Downscaling algorithm proof

Reaching ALS values with the α correction coefficient

Here we show how the α correction coefficient, applied to all tree diameters of a field plot makes it possible to reach the total BA (BA_{ALS}) and the BA proportion of broadleaf trees ($Prop_{BC_{ALS}}$) of the cell to which the plot is associated.

Using the α correction coefficient, the basal area of broadleaf trees (BA_D) of one cell is given by:

$$BA_D = \frac{\pi}{40000} \sum_{Dec.\ trees} \omega.(\alpha.dbh_F)^2$$

According to equation 8 defining ω,

$$BA_D = \frac{\pi}{40000} \sum_{Dec.\ trees} \frac{40000}{\pi} \times \frac{ba_{tree_{ALS,F}}}{(\alpha.dbh_F)^2}.(\alpha.dbh_F)^2$$

$$BA_D = \sum_{Dec.\ trees} ba_{tree_{ALS,F}}$$

According to equation 9 defining $ba_{tree_{ALS,F}}$,

$$BA_D = \sum_{Dec.\ trees} BA_{ALS} \times Prop_{BC_{ALS}} \times Prop_{Sp_F} \times Prop_{tree_F}$$

$$BA_D = BA_{ALS} \times Prop_{BC_{ALS}} \times \sum_{Dec.\ trees} Prop_{Sp_F} \times Prop_{tree_F}$$

As $Prop_{tree_F}$ is the proportion of the trees within Sp and $Prop_{Sp_F}$ is the proportion of species within deciduous species, this sum equals 1. Therefore

$$BA_D = BA_{ALS} \times Prop_{BC_{ALS}}$$

This shows that the BA of broadleaf trees calculated from the trees dbh corrected with the α coefficient equals the broadleaf BA provided by the ALS mapping. The same rational applies for coniferous trees. Thus, the total basal area calculated from individual trees after correction with the α coefficient equals the total BA given by the ALS mapping. This also shows that our downscaling algorithm keeps the broadleaf-coniferous proportion provided by ALS mapping.

Maintaining Dg ratios between species

Here we show how our algorithm maintains the Dg ratios observed on the field plots between the different species.

The Dg of a species in a cell is calculated as

$$Dg_{Sp}^2 = \frac{40000.BA_{Sp}}{\pi.\omega_{Sp}} \tag{12}$$

where Dg_{Sp} is the mean quadratic diameter of the species, BA_{Sp} its basal area, and ω_{Sp} its total stem number, which is given by

$$\omega_{Sp} = \sum_{Sp\ trees} \omega$$

According to equation 8 defining ω,

$$\omega_{Sp} = \sum_{Sp\ trees} \frac{40000}{\pi} \times \frac{ba_{tree_{ALS,F}}}{(\alpha.dbh_F)^2}$$

According to equation 9 defining $ba_{tree_{ALS,F}}$,

$$\omega_{Sp} = \frac{40000}{\pi} \times \sum_{Sp\ trees} \frac{BA_{ALS} \times Prop_{BC_{ALS}} \times Prop_{Sp_F} \times Prop_{tree_F}}{(\alpha.dbh_F)^2}$$

$$\omega_{Sp} = \frac{1}{\alpha^2} \times \frac{40000}{\pi} \times BA_{ALS} \times Prop_{BC_{ALS}} \times Prop_{Sp_F} \times \sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2}$$

$$\omega_{Sp} = \frac{1}{\alpha^2} \times \frac{40000}{\pi} \times BA_{Sp} \times \sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2}$$

Thus, using equation 12, we get

$$Dg_{Sp}^2 = \alpha^2 \times \frac{1}{\sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2}} \tag{13}$$

where $Prop_{tree_F}$ is the BA proportion of trees in species Sp in the field plot given by:

$$Prop_{tree_F} = \frac{\pi}{40000} \frac{n_{tree_F}.dbh_F^2}{BA_{Sp_F}}$$

where n_{tree} is the number of trees in the field data, and BA_{Sp_F} is the basal area of species Sp in the field data. Hence

$$\sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2} = \sum_{Sp\ trees} \frac{\pi}{40000} \frac{n_{tree_F}.dbh_F^2}{BA_{Sp_F}.dbh_F^2}$$

$$\sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2} = \frac{\pi}{40000} \times \frac{1}{BA_{Sp_F}} \times \sum_{Sp\ trees} n_{tree_F}$$

$$\sum_{Sp\ trees} \frac{Prop_{tree_F}}{dbh_F^2} = \frac{\pi}{40000} \times \frac{1}{BA_{Sp_F}} \times N_{Sp_F}$$

where N_{Sp_F} is the number of stems of species Sp in the field plot. Therefore

$$\sum_{Sp\ trees} \frac{Prop_{tree}}{dbh_F^2} = \frac{1}{Dg_{Sp_F}^2}$$

Finally, using equation 13, we get

$$Dg_{Sp}^2 = \alpha^2 \times Dg_{Sp_F}^2 \tag{14}$$

and then

$$Dg_{Sp} = \alpha \times Dg_{Sp_F} \tag{15}$$

As the α coefficient is the same for all trees and all species, the ratio of Dg_{Sp} of two species is equal to their ratio of $\alpha \times Dg_{Sp_F}$. Thus our algorithm maintains the Dg ratios observed on the field plots between the different species.