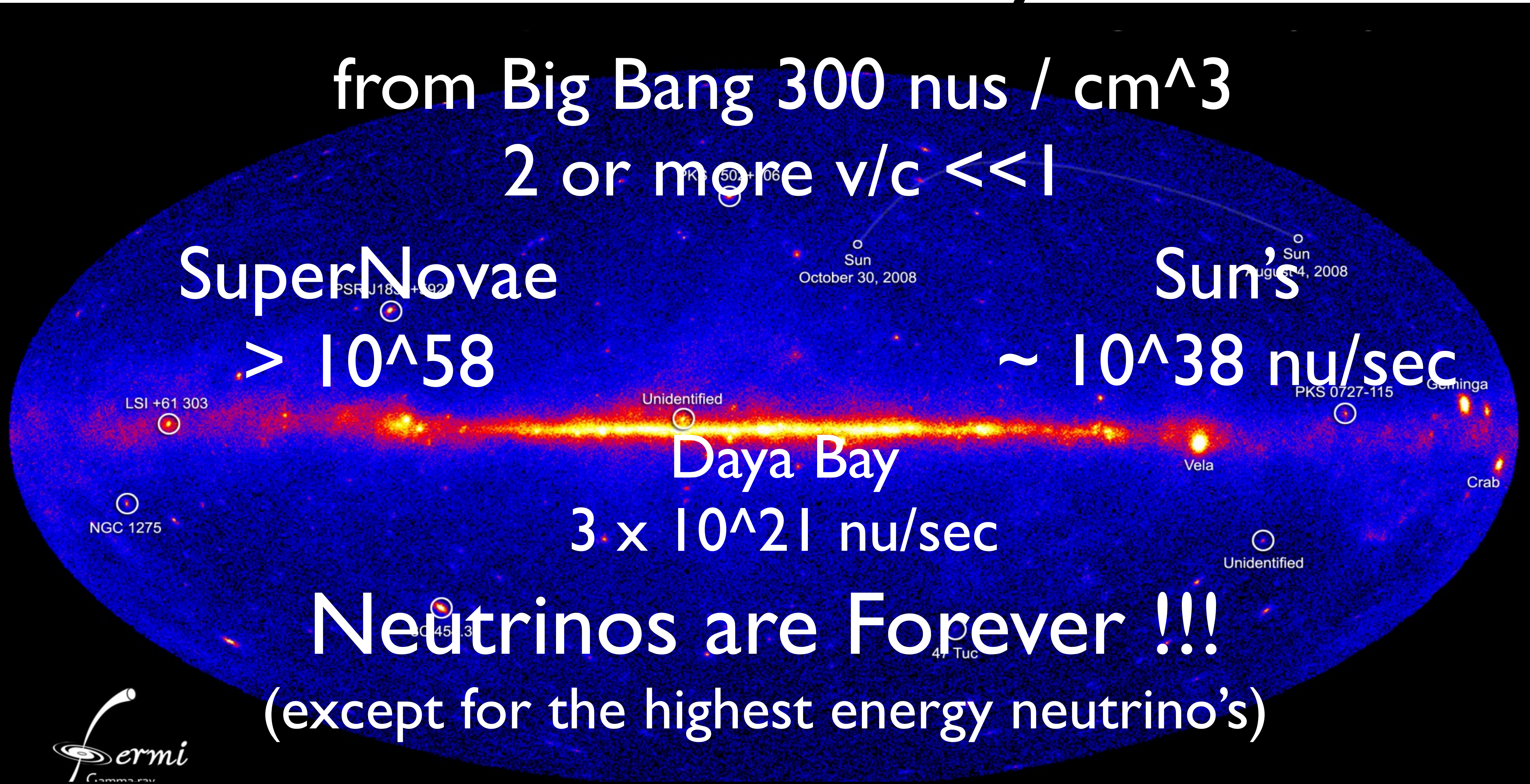
"for the discovery of ***neutrino flavor transformations****,*
which shows that neutrinos have mass"

~ vacuum oscillations

Wolfenstein matter effects dominant flavor transformations

See Smirnov arXiv:1609.02386

Neutrinos are Everywhere !



from Big Bang 300 nus / cm^3

2 or more v/c <<1

SuperNovae

> 10^58

Sun's

~ 10^38 nu/sec

Daya Bay

3 x 10^21 nu/sec

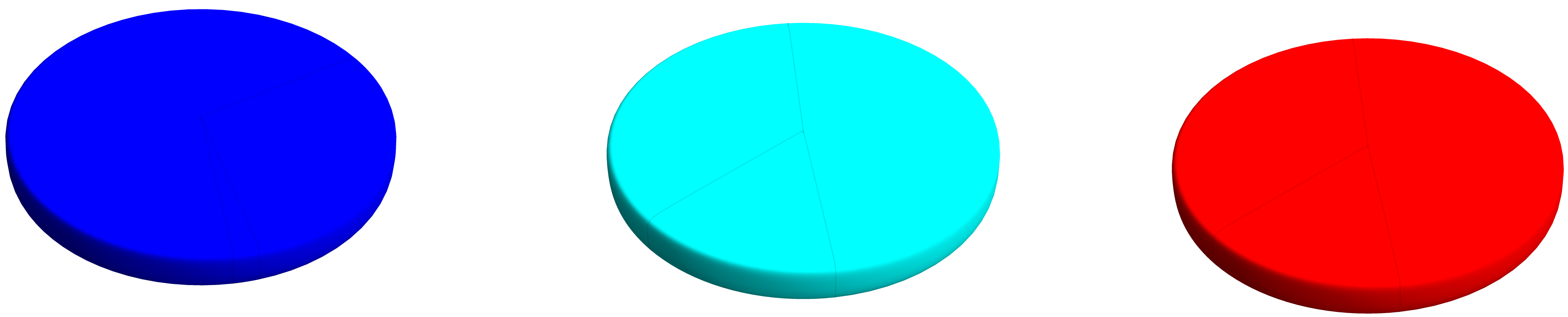
Neutrinos are Forever !!!

(except for the highest energy neutrino's)

therefore in the Universe: $$\frac{\partial N_\nu}{\partial t} > 0$$

Neutrino Flavor or Interaction States:

$W^+ \to e^+ \nu_e$ $\quad$ $W^+ \to \mu^+ \nu_\mu$ $\quad$ $W^+ \to \tau^+ \nu_\tau$



ν_e $\quad$ ν_μ $\quad$ ν_τ

provided $L/E \ll \mathbf{0.5}$ km/MeV $= \mathbf{500}$ km/GeV !!!

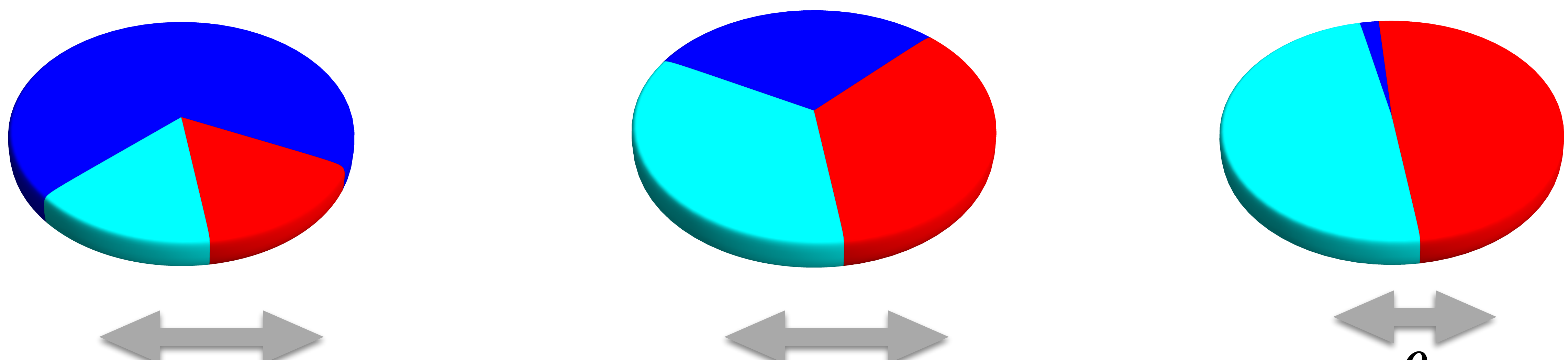
$\sim$ 1 picosecond in Neutrino rest frame !!!

$\approx$ Age of Universe / 10^{26}

Neutrino Mass EigenStates or Propagation States:

$$\text{Propagator } \nu_j \to \nu_k \;= \delta_{jk}\, e^{-i\left(\frac{m_j^2 L}{2E_\nu}\right)}$$

ν_1	ν_2	ν_3
most ν_e		least ν_e



δ,θ_{23}	δ,θ_{23}	θ_{23}

$\nu_e =$

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, …

$\nu_\mu =$

SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

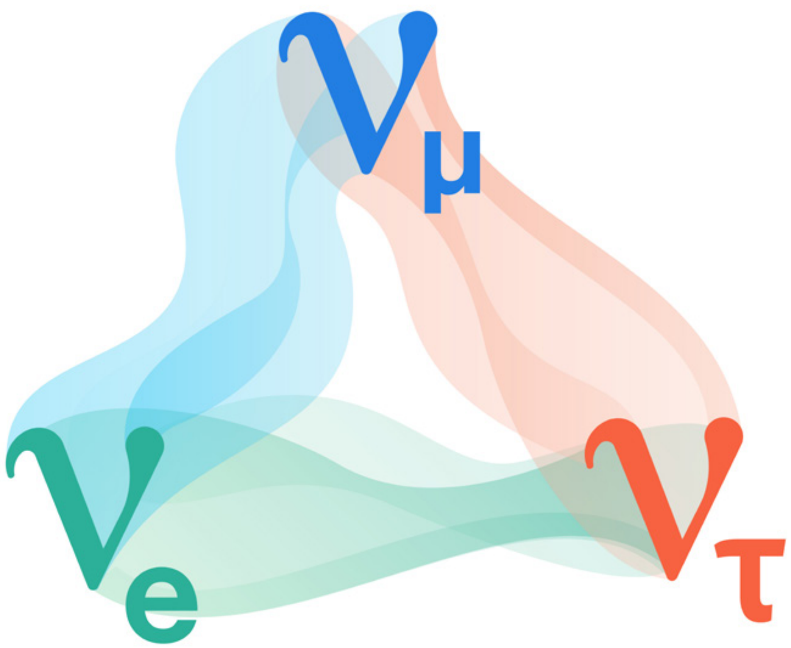
$\nu_\tau =$

Unitarity
SK, Opera
ICECUBE ?

Interactions:

simple

complicated



$= U$

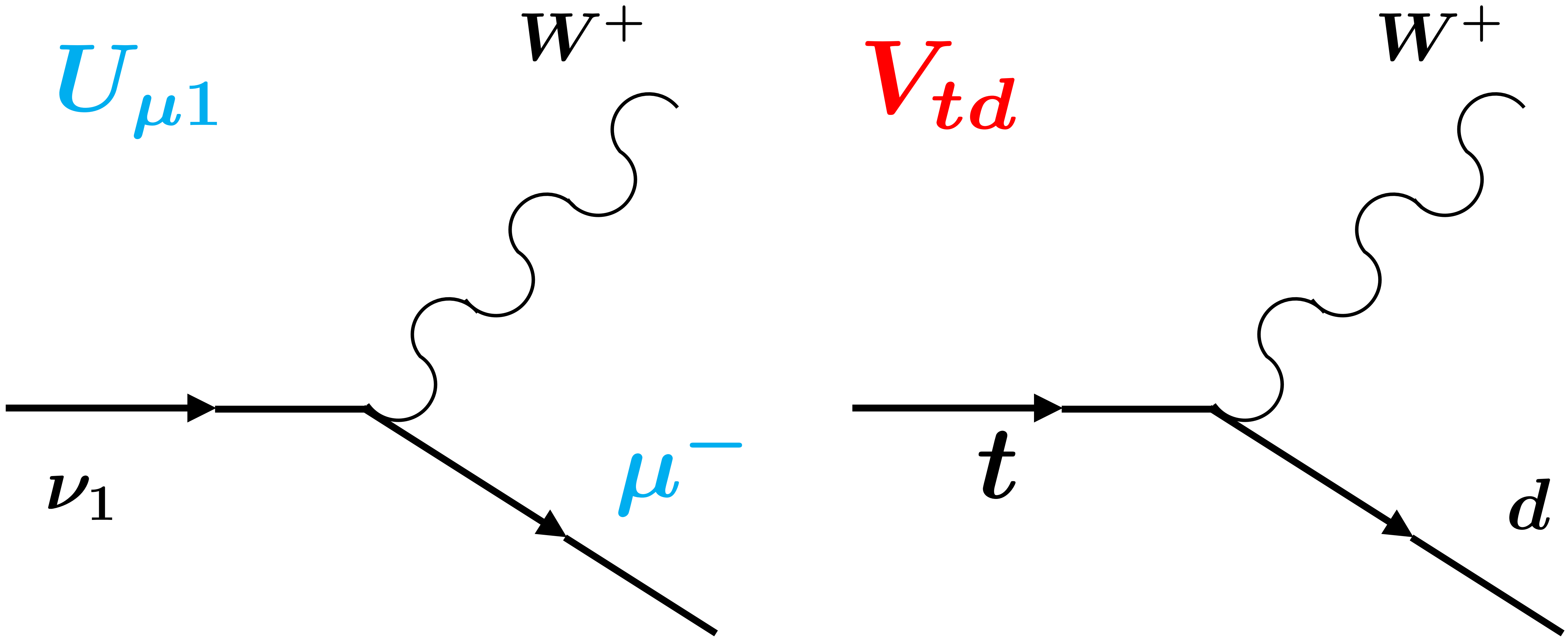
unitary matrix ?

complicated

simple

masses ?

Propagation:



Rates: $|U_{\mu 1}|^2$ & $|V_{td}|^2$

unitary matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

by defn $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$

$$U_{PMNS} = U_{23}(\theta_{23}, 0)\, U_{13}(\theta_{13}, \delta)\, U_{12}(\theta_{12}, 0)$$

Why this order ???

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{+i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

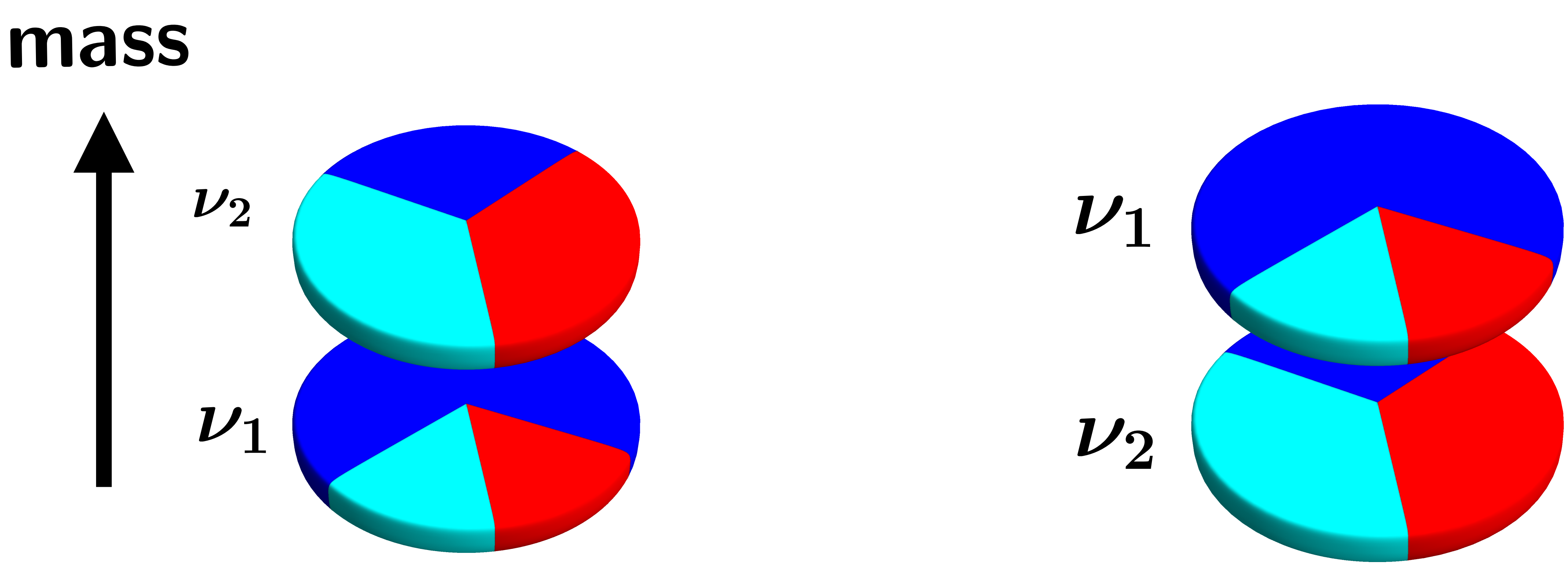
$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$

$$\times\, \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})$$

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$\nu_1, \ \nu_2$ Mass Ordering:

