




# Erratum to: Optimal Repairs in the Description Logic $\mathcal{EL}$ Revisited

Franz Baader<sup>1,2</sup> , Patrick Koopmann<sup>1</sup> , and Francesco Kriegel<sup>1</sup> 

<sup>1</sup> Theoretical Computer Science, Technische Universität Dresden, Dresden, Germany  
`firstname.lastname@tu-dresden.de`

<sup>2</sup> Center for Scalable Data Analytics and Artificial Intelligence (ScaDS.AI)  
Dresden/Leipzig, Germany

In the conference paper [1] and its extended version [2], we gave a brief recap of the definition of optimal repairs in Section 3 and presented on Page 13 of [1] and Page 18 of [2] the following condition on repair types:

**(RT3[1, 2])** If  $C$  is an atom in  $\mathcal{K}$  and  $E \sqsubseteq F$  is a CI in  $\mathcal{T}$  with  $\mathcal{B} \models E(u)^1$  and  $F \sqsubseteq^\emptyset C$ , then there is an atom  $D$  in  $\mathcal{K}$  such that  $E \sqsubseteq^\emptyset D$ .

This version differs from the one given in the original article [3] and its extended version [4]:

**(RT3[3, 4])** If  $C$  is an atom in  $\mathcal{K}$  and  $E \in \text{Sub}(\mathcal{R}, \mathcal{T})$  with  $\mathcal{B} \models E(u)$  and  $E \sqsubseteq^{\mathcal{T}} C$ , then there is an atom  $D$  in  $\mathcal{K}$  such that  $E \sqsubseteq^\emptyset D$ .

Unfortunately, this condition contained a typo w.r.t. the required subsumption relationship, and it is therefore insufficient to prove the results stated in [1, 3].

**Counterexample.** Consider the TBox  $\{C \sqsubseteq \exists r. D, D \sqsubseteq E\}$ , the input qABox  $\exists \emptyset. \{C(a)\}$ , and the repair request  $\{(\exists r. E)(a)\}$ . With respect to (RT3[1, 2]), the set  $\{\exists r. E\}$  is already a repair type for  $a$  since neither  $C$  nor  $\exists r. D$  are mandatorily included by this condition. A problem arises if we now construct the canonical “repair” induced by the repair seed  $\mathcal{S}$  with  $\mathcal{S}_a := \{\exists r. E\}$ . Note that, for the given input, the saturation is the qABox  $\exists \{x\}. \mathcal{B}$  with  $\mathcal{B} = \{C(a), r(a, x), D(x), E(x)\}$ . Then, in the “repair” induced by  $\mathcal{S}$ , the unique  $r$ -successor of  $a$  is the copy of  $x$  that is annotated with the repair type  $\{D, E\}$ . With respect to the empty TBox, the individual name  $a$  is thus no instance of  $\exists r. D$  or  $\exists r. E$ , but is still an instance of  $C$ . Inference with the input TBox then restores the unwanted consequence  $(\exists r. E)(a)$ .

The correct formulation of the above condition must thus use subsumption w.r.t.  $\mathcal{T}$  in place of subsumption w.r.t. the empty TBox, i.e.,  $F \sqsubseteq^\emptyset C$  in (RT3[1, 2]) must be replaced with  $F \sqsubseteq^{\mathcal{T}} C$ :

**(RT3)** If  $C$  is an atom in  $\mathcal{K}$  and  $E \sqsubseteq F$  is a CI in  $\mathcal{T}$  with  $\mathcal{B} \models E(u)$  and  $F \sqsubseteq^{\mathcal{T}} C$ , then there is an atom  $D$  in  $\mathcal{K}$  such that  $E \sqsubseteq^\emptyset D$ .

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<sup>1</sup> Recall that  $\exists Y. \mathcal{B}$  is the saturation of the input qABox  $\exists X. \mathcal{A}$  w.r.t.  $\mathcal{T}$ .

