

# Portland Ice Sheet Modeling School

## Data and Models

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$$\nabla \cdot \sigma(x, y) = \dot{a}(x, y) - \frac{\partial h}{\partial t} \quad (32)$$

$$\sigma_{ij,j} + \rho f_i = m a_i \quad (97)$$

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{u}{\omega} \frac{\partial \omega}{\partial x} + \frac{\partial w}{\partial z} &= 0, \\ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= 0, \\ \epsilon_2 \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} &= \rho g, \end{aligned}$$



Data are needed to apply mass and momentum conservation to ice sheet modeling

It's all fun and games until it's time to get real

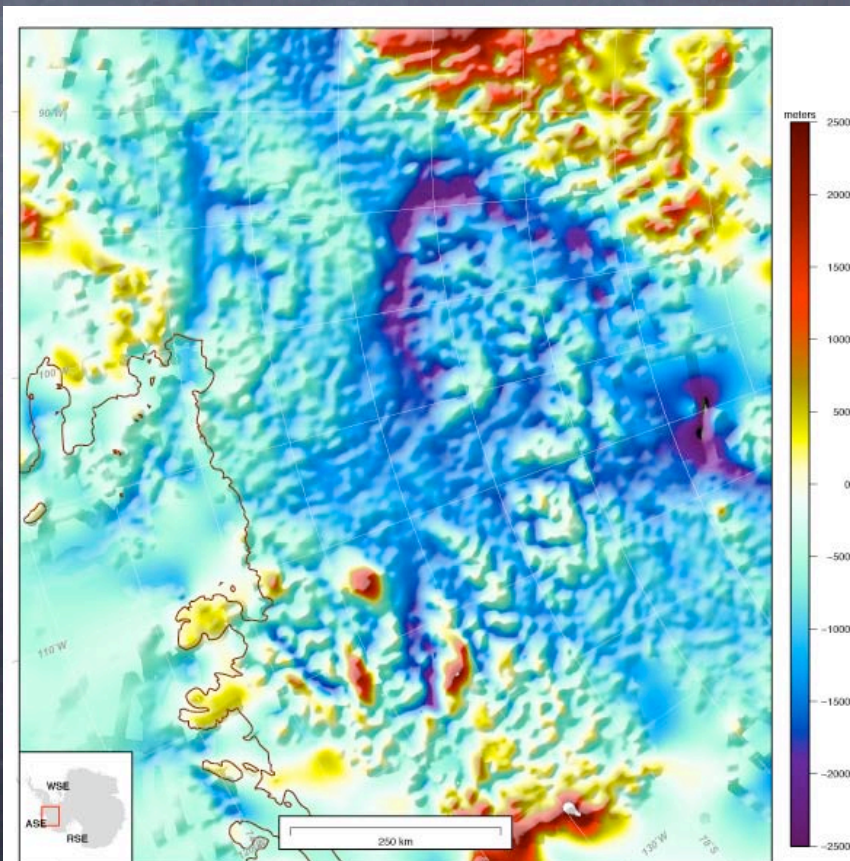


Data are used to 'sculpt' the  
mathematical domain into  
an ice sheet

To turn a system of differential equations into your own home-grown ice sheet:  
A. Initial conditions => all quantities that you need to know at  $t = 0$  just to get through the first time step (*thickness and vels*)

