

MUHAMMAD AL-XORAZMIY
NOMIDAGI TATU FARG'ONA FILIALI
FERGANA BRANCH OF TUIT
NAMED AFTER MUHAMMAD AL-KHORAZMI

“AL-FARG‘ONIIY AVLODLARI”

ELEKTRON ILMIY JURNALI | ELECTRONIC SCIENTIFIC JOURNAL

TA'LIMDAGI ILMIY, OMMABOP VA ILMIY TADQIQOT ISHLARI



4-SON 1(4)
2023-YIL

TATU, FARG'ONA
O'ZBEKISTON



O'ZBEKISTON RESPUBLIKASI RAQAMLI TEXNOLOGIYALAR VAZIRLIGI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI UNIVERSITETI
FARG'ONA FILIALI



Muassis: Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti Farg'ona filiali.

Chop etish tili: O'zbek, ingliz, rus. Jurnal texnika fanlariga ixtisoslashgan bo'lib, barcha shu sohadagi matematika, fizika, axborot texnologiyalari yo'nalishida maqolalar chop etib boradi.

Учредитель: Ферганский филиал Ташкентского университета информационных технологий имени Мухаммада ал-Хоразми.

Язык издания: узбекский, английский, русский. Журнал специализируется на технических науках и публикует статьи в области математики, физики и информационных технологий.

Founder: Fergana branch of the Tashkent University of Information Technologies named after Muhammad al-Khorazmi.

Language of publication: Uzbek, English, Russian. The magazine specializes in technical sciences and publishes articles in the field of mathematics, physics, and information technology.

2023 yil, Tom 1, №4
Vol.1, Iss.4, 2023 y

ELEKTRON ILMIY JURNALI

ELECTRONIC SCIENTIFIC JOURNAL

«Al-Farg'oniylar avlodlari» («The descendants of al-Fargani», «Potomki al-Fargani») O'zbekiston Respublikasi Prezidenti administratsiyasi huzuridagi Axborot va ommaviy kommunikatsiyalar agentligida 2022-yil 21 dekabrda 054493-son bilan ro'yxatdan o'tgan.

Jurnal OAK Rayosatining 2023-yil 30 sentabrdagi 343-sonli qarori bilan Texnika fanlari yo'nalishida milliy nashrlar ro'yxatiga kiritilgan.

Tahririyat manzili:
151100, Farg'ona sh.,
Aeroport ko'chasi 17-uy,
202A-xona
Tel: (+99899) 998-01-42
e-mail: info@al-fargoniy.uz

Qo'lyozmalar taqrizlanmaydi va qaytarilmaydi.

FARG'ONA - 2023 YIL

TAHRIR HAY'ATI

Maxkamov Baxtiyor Shuxratovich,

Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti rektori, iqtisodiyot fanlari doktori, professor

Muxtarov Farrux Muhammadovich,

Muhammad al-Xorazmiy nomidagi Toshkent axborot texnologiyalari universiteti Farg'ona filiali direktori, texnika fanlari doktori

Arjannikov Andrey Vasilevich,

Rossiya Federatsiyasi Sibir davlat universiteti professori, fizika-matematika fanlari doktori

Satibayev Abdugani Djunosovich,

Qirg'iziston Respublikasi, Osh texnologiyalari universiteti, fizika-matematika fanlari doktori, professor

Rasulov Akbarali Maxamatovich,

Muhammad al-Xorazmiy nomidagi TATU Farg'ona filiali Axborot texnologiyalari kafedrasida professori, fizika-matematika fanlari doktori

Yakubov Maksadxon Sultaniyazovich,

Muhammad al-Xorazmiy nomidagi TATU «Axborot texnologiyalari» kafedrasida professori, t.f.d., professor, xalqaro axborotlashtirish fanlari Akademiyasi akademigi

G'ulomov Sherzod Rajaboyevich,

Muhammad al-Xorazmiy nomidagi TATU Kiberxavfsizlik fakulteti dekani, Ph.D., dotsent

G'aniyev Abduxalil Abdjalilovich,

Muhammad al-Xorazmiy nomidagi TATU Kiberxavfsizlik fakulteti, Axborot xavfsizligi kafedrasida t.f.n., dotsent

Zaynidinov Hakimjon Nasritdinovich,

Muhammad al-Xorazmiy nomidagi TATU Kompyuter injiniringi fakulteti, Sun'iy intellekt kafedrasida texnika fanlari doktori, professor

Bo'taboyev Muhammadjon To'ychiyevich,

Farg'ona politexnika instituti, Iqtisod fanlari doktori, professor

Abdullayev Abdujabbor,

Andijon mashinosozlik instituti, Iqtisod fanlari doktori, professor

Qo'ldashev Abbosjon Hakimovich,

O'zbekiston milliy universiteti huzuridagi Yarimo'tkazgichlar fizikasi va mikroelektronika ilmiy-tadqiqot instituti, texnika fanlari doktori, professor

Ergashev Sirojiddin Fayazovich,

Farg'ona politexnika instituti, elektronika va asbobsozlik kafedrasida professori, texnika fanlari doktori, professor

Qoraboyev Muhammadjon Qoraboevich,

Toshkent tibbiyot akademiyasi Farg'ona filiali fizika matematika fanlari doktori, professor, BMT ning maslahatchisi maqomidagi xalqaro axborotlashtirish akademiyasi akademigi

Polvonov Baxtiyor Zaylobiddinovich,

Muhammad al-Xorazmiy nomidagi TATU Farg'ona filiali Ilmiy ishlar va innovatsiyalar bo'yicha direktor o'rinbosari

Zulunov Ravshanbek Mamatovich,

Muhammad al-Xorazmiy nomidagi TATU Farg'ona filiali Dasturiy injiniring kafedrasida dotsenti, fizika-matematika fanlari nomzodi

Saliyev Nabijon,

O'zbekiston jismoniy tarbiya va sport universiteti Farg'ona filiali dotsenti

Abdullaev Temurbek Marufovich,

Muhammad al-Xorazmiy nomidagi TATU Axborot texnologiyalari kafedra mudiri, texnika fanlar bo'yicha falsafa doktori

Zokirov Sanjar Ikromjon o'g'li,

Muhammad al-Xorazmiy nomidagi TATU Farg'ona filiali Ilmiy tadqiqotlar, innovatsiyalar va ilmiy-pedagogik kadrlar tayyorlash bo'limi boshlig'i, fizika-matematika fanlari bo'yicha falsafa doktori

Jurnal quyidagi bazalarda indekslanadi:



Eslatma! Jurnal materiallari to'plamiga kiritilgan ilmiy maqolalardagi raqamlar, ma'lumotlar haqqoniyligiga va keltirilgan iqtiboslar to'g'riligiga mualliflar shaxsan javobgardirlar.