–solar mass ordering



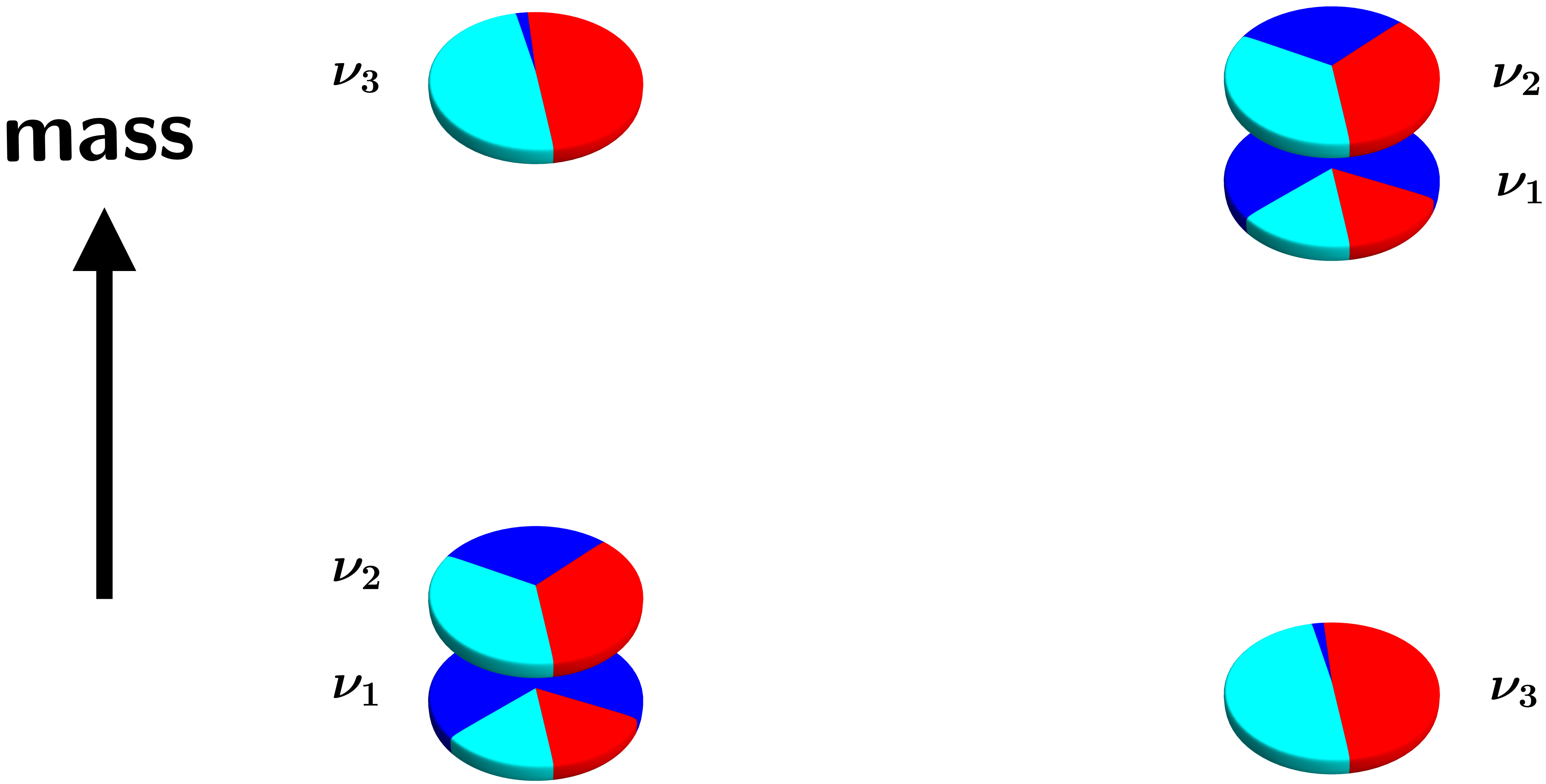
$|\Delta m^2_{21}| = |m^2_2 - m^2_1| = 7.5 \times 10^{-5}\ \mathrm{eV}^2$ $\quad L/E = 15\ \mathrm{km/MeV} = 15,000\ \mathrm{km/GeV}$

SNO $m_2 > m_1$

$\nu_e =$ $\quad \nu_\mu =$ $\quad \nu_\tau =$

ν_3, ν_1/ν_2 Mass Ordering:

–atmospheric mass ordering

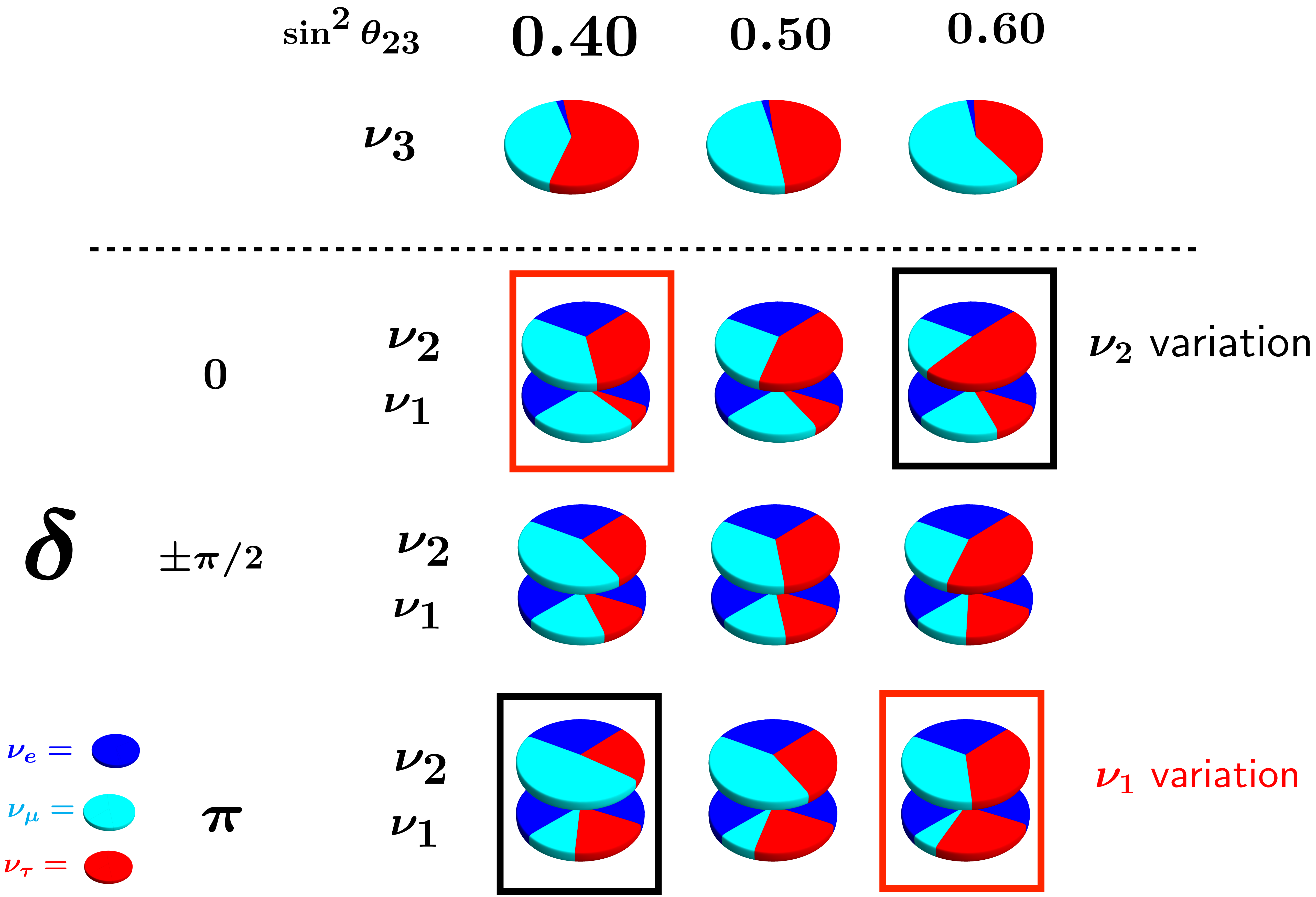


$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3}\ \mathrm{eV}^2$ $\quad$ $L/E = 0.5\ \mathrm{km/MeV} = 500\ \mathrm{km/GeV}$

Unknown: NOνA, JUNO, ICECUBE, DUNE, T2HKK....

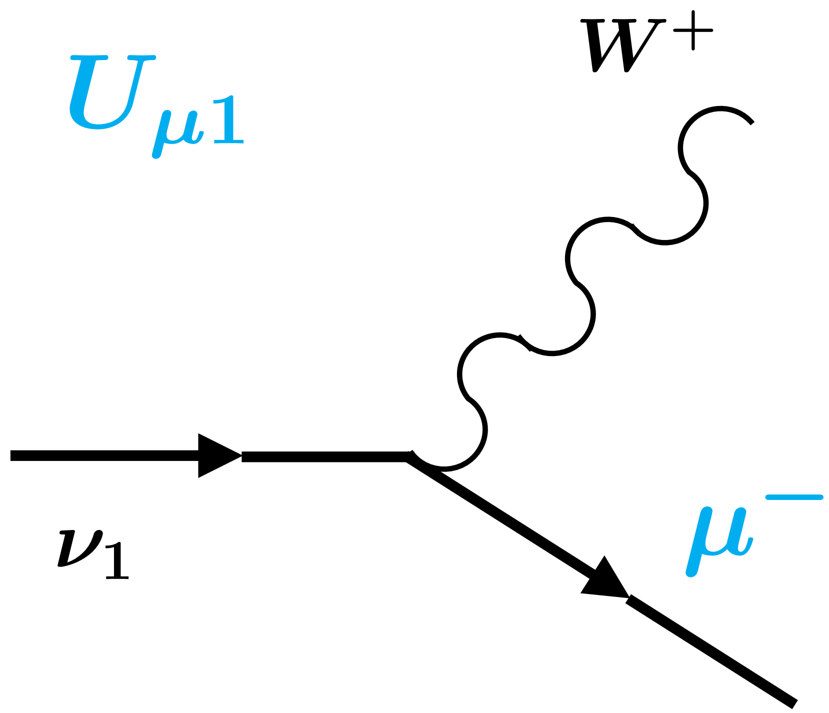
$\nu_e =$ $\quad$ $\nu_\mu =$ $\quad$ $\nu_\tau =$

Summary: Octant of θ_{23}



Leptons:

$U_{\mu 1}$



$$0.08 < |U_{\mu 1}|^2 < 0.24$$

variation in δ only !

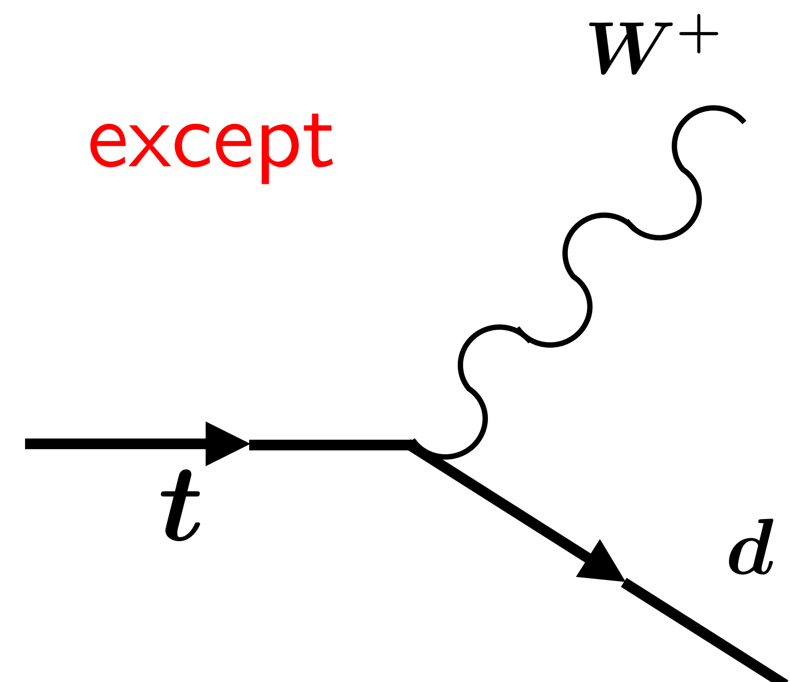
factor of 3 diff.

$$|U_{\mu 3}|^2 = 0.4 - 0.6$$
$$|U_{\mu 2}|^2 = 0.26 - 0.41$$
$$|U_{\mu 1}|^2 = 0.08 - 0.24$$

Quarks:

$|V_{ij}|^2$ essentially independent of δ_q !