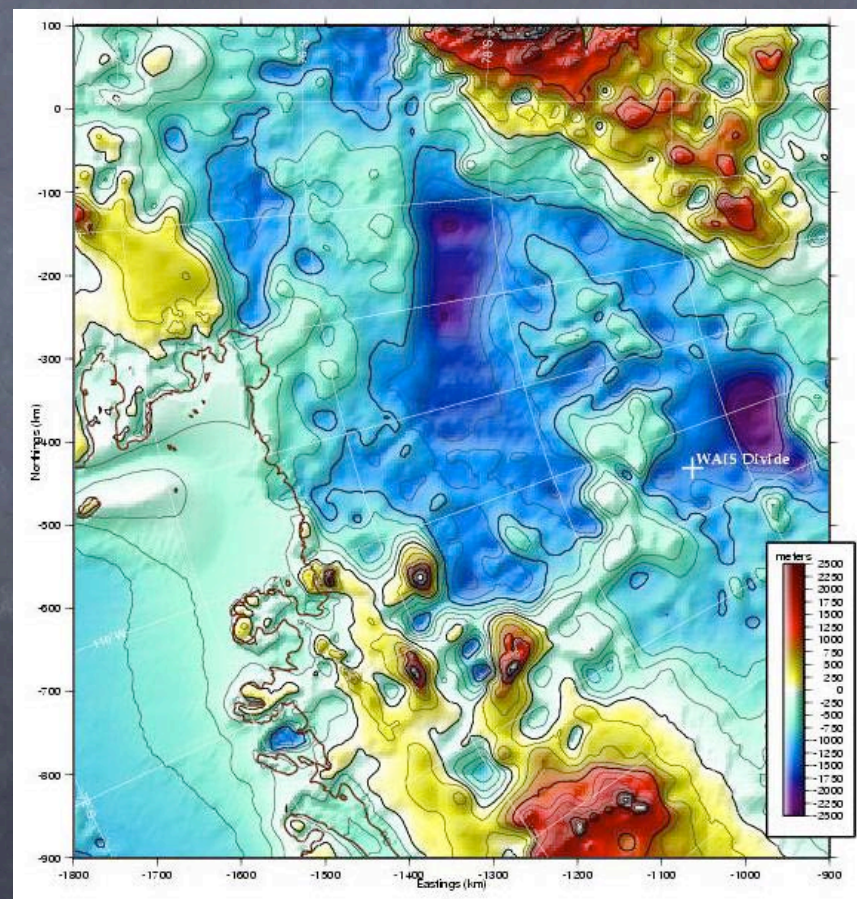
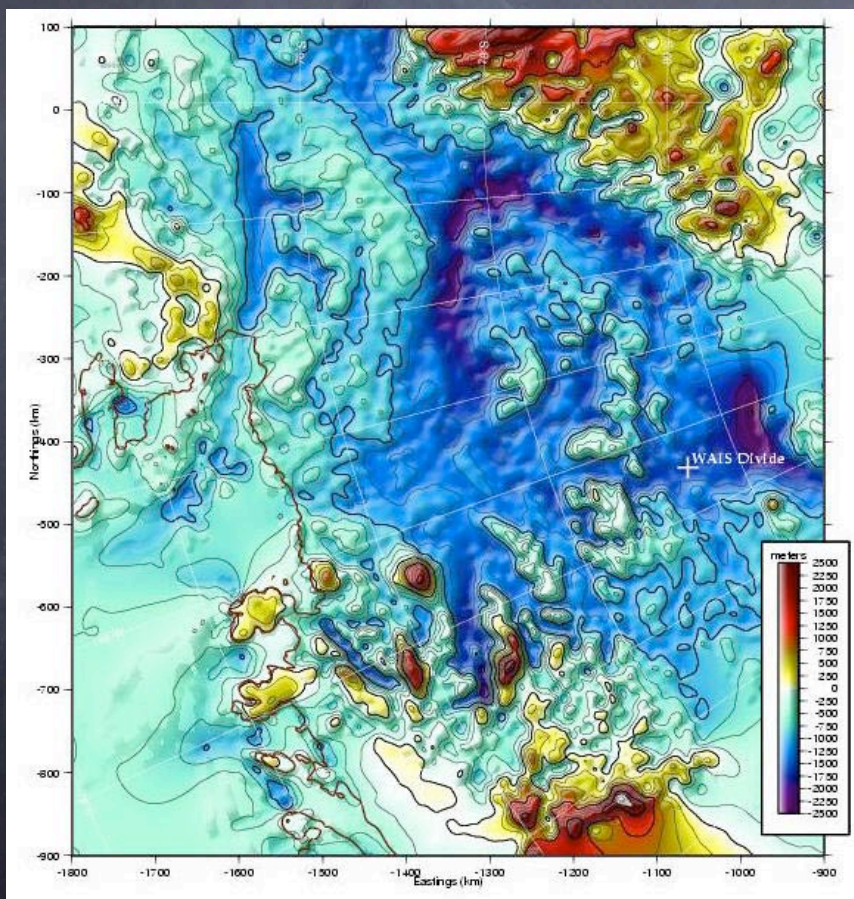
1. Prescribe initial geometry (bed and surface topography from data, or grow from scratch, if you have 10,000 to 100,000 model years)

