MUHAMMAD AL-XORAZMIY
NOMIDAGI TATU FARG'ONA FILIALI

FERGANA BRANCH OF TUIT
NAMED AFTER MUHAMMAD AL-KHORAZMI

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## MUNDARIJA | ОГЛАВЛЕНИЕ | TABLE OF CONTENTS

# Abelning umumlashgan integral tenglamasini yechish uchun Sobolev fazosida optimal kvadratur formulalar

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**Annotatsiya:** Ushbu maqolada Sobolev funksional fazosidagi Abelning umumlashgan integral tenglamasini taqribiy analitik yechish uchun vaznli murakkab optimal kvadratur formula qurilgan. Bu kvadratur formulaning optimal koeffitsientlari topilgan. Bundan tashqari, qurilgan vaznli murakkab optimal kvadratur formula yordamida Abelning umumlashgan integral tenglamasiga doir misollarning sonli natijalari olinib, aniq yechim bilan taqqoslangan.

**Kalit so‘zlar:** Sobolev fazosi, optimal kvadratur formulalar, xatolik funksionali, norma, optimal koeffitsientlar.

**Kirish.** Umumlashgan Abel integral tenglamasi Volterra birinchi tur chiziqli integral tenglamasining xususiy holi hisoblanadi. Umumlashgan Abel tenglamasi fizika, mexanika va boshqa fanlarning qandaydir konkret masalalariga bevosita olib keladigan integral tenglamalardan biridir. Bugungi kunda tabiatshunoslikning ko’plab sohalarida Abel tipidagi chiziqli integral tenglamalarni yechishga olib keladigan masalalar keng tarqalgan. Abel tipidagi tenglamalarga doimo alohida e‘tibor berilgan. Bir qator ishlar ushbu sinf tenglamalari yechimlarining mavjudligi, yagonaligi va turg’unligiga bag’ishlangan [1, 5, 6, 7, 8, 9, 10]. Bugungi kunga kelib, birinchi tur Abel tipidagi integral tenglamalarning sonli yechimlarini olish uchun bir qator yondashuvlar ishlab chiqilgan va keng qo’llanilgan. Turli xil hisoblash algoritmlarining umumiy ko’rinishini, masalan, [1, 2, 3, 4] ishlarda topish mumkin. Ushbu maqolada Sobolev funksional fazosidagi Abelning umumlashgan integral tenglamasini taqribiy analitik yechish uchun vaznli murakkab optimal kvadratur formulalar usulini yaratish bilan shug’ullanamiz. Bundan tashqari, qurilgan vaznli murakkab optimal kvadratur formulalarni kasr integrallarni taqribiy hisoblash uchun qo’llash mumkin. Bizga ma’lumki ushbu

$$f(x)=\int_0^x \frac{\varphi(s)ds}{(x-s)^{\alpha}}, \qquad 0<\alpha<1$$

tenglama Abelning umumlashgan tenglamasi deyiladi. Bu yerda $f(x)$ -ma’lum funksiya, $\varphi(s)$ esa noma’lum funksiya.

## $L_2^{(1)}(0,t)$ optimal kvadratur formula

Ushbu kvadratur formulani $L_2^{(m)}(0,t)$ Sobolev fazosida qaraymiz

$$\int_0^t \frac{\varphi(x)dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^{N}\sum_{\nu=0}^{\rho} C^{(\nu)}[\beta]\varphi^{(\nu)}[\beta] \quad , \quad (1)$$

bu yerda $C^{(\nu)}[\beta]$- kvadratur formula koeffitsientlari,

$[\beta]=h\beta,\ h=\frac{t}{N},\ N=1,2,...,\ 0<\alpha<1,\ t>0.$

$m=1$ holda (1) kvadratur formula xatolik funksionali normasi ushbu ko’rinishda bo’ladi [11]

$$\| l_N \mid L_2^{(1)*} \|^2 = -\sum_{\beta=0}^{N}\sum_{\beta'=0}^{N} C^{(0)}[\beta]C^{(0)}[\beta']\frac{|h\beta-h\beta'|}{2} + 2\sum_{\beta=0}^{N} C^{(0)}[\beta]f_1^{(0)}[\beta] - K_1. \quad (2)$$