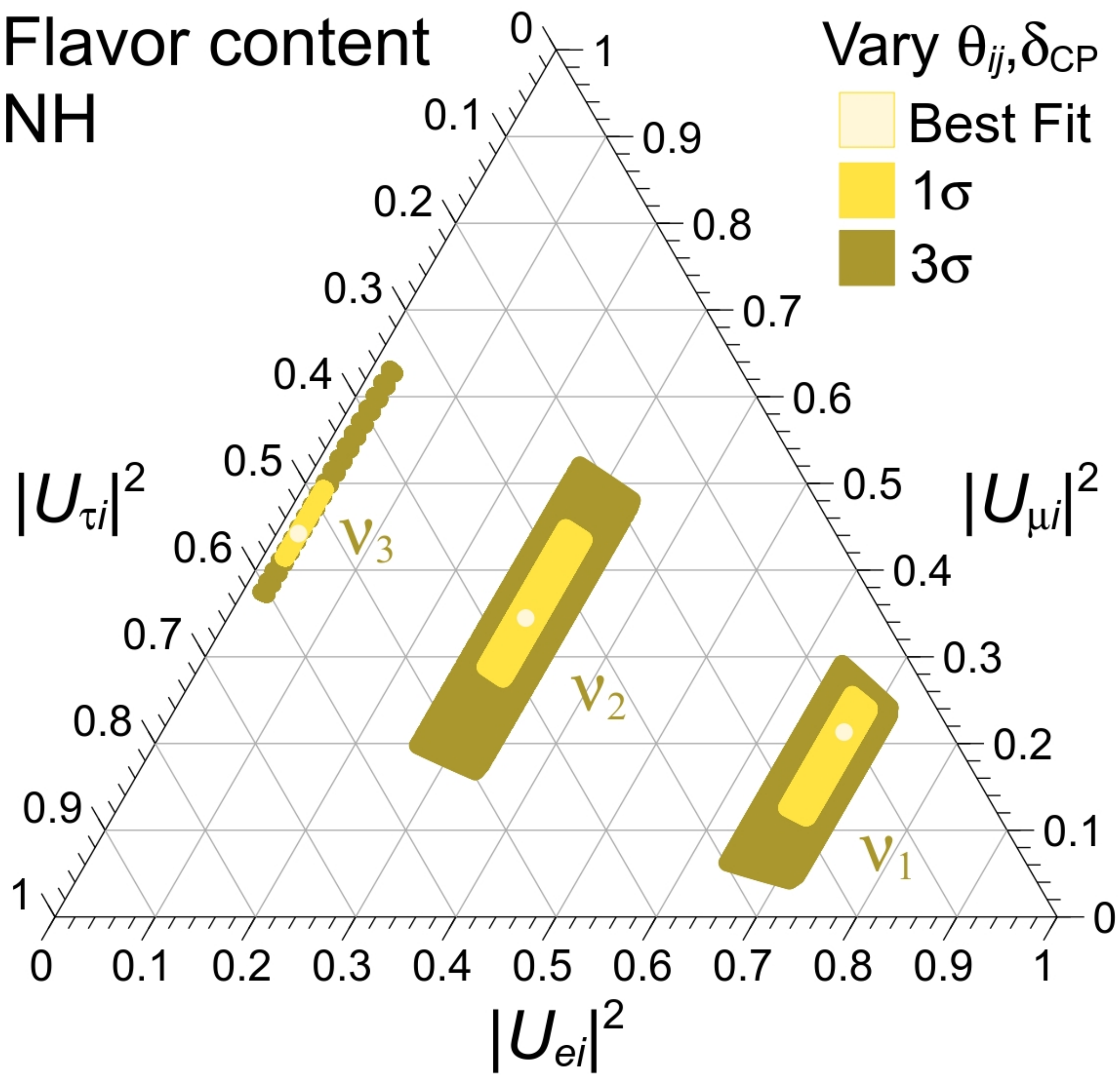
except



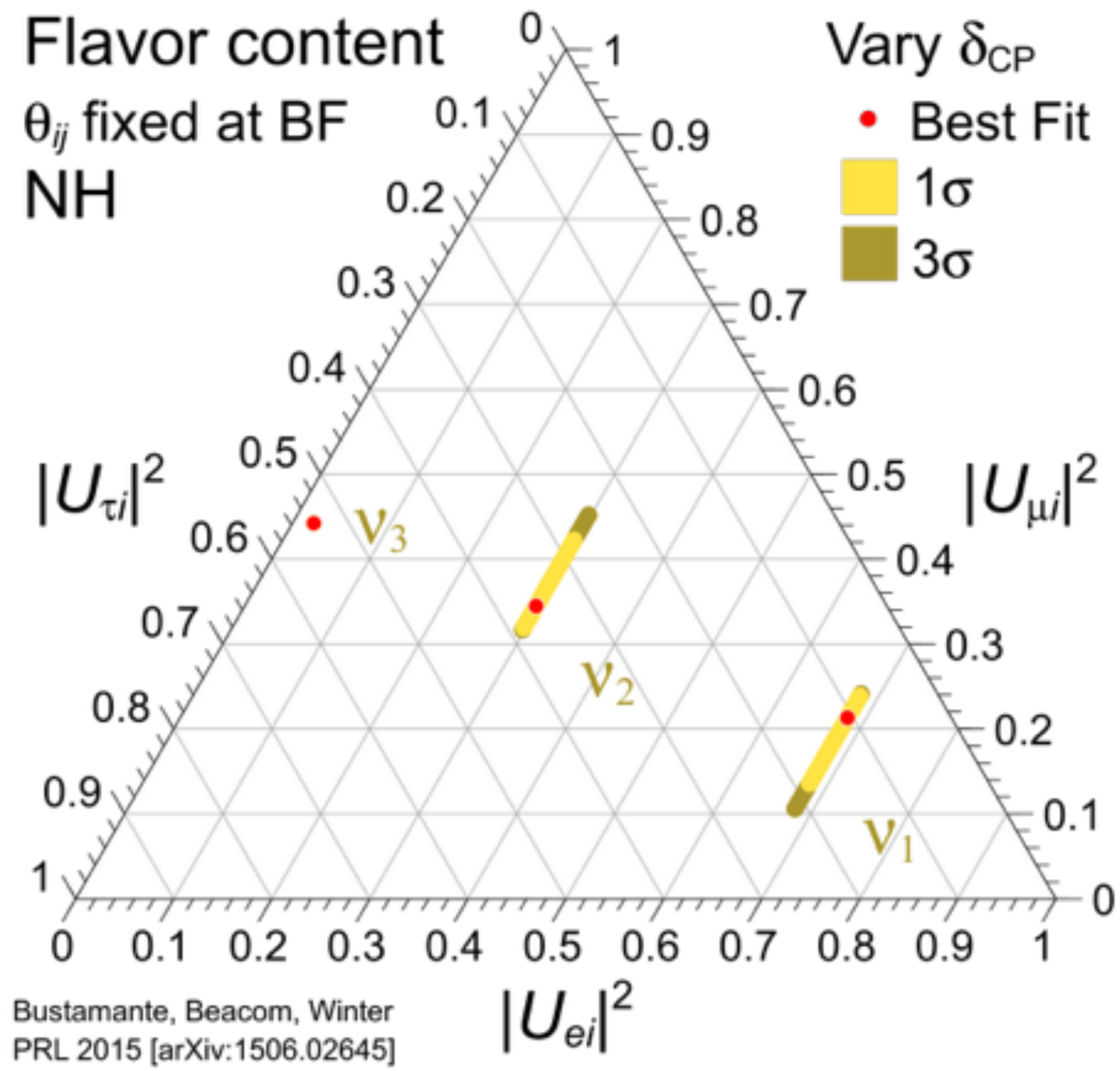
$$V_{td} \approx A\lambda^3(1 - 0.37e^{i\delta_q})$$

$$|V_{td}|^2 \approx 10^{-4}$$

$$|V_{tb}|^2 \approx 1$$
$$|V_{ts}|^2 \sim \lambda^4 \approx 2 \times 10^{-3}$$
δ_q → $$|V_{td}|^2 \sim \lambda^6 \approx 8 \times 10^{-5}$$



δ & θ_{23} uncertainty



no θ_{23} uncertainty

Determine flavor fractions of neutrino mass states

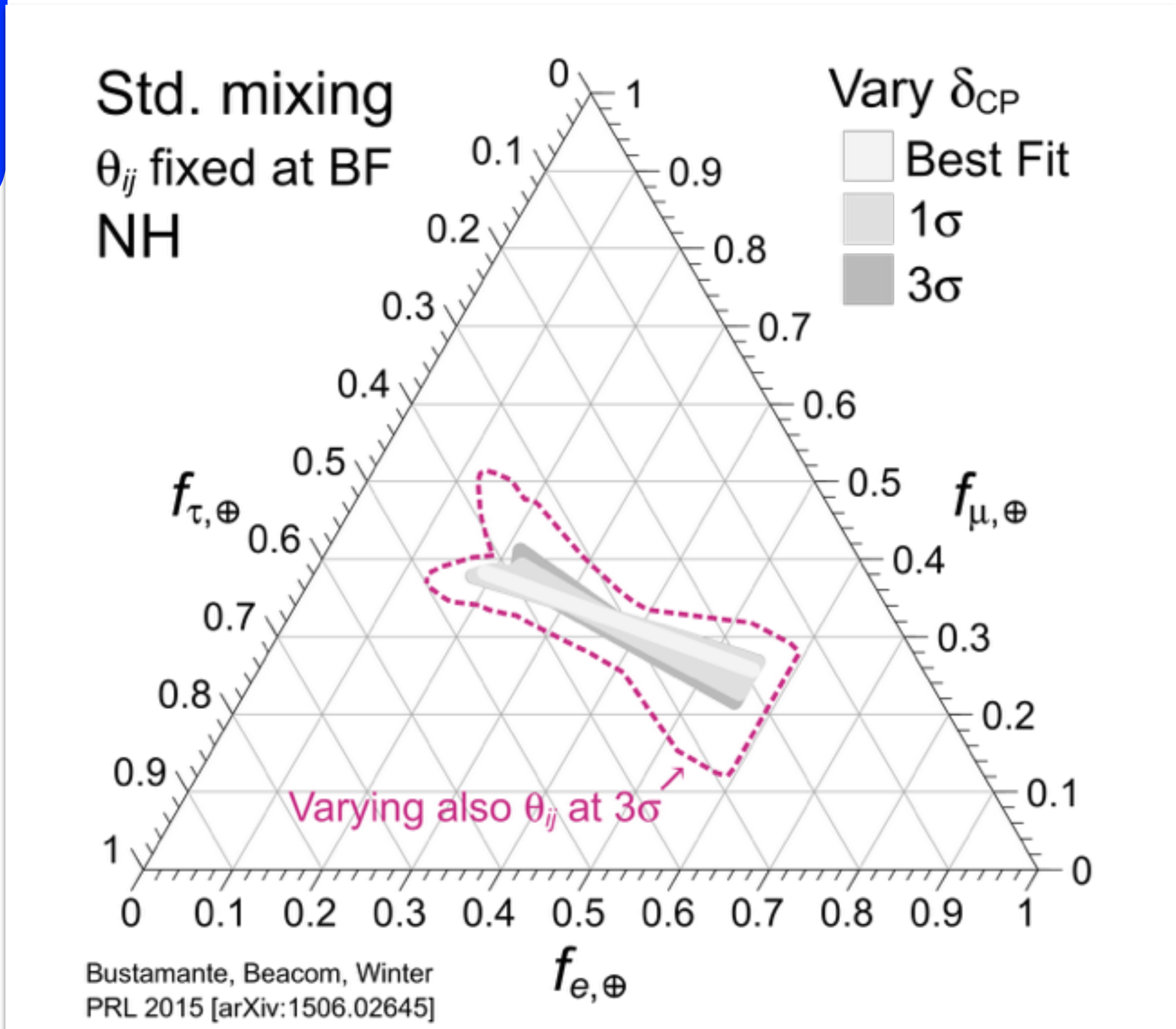
WHY?

Precision Neutrino Measurements:

To discover neutrino BSM, one needs precision predictions for nuSM

Determine flavor fractions of neutrino mass states

Precision Predictions for flavor ratios at ICECUBE.



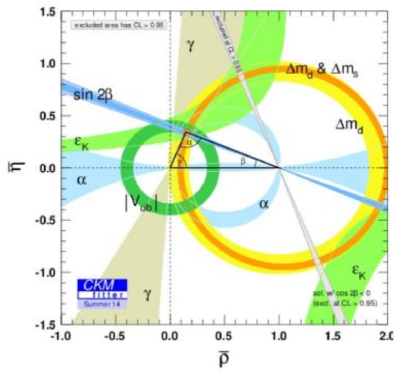
WHY?

Determine flavor fractions of neutrino mass states

Stress Test Neutrino paradigm search for new physics

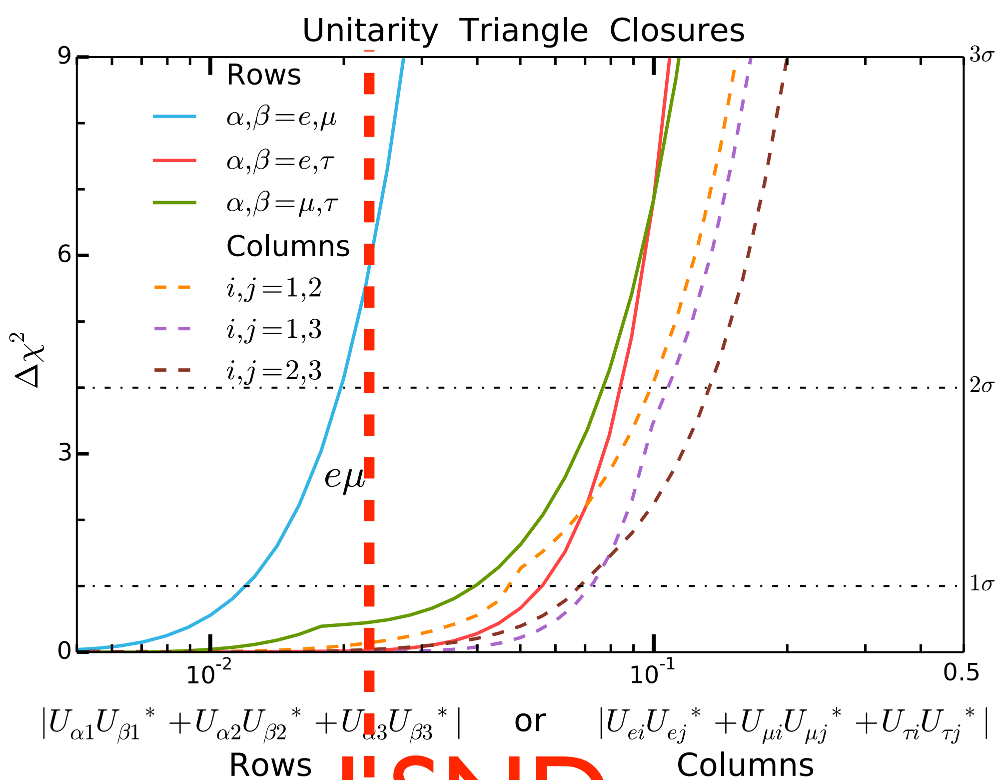
Precision Neutrino Measurements:

Quark



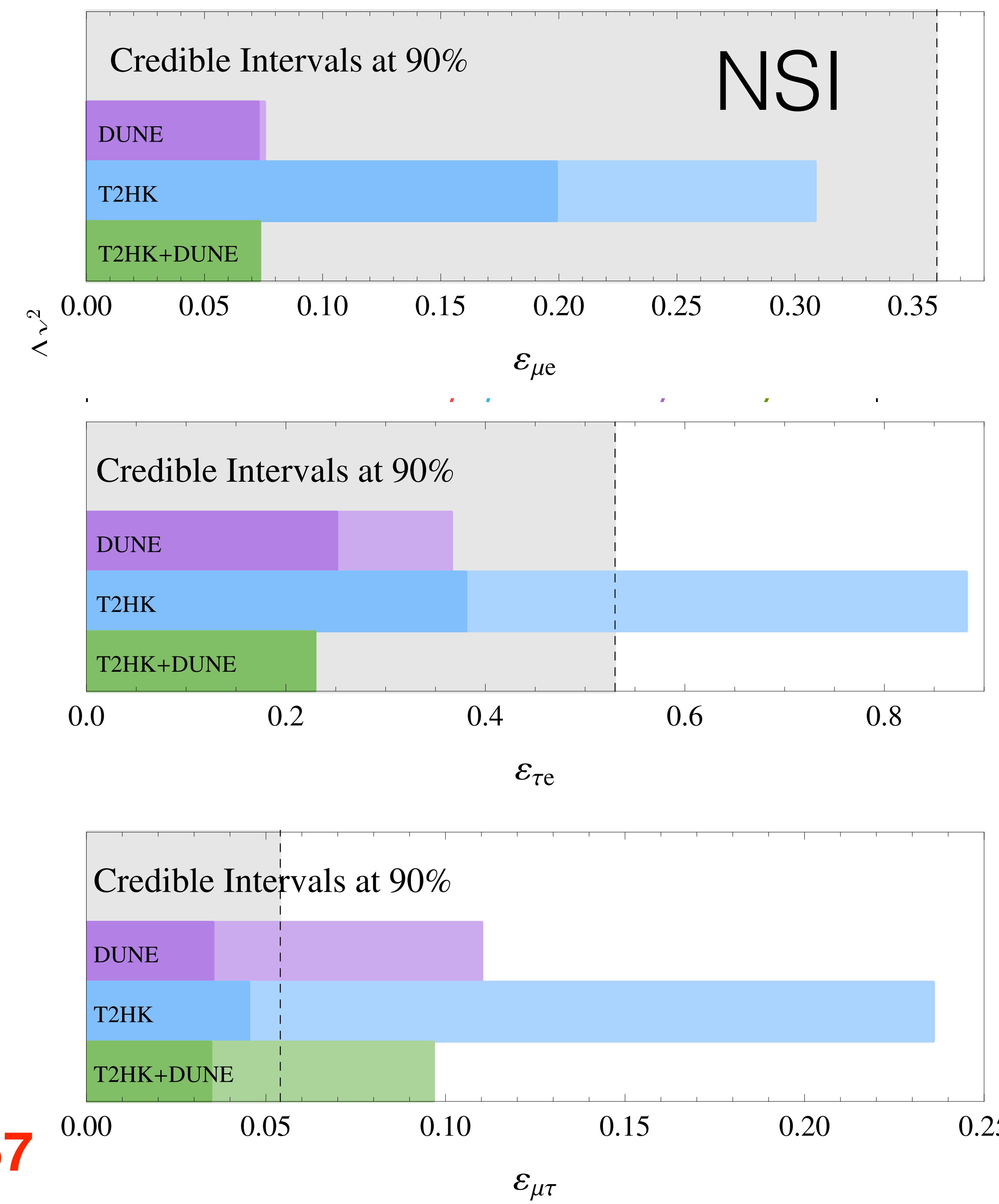
$A\lambda^3$

Stress Test
Neutrino paradigm
search for new physics



LSND

M. Ross-Lonergan + SP
arXiv:1508.05095



P.Coloma
arXiv:1511.06357

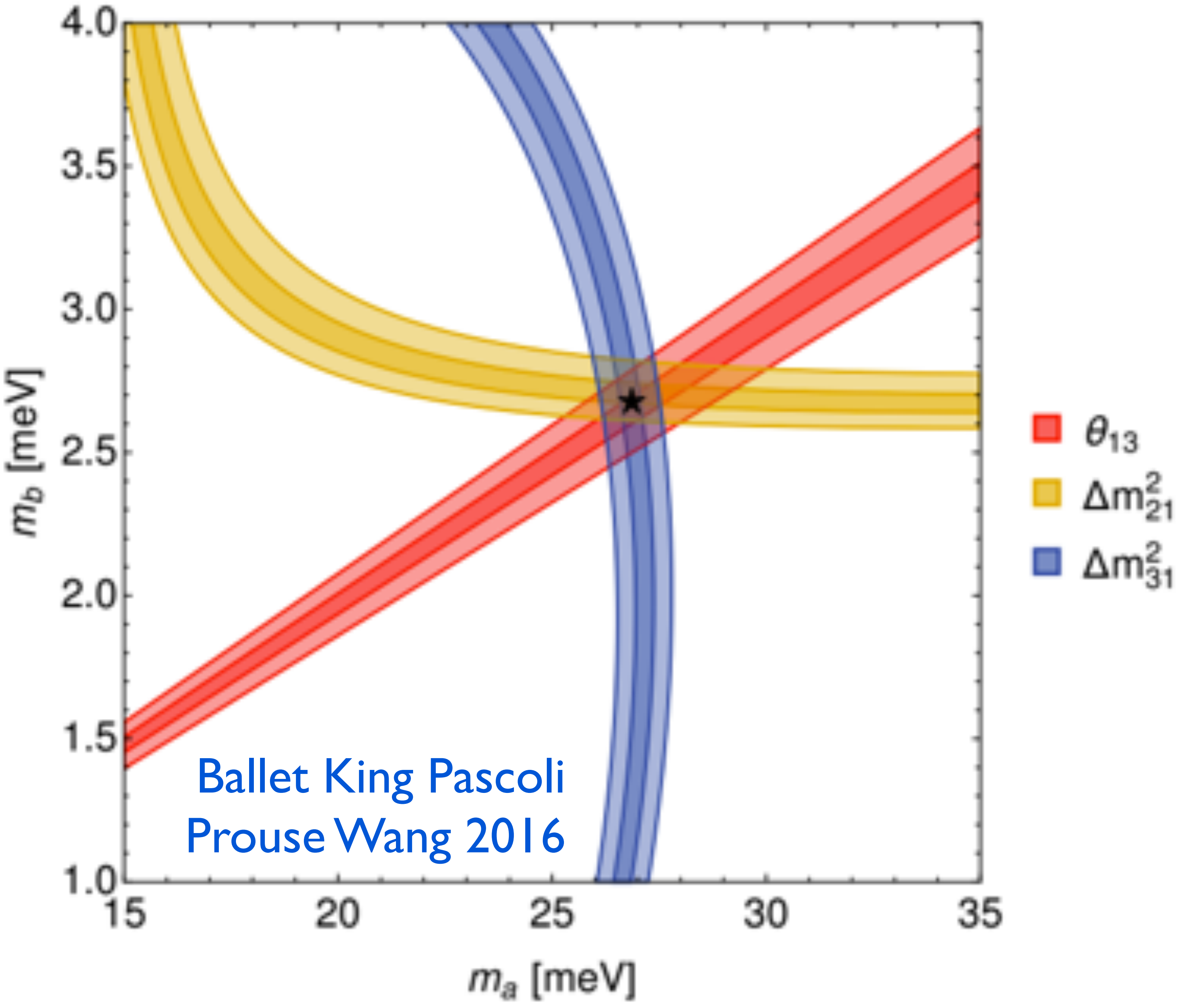
WHY?