Thwaites and Pine Island Glacier systems:

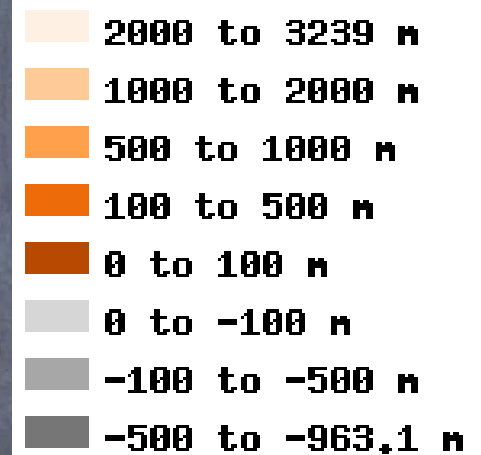
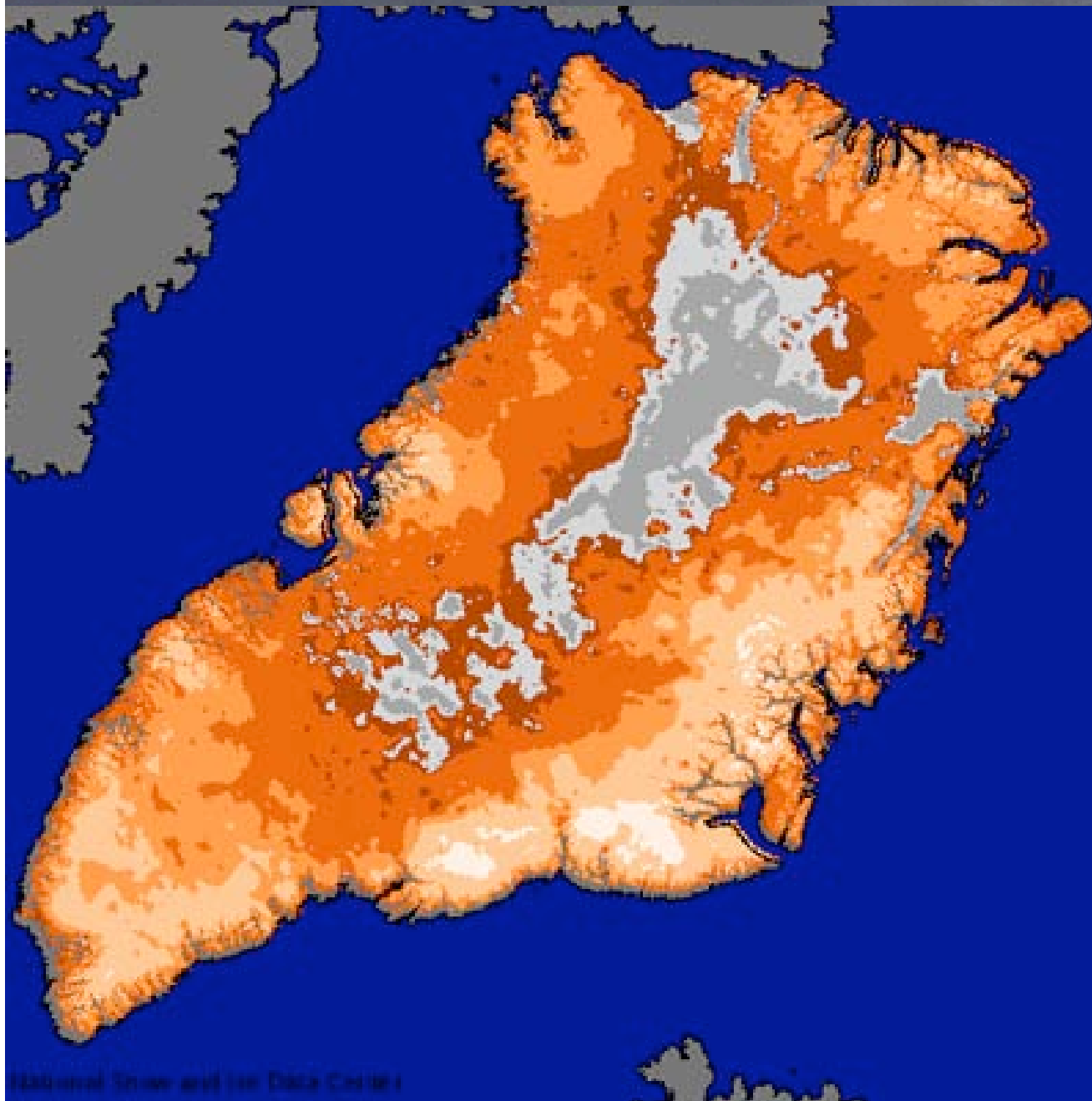