MUNDARIJA | ОГЛАВЛЕНИЕ | TABLE OF CONTENTS

Muxtarov Farrux Muhammadovich, TARMOQ TRAFIGI ANOMALIYALARINI IDENTIFIKATSIYA QILISHNING STATIK USULI	4-7
Daliyev Baxtiyor Sirojiddinovich, Abelning umumlashgan integral tenglamasini yechish uchun Sobolev fazosida optimal kvadratur formulalar	8-14
Umarov Shuxratjon Azizjonovich, KRIPTOBARDOSHLI KRIPTOGRAFIK TIZIMLAR VA ULARNING KLASSIFIKATSIYASI	15-21
Zulunov Ravshanbek Mamatovich, PYTHONDA NEYRON TARMOQNI QURISH VA BASHORAT QILISH	22-26
Djalilov Mamatisa Latibdjanovich, IKKI QATLAMLI NOELASTIK PLASTINKANING KO'NDALANG TEBRANISHI UMUMIY TENGLAMASINI TAHLIL QILISH	27-30
Erkin Uljaev, Azizjon Abdulkhamidov, Utkirjon Ubaydullayev, A Convolutional Neural Network For Classification Cotton Boll Opening Degree	31-36
Seytov Aybek Jumabayevich, Xusanov Azimjon Mamadaliyevich, Magistral kanallarda suv resurslarini boshqarish jarayonlarini modellashtirish algoritmini ishlab chiqish	37-43
Abdullayev Temurbek Marufjonovich, Algorithm of functioning of intellectual information-measuring system	44-49
Odinakhon Sadikovna Rayimjanova, Usmonali Umarovich Iskandarov, Reaserch of highly sensitive deformation semiconductor sensors based on AFV	50-53
S.S.Radjabov, G.R.Mirzayeva, A.O.Tillavoldiyev, J.A.Allayorov, BARG TASVIRI BO'YICHA MADANIY O'SIMLIK LARNING FITOSANITAR HOLATINI ANIQLASH ALGORITMLARI	54-59
Эргашев Отабек Мирзапулатович, Интеллектуальный оптоэлектронный прибор для учета и контроля расходом воды в открытых каналах	60-65
Xomidov Xushnudbek Rapiqjon o'g'li, Nurmatov Sardorbek Xasanboy o'g'li, Yo'ldashev Bilol Iqboljon o'g'li, O'lmasov Farrux Yorqinjon o'g'li, Konus setkali chang tozalovchi qurilma uchun chang namunalarning dispers tarkibi tahlili	66-69
Akhundjanov Umidjon Yunus ugli, VERIFICATION OF STATIC SIGNATURE USING CONVOLUTIONAL NEURAL NETWORK	70-74
Лазарева Марина Викторовна, Горовик Александр Альфредович, Цифровизация и цифровой менеджмент в современном управлении	75-81
D.X.Tojimatov, KIBERTAHDIDLARNI OLDINI OLIHDA KIBERRAZVEDKA AMALIYOTI VA UNING USTUVOR VAZIFALARI	82-85
Muxtarov Farrux Muhammadovich, Rasulov Akbarali Maxamatovich, Ibroximov Nodirbek Ikromjonovich, Kompyuter eksperimenti orqali kam atomli mis klasterlarining geometrik tuzilishini o'rganish	86-89
Umurzakova Dilnoza Maxamadjanovna, BOSHQARISH QONUNLARINI ADAPTATSIYALASH ALGORITMLARINI ISHLAB CHIQLASH	90-94
Muxamedieva Dildora Kabilovna, Muxtarov Farrux Muhammadovich, Sotvoldiev Dilshodbek Marifjonovich, JAMOAT TRANSPORTI MARSHRUTLARINI QURISH INTELLEKTUAL ALGORITMLARI	95-103
Нурдинова Разияхон Абдихаликовна, Перспективы применения элементов с аномальными фотовольтаическими напряжениями	104-108
Bozarov Baxromjon Pخomovich, UCH O'LCHOVLI FAZODAGI SFERADAANIQLANGAN FUNKSIYALARNI TAQRIBIY INTEGRALLASH UCHUN OPTIMAL KUBATUR FORMULALAR	109-113
Улжаев Эркин, Худойбердиев Элёр Фахриддин угли, Нарзуллаев Шохрух Нурали угли, РАЗРАБОТКА КОНСТРУКЦИИ И ФУНКЦИОНАЛЬНОЙ СХЕМЫ ПОЛУЦИЛИНДРИЧЕСКОГО ЁМКОСТНОГО ПОТОЧНОГО ВЛАГОМЕРА	114-122
Mamirov Uktam Farkhodovich, Buronov Bunyod Mamurjon ugli, ALGORITHMS FOR FORMATION OF CONTROL EFFECTS IN CONDITIONS OF UNOBSERVABLE DISTURBANCES	123-127
Sharibayev Nosirjon Yusubjanovich, Jabborov Anvar Mansurjonovich, YURAK-QON TOMIR KASALLIKLARI DIAGNOSTIKASI UCHUN TEXNOLOGIYALAR, ALGORITMLAR VA VOSITALAR	128-136
Marina Lazareva, Estimating development time and complexity of programs	137-141
Asrayev Muhammadmullo, ONLINE HANDWRITING RECOGNITION	142-146
Norinov Muhammadyunus Usibjonovich, SPEKTR ZONALI TASVIRLARGA INTELLEKTUAL ISHLOV BERISH USULLARI TAHLILI	147-152
Xudoynazarov Umidjon Umarjon o'g'li, PARAMETRLI ALGEBRAGA ASOSLANGAN EL-GAMAL SHIFRLASH ALGORITMLARINI GOMOMORFIK XUSUSIYATINI TADQIQ ETISH	153-157
D.M.Okhunov, M.Okhunov, THE ERA OF THE DIGITAL ECONOMY IS AN ERA OF NEW OPPORTUNITIES AND PROSPECTS FOR BUSINESS DEVELOPMENT BASED ON CROWDSOURCING TECHNOLOGIES	158-165