Determine flavor fractions of neutrino mass states

Stress Test Neutrino paradigm search for new physics

Precision Neutrino Measurements:

Connection to Leptogenesis Understanding Universe



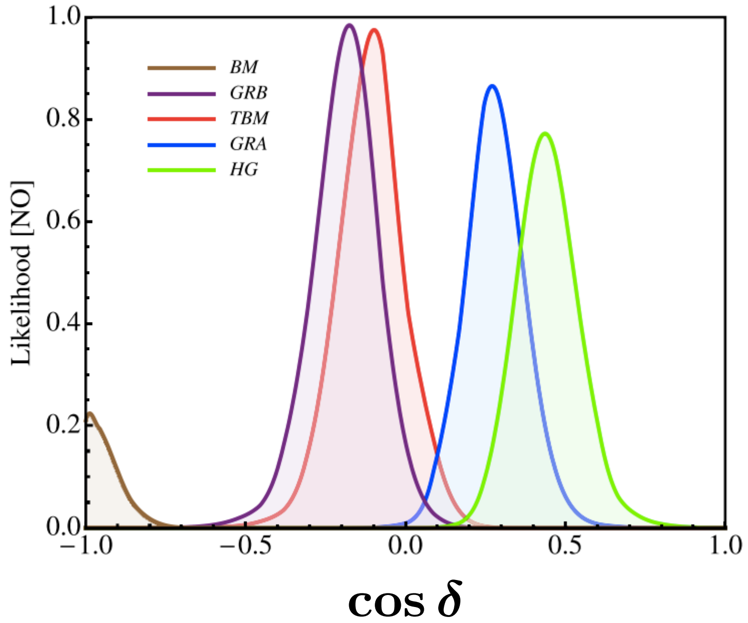
Connection to Leptogenesis
Understanding Universe

WHY?

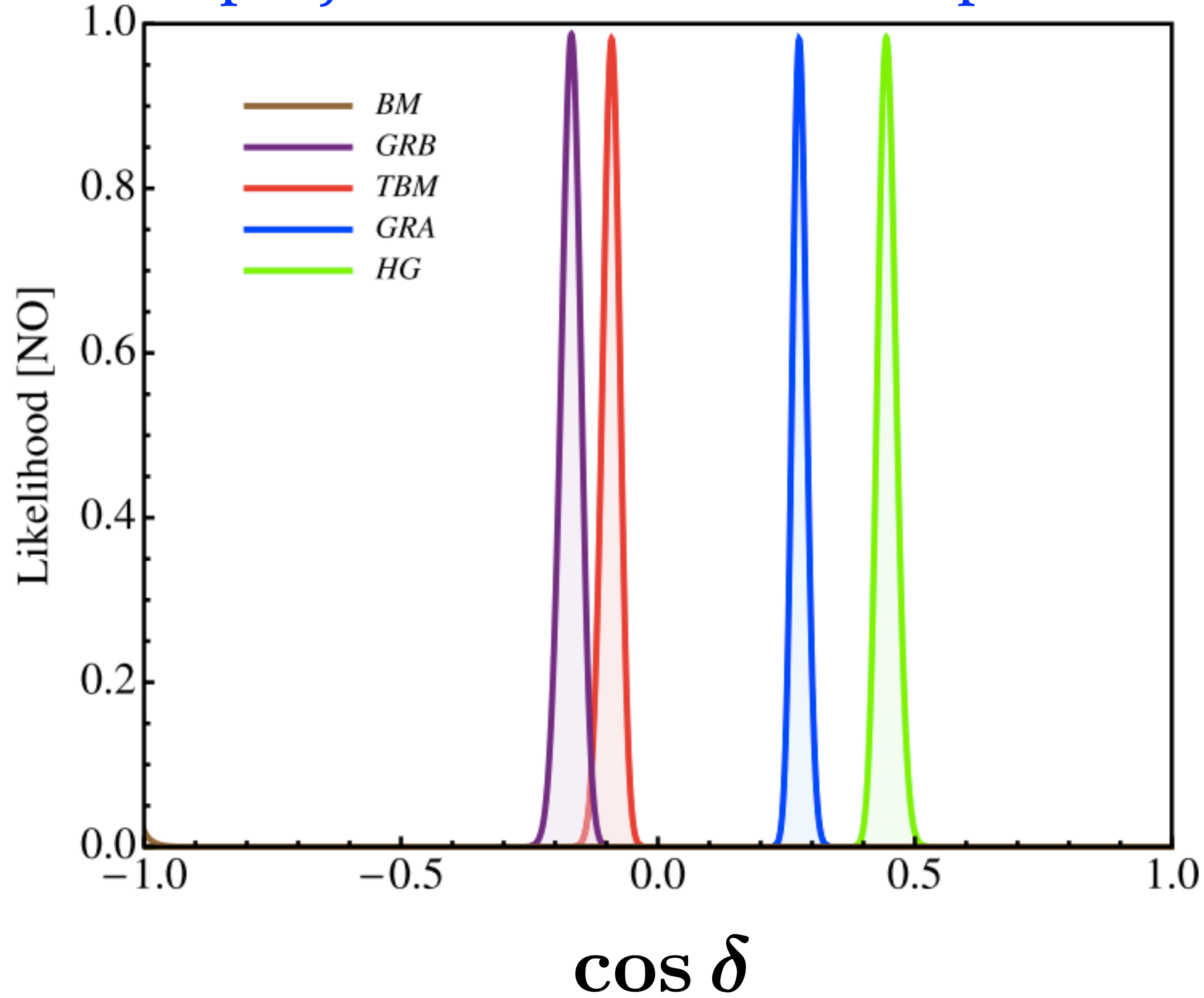
Precision Neutrino Measurements:

- Determine flavor fractions of neutrino mass states
- Stress Test Neutrino paradigm search for new physics
- Test Theoretical Neutrino Models
- Connection to Leptogenesis Understanding Universe

Predictions from flavor symmetry forms
with current measurement precision



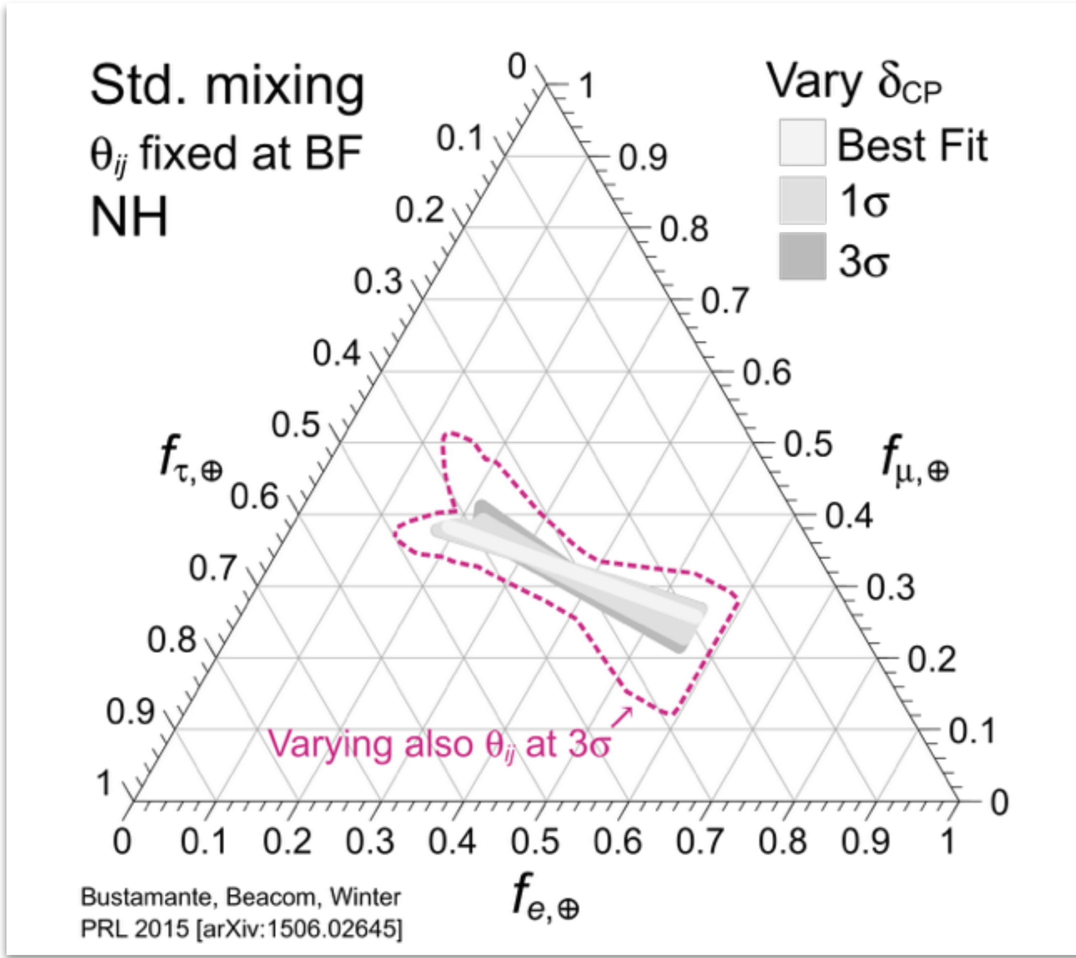
Predictions of flavor symmetry forms with projected measurement precision

