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COMPARISON AND EVALUATION OF BOUNDARY CONDITIONS IN AN FDTD-1D SIMULATION

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Abstract. The FDTD is derived by discretizing Maxwell's equation using the fi-nite difference (FD) method, it is commonly used in solving differential equations [1]. One fundamental challenge faced during implementing FDTD method is the application of boundary conditions, it can be employed to enhance the perfor-mance of the simulation. To evaluate the effectiveness of different boundary con-ditions, a one-dimensional FDTD simulation is carried out in this work. The im-pact of various boundary conditions such as PEC, PMC, 1st order Mur Bounda-ry and PML and CPML, on the plane wave propagation along the z axis are explored. The choice of each boundary depends on the specific problem being solved.

***Keywords*:** Electromagnetic, FDTD, PML, Boundary Condition.

1. Introduction

FDTD (Finite Difference Time Domain) is a numerical method for solving electromagnetic field equations in the time domain. This method is widely used to simulate and analyze the propagation of electromagnetic waves and their interactions with various structures and materials [2] . FDTD computer simulations are frequently used in microwave and photonics design, radar cross-section calculation, and electromagnetic field propagation of antenna radiation.

Applying FDTD to problems where the solution regions are unbounded becomes difficult. We need to establish some sort of limits on the regions of our solutions because no computer can store an unlimited amount of data [3].

Boundary conditions are a way to limit the solution domain and create the digital illusion of infinite space, such as boundary conditions play a crucial role in FDTD simulations because they define how electromagnetic fields interact with the boundaries of the computational domain. There are several FDTD absorbing boundary conditions, the most common are the perfectly matched layer (PML), and Mur’s absorbing boundary, and Perfect electric conductor (PEC) and Perfect Magnetic conductor (PMC). The use of these boundary conditions depend on the specific problem you want to simulate. In order to obtain accurate results, it is important to choose the boundary conditions that best suit your situation. such as The Mur’s absorbing boundary condition is a commonly used method to absorb outgoing waves from the simulation domain in order to prevent reflections. and the Perfectly Matched Layer (PML) is an advanced form of the Mur condition that enables better absorption of electromagnetic waves. Where (PMC) and (PEC) these boundary conditions are used to model objects that are perfect dielectrics. It assumes that the magnetic fields the electric field are zero at the boundary of the simulation domain.

This report describes the governing equation by numerically constructing the Yee Algorithm in 1-Dimensional (1-D) system using MATLAB programming language to describe the distribution of the mode within four types of boundary conditions. First, with (PEC and PMC) boundary condition, second with Absorbing Boundary Condition (ABCs), and third with The Perfectly Matched Layer (PML), and four with Convolutional Perfectly Matched Layer (CPML). The field Ex and Hy are simulated along the z axis in free space region. The wave is excited by a Sinusoidal source modulated by a Gaussian located at a specified position within the simulation domain. and in homogenous and isotropic media, where the conductivity, permeability, and permittivity are all constant throughout time, this mode can spread.

The basic theory FDTD method in 1D along the z axis is shown in Section 2. in Section 3, results of simulations in 1D FDTD with difference case of boundary conditions are presented, the paper concluded with remarks are in section 3.

2. Theory

2.1. *Maxwell's equations*

Maxwell's equations that govern the propagation phenomena are the starting point of the FDTD method. For the case of an anisotropic medium, homogeneous and loss-less, Maxwell's equations under differential form are written.

$$\nabla \times \vec{E} = -\mu \frac{\delta \vec{H}}{\delta t} \qquad (1)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\delta \vec{E}}{\delta t} \qquad (2)$$

where μ and ε respectively, denote the material's permeability and permittivity. Let us consider that neither the physical properties of the medium nor the sources vary along the x- and y-axes.

$$\frac{\delta}{\delta x} = \frac{\delta}{\delta y} = 0 \qquad (3)$$

So, we can write the equation (1) and (2) to be:

E_x / H_y

$$\frac{\delta E_x}{\delta z} = -\mu \frac{\delta H_y}{\delta t} \qquad (4)$$

$$-\frac{\delta H_y}{\delta z} = \varepsilon \frac{\delta E_x}{\delta t} \qquad (5)$$

E_y / H_x

$$-\frac{\delta E_y}{\delta z} = -\mu \frac{\delta H_x}{\delta t} \qquad (6)$$

$$-\frac{\delta E_y}{\delta z} = -\mu \frac{\delta H_x}{\delta t} \qquad (7)$$

While that Ez and Hz, the longitudinal field components, are always zero. and that Maxwell's and equations have decoupled into two sets of two equations. While these modes are physical and would propagate independently, the are numecally the same and will exhibit the same electromagnetic behavior . therefore, it is only necessary to solve one. we will proceed with the Ex/Hy

$$\frac{\delta E_x}{\delta z} = -\mu \frac{\delta H_y}{\delta t} \qquad (8)$$

$$-\frac{\delta H_y}{\delta z} = \varepsilon \frac{\delta E_x}{\delta t} \qquad (9)$$

This is an example of a plane wave moving in the z-direction.