MUNDARIJA | ОГЛАВЛЕНИЕ | TABLE OF CONTENTS

Солиев Бахромжон Набиджонович, Путеводитель по построению веб-API на Django - Шаг за шагом с Django REST framework — от моделей до проверки работоспособности	166-171
Sevinov Jasur Usmonovich, Boborayimov Okhunjon Khushmurod ogli, ALGORITHMS FOR SYNTHESIS OF ADAPTIVE CONTROL SYSTEMS WITH IMPLICIT REFERENCE MODELS BASED ON THE SPEED GRADIENT METHOD	172-176
Mamatov Narzullo Solidjonovich, Jalelova Malika Moyatdin qizi, Tojiboyeva Shaxzoda Xoldorjon qizi, Samijonov Boymirzo Narzullo o'g'li, SUN'IY YO'LDOSHDAN OLINGAN TASVIRDAGI DALA MAYDONI CHEGARALARINI ANIQLASH USULLARI	177-181
Обухов Вадим Анатольевич, Криптография на основе эллиптических кривых (ECC)	182-188
Turdimatov Mamirjon Mirzayevich, Sadirova Xursanoy Xusanboy qizi, AXBOROTNI HIMOYALASHDA CHETLAB O'TISHNING MUMKIN BO'LGAN EHTIMOLLIK XOLATINI BAHOLASH USULLARI	189-193
Musayev Xurshid Sharifjonovich, TRIKOTAJ MAHSULOTLARIDA NUQSONLI TO'QIMALARNING ANIQLASHNING MATEMATIK MODELI VA UNING ALGORITMLARI	194-196
Kodirov Ahkhmadkhon, Umarov Abdumukhtar, Rozaliyev Abdumalikjon, ANALYSIS OF FACIAL RECOGNITION ALGORITHMS IN THE PYTHON PROGRAMMING LANGUAGE	197-205
Suyumov Jorabek Yunusalievich, METHODOLOGICAL PROBLEMS OF QUALIMETRY IN CONDUCT OF PEDAGOGICAL EXPERIMENT-EXAMINATION	206-211
Хаджаев Саидакбар Исмоил угли, АКТУАЛЬНОСТЬ ПРОБЛЕМЫ ЗАЩИТЫ ИНФОРМАЦИОННЫХ СИСТЕМ МАЛОГО И СРЕДНЕГО БИЗНЕСА ОТ КИБЕРАТАК	212-217
M.M.Khalilov, Effect of Heat Treatment on the Photosensitivity of Polycrystalline PbTe Films AND PbS	218-221
Тажибаев Илхом Бахтиёрвич, ПОЛНОСТЬЮ ВОЛОКОННЫЙ СЕНСОР, ОСНОВАННЫЙ НА КОНСТРУКЦИИ ИЗ МАЛОМОДОВОГО ВОЛОКОННОГО СМЕЩЕНИЯ С КАСКАДНЫМ СОЕДИНЕНИЕМ ВОЛОКОННОЙ РЕШЕТКИ С БОЛЬШИМ ИНТЕРВАЛОМ, ИСПОЛЬЗУЕТСЯ ДЛЯ ОПРЕДЕЛЕНИЯ ИСКРИВЛЕНИЯ И ПРОВЕДЕНИЯ АКУСТИЧЕСКИХ ИЗМЕРЕНИЙ	222-225
Sharibaev Nosir Yusubjanovich, Djuraev Sherzod Sobirjanovich, To'xtasinov Davronbek Xoshimjon o'g'li, PRIORITIES IN DETERMINING ELECTRIC MOTOR VIBRATION WITH ADXL345 ACCELEROMETER SENSOR	226-230
Mukhammadjonov A.G., ANALYSIS OF AUTOMATION THROUGH SENSORS OF HEAT AND HUMIDITY OF DIFFERENT DIRECTIONS	231-236
Эрматова Зарина Кахрамоновна, АКТУАЛЬНОСТЬ ПРЕПОДАВАНИЯ ЯЗЫКА ПРОГРАММИРОВАНИЯ C++ В ВЫСШИХ УЧЕБНЫХ ЗАВЕДЕНИЯХ	237-241
Saparbaev Rakhmon, ANALOG TO DIGITAL CONVERSION PROCESS BY MATLAB SIMULINK	242-245
Садикова М.А., Авазова Н.К., САМООБУЧЕНИЕ ИСКУССТВЕННОГО ИНТЕЛЛЕКТА, БАЗОВЫЕ ПРИНЦИПЫ РАБОТЫ ИСКУССТВЕННОГО ИНТЕЛЛЕКТА НА ПРОСТОМ ПРИМЕРЕ	246-250
Abduhafizov Tohirjon Ubaydullo o'g'li, Abdurasulova Dilnoza Botirali kizi, DEVELOPMENT OF ALGORITHMS IN THE ANALYSIS OF DEMAND AND SUPPLY PROCESSES IN ECONOMIC SYSTEMS	251-256
Kayumov Ahror Muminjonovich, CREATING MATHEMATICAL MODELS TO IDENTIFY DEFECTS IN TEXTILE MACHINERY FABRIC	257-261
Mirzakarimov Baxtiyor Abdusalomovich, Xayitov Azizjon Mo'minjon o'g'li, BIOMETRIC METHODS SECURE COMPUTER DATA FROM UNAUTHORIZED ACCESS	262-266
Soliyev B., Odilov A., Abdurasulova Sh., Leveraging Python for Enhanced Excel Functionality: A Practical Exploration	267-271
Жураев Нурмахамад Маматович, Системы Электроснабжения Оборудования Предприятий Связи: Надежность и Эффективность	272-276
Rasulova Feruzaxon Xoshimjon qizi, Isroilov Sharobiddin Mahammadyusufovich, OLIY TA'LIM MUASSASALARIDA MUTAXASSISILIK FANLARINI O'QITISHDA MULTIMEDIALI MOBIL ILOVADANDAN FOYDALANISHNING STATISTIK TAHLILI	277-280
Muxtarov Farrux Muxammadovich, Toshpulatov Sherali Muxamadaliyevich, SUN'IY INTELLEKT YORDAMIDA IJTIMOYIY TARMOQ MONITORINGI TIZIMINI YARATISH, AFZALLIKLARI VA MUHIM JIXATLARI	281-285
Sadikova Munira Alisherovna, APPLICATION OF ARTIFICIAL INTELLIGENCE DEVICES IN MANUFACTURING	286-290
Mamatov Narzullo Solidjonovich, Ibroximov Sanjar Rustam o'g'li, Fayziyev Voxid Orzumurod o'g'li, Samijonov Abdurashid Narzullo o'g'li, SUN'IY INTELLEKT VOSITALARINI TA'LIMNI NAZORAT QILISH VA BAHOLASHDA QO'LLASH	291-297