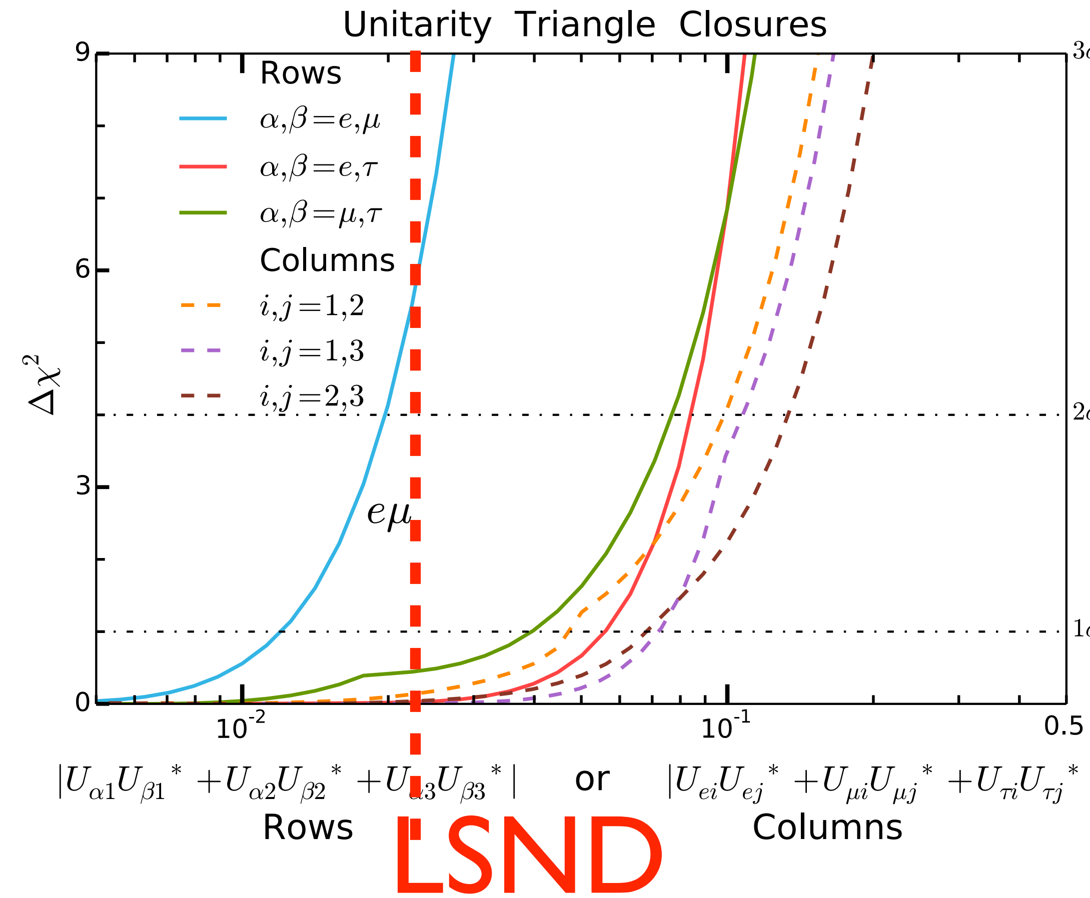


Test Theoretical Neutrino Models

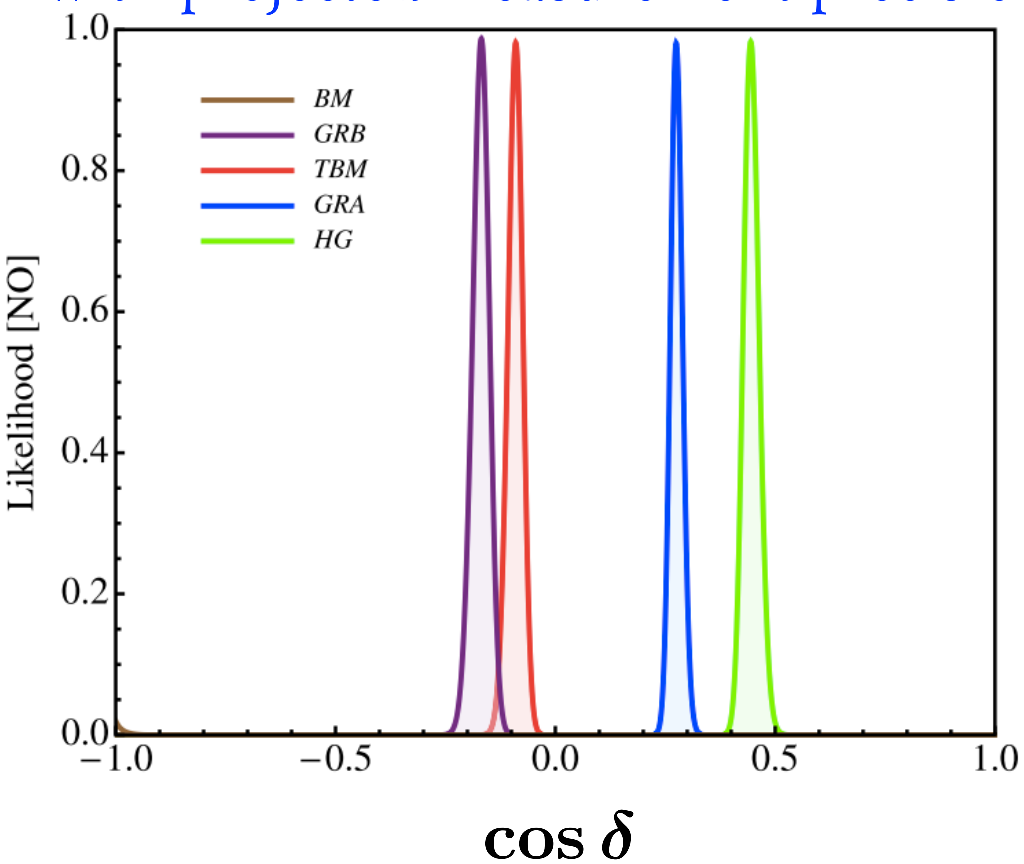
Girardi, Petcov, Titov, arXiv:1410.8056

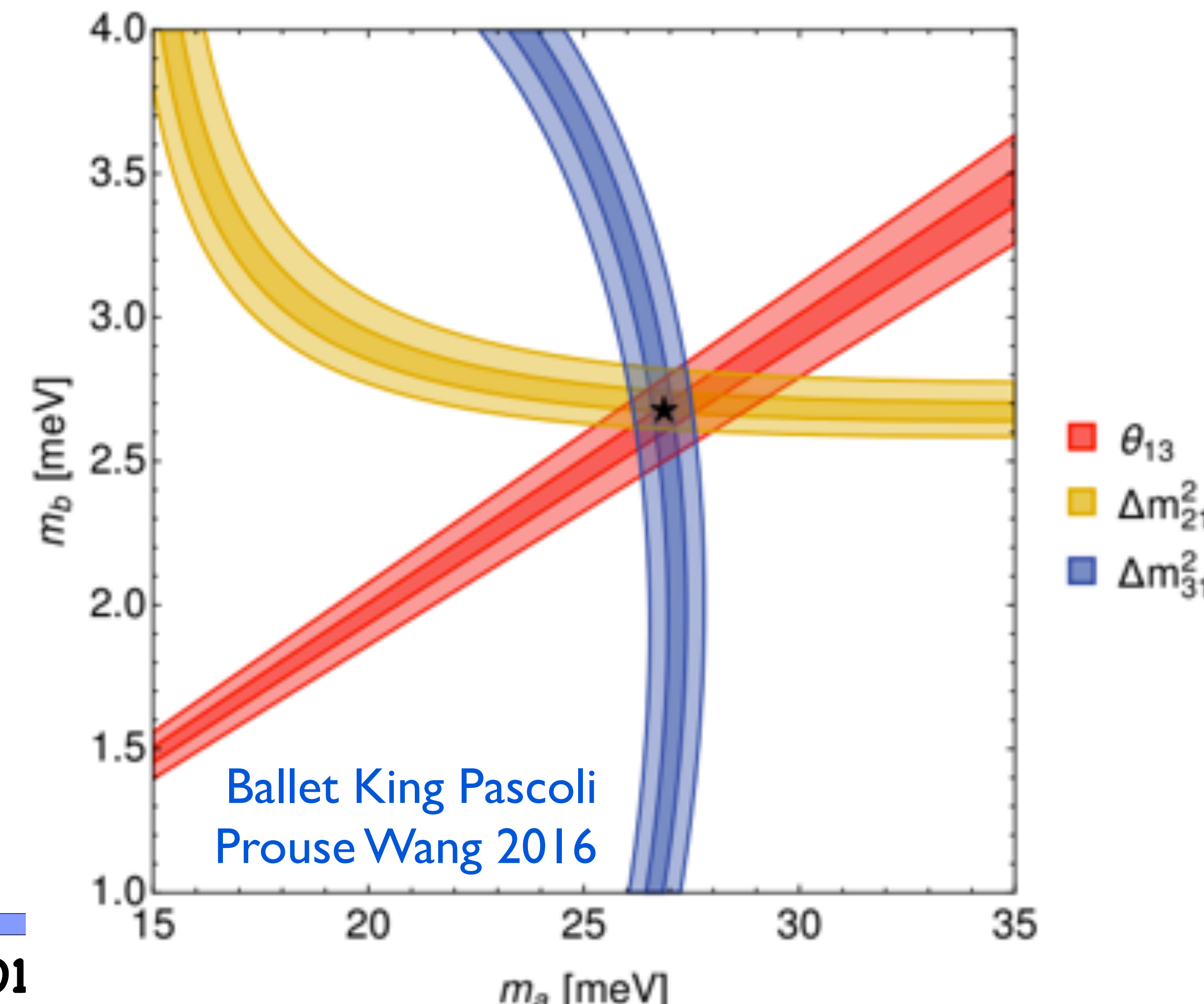
Nucl. Phys. B, Vol. 894, 733-768 (2015)





Predictions of flavor symmetry forms with projected measurement precision





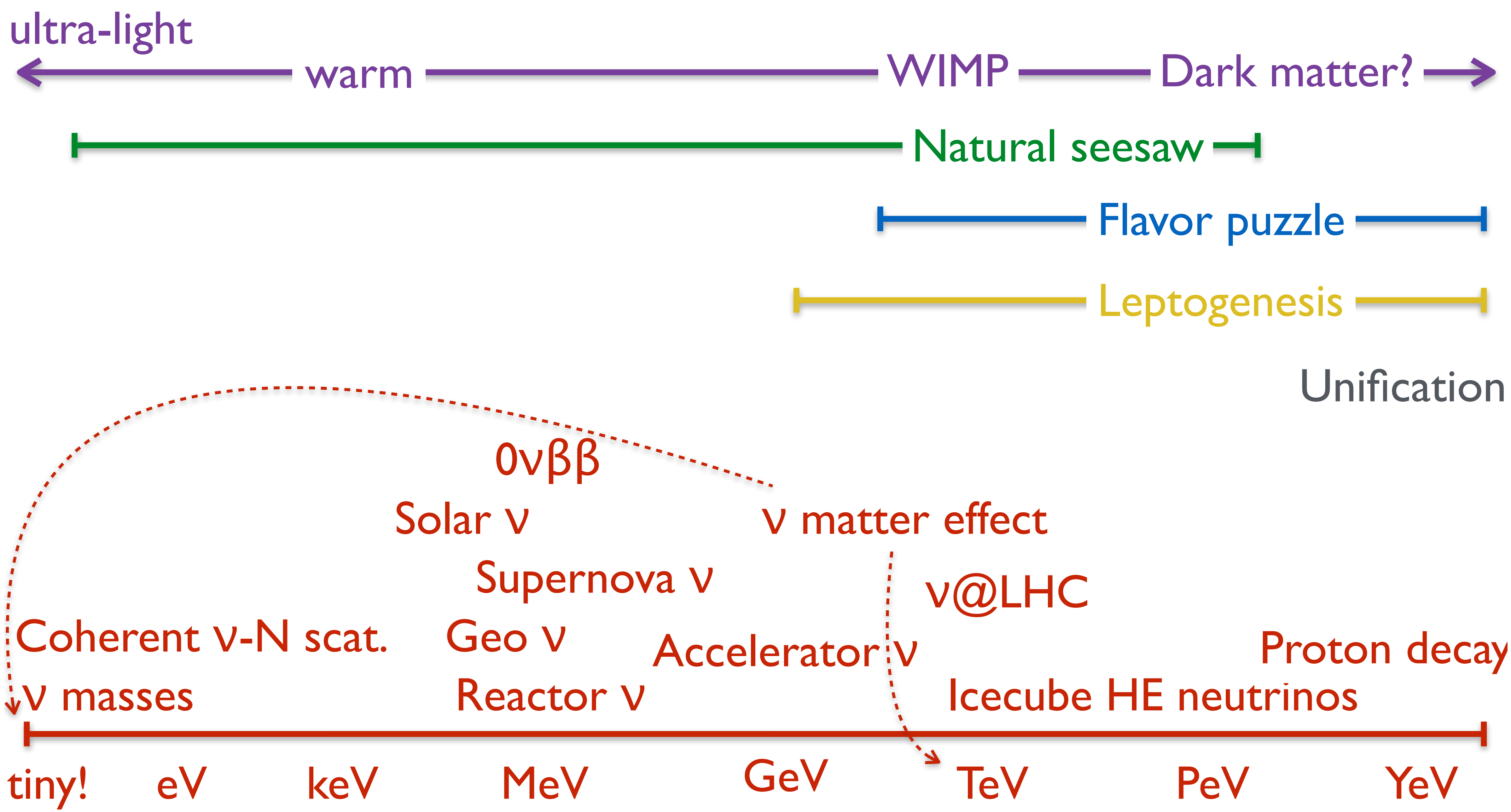
Recent highlights from neutrino theory

Pedro A. N. Machado
Fermilab *soon to be at LANL as junior staff member*

Aug/2017

pmachado@fnal.gov

Neutrinos as a portal to new Physics

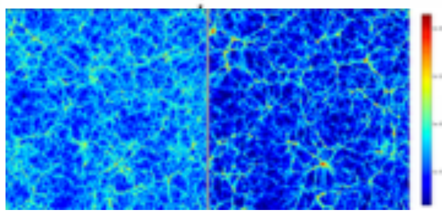


Many many many other fronts!

Neutrino cross sections (NuSTEC effort)

Neutrinos in cosmology
Early universe - BBN

Abazajian, Barbieri, Cirelli, Chizov, Di Bari, Dodelson, Dolgov, Foot, Holanda, Iocco, Kirilova, Kusenko, Mangano, Lesgourges, Pastor, Smirnov, Steigman, Volkas



Secret neutrino interactions

Dasgupta Kopp 2013, Chu Dasgupta Kopp 2015, Lundkvist Archidiacono Hannestad Tram 2016, Ghalsasi McKeen Nelson 2016, Archidiacono Gariazzo Giunti Hannestad Hansen Laveder Tram 2016, Forastieri Lattanzi Mangano Mirizzi Natoli Saviano 2017

Supernova evolution: non-linear effects from collective oscillations

Friedland 2010, Cherry Carlson Friedland Fuller Vlaesnko 2012, Chakraborty Hansen Izaguirre Raffeelt 2016, Capozzi Basudeb Dasgupta 2016, Izaguirre Raffelt Tamborra 2016, Capozzi Dasgupta Lisi Marrone Mirizzi 2017

Chen Ratz Trautner 2015

Cosmic neutrino background: ideas to measure it? Non-thermal component?

Type II, type III and radiative seesaw

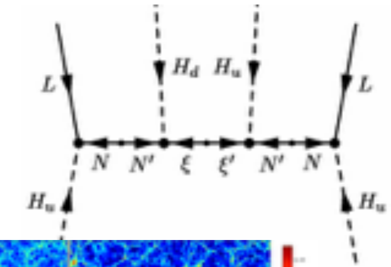
Akhmedov, Bonnet, Babu, Barbieri, Barger, Berezhiani, Ellis, Gaillard, Glashow, Hirsch, Keung, Ma, Mohapatra, Ota, Pakvasa, Schechter, Senjanovic, Valle, Yanagida, Winter, Wolfenstein, Zee, and many others

Flat extra dimensions: light sterile neutrinos

Antoniadis, Arkani-Hamed, Barbieri, Berryman, Davoudiasl, Dimopoulos, Dvali, de Gouvea, Langacker, Machado, Mohapatra, Nandi, Nunokawa, Perelstein, Peres, Perez-Lorenzana, Smirnov, Strumia, Tabrizi, Zukanovich-Funchal, …

Leptogenesis

Barenboim, Davidson, Di Bari, Dolgov, Fukugita, Kuzmin, Rubakov, Servant, Shaposhnikov, Yanagida, Zeldovich, …

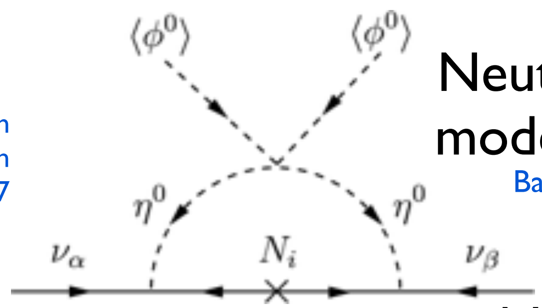


Sterile neutrino in long baseline oscillation experiments

Agarwalla, Bhattacharya, Chaterjee, Dasgupta, Dighe, Donini, Fuki, Klop, Lopez-Pavon, Meloni, Migliozzi, Palazzo, Ray, Tang, Terranova, Thalapillil, Wagner, Yasuda, Winter,…

Dark matter in neutrino detectors: light DM and light mediators

Ballett, Batell, Chen, Coloma, deNiverville, Dobrescu, Frugiuele, Harnik, McKeen, Pascoli, Pospelov, Ritz, Ross-Lonergan

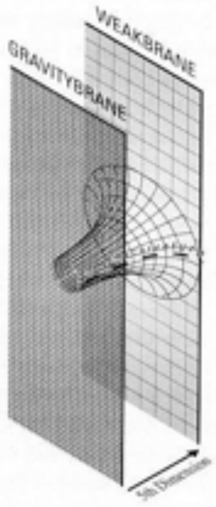


Neutrinos and the standard solar model: CNO cycle and metallicity

Bailey, Busoni, Christensen-Dalsgaard, Krief, Simone, Serenelli, Scott, Vincent, Vilante, Vissani, Vynioli, …

Neutrino magnetic moment

see e.g. Salam 1957, Barbieri Fiorentini 1988, Barbieri Mohapatra 1989, Babu Chang Keung Phillips 1992, Tarazona Diaz Morales Castillo 2015 Cañas Miranda Parada Tortola Valle 2015, Barranco Delepine Napsuciale Yebra 2017 Coloma Machado Martinez-Soler Shoemaker 2017

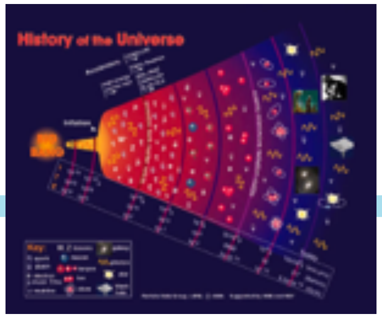


Discrete symmetries with non-zero θ_{13}

Feruglio Hagedorn Toroop 2011, Lam 2012, Lam 2013, Holthausen Lim Lindner2012, Neder King Stuart 2013, Hagedorn Meroni Vitale 2013 King Neder 2014, Ishimori King Okada Tanimoto 2014, Yao Ding 2015 , …

Effective operator approach to neutrino masses and collider/low scale pheno

de Gouvea Jenkins 2007, Boucenna Morisi Valle 2014, Nath Syed 2015, Geng Tsai Wang 2015, Chiang Huo 2015, Bhattacharya Wudka 2015, Geng Huang 2016, Quintero 2016, Mohapatra 2016, Kobach 2016



New physics in neutrinoless double beta decay, lepton number violation at the LHC, left-right models, RS models and neutrino masses, neutrinos as dark matter, and much more!

Fermilab

pmachado@fnal.gov

Towards a better understanding of Osc. Prob.

Globes,
while a very useful tool,
is not enough !

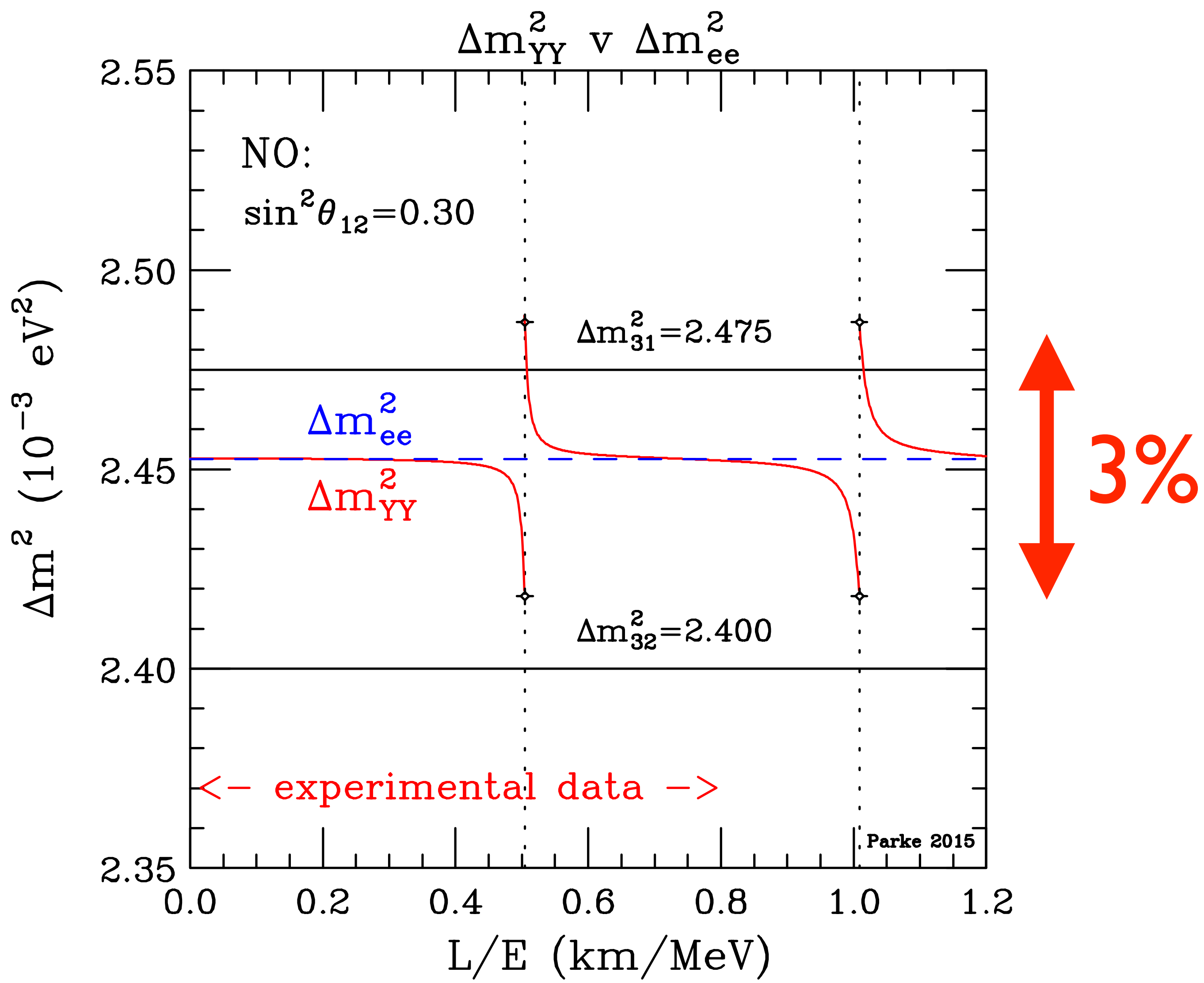
$\bar{\nu}_e \rightarrow \bar{\nu}_e$

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$ $\quad \Delta \equiv \Delta m^2 L/4E$

$+\ \mathcal{O}(0)$ $\quad \Delta m^2_{YY} \equiv \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})}\right]$

OR

$+\ \mathcal{O}(10^{-4})$ $\quad \Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$



ν_e average !