2.2. *Discretization of Maxwell's Equations*

for stable finit-diffrence equation Each term in a finite-difference equation must exist at the same point in time and space[4] . For this reason, Yee proposes to shift the calculation of the fields by half a time step[5], the so-called "leapfrog time step" :
H is given at times (n - 1/2)Δt, (n + 1/2)Δt, (n + 3/2)Δt ... etc.
and E to the moments (nΔt, (n + 1)Δt, (n + 2)Δt … etc.

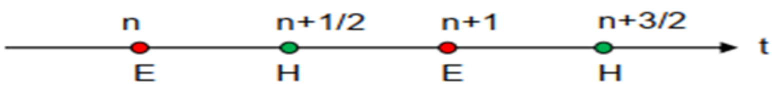

Fig. 1. Temporal evolution of electromagnetic fields.

The discretization of Equations (8) and (9), respectively, is now being considered individually, for Ex/Hy mode using central difference:
the equation (8) become:

$$\frac{E_x^t(k+1)-E_x^t(k)}{\Delta z}=-\mu\frac{H_y^{t+\frac{\Delta t}{2}}(k)-H_y^{t-\frac{\Delta t}{2}}(k)}{\Delta t} \tag{10}$$

the equation (9) become:

$$-\frac{H_y^{t+\frac{\Delta t}{2}}(k)-H_y^{t+\frac{\Delta t}{2}}(k-1)}{\Delta z}=\varepsilon\frac{E_x^{t+\Delta t}(k)-E_x^t(k)}{\Delta t} \tag{11}$$

From (10) and (11) one can deduce the explicit FDTD equation and get:

$$H_y^{t+\frac{\Delta t}{2}}(k)=H_y^{t-\frac{\Delta t}{2}}(k)-(\frac{\Delta t}{\mu})(\frac{E_x^t(k+1)-E_x^t(k-1)}{\Delta z}) \tag{12}$$

$$E_x^{t+\Delta t}(k)=E_x^t(k)-(\frac{\Delta t}{\varepsilon})(\frac{H_y^{t+\frac{\Delta t}{2}}(k)-H_x^{t+\frac{\Delta t}{2}}(k-1)}{\Delta z}) \tag{13}$$

2.3. *stability*

The Courant-Friedrichs-Lewy (CFL) condition, a stability criterion, sets a limit on the time increment Δt in FDTD [6].
For the sake of stability

$$v\frac{\Delta t}{\Delta z}\le 1 \tag{14}$$

The stability criterion restricts the amount of time that can be added to the FDTD time increment, where:

$$v=\frac{1}{\sqrt{\mu\varepsilon}}$$

is the speed at which electromagnetic waves can travel through the medium.

So

$$\Delta t\le\frac{\Delta z}{\sqrt{\mu\varepsilon}} \tag{15}$$

3. Simulation and results with difference boundary

To validate the boundary conditions in the FDTD simulation, the reflection and transmission behavior of the sinusoidal source modulated by the boundary conditions are presented in the free space region along z direction z=12,5.
where the wave collide the boundary at t= 30.10e-15s and the origin of the source on the z axis is z=3 in the case the PMC, PEC, ABC boundary and z=250 in the case the perfect matched layer PML boundary. The explicit FDTD algorithm is used and calculated by using the original MATLAB codes.
the Sinusoidal source modulated by a Gaussian. as the initial pulse at t=0

$$E_x(t) = \sin(2\pi f_0 t) * e^{-\left(\frac{t-tc}{2\tau}\right)^2} \tag{16}$$

Initialization ought to be carried out at the start of a simulation.
So, the equation (12) and (13) can be simulate in MTLAB[8].

3.1. *PEC Boundary*

Equations in this section we apply the PEC to Z=Zmax=12.5µm or K=M=500 and let the code run long enough.
Or, Perfect electric conductor (PEC) boundary are specified by simply setting the boundary electric field node Ex(M+1)=0.