This version of Condition (RT3) is stronger than (RT3[1, 2]), but weaker than the original version (RT3[3, 4]). Due to the latter fact, it enables more efficient computation of the optimized repairs, which are equivalent to the canonical repairs (see Section 5 in [3, 4]). The reason is that all minimal repair types covering a given set of concept descriptions can be computed more efficiently, since fewer steps are necessary to close a set under the implication in (RT3), and thus these repair types are smaller in size.

For the strongest condition (RT3[3, 4]), we have proved in [3, 4] that the canonical repairs induced by repair seed are indeed repairs, and that they cover all repairs in the sense that every repair is entailed by a canonical repair. We have just seen that, for the weakest condition (RT3[1, 2]), the canonical “repairs” need not be repairs w.r.t. the TBox. To show that the modified condition (RT3) introduced above suffices to obtain the results shown in [3, 4], we will re-prove all the auxiliary results in [3] and its extended version [4] that rely on this condition, namely Lemmas XI, XII, and XIII.

**Lemma XI.** *If  $\mathcal{K}$  is a repair pre-type for  $u$  and no atom in  $\mathcal{K}$  subsumes  $C$  w.r.t.  $\mathcal{T}$ , then there is a repair type for  $u$  that covers  $\mathcal{K}$  and that does not contain an atom subsuming  $C$  w.r.t.  $\mathcal{T}$ .*

*Proof.* If  $\mathcal{K}$  satisfies Condition (RT3), then it is a repair type and we are done. Otherwise, there is a CI  $E \sqsubseteq F$  in  $\mathcal{T}$  with  $\mathcal{B} \models E(u)$  and there is an atom  $D$  in  $\mathcal{K}$  with  $F \sqsubseteq^{\mathcal{T}} D$ , but no atom in  $\mathcal{K}$  subsumes  $E$  w.r.t.  $\emptyset$ . Since no atom in  $\mathcal{K}$  subsumes  $C$  w.r.t.  $\mathcal{T}$ , we have  $C \not\sqsubseteq^{\mathcal{T}} D$ . It follows that  $C \not\sqsubseteq^{\mathcal{T}} E$  and so there is  $G \in \text{Conj}(E)$  with  $C \not\sqsubseteq^{\mathcal{T}} G$ . We then replace  $\mathcal{K}$  with  $\text{Max}(\mathcal{K} \cup \{G\})$ . Obviously, the new  $\mathcal{K}$  covers the old  $\mathcal{K}$ . After finitely many iterations the Condition (RT3) must be fulfilled, and then we have reached the desired repair type.  $\square$

**Lemma XII.** *Consider a repair seed  $\mathcal{S}$  and an  $\mathcal{EL}$  concept description  $C$ .*

1. *If the matrix of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$  entails  $C(\langle\langle u, \mathcal{K} \rangle\rangle)$ , then the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entails  $C(u)$  and no atom in  $\mathcal{K}$  subsumes  $C$  w.r.t.  $\emptyset$ .*
2. *If the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entails  $C(u)$  and no atom in  $\mathcal{K}$  subsumes  $C$  w.r.t.  $\mathcal{T}$ , then the matrix of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$  entails  $C(\langle\langle u, \mathcal{K} \rangle\rangle)$ .*

*Proof.* The first statement above is still the same as the only-if direction in the original Lemma XII and thus the same proof still works. We proceed with the second statement, by induction on  $C$ . Denote by  $\mathcal{B}$  the matrix of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$ . Let the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entail  $C(u)$  and assume that  $\mathcal{K}$  does not contain an atom subsuming  $C$ .

- The case where  $C = \top$  is trivial.
- Assume that  $C = A$  for a concept name  $A$ . Since no atom in  $\mathcal{K}$  subsumes  $A$  w.r.t.  $\mathcal{T}$ , we infer that  $A \notin \mathcal{K}$  and so it follows from the very definition of  $\mathcal{B}$  that the concept assertion  $A(\langle\langle u, \mathcal{K} \rangle\rangle)$  is contained in  $\mathcal{B}$ , i.e.,  $\mathcal{B} \models A(\langle\langle u, \mathcal{K} \rangle\rangle)$ .