Abelning umumlashgan integral tenglamasini yechish uchun Sobolev fazosida optimal kvadratur formulalar

Daliyev Baxtiyor Sirojiddinovich

Toshkent axborot texnologiyalari universiteti Farg'ona filiali
e-mail: bahtiyordaliyev@gmail.com

Annotatsiya: Ushbu maqolada Sobolev funksional fazosidagi Abelning umumlashgan integral tenglamasini taqribiy analitik yechish uchun vaznli murakkab optimal kvadratur formula qurilgan. Bu kvadratur formulaning optimal koeffitsientlari topilgan. Bundan tashqari, qurilgan vaznli murakkab optimal kvadratur formula yordamida Abelning umumlashgan integral tenglamasiga doir misollarning sonli natijalari olinib, aniq yechim bilan taqqoslangan.

Kalit so'zlar: Sobolev fazosi, optimal kvadratur formulalar, xatolik funksionali, norma, optimal koeffitsientlar.

Kirish. Umumlashgan Abel integral tenglamasi Volterra birinchi tur chiziqli integral tenglamasining xususiy holi hisoblanadi. Umumlashgan Abel tenglamasi fizika, mexanika va boshqa fanlarning qandaydir konkret masalalariga bevosita olib keladigan integral tenglamalardan biridir. Bugungi kunda tabiatshunoslikning ko'plab sohalarida Abel tipidagi chiziqli integral tenglamalarni yechishga olib keladigan masalalar keng tarqalgan. Abel tipidagi tenglamalarga doimo alohida e'tibor berilgan. Bir qator ishlar ushbu sinf tenglamalari yechimlarining mavjudligi, yagonaligi va turg'unligiga bag'ishlangan [1, 5, 6, 7, 8, 9, 10]. Bugungi kunga kelib, birinchi tur Abel tipidagi integral tenglamalarning sonli yechimlarini olish uchun bir qator yondashuvlar ishlab chiqilgan va keng qo'llanilgan. Turli xil hisoblash algoritmlarining umumiy ko'rinishini, masalan, [1, 2, 3, 4] ishlarda topish mumkin. Ushbu maqolada Sobolev funksional fazosidagi Abelning umumlashgan integral tenglamasini taqribiy analitik yechish uchun vaznli murakkab optimal kvadratur formulalar usulini yaratish bilan shug'ullanamiz. Bundan tashqari, qurilgan vaznli murakkab optimal kvadratur formulalarni kasr integrallarni taqribiy hisoblash uchun qo'llash mumkin. Bizga ma'lumki ushbu

$$f(x) = \int_0^x \frac{\varphi(s) ds}{(x-s)^\alpha}, \quad 0 < \alpha < 1$$

tenglama Abelning umumlashgan tenglamasi deyiladi. Bu yerda $f(x)$ -ma'lum funksiya, $\varphi(s)$ esa noma'lum funksiya.

$L_2^{(1)}(0, t)$ optimal kvadratur formula

Ushbu kvadratur formulani $L_2^{(m)}(0, t)$ Sobolev fazosida qaraymiz

$$\int_0^t \frac{\varphi(x) dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^N \sum_{\nu=0}^p C^{(\nu)}[\beta] \varphi^{(\nu)}[\beta], \quad (1)$$

bu yerda $C^{(\nu)}[\beta]$ - kvadratur formula koeffitsientlari,

$$[\beta] = h\beta, \quad h = \frac{t}{N}, \quad N = 1, 2, \dots, \quad 0 < \alpha < 1, \quad t > 0.$$

$m = 1$ holda (1) kvadratur formula xatolik funksionali normasi ushbu ko'rinishda bo'ladi [11]

$$\|l_N\|_{L_2^{(1)*}}^2 = - \sum_{\beta=0}^N \sum_{\beta'=0}^N C^{(0)}[\beta] C^{(0)}[\beta'] \frac{|h\beta - h\beta'|}{2} + 2 \sum_{\beta=0}^N C^{(0)}[\beta] f_1^{(0)}[\beta] - K_1. \quad (2)$$

Bu yerda

$$f_1^{(0)}[\beta] = \int_0^t \frac{|x - h\beta| dx}{2(t-x)^{1-\alpha}} = \frac{h\beta t^\alpha}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},$$

$$K_1 = \frac{t^{\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}.$$

Bu holda $(l_N(x), x^k) = 0, \quad k = 0, 1, \dots, m-1$ shartlar ko'rinishi quyidagicha



$$\sum_{\beta=0}^N C^{(0)}[\beta] = \frac{t^\alpha}{\alpha} \quad (3)$$

Endi (3) shartlar asosida (2) ni minimumlashtiramiz. Buning uchun Lagranj funksiyasini tuzamiz

$$\Lambda(C^{(0)}[\beta], \lambda_0) = \|l_N | L_2^{(1)*} \|^2 + 2\lambda_0 \left(\sum_{\beta=0}^N C^{(0)}[\beta] - \frac{t^\alpha}{\alpha} \right).$$

$$\frac{\partial \Lambda}{\partial C^{(0)}[\beta]}, \frac{\partial \Lambda}{\partial \lambda_0} \text{ xususiylarini hisoblab va}$$

bu hosilalarni nolga tenglab, $\dot{C}^{(0)}[\beta]$ va $\dot{\lambda}$ noma'lumlarni topish uchun quyidagi sistemani olamiz

$$\sum_{\gamma=0}^N \dot{C}^{(0)}[\gamma] \frac{|h\beta - h\gamma|}{2} + \dot{\lambda}_0 = \frac{h\beta t^\alpha}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \quad (4)$$