Bu yerda

$$f_1^{(0)}[\beta]=\int_0^t \frac{|x-h\beta|\,dx}{2(t-x)^{1-\alpha}} = \frac{h\beta t^{\alpha}}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},$$

$$K_1 = \frac{t^{\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}.$$

Bu holda $\left(l_N(x),x^k\right)=0,\ k=0,1,...,m-1$ shartlar ko’rinishi quyidagicha

$$\sum_{\beta=0}^{N} C^{(0)}[\beta] = \frac{t^\alpha}{\alpha}. \qquad (3)$$

Endi (3) shartlar asosida (2) ni minimumlashtiramiz. Buning uchun Lagranj funksiyasini tuzamiz

$$\Lambda\left(C^{(0)}[\beta], \lambda_0\right) = \| l_N \,|\, L_2^{(1)*} \|^2 + 2\lambda_0 \left( \sum_{\beta=0}^{N} C^{(0)}[\beta] - \frac{t^\alpha}{\alpha} \right).$$

$\frac{\partial \Lambda}{\partial C^{(0)}[\beta]}$, $\frac{\partial \Lambda}{\partial \lambda_0}$ xususiy hosilalarni hisoblab va bu hosilalarni nolga tenglab, $\overset{\circ}{C}{}^{(0)}[\beta]$ va $\overset{\circ}{\lambda}$ noma’lumlarni topish uchun quyidagi sistemani olamiz

$$\sum_{\gamma=0}^{N} \overset{\circ}{C}{}^{(0)}[\gamma] \frac{|h\beta - h\gamma|}{2} + \overset{\circ}{\lambda}_0 = \frac{h\beta t^\alpha}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \qquad (4)$$

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] = \frac{t^\alpha}{\alpha}. \qquad (5)$$

$h\gamma \notin [0,t]$ da $\overset{\circ}{C}{}^{(0)}[\gamma] = 0$ deb olib, ya’ni $\gamma = \dots, -3, -2, -1$ va $\gamma = N+1, N+2, \dots$ (4) va (5) sistemani svyortka tenglama ko’rinishda yozamiz

$$G_1[\beta] * \overset{\circ}{C}{}^{(0)}[\beta] + \overset{\circ}{\lambda}_0 = f_1^{(0)}[\beta], \ [\beta] \in [0,t] \qquad (6)$$

$$\overset{\circ}{C}{}^{(0)}[\beta] = 0, \ [\beta] \notin [0,t] \qquad (7)$$

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] = \frac{t^\alpha}{\alpha}. \qquad (8)$$

Bu yerda

$$G_1[\beta] = \frac{|h\beta|}{2} = \frac{(h\beta) sign(h\beta)}{2} = \frac{[\beta] sign[\beta]}{2}, [\beta] = h\beta, \qquad (9)$$

$$f_1^{(0)}[\beta] = \frac{t^\alpha[\beta]}{2\alpha} + \frac{(t-[\beta])^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \qquad (10)$$

$h = \frac{1}{N}$, $N = 1, 2, \dots$, $\lambda_0$ - noma’lum parametr.

$U[\beta] = G_1[\beta] * \overset{\circ}{C}{}^{(0)}[\beta] + \overset{\circ}{\lambda}_0$ belgilash kiritamz. (6) dan $[0,t]$ kesmada $U[\beta] = f_1^{(0)}[\beta]$ diskret funksiya ekanligi kelib chiqadi, ya’ni

$$U[\beta] = \frac{t^\alpha[\beta]}{2\alpha} + \frac{(t-[\beta])^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \ [\beta] \in [0,t].$$

Endi $U[\beta]$ ni $[0,t]$ kesmadan tashqarida aniqlaymiz, ya’ni $h\beta \notin [0,t]$. $h\beta < 0$ bo’lsin yoki $\beta = -1, -2, -3, \dots$, u holda (7) va (9) formulalarga asosan

$$U[\beta] = G_1[\beta] * \overset{\circ}{C}{}^{(0)}[\beta] + \overset{\circ}{\lambda}_0 = -\frac{h\beta}{2} \sum_{\gamma=0}^{N} \overset{\circ}{C}{}^{(0)}[\gamma] + \frac{1}{2} \sum_{\gamma=0}^{N} \overset{\circ}{C}{}^{(0)}[\gamma](h\gamma) + \overset{\circ}{\lambda}_0.$$

bundan (8) ni hisobga olib, quyidagiga ega bo’lamiz

$$U[\beta] = -\frac{t^\alpha h\beta}{2\alpha} + a_0^-, \qquad (11)$$

bu yerda

$$a_0^- = \frac{1}{2} \sum_{\gamma=0}^{N} \overset{\circ}{C}{}^{(0)}[\gamma](h\gamma) + \overset{\circ}{\lambda}_0. \qquad (12)$$