Airborne data are helping (UTIG, LDEO, CReSIS, BAS, e.g. Holt et al., 2006)



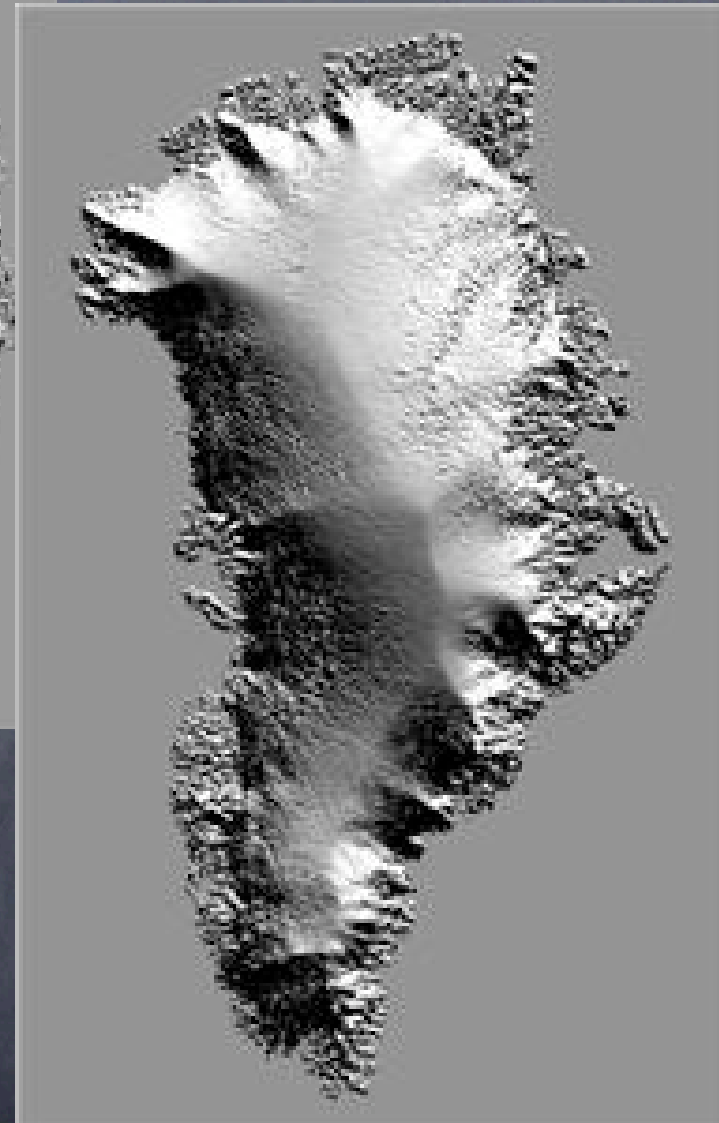
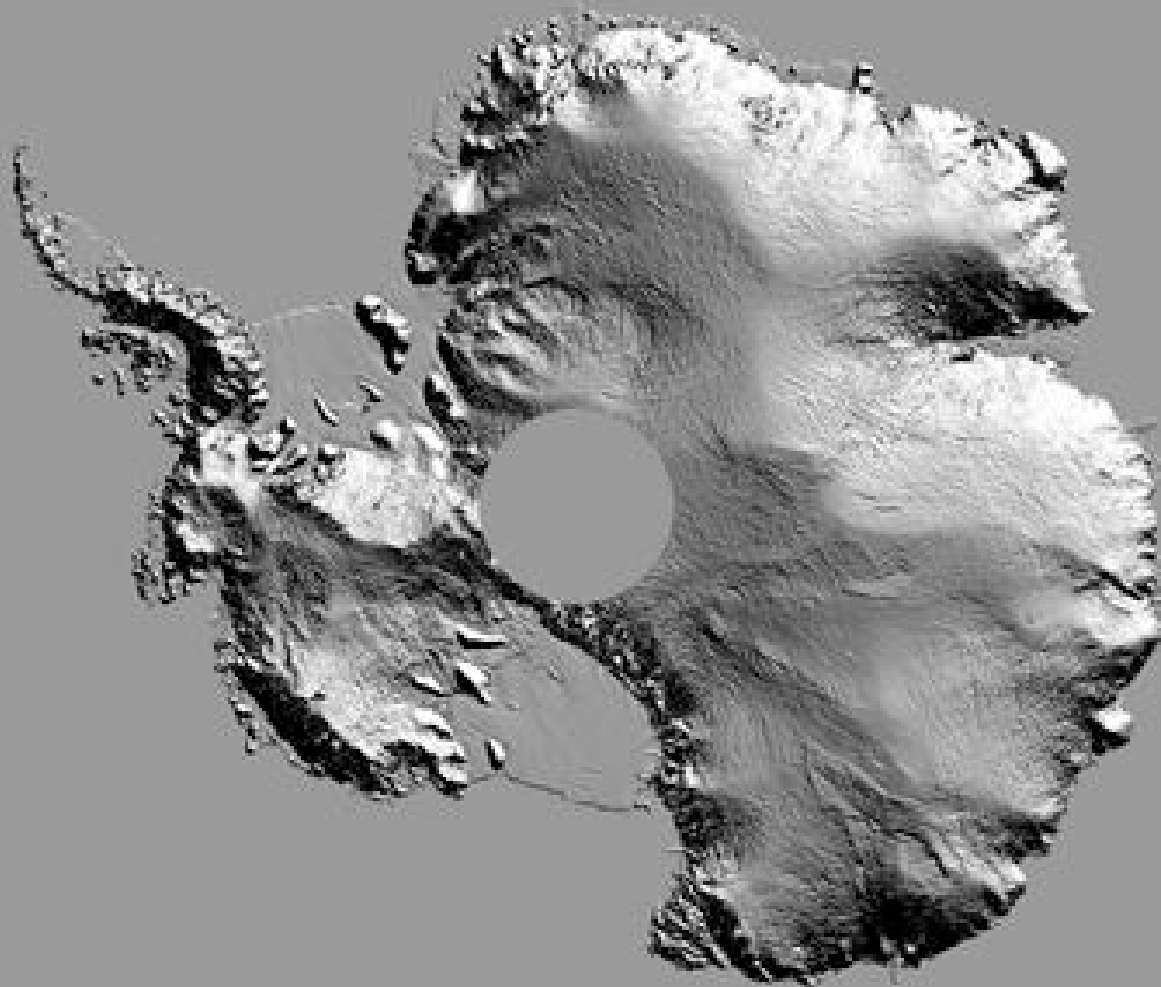
# Greenland bed topo - NSIDC, 5km horizontal resolution





For comparison, this is how well we mapped out topography of Mars, about two orders of magnitude higher horizontal resolution