$$\sum_{\beta=0}^N \dot{C}^{(0)}[\beta] = \frac{t^\alpha}{\alpha} \quad (5)$$

$h\gamma \notin [0, t]$ da $\dot{C}^{(0)}[\gamma] = 0$ deb olib, ya'ni $\gamma = \dots, -3, -2, -1$ va $\gamma = N+1, N+2, \dots$ (4) va (5) sistemani svyortka tenglama ko'rinishda yozamiz

$$G_1[\beta] * \dot{C}^{(0)}[\beta] + \dot{\lambda}_0 = f_1^{(0)}[\beta], \quad [\beta] \in [0, t] \quad (6)$$

$$\dot{C}^{(0)}[\beta] = 0, \quad [\beta] \notin [0, t] \quad (7)$$

$$\sum_{\beta=0}^N \dot{C}^{(0)}[\beta] = \frac{t^\alpha}{\alpha} \quad (8)$$

Bu yerda

$$G_1[\beta] = \frac{|h\beta|}{2} = \frac{(h\beta)\text{sign}(h\beta)}{2} = \frac{[\beta]\text{sign}[\beta]}{2}, \quad [\beta] = h\beta, \quad (9)$$

$$f_1^{(0)}[\beta] = \frac{t^\alpha[\beta]}{2\alpha} + \frac{(t-[\beta])^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \quad (10)$$

$$h = \frac{1}{N}, \quad N = 1, 2, \dots, \quad \lambda_0 \text{ - noma'lum parametr.}$$

$$U[\beta] = G_1[\beta] * \dot{C}^{(0)}[\beta] + \dot{\lambda}_0 \quad \text{belgilash}$$

kiritamz. (6) dan $[0, t]$ kesmada $U[\beta] = f_1^{(0)}[\beta]$ diskret funksiya ekanligi kelib chiqadi, ya'ni

$$U[\beta] = \frac{t^\alpha[\beta]}{2\alpha} + \frac{(t-[\beta])^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \quad [\beta] \in [0, t].$$

Endi $U[\beta]$ ni $[0, t]$ kesmadan tashqarida aniqlaymiz, ya'ni $h\beta \notin [0, t]$. $h\beta < 0$ bo'lsin yoki $\beta = -1, -2, -3, \dots$, u holda (7) va (9) formulalarga asosan

$$U[\beta] = G_1[\beta] * \dot{C}^{(0)}[\beta] + \dot{\lambda}_0 = -\frac{h\beta}{2} \sum_{\gamma=0}^N \dot{C}^{(0)}[\gamma] + \frac{1}{2} \sum_{\gamma=0}^N \dot{C}^{(0)}[\gamma](h\gamma) + \dot{\lambda}_0.$$

bundan (8) ni hisobga olib, quyidagiga ega bo'lamiz

$$U[\beta] = -\frac{t^\alpha h\beta}{2\alpha} + a_0^-, \quad (11)$$

bu yerda

$$a_0^- = \frac{1}{2} \sum_{\gamma=0}^N \dot{C}^{(0)}[\gamma](h\gamma) + \dot{\lambda}_0. \quad (12)$$

Xuddi shunday $U[\beta]$ ni $h\beta > t$ da topamiz, ya'ni $\beta = N+1, N+2, \dots$.

$$U[\beta] = \frac{t^\alpha h\beta}{2\alpha} + a_0^+, \quad (13)$$

bu yerda

$$a_0^+ = -\frac{1}{2} \sum_{\gamma=0}^N \dot{C}^{(0)}[\gamma](h\gamma) + \dot{\lambda}_0. \quad (14)$$

(14)

(12) va (14) dan



$$\lambda_0 = \frac{a_0^- + a_0^+}{2}. \quad (15)$$

Demak $\beta \in Z$ butun sonlar to'plamida, $U[\beta]$ ni ko'rinishi quyidagicha bo'ladi

$$U[\beta] = \begin{cases} -\frac{t^\alpha h \beta}{2\alpha} + a_0^-, & \beta < 0, \\ \frac{t^\alpha h \beta}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, & \beta = 0, 1, \dots, N, \\ \frac{t^\alpha h \beta}{2\alpha} + a_0^+, & \beta > N. \end{cases} \quad (16)$$

Endi a_0^- va a_0^+ noma'lum koeffitsientlarni aniqlaymiz. Buning uchun ma'lum svyortka operatoridan foydalanamiz [12]

$$D_1[\beta] = \begin{cases} 0, & |\beta| \geq 2, \\ h^{-2}, & |\beta| = 1, \\ -2h^{-2}, & \beta = 0. \end{cases} \quad (17)$$

Bu operator ushbu tenglikni qanoatlantiradi

$$hD_1[\beta] * \frac{|h\beta|}{2} = \delta[\beta], \quad (18)$$

bu yerda $\delta[\beta]$ -diskret delta funksiya

$$\delta[\beta] = \begin{cases} 1, & \beta = 0, \\ 0, & \beta \neq 0. \end{cases} \quad (19)$$

Bundan va $U[\beta]$ ning ta'rifidan

$$\dot{C}^{(0)}[\beta] = hD_1[\beta] * U[\beta], \quad \beta = 0, 1, \dots, N. \quad (20)$$

Biroq $h\beta \notin [0, t]$ bo'lganda $\dot{C}^{(0)}[\beta] = 0$ shartlardan quyidagi kelib chiqadi

$$D_1[\beta] * U[\beta] = 0, \quad h\beta \notin [0, t] \quad (21)$$

(21) da svyortkani hisoblaymiz va (17) dan foydalanib, ushbuni hosil qilamiz

$$D_1[\beta] * U[\beta] = \sum_{\gamma=-\infty}^{\infty} D_1[\beta-\gamma]U[\gamma] = h^{-2}U[\beta-1] - 2h^{-2}U[\beta] + h^{-2}U[\beta+1].$$

Bundan (21) ga asosan $\beta = -1$ va $\beta = N+1$ bo'lganda a_0^- va a_0^+ noma'lum koeffitsientlarni topish uchun quyidagi chiziqli tenglamalar sistemasini hosil qilamiz

$$\begin{cases} U[-2] - 2U[-1] + U[0] = 0, \\ U[N] - 2U[N+1] + U[N+2] = 0. \end{cases} \quad (22)$$