Xuddi shunday $U[\beta]$ ni $h\beta > t$ da topamiz, ya’ni $\beta = N+1, N+2, \dots$.

$$U[\beta] = \frac{t^\alpha h\beta}{2\alpha} + a_0^+, \qquad (13)$$

bu yerda

$$a_0^+ = -\frac{1}{2} \sum_{\gamma=0}^{N} \overset{\circ}{C}{}^{(0)}[\gamma](h\gamma) + \overset{\circ}{\lambda}_0. \qquad (14)$$

(12) va (14) dan

$$\lambda_0 = \frac{a_0^- + a_0^+}{2}. \quad (15)$$

Demak $\beta \in Z$ butun sonlar to'plamida, $U[\beta]$ ni ko'rinishi quyidagicha bo'ladi

$$U[\beta] = \begin{cases} -\dfrac{t^\alpha h\beta}{2\alpha} + a_0^-, & \beta < 0, \\ \dfrac{t^\alpha h\beta}{2\alpha} + \dfrac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \dfrac{t^{\alpha+1}}{2\alpha(\alpha+1)}, & \beta = 0,1,...,N, \\ \dfrac{t^\alpha h\beta}{2\alpha} + a_0^+, & \beta > N. \end{cases}$$
(16)

Endi $a_0^-$ va $a_0^+$ noma'lum koeffitsientlarni aniqlaymiz. Buning uchun ma'lum svyortka operatoridan foydalanamiz [12]

$$D_1[\beta] = \begin{cases} 0, & |\beta| \ge 2, \\ h^{-2}, & |\beta| = 1, \\ -2h^{-2}, & \beta = 0. \end{cases}$$
(17)

Bu operator ushbu tenglikni qanoatlantiradi

$$hD_1[\beta] * \frac{|h\beta|}{2} = \delta[\beta], \quad (18)$$

bu yerda $\delta[\beta]$ -diskret delta funksiya

$$\delta[\beta] = \begin{cases} 1, & \beta = 0, \\ 0, & \beta \ne 0. \end{cases} \quad (19)$$

Bundan va $U[\beta]$ ning ta'rifidan

$$\overset{\circ}{C}{}^{(0)}[\beta] = hD_1[\beta] * U[\beta], \ \beta = 0,1,...,N. \quad (20)$$

Biroq $h\beta \notin [0,t]$ bo'lganda $\overset{\circ}{C}{}^{(0)}[\beta] = 0$ shartlardan quyidagi kelib chiqadi

$$D_1[\beta] * U[\beta] = 0, \ \ h\beta \notin [0,t] \quad (21)$$

(21) da svyortkani hisoblaymiz va (17) dan foydalanib, ushbuni hosil qilamiz

$$D_1[\beta] * U[\beta] = \sum_{\gamma=-\infty}^{\infty} D_1[\beta-\gamma]U[\gamma] = h^{-2}U[\beta-1] - 2h^{-2}U[\beta] + h^{-2}U[\beta+1].$$

Bundan (21) ga asosan $\beta = -1$ va $\beta = N+1$ bo'lganda $a_0^-$ va $a_0^+$ noma'lum koeffitsientlarni topish uchun quydagi chiziqli tenglamalar sistemasini hosil qilamiz

$$\begin{cases} U[-2] - 2U[-1] + U[0] = 0, \\ U[N] - 2U[N+1] + U[N+2] = 0. \end{cases}$$
(22)

(16) dan

$$U[-2] = \frac{t^\alpha h}{\alpha} + a_0^-, \ U[-1] = \frac{t^\alpha h}{2\alpha} + a_0^-, \ U[0] = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},$$

$$U[N] = \frac{t^{\alpha+1}}{2(\alpha+1)}, \ U[N+1] = \frac{(t+h)t^\alpha}{2\alpha} + a_0^+, \ U[N+2] = (\frac{t}{2}+h)\frac{t^\alpha}{\alpha} + a_0^+.$$