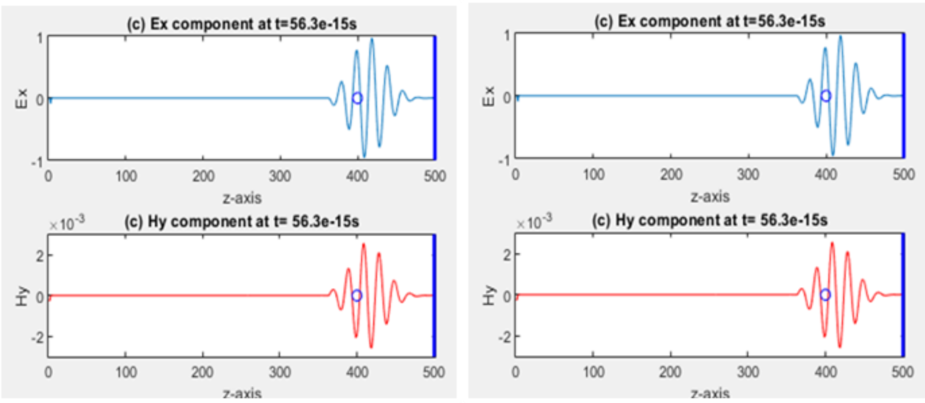

Fig. 2. the Ex and Hy at (a) t=20.3e-15s and (b) t=56.3e-15s

This in Fig. 2 (a), this outcome is displayed. We can observe from the simulation that Ex and Hy were apart before they collided (K=500). The Ex and Hy share the same phase. is irrelevant because it serves as the initial condition, which we define. The circumstance under which the wave collides with the border is crucial. We can see from the simulation in Fig. 2(b) that Ex and Hy eventually collide the boundary. The interference appears to be constructive in Ex but destructive in Hy due to the 180° phase difference between themes. It is reflected in the same magnitude but the opposite phase.

3.2. *PMC Boundary*

In this section we apply the PMC to Z=Zmax=12.5µm or K=M=500 and let the code run long enough.
Or, Perfect Magnetic conductor (PMC) boundary are specified by simply setting the boundary magnetic field node Hy(M)=0.

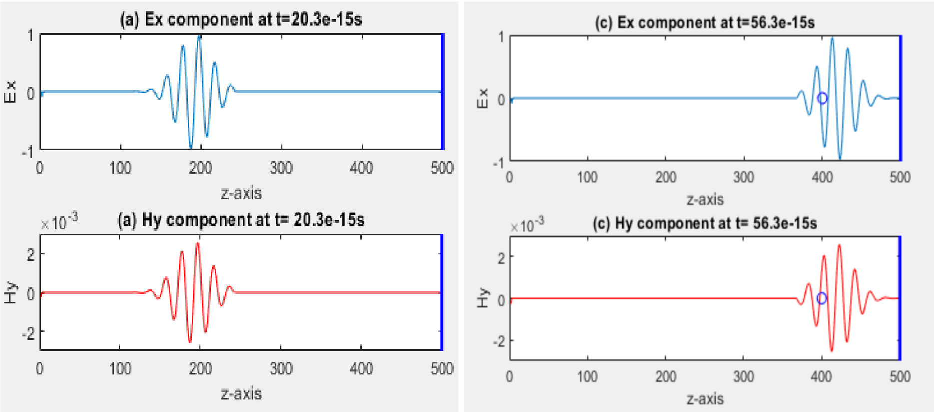

Fig. 3. the Ex and Hy at (a) t=20.3e-15s and (b) t=56.3e-15s.

From the Fig. 3 (b) During the collision, part of the incident pulse reflects off the PMC and interferes with its own trailing tail, this interference happens to be constructive in Hy but destructive in Ex due to the phase difference of π between them. That means that the boundary is acting like an operator for changing the phase but still preserve the magnitude.