(16) dan

$$U[-2] = \frac{t^\alpha h}{\alpha} + a_0^-, \quad U[-1] = \frac{t^\alpha h}{2\alpha} + a_0^-, \quad U[0] = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},$$

$$U[N] = \frac{t^{\alpha+1}}{2(\alpha+1)}, \quad U[N+1] = \frac{(t+h)t^\alpha}{2\alpha} + a_0^+, \quad U[N+2] = \frac{(t+h)t^\alpha}{2} + a_0^+.$$

U holda (22) sistemaning ko'rinishi quyidagicha bo'ladi

$$\begin{cases} a_0^- - 2a_0^- + \frac{t^{\alpha+1}}{2\alpha(\alpha+1)} = 0, \\ a_0^+ - 2a_0^+ - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)} = 0. \end{cases}$$

Bundan

$$a_0^- = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \quad a_0^+ = -\frac{t^{\alpha+1}}{2\alpha(\alpha+1)}. \quad (23)$$

Bu yerdan va (15) ga asosan

$$\dot{\lambda}_0 = 0. \quad (24)$$

Endi $L_2^{(1)}(0, t)$ fazoda (1) ko'rinishdagi kvadratur formula koeffitsientlarini $\rho = 0$ bo'lganda hisoblashga o'tamiz. $h\beta \in [0, t]$ bo'lganda $\dot{C}^{(0)}[\beta]$ optimal koeffitsientlar quyidagi formula bo'yicha hisoblanadi



$$\overset{\circ}{C}^{(0)}[\beta] = hD_1[\beta] * U[\beta] = h \sum_{\beta=-\infty}^{\infty} D_1[\beta - \gamma] U[\gamma], \text{ at } \beta = 0, 1, \dots, N$$

$\beta = 0$ va $\beta = N$ bo'lganda optimal koefitsientlar alohida hisoblanadi.

$\beta = 0$ bo'lsin, u holda

$$\overset{\circ}{C}^{(0)}[0] = h(D_1[-1]U[-1] + D_1[0]U[0] + D_1[1]U[1]).$$

(16) va (24) dan

$$U[-1] = \frac{t^\alpha h}{2\alpha} + \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, U[0] = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, U[1] = \frac{t^\alpha h}{2\alpha} + \frac{(t-h)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}.$$

Demak

$$\overset{\circ}{C}^{(0)}[0] = \frac{t^\alpha}{\alpha} + \frac{(t-h)^{\alpha+1} - t^{\alpha+1}}{h\alpha(\alpha+1)}.$$

$\beta = N$ bo'lsin, u holda

$$\overset{\circ}{C}^{(0)}[N] = h(D_1[-1]U[N+1] + D_1[0]U[N] + D_1[1]U[N-1]).$$

Shunga o'xshash (16) va (24) dan

$$U[N+1] = \frac{t^\alpha h}{2\alpha} + \frac{t^{\alpha+1}}{2(\alpha+1)}, U[N] = \frac{t^{\alpha+1}}{2(\alpha+1)}, U[N-1] = \frac{t^{\alpha+1}}{2(\alpha+1)} - \frac{t^\alpha h}{2\alpha} + \frac{h^{\alpha+1}}{\alpha(\alpha+1)}.$$

Bundan optimal koefitsientlar ushbu formula bo'yicha hisoblanishini hosil qilamiz

$$\overset{\circ}{C}^{(0)}[N] = \frac{h^\alpha}{\alpha(\alpha+1)}.$$

Endi (1) ko'rinishdagi kvadratur formula koefitsientlarini $\rho = 0$ va $\beta = 1, 2, \dots, N-1$ bo'lganda hisoblashga o'tamiz:

$$\overset{\circ}{C}^{(0)}[\beta] = h \sum_{\gamma=-\infty}^{\infty} D_1[\beta - \gamma] U[\gamma] = h(D_1[-1]U[\beta+1] + D_1[0]U[\beta] + D_1[1]U[\beta-1]).$$

Demak

$$U[\beta] = f_1^{(0)}[\beta] = \frac{h\beta t^\alpha}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \quad \beta = 0, 1, \dots, N,$$

U holda ayrim soddalashtirishlardan so'ng quyidagini hosil qilamiz

$$\overset{\circ}{C}^{(0)}[\beta] = \frac{h^{-1}}{\alpha(\alpha+1)} [(t-h(\beta+1))^{\alpha+1} - 2(t-h\beta)^{\alpha+1} + (t-h(\beta-1))^{\alpha+1}], \quad \beta = 1, 2, \dots, N-1.$$

Shunday qilib biz quyidagi teoremani isbotladik.

1-teorema. $L_2^{(1)}(0, t)$ fazoda (1) ko'rinishdagi optimal kvadratur formulani koefitsientlari $\rho = 0$ bo'lganda quyidagicha aniqlanadi

$$\overset{\circ}{C}^{(0)}[0] = \frac{t^\alpha}{\alpha} + \frac{h^{-1}}{\alpha(\alpha+1)} ((t-h)^{\alpha+1} - t^{\alpha+1}), \quad (25)$$

$$\overset{\circ}{C}^{(0)}[\beta] = \frac{h^{-1}}{\alpha(\alpha+1)} [(t-h(\beta+1))^{\alpha+1} - 2(t-h\beta)^{\alpha+1} + (t-h(\beta-1))^{\alpha+1}], \quad (26)$$

$\beta = 1, 2, \dots, N-1.$

$$\overset{\circ}{C}^{(0)}[N] = \frac{h^\alpha}{\alpha(\alpha+1)}. \quad (27)$$

Yana 1-teoremani quyidagicha ifodalash mumkin.

2-teorema. Sobolevning $L_2^{(1)}(0, t)$ fazosida $\overset{\circ}{C}^{(0)}[\beta]$ koefitsientlari (25)-(27) formulalar bilan aniqlanuvchi ushbu yagona optimal kvadratur formula mavjud

$$\int_0^t \frac{\varphi(x) dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] \varphi[\beta]. \quad (28)$$

(28) optimal kvadratur formulani qurishda, bu formulaning konstantaga aniqligi kerak bo'ladi, ya'ni (5) shartning bajarilishi.

Endi biz (28) optimal kvadratur formulaning x bixadga aniqligini isbotlaymiz.