U holda (22) sistemaning ko'rinishi quyidagicha bo'ladi

$$\begin{cases} a_0^- - 2a_0^- + \dfrac{t^{\alpha+1}}{2\alpha(\alpha+1)} = 0, \\ a_0^+ - 2a_0^+ - \dfrac{t^{\alpha+1}}{2\alpha(\alpha+1)} = 0. \end{cases}$$

Bundan

$$a_0^- = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}, \ a_0^+ = -\frac{t^{\alpha+1}}{2\alpha(\alpha+1)}.$$
(23)

Bu yerdan va (15) ga asosan

$$\overset{\circ}{\lambda}_0 = 0. \quad (24)$$

Endi $L_2^{(1)}(0,t)$ fazoda (1) ko'rinishdagi kvadratur formula koeffitsientlarini $\rho = 0$ bo'lganda hisoblashga o'tamiz. $h\beta \in [0,t]$ bo'lganda $\overset{\circ}{C}{}^{(0)}[\beta]$ optimal koeffitsientlar quyidagi formula bo'yicha hisoblanadi

$$\mathring{C}^{(0)}[\beta] = hD_1[\beta]*U[\beta] = h\sum_{\beta=-\infty}^{\infty} D_1[\beta-\gamma]U[\gamma],\ \ at\ \ \beta = 0,1,...,N$$

$\beta = 0$ va $\beta = N$ bo’lganda optimal koeffitsientlar alohida hisoblanadi.

$\beta = 0$ bo’lsin, u holda

$$\mathring{C}^{(0)}[0] = h\left(D_1[-1]U[-1] + D_1[0]U[0] + D_1[1]U[1]\right).$$

(16) va (24) dan

$$U[-1] = \frac{t^\alpha h}{2\alpha} + \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},\ U[0] = \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},\ U[1] = \frac{t^\alpha h}{2\alpha} + \frac{(t-h)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}.$$

Demak

$$\mathring{C}^{(0)}[0] = \frac{t^\alpha}{\alpha} + \frac{(t-h)^{\alpha+1} - t^{\alpha+1}}{h\alpha(\alpha+1)}.$$

$\beta = N$ bo’lsin, u holda

$$\mathring{C}^{(0)}[N] = h\left(D_1[-1]U[N+1] + D_1[0]U[N] + D_1[1]U[N-1]\right).$$

Shunga o’xshash (16) va (24) dan

$$U[N+1] = \frac{t^\alpha h}{2\alpha} + \frac{t^{\alpha+1}}{2(\alpha+1)},\ U[N] = \frac{t^{\alpha+1}}{2(\alpha+1)},\ U[N-1] = \frac{t^{\alpha+1}}{2(\alpha+1)} - \frac{t^\alpha h}{2\alpha} + \frac{h^{\alpha+1}}{\alpha(\alpha+1)}.$$

Bundan optimal koeffitsientlar ushbu formula bo’yicha hisoblanishini hosil qilamiz

$$\mathring{C}^{(0)}[N] = \frac{h^\alpha}{\alpha(\alpha+1)}.$$

Endi (1) ko’rinishdagi kvadratur formula koeffitsientlarini $\rho = 0$ va $\beta = 1,2,...,N-1$ bo’lganda hisoblashga o’tamiz:

$$\mathring{C}^{(0)}[\beta] = h\sum_{\gamma=-\infty}^{\infty} D_1[\beta-\gamma]U[\gamma] = h\left(D_1[-1]U[\beta+1] + D_1[0]U[\beta] + D_1[1]U[\beta-1]\right).$$

Demak

$$U[\beta] = f_1^{(0)}[\beta] = \frac{h\beta t^\alpha}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)},\quad \beta = 0,1,...,N,$$

U holda ayrim soddalashtirishlardan so’ng quyidagini hosil qilamiz

$$\mathring{C}^{(0)}[\beta] = \frac{h^{-1}}{\alpha(\alpha+1)}\left[(t-h(\beta+1))^{\alpha+1} - 2(t-h\beta)^{\alpha+1} + (t-h(\beta-1))^{\alpha+1}\right],\quad \beta = 1,2,...,N-1.$$

Shunday qilib biz quyidagi teoremani isbotladik.