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal,
"Another possible way to determine the neutrino mass hierarchy,"
Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

SP arXiv:1601.07464

$\nu_\mu \to \nu_\mu$

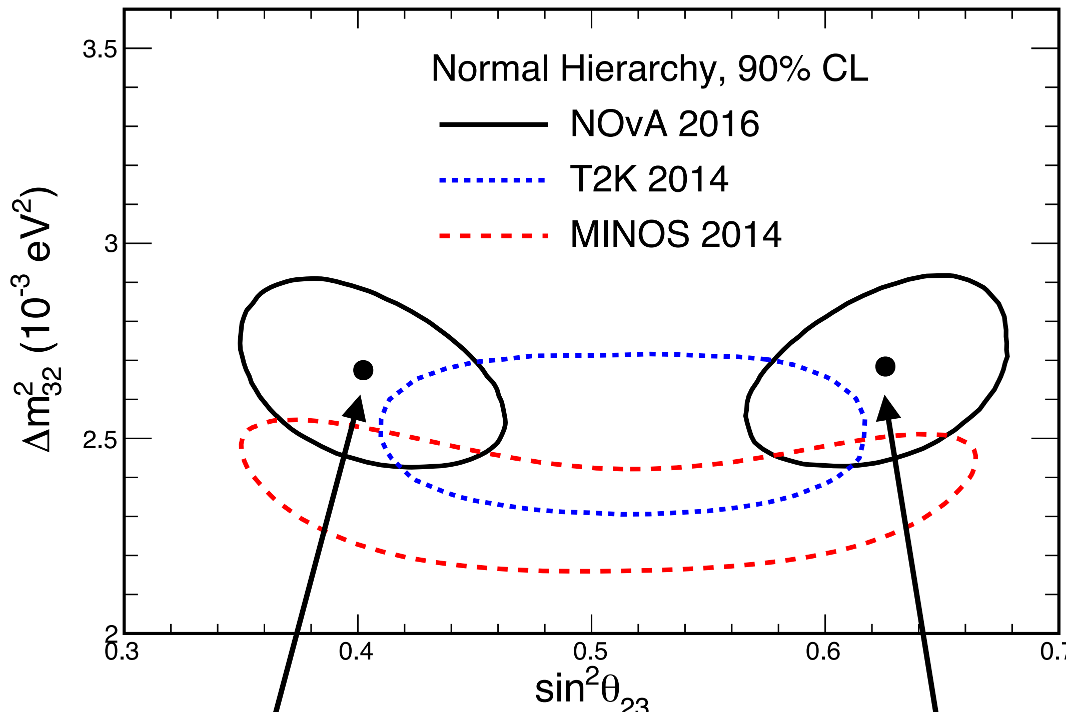
$$1 - P(\nu_\mu \to \nu_\mu) \approx 4\,|U_{\mu 3}|^2\,(\,1 - |U_{\mu 3}|^2\,)\ \sin^2 \Delta_{\mu\mu}$$

Amplitude of Oscillation:

ν_μ average !

$$c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}$$

$$= \quad 4\, c_{13}^2 s_{23}^2\, (\,1 - c_{13}^2 s_{23}^2\,)$$



for every $(s_{23}^2)_1$ point,

$$(s_{23}^2)_2 \;=\; 1/c_{13}^2 - (s_{23}^2)_1 \approx (1 + s_{13}^2) - (s_{23}^2)_1$$

has approx. same χ^2

and $(s_{23}^2)_1 + (s_{23}^2)_2 \;\approx\; (1 + s_{13}^2)$

Symmetry about $s_{23}^2 \approx \frac{1}{2}(1 + s_{13}^2) \approx 0.51$

$(s_{23}^2)_1$ $(s_{23}^2)_2$

Neutrino Oscillation Amplitudes

$$P(\nu_\alpha \to \nu_\beta) \quad = \quad |\mathcal{A}_{\alpha\beta}|^2$$

Two Flavors:

$$\mathcal{A}_{\alpha\alpha} = 1 + (2i)\; s_\theta^2\; e^{+i\Delta}\;\; \sin\Delta$$

and $\mathcal{A}_{\alpha\beta} = (2i)\; s_\theta c_\theta\; e^{-i\Delta}\;\; \sin\Delta$

$$\Delta \equiv \Delta m^2 L/4E$$

Neutrino Oscillation Amplitudes in vacuum:

"the billion $ process"

$$P(\nu_\mu \to \nu_e) = |\mathcal{A}_{\mu e}|^2$$

$$\mathcal{A}_{\mu e} = (2i)\,[\,(s_{23}s_{13}c_{13})\,[\,c_{12}^2 e^{-i\Delta_{32}} \sin\Delta_{31} + s_{12}^2 e^{-i\Delta_{31}} \sin\Delta_{32}\,]$$
$$+ (c_{23}c_{13}s_{12}c_{12})\, e^{i\delta} \sin\Delta_{21}\,]$$

maintain the symmetry: $m_1^2 \leftrightarrow m_2^2$ with $\theta_{12} \to \theta_{12} \pm \pi/2$

Denton, Minakata, SP arXiv:1604.08167

$$\Delta P_{CP} = 8\,(s_{23}s_{13}c_{13})\,(c_{23}c_{13}s_{12}c_{12})\,\sin\delta\,\sin\Delta_{21}\,\sin\Delta_{31}\,\sin\Delta_{32}$$

J

$$\Delta_{32} \approx \Delta_{31}$$

$$\mathcal{A}_{\mu e} \approx (2i)\,[\,(s_{23}s_{13}c_{13})\,\sin\Delta_{31} + (c_{23}c_{13}s_{12}c_{12})\,e^{i(\delta+\Delta_{31})}\,\sin\Delta_{21}\,]$$

$\nu_\mu \rightarrow \nu_e$

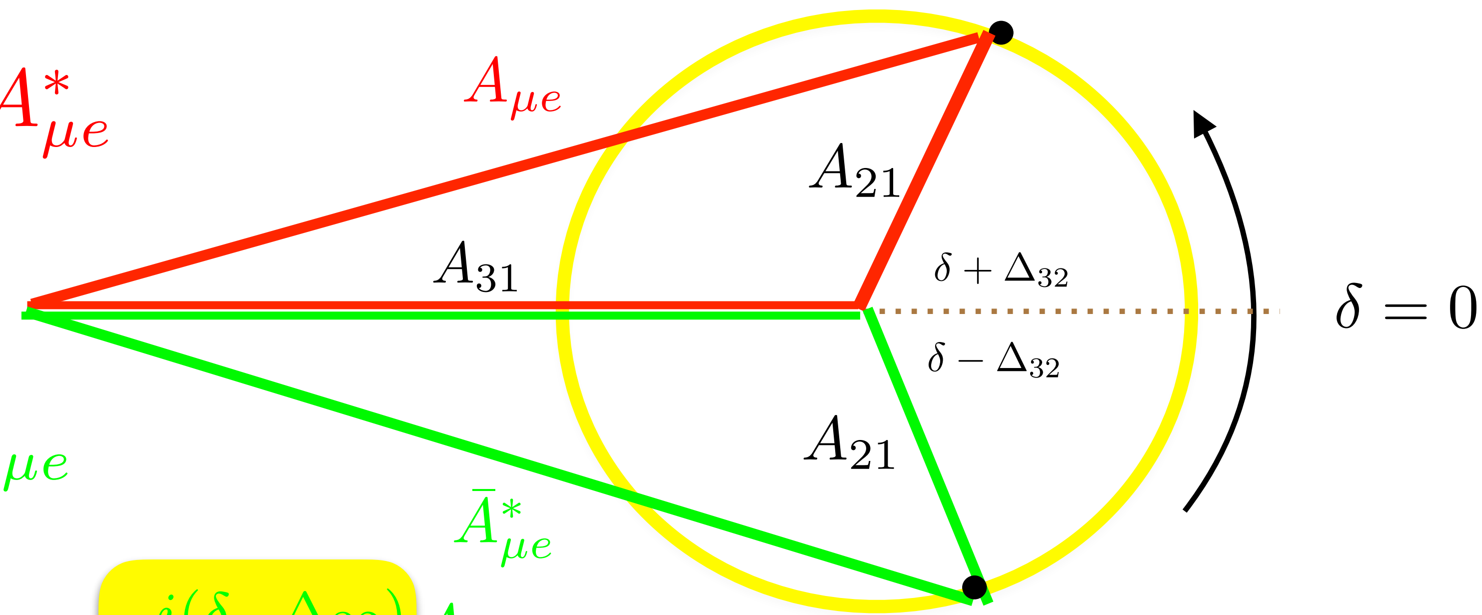
$A_{31} = 2 s_{23} s_{13} c_{13} \sin \Delta_{31}$

$A_{21} = 2 c_{13} c_{23} s_{12} c_{12} \sin \Delta_{21}$

$$A_{\mu e} = A_{31} + e^{i(\delta+\Delta_{32})} A_{21}$$

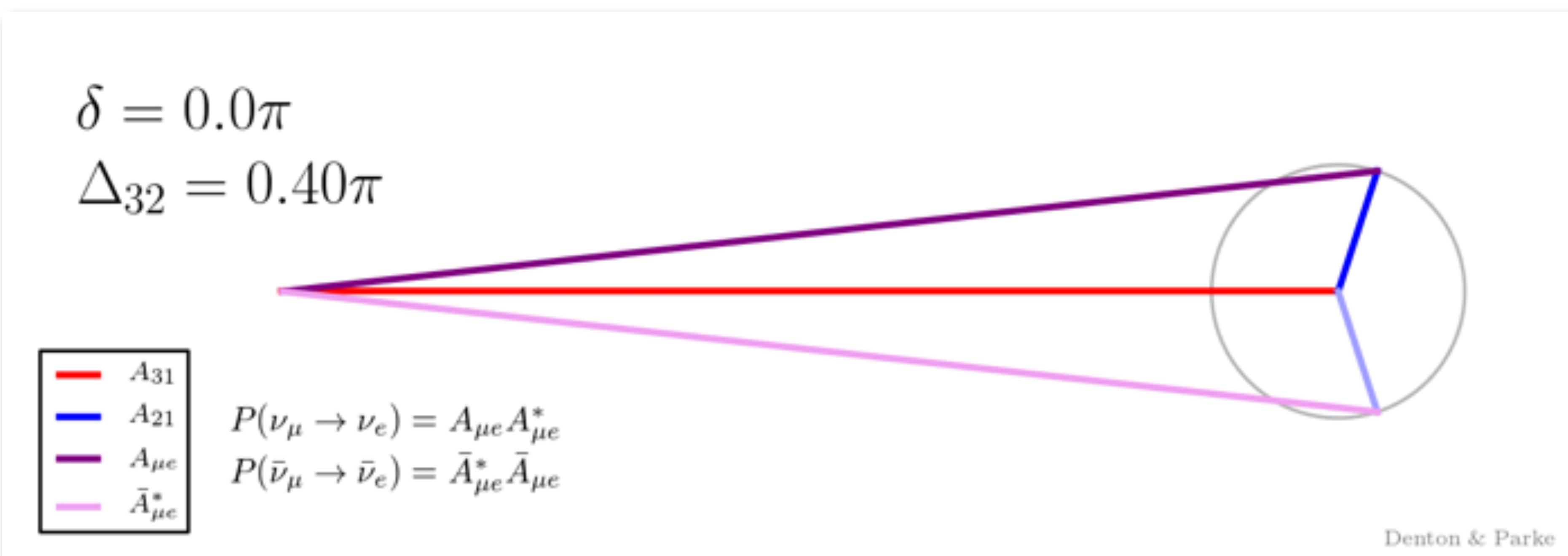
$\boldsymbol{\Delta_{ij} = \Delta m^2_{ij} L / 4E}$

$$P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A^*_{\mu e}$$

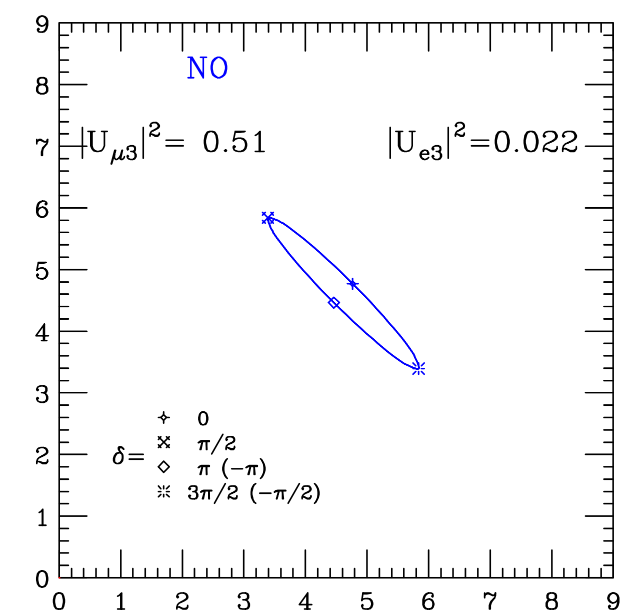


$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}^*_{\mu e} \bar{A}_{\mu e}$$

$$\bar{A}^*_{\mu e} = A_{31} + e^{i(\delta-\Delta_{32})} A_{21}$$



$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = |\bar{A}_{\mu e}|^2$



$P(\nu_\mu \rightarrow \nu_e) = |\bar{A}_{\mu e}|^2$

Matter Effects:

2 flavor mixing in matter

$$ax^2 + bx + c = 0$$

simple, intuitive, useful

3 flavor mixing in matter

$$ax^3 + bx^2 + cx + d = 0$$

complicated, counter intuitive, ...

