Mode Coupling Analysis of Hollow Ring-Core Fibers for OAM Transmission

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Abstract   *We present an analytical and numerical description of coupling between OAM modes in hollow ring-core fibers affected by stress birefringence and ellipticity. The analysis paves the way to a better modeling of propagation in these fibers.*

Introduction

Mode division multiplexing (MDM) can greatly expand the capacity of future optical transmission systems, however, complex receiver-side multiple-input, multiple-output (MIMO) processing is required in most demonstrated linearly polarized MDM systems. Orbital angular momentum (OAM) modes for MDM have the potential for MIMO-free or reduced-MIMO signal processing. In fibers where the effective indices of OAM modes of different order are sufficiently separated, crosstalk among OAM modes is low.[1,2] Air core fibers support a high number of OAM modes due to the large refractive index contrast at the air/glass interface, with air core OAM fibers supporting 12 and 36 OAM modes demonstrated.[3,4] Recently MIMO-free data transmission over a 1.2 km air core fiber supporting 12 OAM modes was demonstrated.[5]

Crosstalk induced by coupling among propagating OAM modes constrains capacity increases. Coupling in propagation could be the result of physical perturbations such as fiber core ellipticity, birefringence, mechanical stress, micro bending, etc. The design and fabrication of fibers with minimum OAM mode coupling requires an understanding of the impact of these coupling mechanisms. Current statistical models describing propagation impairments in non-ideal fibers are limited to the strong mode coupling regime and provide only asymptotic results.[6] A numerical study showed these asymptotic properties will be observed for few mode fiber only after thousands of kilometers.[7] These models are therefore doubly inappropriate for OAM fibers optimized for minimum coupling, and designed for short reach applications. In this paper, we examine mode coupling in hollow ring-core (air core) OAM fibers to develop models for mode propagation. We discuss the interactions among OAM modes, as well as their interactions with parasitic TE and TM modes. In particular, we study the effect of fiber core ellipticity and birefringence, and we discuss coupling among modes of different order due to each of these impairments.

Composition of OAM modes

Propagation in any cylindrical fiber with an arbitrary number of concentric rings of homogeneous lossless material is described by the complete set of hybrid (or vector) modes, $TE_{0,p}$, $TM_{0,p}$, $HE_{\ell,p}$ and $EH_{\ell,p}$, where $\ell \neq 0$ and the arbitrary sign of ℓ corresponds to the 2-fold degeneracy of HE and EH modes.[8] With respect to basis of circular polarizations, the electric field of hybrid modes can be expressed as

$$\begin{aligned} \boldsymbol{E} = &[f_\ell(r) + jg_\ell(r)]e^{j(\ell+1)\varphi}\,\hat{R}+ \\ &[f_\ell(r) - jg_\ell(r)]e^{j(\ell-1)\varphi}\,\hat{L} + jh_\ell(r)e^{j\ell\varphi}\,\hat{z}, \quad (1) \end{aligned}$$

where f_ℓ, g_ℓ and h_ℓ are mode-dependent real functions, $\hat{z} = (0,0,1)$, and $\hat{R} = (1,-j,0)/\sqrt{2}$ and $\hat{L} = (1,j,0)/\sqrt{2}$ represent right- and left-handed circular polarization respectively. This expression shows that the transverse field of hybrid modes is a combination of two OAM "beams" of orders $\ell+1$ and $\ell-1$. In weakly-guiding fibers one of the two "beams" becomes negligible; therefore, hereinafter we call *dominant* the transverse component that survives in the weak guidance limit and *secondary* the transverse component that would not be present in a weakly-guiding fiber, but does exist in OAM fiber with strong index contrast. Note that also the longitudinal component tends to be negligible in the weak guidance limit.