3.3. *1st order Mur Boundary*

gives the equation describing the first-order Mur limit In Note [3]. We will update these equations so that they agree with the notation used here. Place mur radiation limit at node 1 or first electric field node on the left

$$E_x^{n+1}(1) = E_x^n(2) - \left(\frac{c\Delta t - \Delta z}{c\Delta t + \Delta z}\right)\left(E_x^{n+1}(2) - E_x^n(1)\right) \tag{17}$$

Place Mur Radiation limit at node M or last electric field node on the right

$$E_x^{n+1}(M) = E_x^n(M-1) - \left(\frac{c\Delta t - \Delta z}{c\Delta t + \Delta z}\right)\left(E_x^{n+1}(M) - E_x^n(M-1)\right) \tag{18}$$

If cΔt-Δx =0, the second term on the right hand side of Equations (17) and (18) will vanish.
Under the condition Δt =Δx/c The absorbing boundary condition for the 1-D case can be therefore expressed by
For z=1

$$E_x^{n+1}(1) = E_x^n(2)$$

For z=M

$$E_x^{n+1}(M) = E_x^n(M-1)$$

which is the same as the result obtained by common sense approach.
Finally, we simulate the equation (19) and (20) using MATLAB

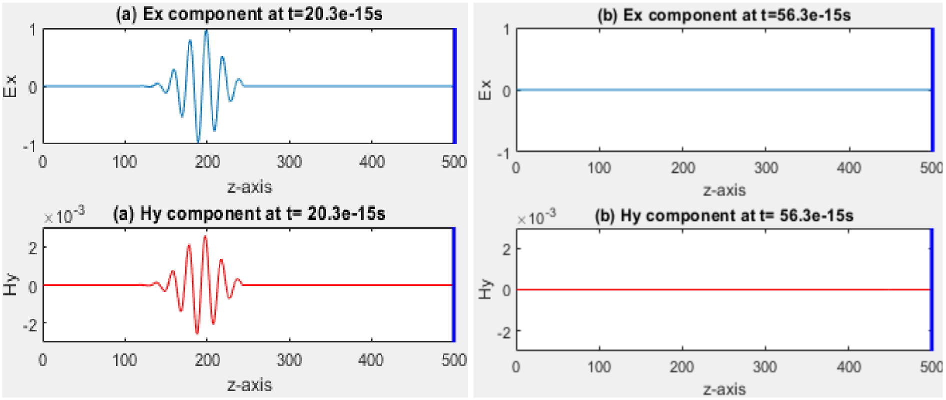

Fig. 4. the Ex and Hy at (a) t=20.3e-15s and (b) t=56.3e-15s

From the figure.4. (b), The wave after collides with the boundary, there are no reflected wave. It is called as ABCs boundary condition which can absorb the magnitude of both TE and TM. So Mur's ABC is a simple absorbing boundary condition that can be used to model free-space boundaries. It approximates an open boundary by setting the fields to zero at the boundary points, preventing reflections

3.4. *Perfect matched layer PML*

The most popular ABC today is the perfectly matched layer (PML) that was introduced by Jean-Pierre Berenger in 1994[6].
the problem statement for the PML is we need a boundary region that provides 100 % transmission so absorbing all waves and 0% reflection for all waves independent of the polarization, incidence angle, and frequency. Specifically,
for normally incident electromagnetic waves, when the following three relations are satisfied,
we can get Γ= 0 : $\mu 1 = \mu 2$ and $\varepsilon 1 = \varepsilon 2$, and

$$\frac{\sigma^*}{\sigma} = \frac{\mu_2}{\varepsilon_2}$$

But how do we include this PML region into our simulation space now we know the parameters that we have to implement .
the answer is we put them into maxwell's equation so we modify maxwell's equations to include the PML conductivities .
we modify Maxwell's equations to include the PML conductivities

$$\nabla \times \vec{E} = -\mu_0 \frac{\delta \vec{H}}{\delta t} + \sigma^* \vec{H} \qquad (19)$$

$$\nabla \times \vec{H} = \varepsilon_0 \frac{\delta \vec{E}}{\delta t} + \sigma \vec{E} \qquad (20)$$

gives the equation describing The polynomial grading of conductivity:

$$\sigma(z) = \sigma_{\max} * \left(\frac{z}{d}\right)^M \qquad (21)$$

Where z is the thickness for nth layer within the PML. Symbol d is the total thickness of the PML, M is the order of the polynomial, σ_max is a maximum conductivity when z=d.
Finally, we simulate the equation (19) and (20) by using PML with d=55 and M=3 and $\sigma max = -(M + 1) * \log(R)/(2 * neta * d * dz)$, where R is required reflectivity
And neta is impedance in free space