*1-teorema.* $L_2^{(1)}(0,t)$ fazoda (1) ko’rinishdagi optimal kvadratur formulani koeffitsientlari $\rho = 0$ bo’lganda quyidagicha aniqlanadi

$$\mathring{C}^{(0)}[0] = \frac{t^\alpha}{\alpha} + \frac{h^{-1}}{\alpha(\alpha+1)}\left((t-h)^{\alpha+1} - t^{\alpha+1}\right), \tag{25}$$

$$\mathring{C}^{(0)}[\beta] = \frac{h^{-1}}{\alpha(\alpha+1)}\left[(t-h(\beta+1))^{\alpha+1} - 2(t-h\beta)^{\alpha+1} + (t-h(\beta-1))^{\alpha+1}\right], \tag{26}$$

$\beta = 1,2,...,N-1.$

$$\mathring{C}^{(0)}[N] = \frac{h^\alpha}{\alpha(\alpha+1)}. \tag{27}$$

Yana 1-teoremani quydagicha ifodalash mumkin.

*2-teorema.* Sobolevning $L_2^{(1)}(0,t)$ fazosida $\mathring{C}^{(0)}[\beta]$ koeffitsientlari (25)-(27) formulalar bilan aniqlanuvchi ushbu yagona optimal kvadratur formula mavjud

$$\int_0^t \frac{\varphi(x)dx}{(t-x)^{1-\alpha}} \cong \sum_{\beta=0}^{N} \mathring{C}^{(0)}[\beta]\varphi[\beta]. \tag{28}$$

(28) optimal kvadratur formulani qurishda, bu formulaning konstantaga aniqligi kerak bo’ladi, ya’ni (5) shartning bajarilishi.

Endi biz (28) optimal kvadratur formulaning $x$ birxadga aniqligini isbotlaymiz.

*1-lemma.* (28) optimal kvadratur formula $x$ birxadni va konstantani aniq integrallaydi, ya’ni ushbu tengliklar o’rinli

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] = \frac{t^{\alpha}}{\alpha}, \qquad (29)$$

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta](h\beta) = \frac{t^{\alpha+1}}{\alpha(\alpha+1)}. \qquad (30)$$

*Lemmaning isboti.*

(25)-(27) formulalardan foydalanib va $h = \frac{t}{N}$ ni hisobga olib, yig’indini quyidagi ko’rinishga keltiramiz

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] = \frac{h^{\alpha}}{\alpha(\alpha+1)}[1+\sum_{\beta=1}^{N-1}((N-(\beta+1))^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-(\beta-1))^{\alpha+1})] +$$

$$+\frac{t^{\alpha}}{\alpha} + \frac{h^{\alpha}(N-1)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha}N}{\alpha(\alpha+1)}.$$

Yig’indi tartibini o’zgartiramiz

$$\sum_{\beta=1}^{N-1}\left[(N-(\beta+1))^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-(\beta-1))^{\alpha+1}\right] =$$

$$= \sum_{\beta=2}^{N}(N-\beta)^{\alpha+1} - 2\sum_{\beta=1}^{N-1}(N-\beta)^{\alpha+1} + \sum_{\beta=0}^{N-2}(N-\beta)^{\alpha+1} =$$

$$= N^{\alpha+1} - (N-1)^{\alpha+1} - 1.$$

U holda

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] = \frac{h^{\alpha}}{\alpha(\alpha+1)}\left(N^{\alpha+1} - (N-1)^{\alpha+1}\right) + \frac{t^{\alpha}}{\alpha} + \frac{h^{\alpha}(N-1)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha}N}{\alpha(\alpha+1)} = \frac{t^{\alpha}}{\alpha}.$$

Bu (29) formulaning to’g’riligini ko’rsatadi. Endi (30) tenglikni isbotlashga o’tamiz. Buning uchun (25)-(27) formulalardan foydalanib va $h = \frac{t}{N}$ ni hisobga olib, quyidagini hosil qilamiz

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta](h\beta) = h\sum_{\beta=1}^{N-1} \overset{\circ}{C}{}^{(0)}[\beta]\beta + t\,\overset{\circ}{C}{}^{(0)}[N] =$$

$$= \frac{t^{\alpha+1}}{N^{\alpha}\alpha(\alpha+1)} + \frac{t^{\alpha+1}}{N^{\alpha+1}\alpha(\alpha+1)}[\sum_{\beta=1}^{N-1}\left((N-\beta-1)^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-\beta+1)^{\alpha+1}\right)\beta].$$