When expressed as in (1), hybrid modes are directly related to OAM modes; so according to the customary definition,[3] we can write

$$OAM^{\pm}_{\pm n,p} \triangleq HE_{\pm(n+1),p} \qquad \text{for} \qquad n \geq 0 \quad (2)$$

$$OAM^{\mp}_{\pm n,p} \triangleq EH_{\pm(n-1),p} \qquad \text{for} \qquad n \geq 2. \quad (3)$$

This definition is based on the dominant component of the field; the secondary component is considered a spurious term of an otherwise pure OAM mode.[9] While TE and TM modes are not

	OAM^+_{+n}	OAM^-_{-n}	OAM^-_{+m}	OAM^+_{-m}	TE	TM
$\hat{R}$	$n+2$	$-n$	m	$-m+2$	1	1
$\hat{L}$	n	$-n-2$	$m-2$	$-m$	-1	-1
$\hat{z}$	$n+1$	$-n-1$	$m-1$	$-m+1$	---	0

Tab. 1: Orders and strength of the components of OAM, TE and TM modes ($n \geq 0$, $m \geq 2$). White, pink and green cells indicate the dominant, secondary and longitudinal components of the field, respectively. Some indices are omitted for brevity.

useful in OAM-multiplexed transmission systems, these modes must be taken into account in the coupled mode analysis as long as they can propagate. It is also worthwhile to identify dominant and secondary components by their azimuthal order; this can be done by comparison with LP modes in the weak guidance limit and the result is summarized in Tab. 1.

Coupled Mode Analysis
According to coupled mode theory,[10] the complex amplitudes of the N propagating modes, represented by the vector c, vary along z as

$$\frac{dc}{dz} = -j(\beta + K)c,\qquad(4)$$

where $\beta = \mathrm{diag}(\beta_1, ..., \beta_N)$ is the diagonal matrix of propagation constants and the $N \times N$ matrix $K(z)$ represents the coupling per unit length. For small perturbation the elements of K are[10]

$$K_{\mu,\nu} = \omega \int_0^\infty \int_0^{2\pi} r E^*_\mu \, \tilde{\epsilon} \, E_\nu \, d\varphi \, dr = K^*_{\nu,\mu}\,,\quad(5)$$

where r and φ are the polar coordinates, $\tilde{\epsilon}$ is the 3×3 matrix of dielectric perturbation, and the 3-dimensional vectors E_μ and E_ν are the electric fields of modes μ and ν, properly normalized.[10]

The complete evaluation of the overlap integral (5) is generally quite involved; however, at least the integration with respect to φ can be performed easily.[11,12] Actually, in almost every case of practical interest the elements of the dielectric perturbation matrix $\tilde{\epsilon}(r,\varphi)$ can be expressed as the product of two functions each depending on either r or φ. Moreover, since $\tilde{\epsilon}(r,\varphi)$ has to be single-valued, the dependence on φ can be expressed by terms like $e^{jk\varphi}$, where $k = 0, \pm 1, \ldots$ are the azimuthal orders of the perturbation. Given that the fields have a similar dependence on φ [see (1)], the integration over φ is nonzero only if specific "resonances" occur.[11] Specifically, let n_μ be the azimuthal order of a component of mode E_μ and similarly let n_ν be the azimuthal order of a component of E_ν. Then, we can say that these two modes can couple through the two respective components only if

$$n_\mu - n_\nu = \pm k,\qquad(6)$$

where the minus sign in the right term stems from the fact that one of the fields in (5) is conjugated.

	Air hole	Ring core	Trench	Cladding
Radii [µm]	≤ 9.1	≤ 11.3	≤ 16.2	≤ 25.0
Refr. Indx.	1.000	1.473	1.438	1.444

Tab. 2: Parameters of the simulated fiber.

Note that condition (6) is *necessary* but *not sufficient* for coupling to occur, because in some cases (5) can be zero after integration over r. Moreover, as clarified in the following, the structure of $\tilde{\epsilon}$ selects which field components can originate the coupling.

We start considering stress birefringence. With respect to the basis of circular polarizations, the corresponding dielectric perturbation is[12]

$$\tilde{\epsilon} \approx \varepsilon_{co} \, \Delta n \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\qquad(7)$$

where ε_{co} is the dielectric permittivity of the core and Δn the birefringence. The azimuthal order is $k = 0$, hence coupling may occur only between components of the same azimuthal order ($n_\mu = n_\nu$) and, given the structure, only between orthogonally polarized transverse components ($\hat{R} \leftrightarrow \hat{L}$). Consulting Tab. 1 we can conclude, for example, that modes $OAM^+_{-m,p}$ and $OAM^-_{m,p}$ cannot couple because of birefringence; differently, coupling may occur between modes $OAM^+_{m,p}$ and $OAM^-_{m,p}$, through their dominant components, and between modes $OAM^+_{m,p}$ and $OAM^-_{m+4,p}$, through the secondary ones.