- Let  $C = C_1 \sqcap \dots \sqcap C_n$  be a conjunction of atoms  $C_1, \dots, C_n$  where  $n \geq 2$ . The preconditions immediately imply that, for each index  $i$ , the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entails  $C_i(u)$  and  $\mathcal{K}$  does not contain an atom subsuming  $C_i$  w.r.t.  $\mathcal{T}$  (otherwise there would be an atom subsuming  $C$  since  $C \sqsubseteq^{\emptyset} C_i$ ). The induction hypothesis yields that  $\mathcal{B} \models C_i(\langle\langle u, \mathcal{K} \rangle\rangle)$  for each  $i$ , and thus it follows that  $\mathcal{B} \models C(\langle\langle u, \mathcal{K} \rangle\rangle)$ .
- Consider the last case where  $C = \exists r.D$  is an existential restriction. According to Lemma II, it follows from the preconditions that there exists some object  $v$  such that the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  contains  $r(u, v)$  and entails  $D(v)$ . Since  $\exists r.D$  is not subsumed by an atom in  $\mathcal{K}$  w.r.t.  $\mathcal{T}$ , it follows that  $D \not\sqsubseteq^{\mathcal{T}} E$  for each  $\exists r.E \in \mathcal{K}$ . Thus for each  $\exists r.E \in \mathcal{K}$ , there is some atom  $F_E \in \text{Conj}(E)$  such that  $D \not\sqsubseteq^{\mathcal{T}} F_E$ . According to Lemma XI there exists a repair type  $\mathcal{L}$  for  $v$  that covers the repair pre-type  $\text{Max}_{\sqsubseteq^{\emptyset}}(\{ F_E \mid \exists r.E \in \mathcal{K} \text{ and the matrix of } \text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}) \text{ entails } E(v) \})$  and that does not contain an atom subsuming  $D$  w.r.t.  $\mathcal{T}$ . Applying the induction hypothesis then yields that  $\mathcal{B} \models D(\langle\langle v, \mathcal{L} \rangle\rangle)$ . By the very construction of  $\mathcal{L}$ , it follows that the matrix  $\mathcal{B}$  contains the role assertion  $r(\langle\langle u, \mathcal{K} \rangle\rangle, \langle\langle v, \mathcal{L} \rangle\rangle)$ . Thus, we conclude that  $\mathcal{B} \models C(\langle\langle u, \mathcal{K} \rangle\rangle)$ .  $\square$

**Lemma XIII.** *For each repair seed  $\mathcal{S}$ , the canonical repair induced by  $\mathcal{S}$  equals its saturation, i.e.,  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S}) = \text{sat}_{\text{QL}}^{\mathcal{T}}(\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S}))$ .*

*Proof.* Since, for both query languages IQ and CQ, the  $\sqsubseteq$ -rule employed for constructing the saturations is the same, the following argumentation applies to both choices. We show that the  $\sqsubseteq$ -rule is not applicable to  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$ . It is trivial that none of the other two rules is applicable, since the matrix  $\mathcal{B}$  can never contain a concept assertion involving a complex concept description.

Consider an object  $\langle\langle u, \mathcal{K} \rangle\rangle$  of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$  and a concept inclusion  $C \sqsubseteq D$  in  $\mathcal{T}$ . Further assume that the matrix of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$  entails  $C(\langle\langle u, \mathcal{K} \rangle\rangle)$ . Lemma XII implies that the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entails  $C(u)$  and no atom in  $\mathcal{K}$  subsumes  $C$  w.r.t.  $\emptyset$ . Since the  $\sqsubseteq$ -rule is not applicable to  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$ , it follows that the matrix of  $\text{sat}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A})$  entails  $D(u)$ . Since  $\mathcal{K}$  satisfies Condition (RT3), no atom in  $\mathcal{K}$  subsumes  $D$  w.r.t.  $\mathcal{T}$ . A further application of Lemma XII yields that the matrix of  $\text{rep}_{\text{QL}}^{\mathcal{T}}(\exists X.\mathcal{A}, \mathcal{S})$  entails  $D(\langle\langle u, \mathcal{K} \rangle\rangle)$ .  $\square$

An implementation that employs this more efficient version of Condition (RT3) is available from <https://github.com/francesco-kriegel/interactive-optimal-repairs>.

## References

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