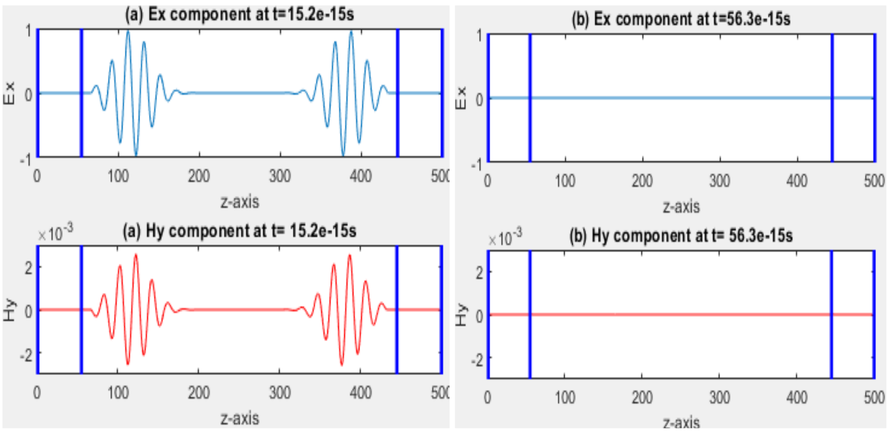

Fig. 5. the Ex and Hy at (a) t=15.2e-15s and (b) t=56.3e-15s

From the figure.5. (b) above, the wave after collide with the boundary, there are no reflected wave it mains 0% reflexion and 100% transmission .so PML is an absorbing boundary condition that effectively absorbs outgoing waves. It is widely used in FDTD simulations to model open boundaries. PML layers are added to the edges of the computational domain, and the fields are attenuated within these layers.

3.5. *Convolutional Perfectly Matched Layer (CPML)*

For the CPML boundary it is specifically designed to improve the absorption of outgoing waves and reduce reflections at the boundaries. And it is an extension of the Perfectly Matched Layer (PML) technique.

The CPML parameters are scaled as follows: kappa =1+(sigma *dt)/(2 * ep)
And alpha= sigma./(kappa* ep) and $\sigma max = -(\mathrm{M} + 1) * \log(\mathrm{R})/(2 * \mathrm{neta} * \mathrm{d} * \mathrm{dz})$.
Where polynomial scaling M= 4. and updating the equations (32) and (33) for auxiliary parameters and electric and magnetic field components.

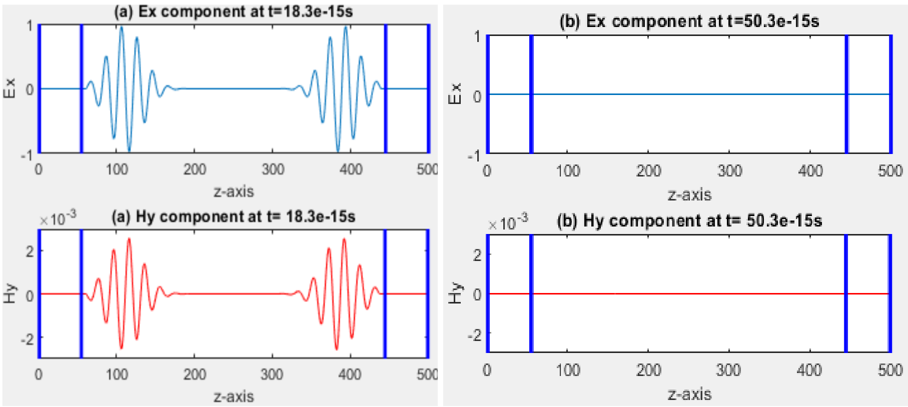

Fig. 6. the Ex and Hy at (a) t=18.3e-15s and (b) t=50.3e-15s .

The results of CPML have shown in Fig. 6. (b) that it has negligible reflection , We see also resulting in faster simulations compared to PML. So CPML's convolutional update reduces the number of calculations required compared to PML, and resulting in faster simulations.

4. Conclusion

The FDTD approach is popular because of its ease of numerical solution. It uses finite differences to discretize Maxwell's time-dependent curl equations in order to solve them. This report outlines the 1-FDTD simulation design for the Ex/Hy mode in the free space area. Additionally, this paper effectively presents a 1D-FDTD code that implements PEC, PMC, and 1st order Mur Boundary and perfect matched layer (PML) and CPML. To this end, the specific choice of boundary condition depends on the nature of the problem being solved and the desired simulation outcomes. For example if we are modeling an interface between two different materials, we may need to use a boundary condition that accurately represents the reflection and transmission of waves at that interface, such as the Perfect Electric Conductor (PEC) or Perfect Magnetic Conductor (PMC) boundary conditions and if we want to minimize reflections, we may consider using the Mur Absorbing Boundary Condition or perfect matched layer PML.

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