To verify these conclusions, we numerically evaluated the coupling coefficients (5) for a hollow ring-core fiber supporting (at 1550 nm) 26 information bearing modes and 2 parasitic TE/TM modes;[3] the parameters are given in Tab. 2; field distributions have been calculated with ComSol[TM]. The resulting coupling coefficients for birefringence are reported in Fig. 1, normalized to the largest one; for this case the strongest coupling occurs between modes $OAM^\pm_{\pm 6}$ and $OAM^\mp_{\pm 6}$ and the respective coefficient is about $(2.9 \times 10^6)\Delta n$ rad/m. The dots reported on the figure are the couplings predicted by the above theoretical analysis. As we see there is complete agreement, that is, all non-zero values are anticipated by theory.

In the case of slight ellipticity in a cylindrical fiber made of M concentric homogeneous rings, the dielectric perturbation is given by[12]

$$\tilde{\epsilon} = \varepsilon_0 \left[\sum_{k=1}^{M-1} \gamma_k (n_k^2 - n_{k+1}^2) \delta(r - r_k) \right] \cos 2\varphi\,,\quad(8)$$

where r_k is the ideal outer radius of the kth ring, n_k its refractive index and $\gamma_k \geq 0$ is the maximum radius variation. The dielectric perturbation is scalar, hence coupling can occur only between
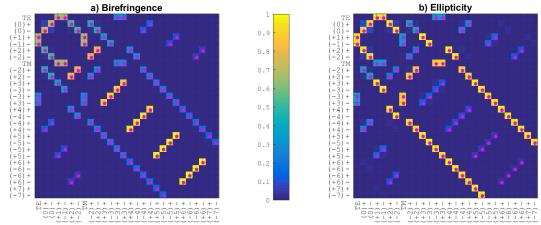
Fig. 1: Normalized magnitudes of coupling coefficients for (a) birefringence and (b) ellipticity, numerically evaluated for the fiber described in Tab. 2. Dots mark coupling predicted by theory. TE/TM modes are of order $(0,1)$; "$(\pm k)s$" indicates $\mathrm{OAM}_{\pm k,1}^{s}$.

equal polarizations of the transverse components ($\hat{R} \leftrightarrow \hat{R}$, $\hat{L} \leftrightarrow \hat{L}$) or between longitudinal components ($\hat{z} \leftrightarrow \hat{z}$), and has azimuthal order $k = 2$ ($n_\mu - n_\nu = \pm 2$). For example, from Tab. 1 we see that these conditions allow coupling between $\mathrm{OAM}_{n,p}^{-}$ and $\mathrm{OAM}_{-m+2,p}^{+}$ only for $n = m = 2$. Most importantly, there is always strong coupling between OAM modes of the same kind, but with orders $\pm n$ and $\pm(n+2)$.

Also in this case we performed the numerical analysis, assuming $\gamma_k = \gamma$ for all interfaces; results are shown in Fig. 1 and are normalized to the coupling coefficients between modes $\mathrm{OAM}_{\pm 4}^{\pm}$ and $\mathrm{OAM}_{\pm 2}^{\pm}$; the respective coefficient is about $(1.25 \times 10^4)\gamma$ rad/m (with γ expressed in meters). As for previous case there is complete agreement with the theoretical analysis.

Quite surprisingly, birefringence and ellipticity give the same coupling although with different strengths. It can be shown that this is a consequence of the fact that both cause coupling only between hybrid modes whose orders ℓ_1 and ℓ_2 are such that $|\ell_1 - \ell_2| = 2$.

Conclusions

The analysis presented here is a preliminary but promising step towards a better description and deeper understanding of coupling mechanism in ring-core fibers for OAM transmissions, such as those observed experimentally between TE/TM and $\mathrm{OAM}_{\pm 1}$.[13] The tables shown in Fig. 1 provide rich quantitative information about coupling mechanisms, some of which, like the critical coupling with TE and TM modes, have not been discussed due to the limited space. This kind of analysis is fundamental for the assessment of the effect of mode coupling, which also depends on the difference between the propagation constants of the modes and the distance of propagation. Further analyses to include these aspects as well as the other sources of coupling (such as bending or twisting) are needed.

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