1-lemma. (28) optimal kvadratur formula x bixadni va konstantani aniq integrallaydi, ya'ni ushbu tengliklar o'rinli



$$\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] = \frac{t^\alpha}{\alpha}, \quad (29)$$

$$\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta](h\beta) = \frac{t^{\alpha+1}}{\alpha(\alpha+1)}. \quad (30)$$

Lemmaning isboti.

(25)-(27) formulalardan foydalanib va $h = \frac{t}{N}$ ni hisobga olib, yig'indini quyidagi ko'rinishga keltiramiz

$$\begin{aligned} \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] &= \frac{h^\alpha}{\alpha(\alpha+1)} [1 + \sum_{\beta=1}^{N-1} ((N-(\beta+1))^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-(\beta-1))^{\alpha+1})] + \\ &+ \frac{t^\alpha}{\alpha} + \frac{h^\alpha (N-1)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^\alpha N}{\alpha(\alpha+1)}. \end{aligned}$$

Yig'indi tartibini o'zgartiramiz

$$\begin{aligned} \sum_{\beta=1}^{N-1} [(N-(\beta+1))^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-(\beta-1))^{\alpha+1}] &= \\ = \sum_{\beta=2}^N (N-\beta)^{\alpha+1} - 2 \sum_{\beta=1}^{N-1} (N-\beta)^{\alpha+1} + \sum_{\beta=0}^{N-2} (N-\beta)^{\alpha+1} &= \\ = N^{\alpha+1} - (N-1)^{\alpha+1} - 1. \end{aligned}$$

U holda

$$\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] = \frac{h^\alpha}{\alpha(\alpha+1)} (N^{\alpha+1} - (N-1)^{\alpha+1}) + \frac{t^\alpha}{\alpha} + \frac{h^\alpha (N-1)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^\alpha N}{\alpha(\alpha+1)} = \frac{t^\alpha}{\alpha}.$$

Bu (29) formulaning to'g'riligini ko'rsatadi. Endi (30) tenglikni isbotlashga o'tamiz. Buning uchun

(25)-(27) formulalardan foydalanib va $h = \frac{t}{N}$ ni hisobga olib, quyidagini hosil qilamiz

$$\begin{aligned} \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta](h\beta) &= h \sum_{\beta=1}^{N-1} \overset{\circ}{C}^{(0)}[\beta] \beta + t \overset{\circ}{C}^{(0)}[N] = \\ &= \frac{t^{\alpha+1}}{N^\alpha \alpha(\alpha+1)} + \frac{t^{\alpha+1}}{N^{\alpha+1} \alpha(\alpha+1)} \left[\sum_{\beta=1}^{N-1} ((N-\beta-1)^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-\beta+1)^{\alpha+1}) \beta \right]. \end{aligned}$$

Ko'rish mumkinki

$$\sum_{\beta=1}^{N-1} ((N-\beta-1)^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-\beta+1)^{\alpha+1}) \beta =$$

$$\begin{aligned} &= \sum_{\beta=2}^N (N-\beta)^{\alpha+1} (\beta-1) - 2 \sum_{\beta=1}^{N-1} (N-\beta)^{\alpha+1} \beta + \sum_{\beta=0}^{N-2} (N-\beta)^{\alpha+1} (\beta+1) = \\ &= \sum_{\beta=2}^N (N-\beta)^{\alpha+1} \beta - 2 \sum_{\beta=1}^{N-1} (N-\beta)^{\alpha+1} \beta + \sum_{\beta=0}^{N-2} (N-\beta)^{\alpha+1} \beta - \\ &- \sum_{\beta=2}^N (N-\beta)^{\alpha+1} + \sum_{\beta=0}^{N-2} (N-\beta)^{\alpha+1} = N^{\alpha+1} - N. \end{aligned}$$

Olingan tenglikka ko'ra

$$\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta](h\beta) = \frac{t^{\alpha+1}}{N^\alpha \alpha(\alpha+1)} + \frac{t^{\alpha+1}}{N^{\alpha+1} \alpha(\alpha+1)} (N^{\alpha+1} - N) = \frac{t^{\alpha+1}}{\alpha(\alpha+1)}.$$

Bu (30) formulaning to'g'riligini ko'rsatadi. Biz 1-lemmani to'liq isbotladik.

3-teorema. $L_2^{(1)}(0, t)$ fazoda (28) o'timaal kvadratur formulaning xatolik funksionali normasining kvadrati ushbu tenglik bilan aniqlanadi

$$\|\ell_N | L_2^{(1)*} \|^2 = \frac{1}{\alpha(\alpha+1)} \left[\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] (t-h\beta)^{\alpha+1} - \frac{t^{2\alpha+1}}{2\alpha+1} \right].$$

Bu yerda $\overset{\circ}{C}^{(0)}[\beta]$ (25)-(27) formulalar bilan aniqlanadi.

Isboti.

(24) ga asosan (4) ni quyidagicha yozamiz

$$\sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] \frac{|h\beta - h\beta'|}{2} = f_1^{(0)}[\beta], \quad \beta = 0, 1, \dots, N,$$

(31)

bu yerda

$$f_1^{(0)}[\beta] = \frac{t^\alpha h \beta}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}.$$

U holda normaning kvadrati (2) ifoda uchun quyidagiga egamiz



$$\begin{aligned} \|\ell_N | L_2^{(1)*} \|^2 &= -\sum_{\beta'=0}^N \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta'] \overset{\circ}{C}^{(0)}[\beta] \frac{|h\beta - h\beta'|}{2} + \\ &+ 2 \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] f_1^{(0)}[\beta] - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)} = \\ &= \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] f_1^{(0)}[\beta] - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)} = \\ &= \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] \left(\frac{t^\alpha h\beta}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)} \right) - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}. \end{aligned}$$

Bundan (29) va (30) ga asosan, quyidagini hosil qilamiz

$$\|\ell_N | L_2^{(1)*} \|^2 = \sum_{\beta=0}^N \overset{\circ}{C}^{(0)}[\beta] \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}.$$

Bu esa 3-teoremani isboti.

Sonli natijalar

1-misol. Quyidagi umumlashgan Abel integral tenglamasini yeching

$$\frac{128}{231} x^{\frac{11}{4}} = \int_0^x \frac{1}{(x-t)^{\frac{1}{4}}} \varphi(t) dt.$$

Bu yerda $\alpha = \frac{1}{4}, f(x) = \frac{128}{231} x^{\frac{11}{4}}$.