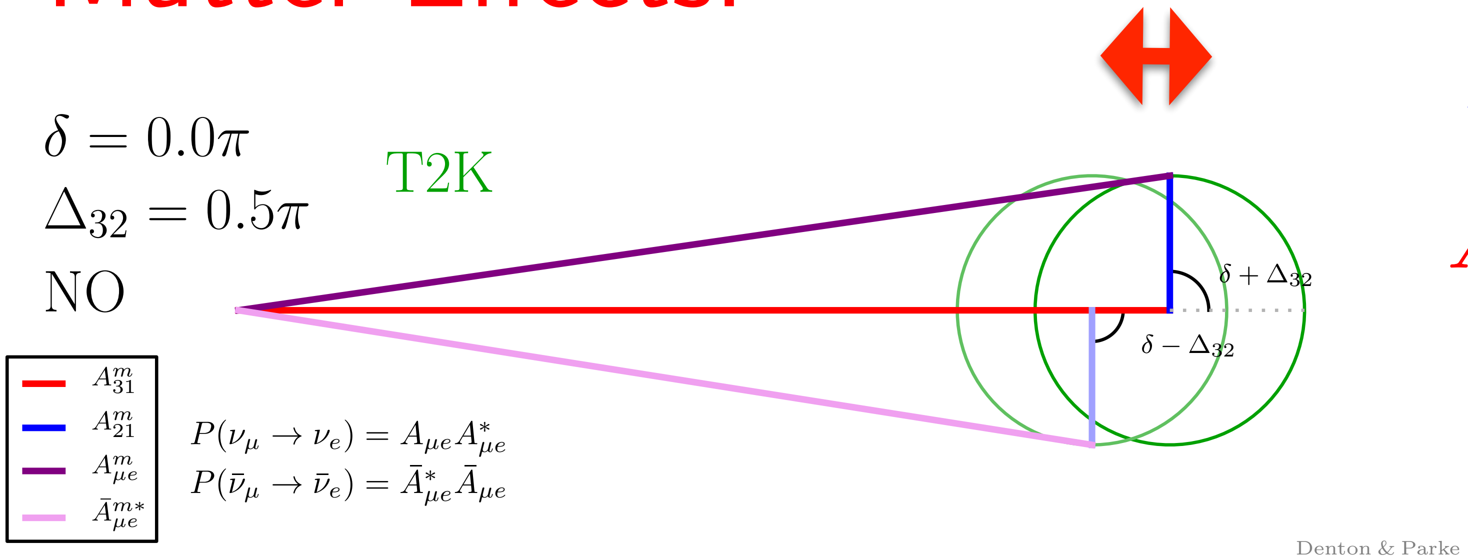
Matter Effects:

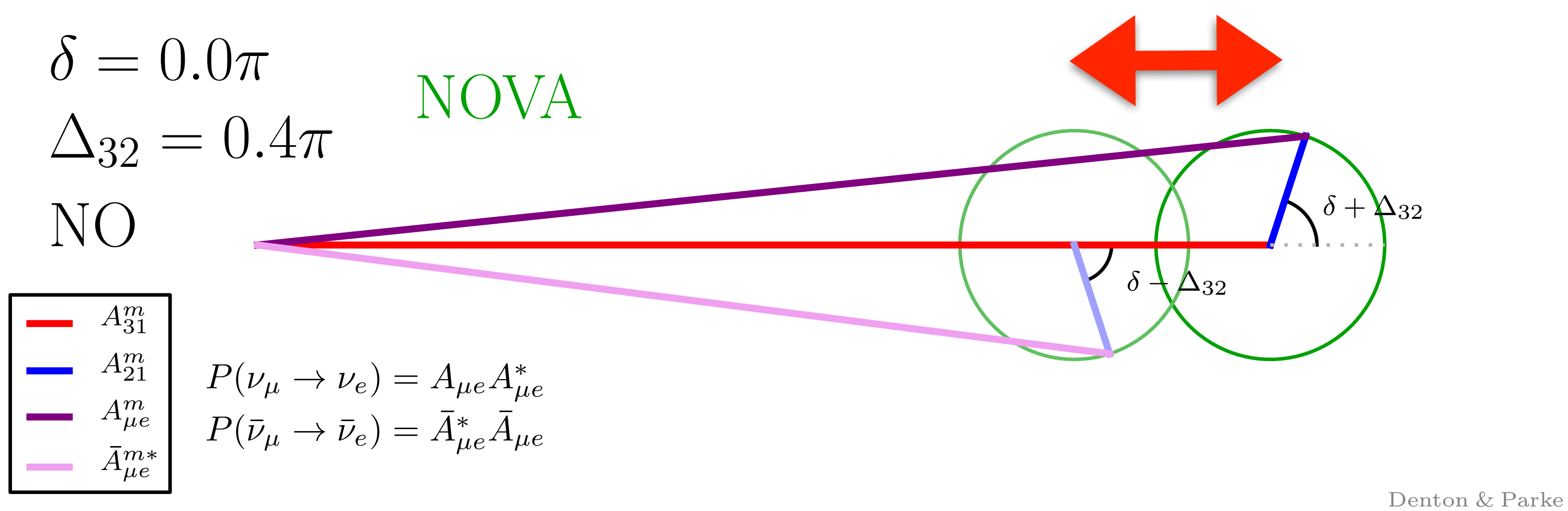
$$A_{31} + e^{i(\Delta_{32} \pm \delta)} A_{21}$$

$$A_{31} = 2 s_{23} s_{13} c_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)} \Delta_{31}$$

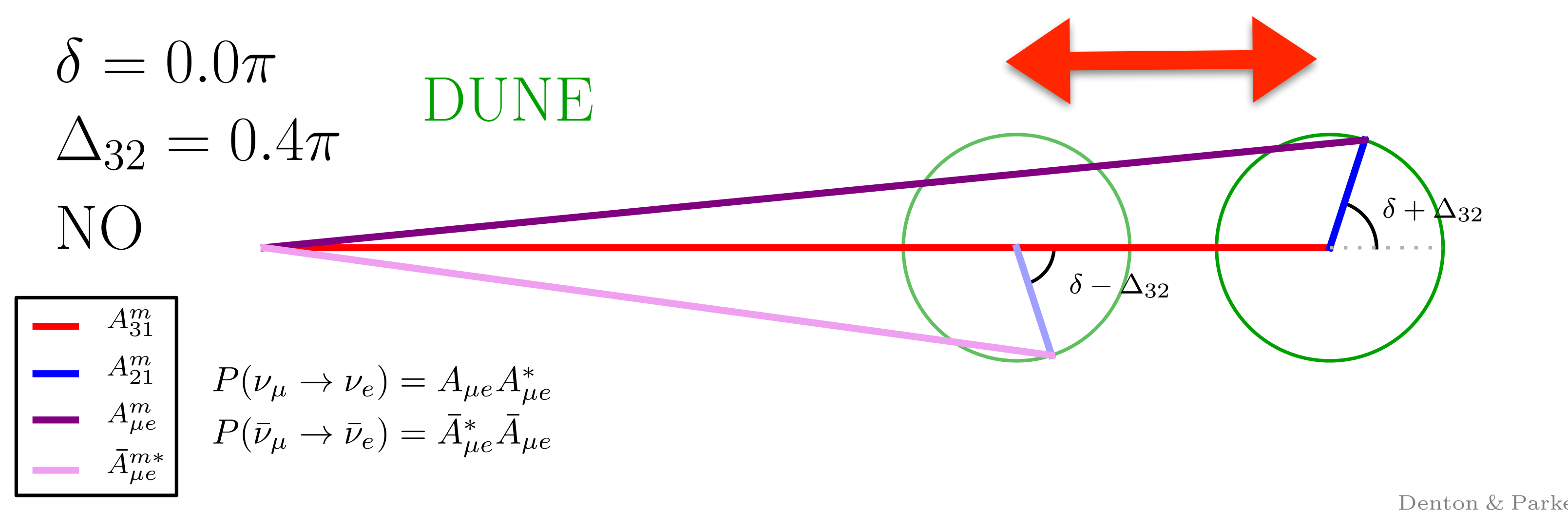
$$A_{21} = 2 c_{13} c_{23} s_{12} c_{12} \frac{\sin(aL)}{(aL)} \Delta_{21}$$

$$a = G_F N_e / \sqrt{2} = (4000\ km)^{-1},$$





$$\propto \rho L \ \sin^2 \theta_{23}$$



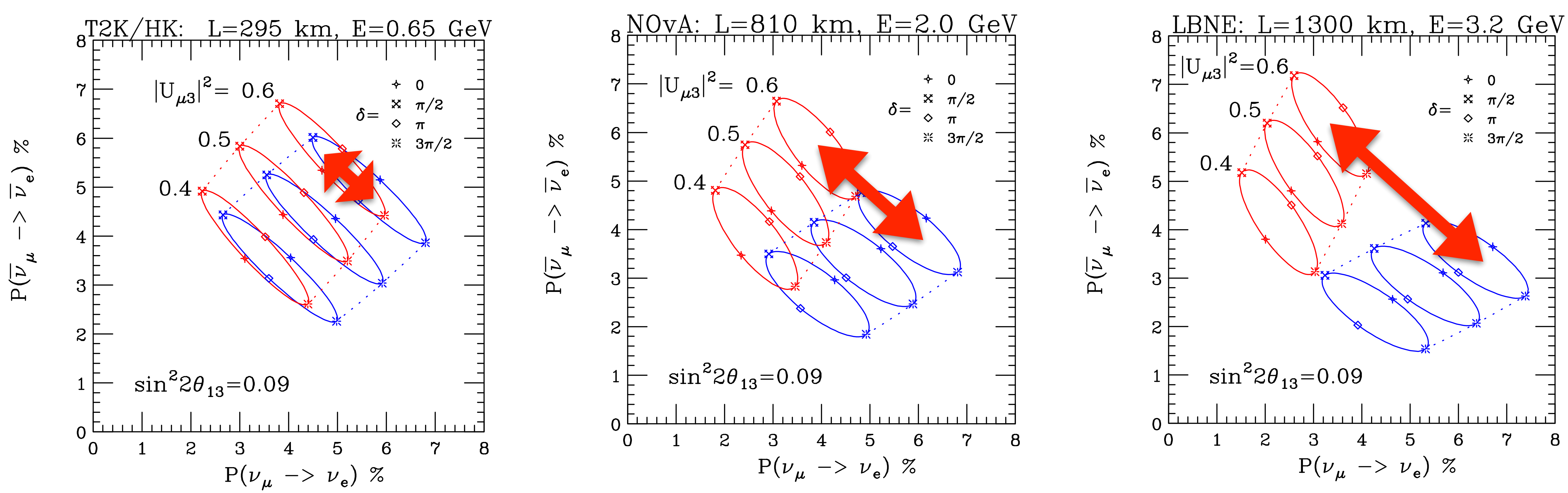
Correlations between $\nu_\mu \to \nu_e \quad \bar{\nu}_\mu \to \bar{\nu}_e$

Normal Ordering — Inverted Ordering

$\nu_\mu \to \nu_\mu$ gives: $\sin^2 2\theta_{\mu\mu} \equiv 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2) = 0.96 \; - \; 1.00$

$|U_{\mu 3}|^2 \leftrightarrow (1-|U_{\mu 3}|^2)$ degeneracy !

T2K/HK — NOvA — DUNE (Same L/E as NOνA)

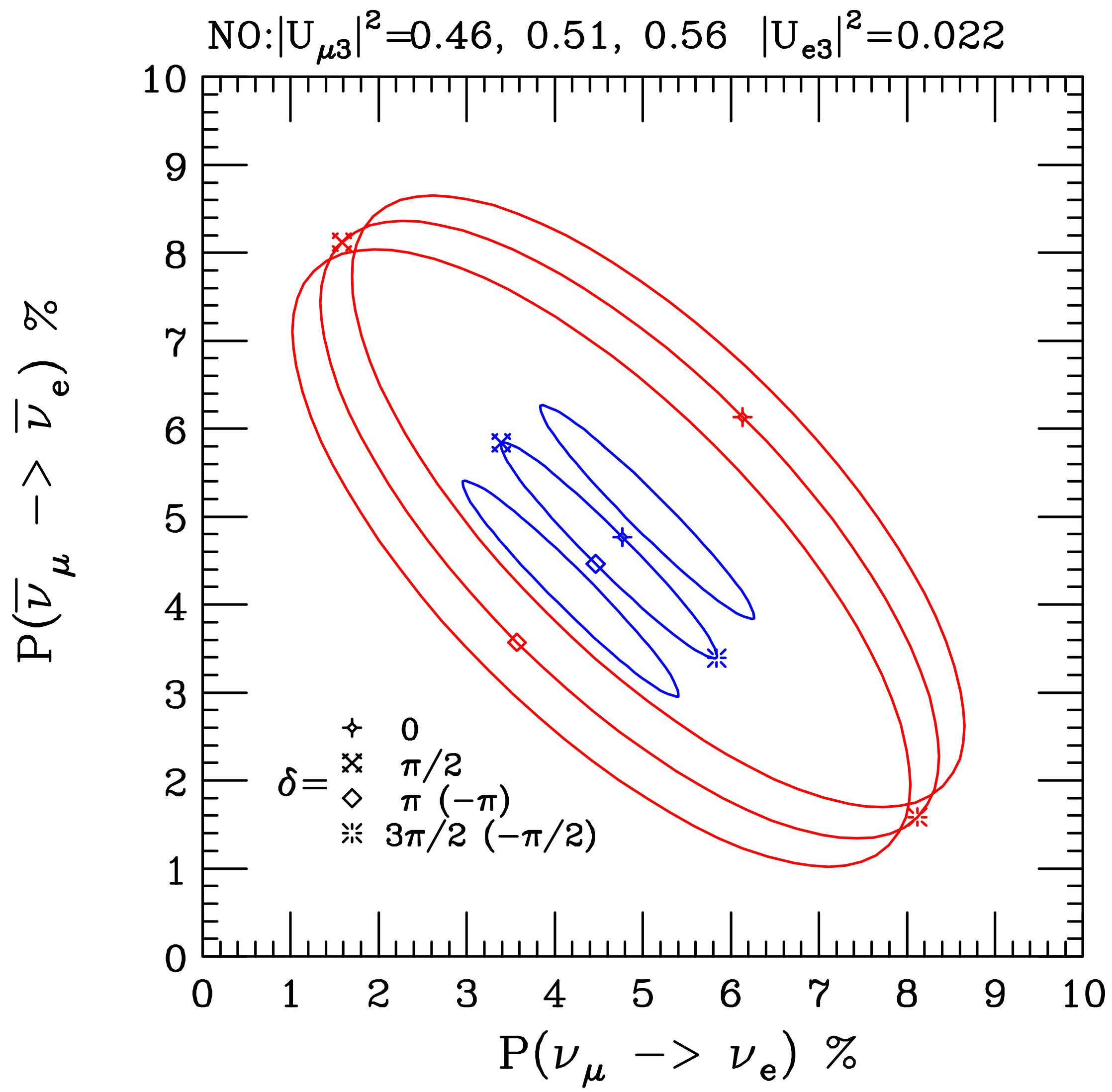


$\propto \rho L \; \sin^2 \theta_{23}$

$$\sin \delta_{NO} - \sin \delta_{IO} = \tan \theta_{23} \times \begin{cases} 0.48 & \text{T2K} \\ 1.62 & \text{NO}\nu\text{A} \\ 2.60 & \text{DUNE} \end{cases}$$

O. Mena & SP hep-ph/0408070

2nd Osc Max: (vacuum)



$$A \equiv \frac{\bar{P}-P}{\bar{P}+P}$$

$$A_1 \approx 0.30 \sin\delta$$

$$A_2 \approx 0.75 \sin\delta$$

$$\frac{A_2}{A_1} \approx 2.5 \qquad \frac{\sqrt{N_2}}{\sqrt{N_1}} \approx \frac{1}{3}$$

Approximately same uncertainty on δ
until systematic uncertainities dominate at 1st OM !