Ko’rish mumkinki

$$\sum_{\beta=1}^{N-1}\left((N-\beta-1)^{\alpha+1} - 2(N-\beta)^{\alpha+1} + (N-\beta+1)^{\alpha+1}\right)\beta =$$

$$= \sum_{\beta=2}^{N}(N-\beta)^{\alpha+1}(\beta-1) - 2\sum_{\beta=1}^{N-1}(N-\beta)^{\alpha+1}\beta + \sum_{\beta=0}^{N-2}(N-\beta)^{\alpha+1}(\beta+1) =$$

$$= \sum_{\beta=2}^{N}(N-\beta)^{\alpha+1}\beta - 2\sum_{\beta=1}^{N-1}(N-\beta)^{\alpha+1}\beta + \sum_{\beta=0}^{N-2}(N-\beta)^{\alpha+1}\beta -$$

$$-\sum_{\beta=2}^{N}(N-\beta)^{\alpha+1} + \sum_{\beta=0}^{N-2}(N-\beta)^{\alpha+1} = N^{\alpha+1} - N.$$

Olingan tenglikka ko’ra

$$\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta](h\beta) = \frac{t^{\alpha+1}}{N^{\alpha}\alpha(\alpha+1)} + \frac{t^{\alpha+1}}{N^{\alpha+1}\alpha(\alpha+1)}(N^{\alpha+1} - N) = \frac{t^{\alpha+1}}{\alpha(\alpha+1)}.$$

Bu (30) formulaning to’g’riligini ko’rsatadi. Biz 1-lemmani to’liq isbotladik.

*3-teorema.* $L_2^{(1)}(0,t)$ fazoda (28) o’timaal kvadratur formulaning xatolik funksionali normasining kvadrati ushbu tenglik bilan aniqlanadi

$$\|\ell_N \,|\, L_2^{(1)*}\|^2 = \frac{1}{\alpha(\alpha+1)}\left[\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta](t-h\beta)^{\alpha+1} - \frac{t^{2\alpha+1}}{2\alpha+1}\right].$$

Bu yerda $\overset{\circ}{C}{}^{(0)}[\beta]$ (25)-(27) formulalar bilan aniqlanadi.

*Isboti.*

(24) ga asosan (4) ni quyidagicha yozamiz

$$\sum_{\beta'=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta]\frac{|h\beta - h\beta'|}{2} = f_1^{(0)}[\beta], \quad \beta = 0,1,...,N,$$

(31)

bu yerda

$$f_1^{(0)}[\beta] = \frac{t^{\alpha}h\beta}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)}.$$

U holda normaning kvadrati (2) ifoda uchun quyidagiga egamiz

$$\| \ell_N | L_2^{(1)*} \|^2 = -\sum_{\beta'=0}^{N}\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta'] \overset{\circ}{C}{}^{(0)}[\beta] \frac{|h\beta - h\beta'|}{2} +$$

$$+2\sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] f_1^{(0)}[\beta] - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)} =$$

$$= \sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] f_1^{(0)}[\beta] - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)} =$$

$$= \sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] \left( \frac{t^{\alpha} h\beta}{2\alpha} + \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{\alpha+1}}{2\alpha(\alpha+1)} \right) - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}.$$

Bundan (29) va (30) ga asosan, quyidagini hosil qilamiz

$$\| \ell_N | L_2^{(1)*} \|^2 = \sum_{\beta=0}^{N} \overset{\circ}{C}{}^{(0)}[\beta] \frac{(t-h\beta)^{\alpha+1}}{\alpha(\alpha+1)} - \frac{t^{2\alpha+1}}{\alpha(\alpha+1)(2\alpha+1)}.$$

Bu esa 3-teoremani isboti.

**Sonli natijalar**

**1-misol.** Quyidagi umumlashgan Abel integral tenglamasini yeching

$$\frac{128}{231} x^{\frac{11}{4}} = \int_0^x \frac{1}{(x-t)^{\frac{1}{4}}} \varphi(t)dt.$$

Bu yerda $\alpha = \frac{1}{4}, \quad f(x) = \frac{128}{231} x^{\frac{11}{4}}$.

Mahlumki, yechim $\varphi(x) = x^2$ ko’rinishda bo’ladi.