Mahlumki, yechim $\varphi(x) = x^2$ ko'rinishda bo'ladi.

$m=1$ da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

t_i	$N=1$	$N=10$	$N=100$	Aniq yechim	Xatolik $\Delta_{N=100}$
0.1	0.01097528461	0.01001266007	0.01000014441	0.009999999996	1.44(-7)

0.2	0.04390113845	0.04005064032	0.04000057578	0.03999999998	5.75(-7)
0.3	0.09877756149	0.09011394066	0.09000129110	0.08999999995	1.29(-6)

Jadvallardan ko'rish mumkinki, taqribiy yechim $x = 0.1, 0.2$ va 0.3 lar uchun mos ravishda maksimal sonli $1.44(-7), 5.75(-7)$ va $1.29(-6)$ xatoliklar bilan olingan.

2-misol. Quyidagi umumlashgan Abel integral tenglamasini yeching

$$\frac{432}{935} x^{\frac{17}{6}} = \int_0^x \frac{1}{(x-t)^{\frac{1}{6}}} \varphi(t) dt.$$

Bu yerda $\alpha = \frac{1}{6}, f(x) = \frac{432}{935} x^{\frac{17}{6}}$.

Mahlumki, yechim $\varphi(x) = x^2$ ko'rinishda bo'ladi.

$m=1$ da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

t_i	$N=1$	$N=10$	$N=100$	Aniq yechim	Xatolik $\Delta_{N=100}$
0.1	0.01071505487	0.01001008169	0.01000012197	0.01000000000	1.2(-7)
0.2	0.04286021946	0.04004032676	0.04000048652	0.04000000001	4.86(-7)
0.3	0.09643549377	0.09009073517	0.09000109981	0.08999999998	1.09(-6)

Bu yerda taqribiy yechim $x = 0.1, 0.2$ va 0.3 lar uchun mos ravishda maksimal sonli $1.2(-7), 4.86(-7)$ va $1.09(-6)$ xatoliklar bilan olingan.

3-misol. Quyidagi Abel integral tenglamasini yeching

$$e^x - 1 = \int_0^x \frac{1}{(x-t)^{\frac{1}{2}}} \varphi(t) dt.$$

Bu yerda $\alpha = \frac{1}{2}, f(x) = e^x - 1$.



Ma'lumki, yechim

$$\varphi(x) = \frac{e^x}{\sqrt{\pi}} \operatorname{erf}(\sqrt{x}), \quad \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t} dt$$

ko'rinishda bo'ladi.

$m=1$ da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

t_i	$N=1$	$N=10$	$N=100$	Aniq yechim	Xatolik $\Delta_{N=100}$
0.1	.21543196 69327	.215292176 4170	.215290519 7120	.215290502 1493	1.76(-8)
0.2	.32672800 14382	.325894187 8189	.325884182 5865	.325884076 3232	1.062(-7)
0.3	.43001942 38321	.427595430 2070	.427565971 0321	.427565657 5623	3.13(-7)

Bu yerda ham taqribiy yechim $x = 0.1, 0.2$ va 0.3 lar uchun mos ravishda maksimal sonli 1.76(-8), 1.062(-7) i 3.13(-7) xatoliklar bilan olingan.

Xulosa. Abel tipidagi singulyar integrallarni taqribiy hisoblash uchun murakkab optimal kvadratur formulari aniq qurilgan. Bu yerda $L_2^{(1)}(0, t)$ fazoda murakkab kvadratur formulaning optimal koeffitsientlari ham to'ilgan. Keyin, $L_2^{(1)}(0, t)$ fazoda optimal kvadratur formulalarning xatolik funksionali normasining kvadrati hisoblangan.

Sobolev fazosida Abel tipidagi singulyar integrallar uchun qurilgan yangi optimal kvadratur formulalarning sonli yaqinlashishini tasdiqlovchi sonli natijalar keltirilgan. Olingan sonli natijalar aniq natijalar bilan taqqoslangan.

Adabiyotlar

- Gorenflo R., Vessella S. Abe lintegral equations. Analysis and applications. — Berlin: Springer-Verlag, 1991.
- Anderssen R. S. Application and numerical solution of Abel-type integral equation // Technical Summary Report № 1787, September 1977.—

Madison: University of Wisconsin-Madison, Mathematics Research Center.

3. Преображенский Н. Г., Пикалов В. В. Неустойчивые задачи диагностики плазмы. — Новосибирск: Наука, 1982.

4. Воскобойников Ю. Е., Преображенский Н. Г., Седельников А. И. Математическая обработка эксперимента в молекулярной газодинамике. — Новосибирск: Наука, 1984.

5. Бухгейм А. Л. "Уравнения Вольтерра и обратные задачи. — Новосибирск: Наука, 1983.

6. Vessella S. Stability results for Abel equation // Journal of Integral Equations. 1985. - Vol. 9. - P. 125-135.

7. Грынъ В. И. О существовании, единственности и устойчивости решений уравнения Абеля // Рукопись деп. в ВИНТИ 09.06.1995, № 1715-B95.

8. Gorenflo R., Yamamoto M. Operator Theoretic Treatment of Linear Abel Integral Equations of First Kind // Japan Journal of Industrial and Applied Mathematics.-1999.-Vol. 16.-P. 137-161.

9. Baker C.T. H. A perspective on the numerical treatment of Volterra equations // Journal of Computational and Applied Mathematics. — 2000. — Vol. 125.-P. 217-249.

10. Minerbo G. N., Levy M. E. Inversion on Abel's integral equation by means of orthogonal polynomials // SIAM Journal on Numerical Analysis. — 1969. — Vol. 6, JV* 4.-P. 598-616.

11. Шадиметов Х.М., Далиев Б.С. Экстремальная функция квадратурных формул для приближённого решения обобщённого интегрального уравнения Абеля // Проблемы вычислительной и прикладной математики. — Ташкент, 2019. - № 2. -С. 88-96.

12. K. M. Shadimetov, A. R. Hayotov, and F. A. Nuraliev, "Optimal interpolation formulas with derivative in the space $L_2^{(m)}(0,1)$," Published by Faculty of Sciences and Mathematics, University of Nis, Serbia , 5661–5675 (2019).