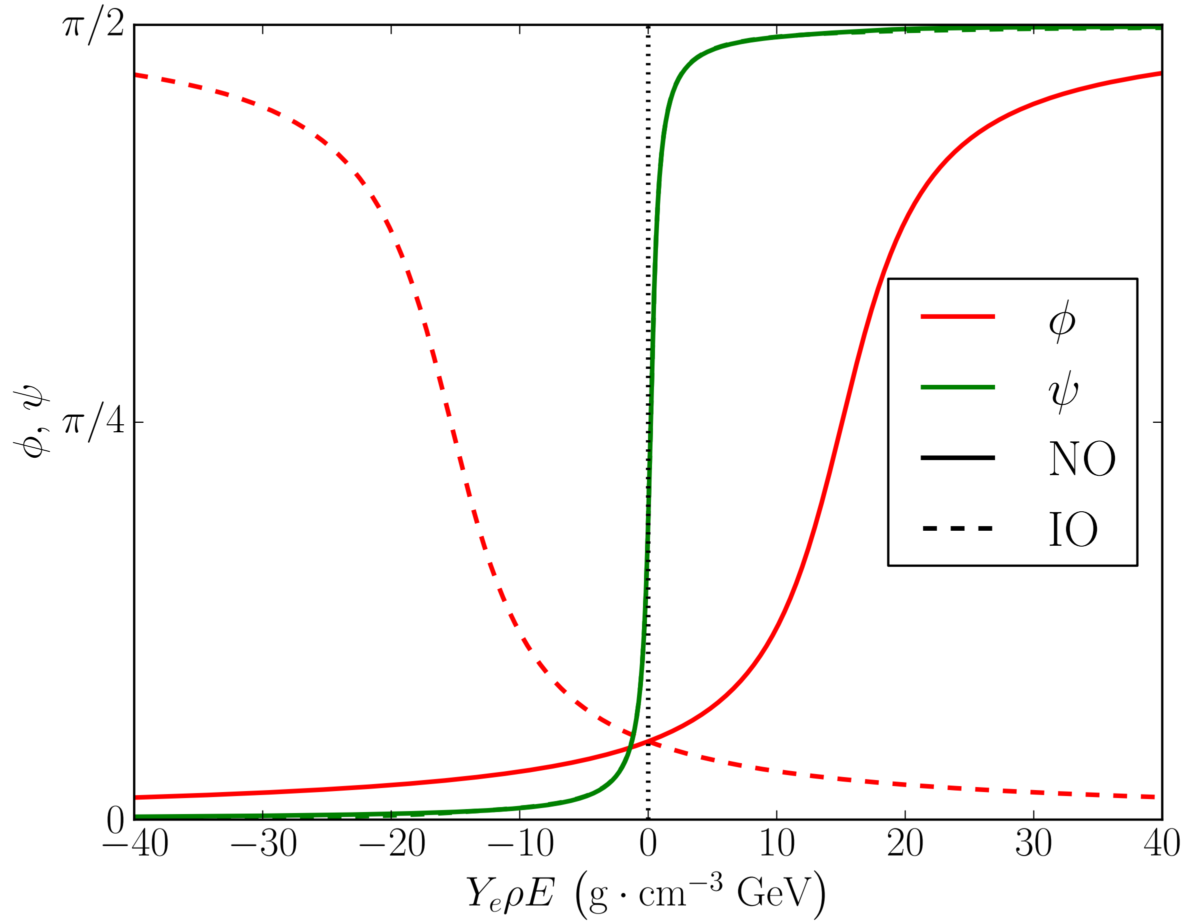
ESSnuSB, T2HKK

New Perturbation Theory for Osc. Probabilities

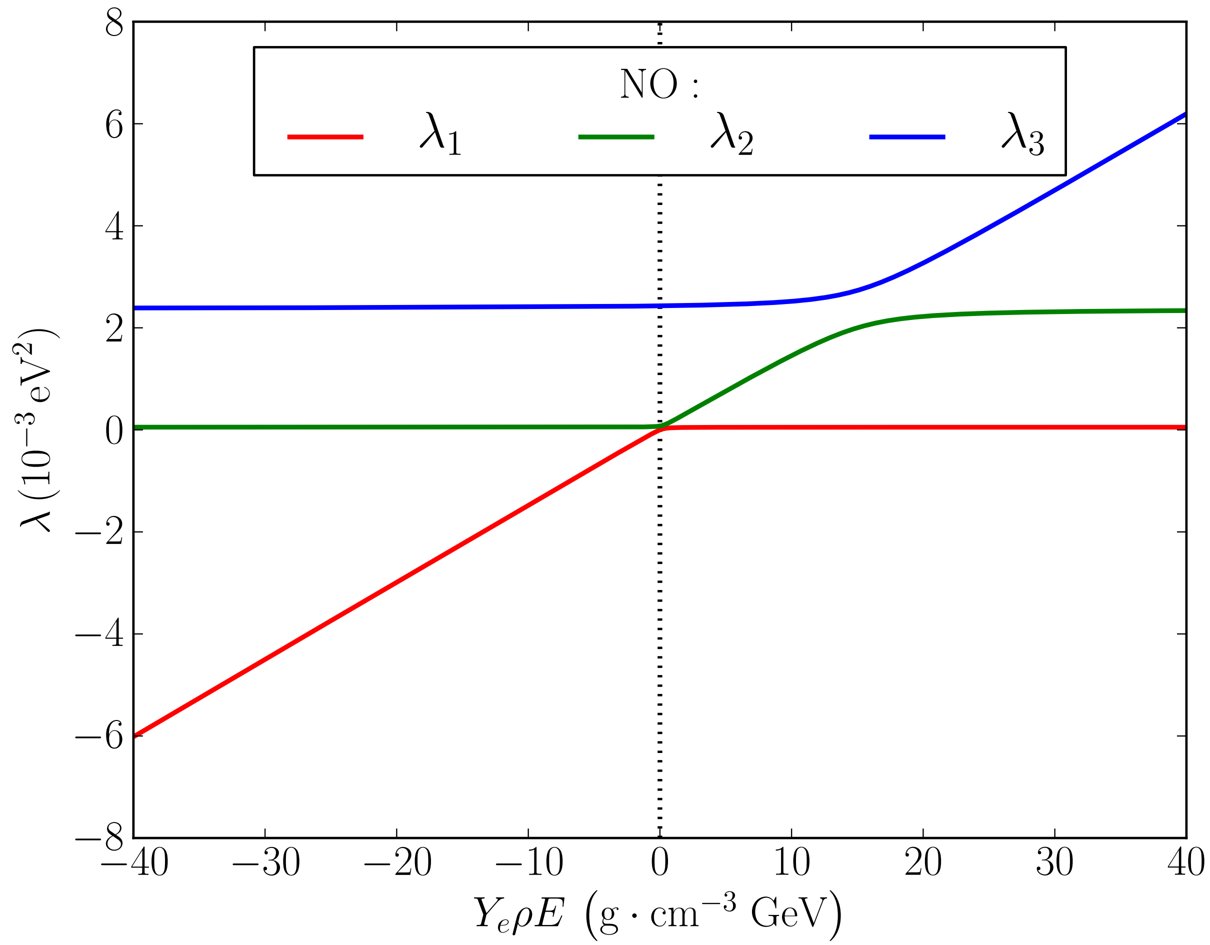
Denton, Minakata, SP arXiv:1604.08167

$$\mathcal{A}_{\mu e} \;=\; (2i)\,[\,(s_{23}s_{13}c_{13})\,[\,c_{12}^2 e^{-i\Delta_{32}} \sin\Delta_{31} + s_{12}^2 e^{-i\Delta_{31}} \sin\Delta_{32}\,]$$
$$+\,(c_{23}c_{13}s_{12}c_{12})\; e^{i\delta}\;\sin\Delta_{21}\,]$$

mixing angles in matter $\theta_{13} \to \phi$ and $\theta_{12} \to \psi$

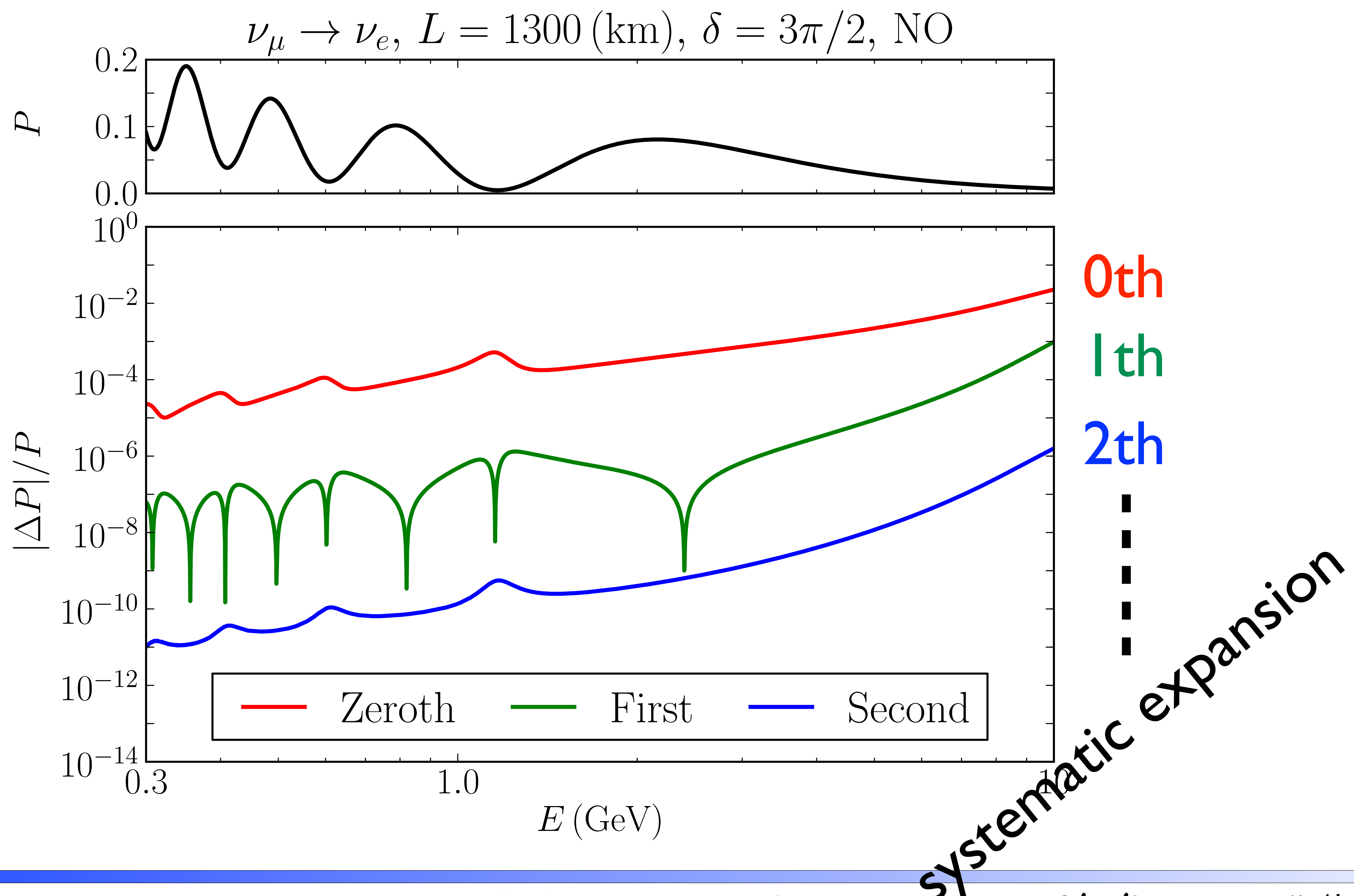


mass eigenvalues in matter $m_i^2 \to \lambda_i$

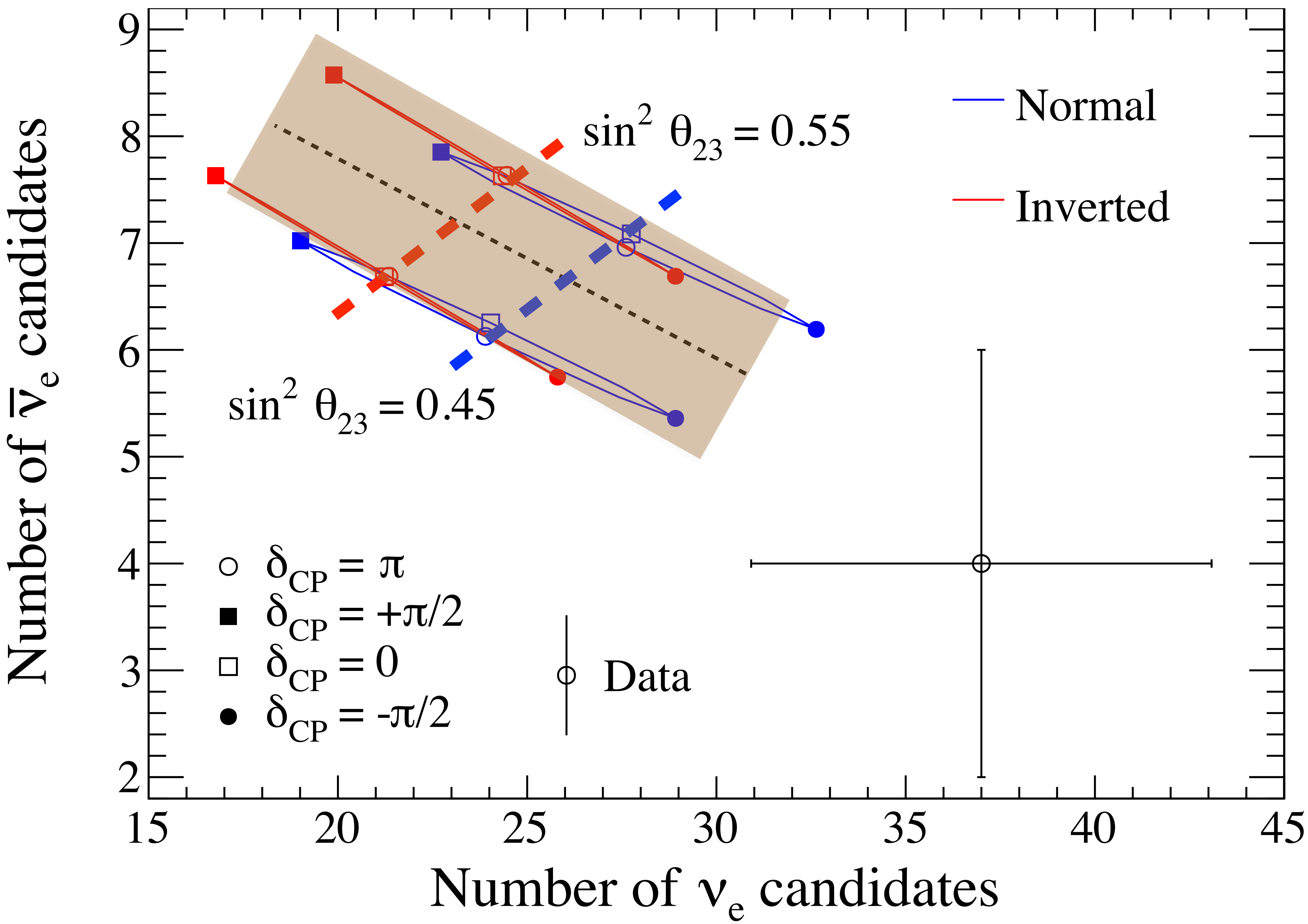


Intuitive and Analytically simple !

New Perturbation Theory for Osc. Probabilities



T2K



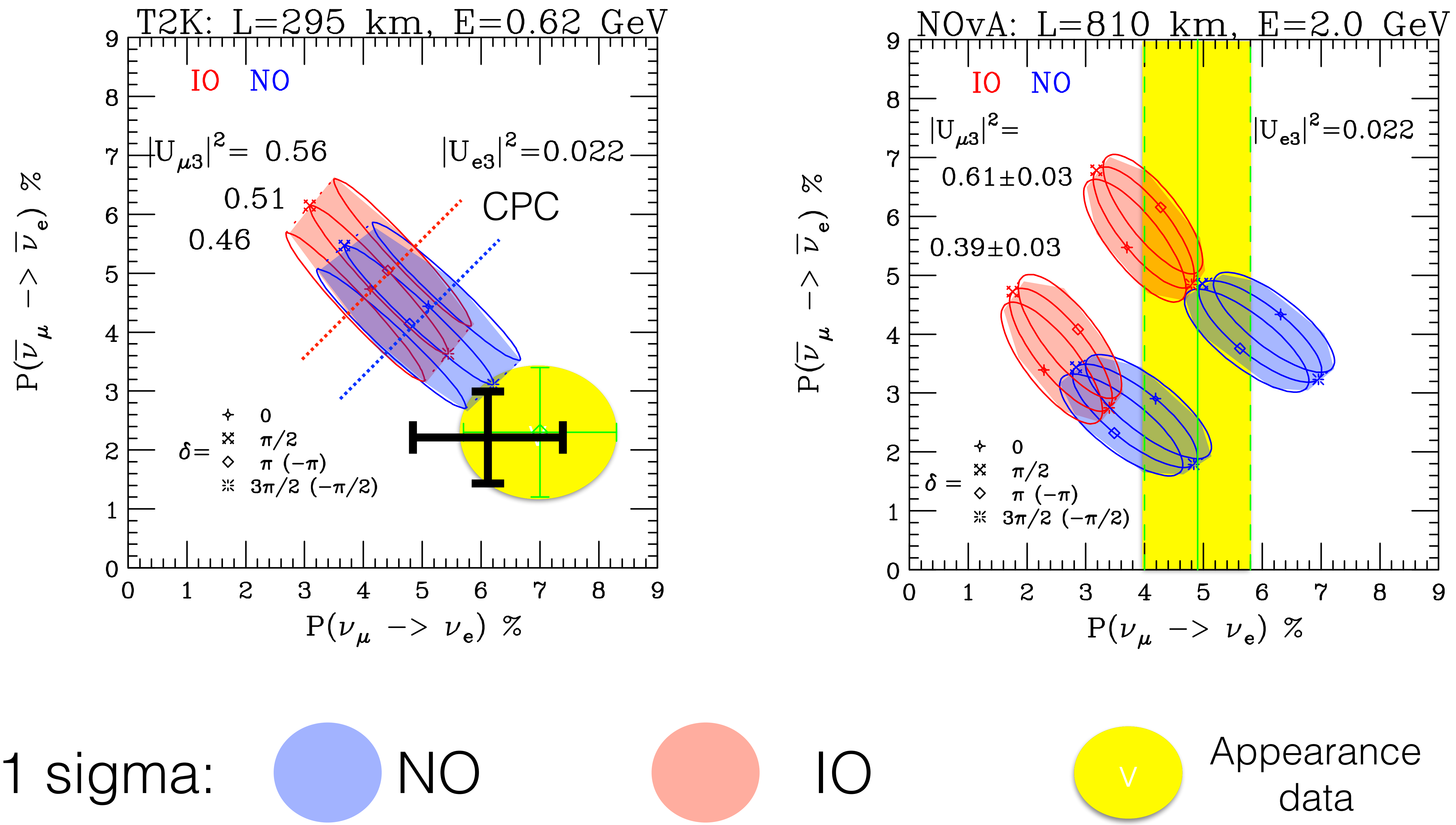
CPC - IO

CPC - NO

All except appearance !

T2K & NOvA

Number of Events proportional to Oscillation Probability



Summary:

- from Nu1998 to now, tremendous exp. progress on Neutrino SM: more at Nu2018
- LSND Sterile Nu's neither confirmed or ruled out at acceptable CL: - ultra short baseline reactor exp.
- Great Theoretical progress on understand many aspects of Quantum Neutrino Physics: - Oscillations, Decoherence, Osc. Probabilities in Matter, Leptogenesis, ...
- Still searching for convincing model of Neutrino masses and mixings: with testable and confirmed predictions !