$m = 1$da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

| $t_i$ | $N=1$ | $N=10$ | $N=100$ | Aniq yechim | Xatolik $\Delta_{N=100}$ |
|---|---|---|---|---|---|
| 0.1 | 0.01097528461 | 0.01001266007 | 0.01000014441 | 0.009999999996 | 1.44(-7) |
| 0.2 | 0.04390113845 | 0.04005064032 | 0.04000057578 | 0.03999999998 | 5.75(-7) |
| 0.3 | 0.09877756149 | 0.09011394066 | 0.09000129110 | 0.08999999995 | 1.29(-6) |

Jadvallardan ko’rish mumkinki, taqribiy yechim x = 0.1, 0.2 va 0.3 lar uchun mos ravishda maksimal sonli 1.44(-7), 5.75(-7) va 1.29(-6) xatoliklar bilan olingan.

**2-misol.** Quyidagi umumlashgan Abel integral tenglamasini yeching

$$\frac{432}{935} x^{\frac{17}{6}} = \int_0^x \frac{1}{(x-t)^{\frac{1}{6}}} \varphi(t)dt.$$

Bu yerda $\alpha = \frac{1}{6}, \quad f(x) = \frac{432}{935} x^{\frac{17}{6}}$.

Mahlumki, yechim $\varphi(x) = x^2$ ko’rinishda bo’ladi.

$m = 1$da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

| $t_i$ | $N=1$ | $N=10$ | $N=100$ | Aniq yechim | Xatolik $\Delta_{N=100}$ |
|---|---|---|---|---|---|
| 0.1 | 0.01071505487 | 0.01001008169 | 0.01000012197 | 0.01000000000 | 1.2(-7) |
| 0.2 | 0.04286021946 | 0.04004032676 | 0.04000048652 | 0.04000000001 | 4.86(-7) |
| 0.3 | 0.09643549377 | 0.09009073517 | 0.09000109981 | 0.08999999998 | 1.09(-6) |

Bu yerda taqribiy yechim x = 0.1, 0.2 va 0.3 lar uchun mos ravishda maksimal sonli 1.2(-7), 4.86(-7) va 1.09(-6) xatoliklar bilan olingan.

**3-misol.** Quyidagi Abel integral tenglamasini yeching

$$e^x - 1 = \int_0^x \frac{1}{(x-t)^{\frac{1}{2}}} \varphi(t)dt.$$

Bu yerda $\alpha = \frac{1}{2}, \quad f(x) = e^x - 1$.

Ma'lumki, yechim

$$\varphi(x) = \frac{e^x}{\sqrt{\pi}} erf(\sqrt{x}), \qquad erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t} dt$$

ko'rinishda bo'ladi.

$m=1$da optimal kvadratur formulalar usuli bilan olingan sonli natijalar

| $t_i$ | $N=1$ | $N=10$ | $N=100$ | Aniq yechim | Xatolik $\Delta_{N=100}$ |
|---|---|---|---|---|---|
| 0.1 | .2154319669327 | .2152921764170 | .2152905197120 | .2152905021493 | 1.76(-8) |
| 0.2 | .3267280014382 | .3258941878189 | .3258841825865 | .3258840763232 | 1.062(-7) |
| 0.3 | .4300194238321 | .4275954302070 | .4275659710321 | .4275656575623 | 3.13(-7) |

Bu yerda ham taqribiy yechim x = 0.1, 0.2 va 0.3 lar uchun mos ravishda maksimal sonli 1.76(-8), 1.062(-7) i 3.13(-7) xatoliklar bilan olingan.

**Xulosa.** Abel tipidagi singulyar integrallarni taqribiy hisoblash uchun murakkab optimal kvadratur formulalari aniq qurilgan. Bu yerda $L_2^{(1)}(0,t)$ fazoda murakkab kvadratur formulaning optimal koeffitsientlari ham to'ilgan. Keyin, $L_2^{(1)}(0,t)$ fazoda optimal kvadratur formulalarning xatolik funksionali normasining kvadrati hisoblangan.

Sobolev fazosida Abel tipidagi singulyar integrallar uchun qurilgan yangi optimal kvadratur formulalarning sonli yaqinlashishini tasdiqlovchi sonli natijalar keltirilgan. Olingan sonli natijalar aniq natijalar bilan taqqoslangan.

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