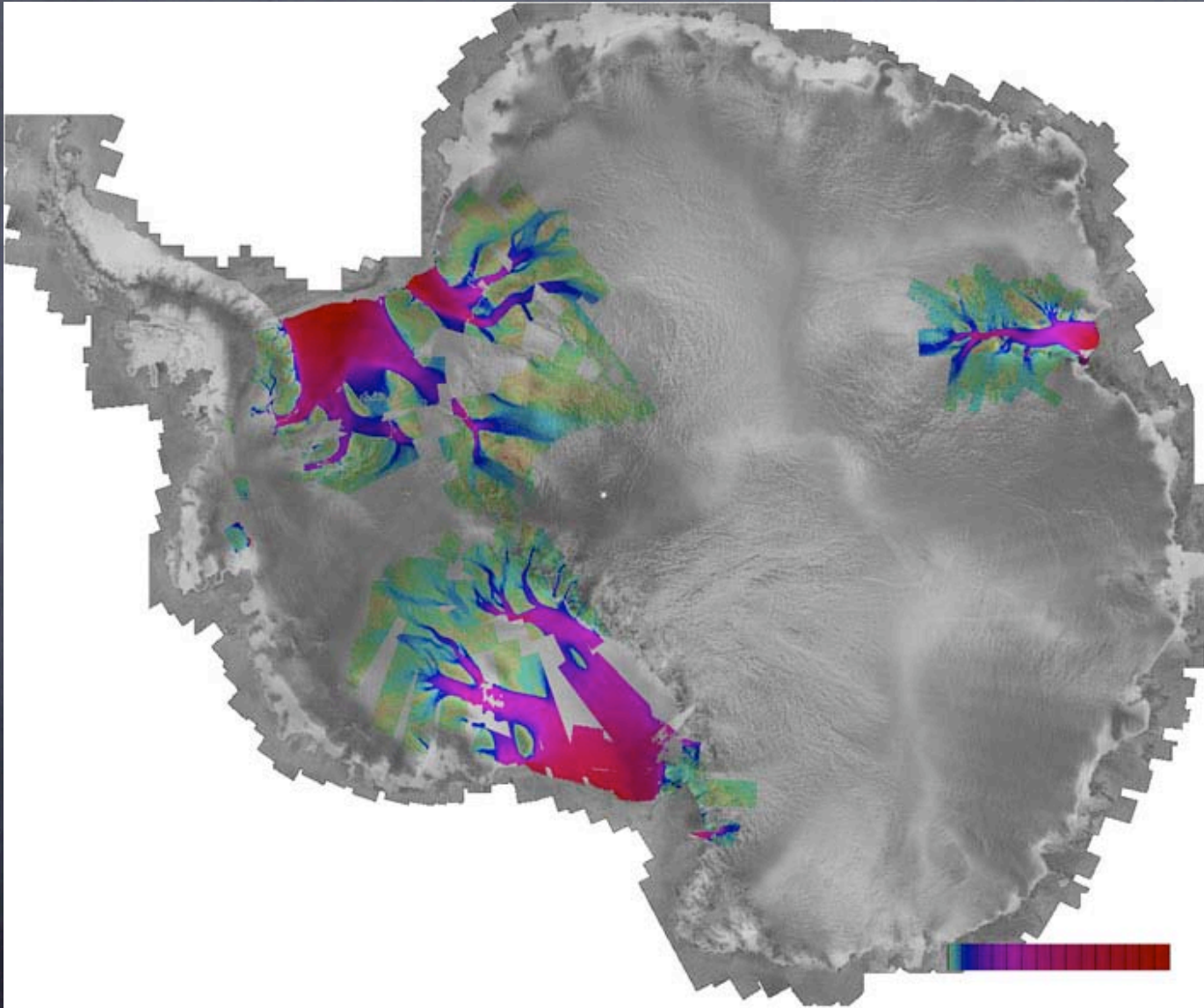




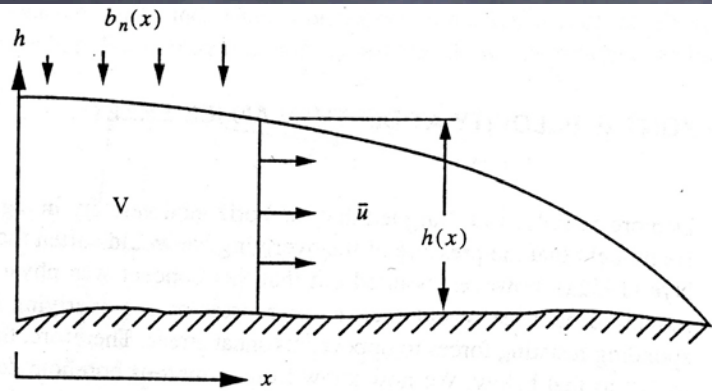
IceSat surface topo data - NSIDC  
has interpolated to 0.5 and 1km

A. Initial conditions continued:

2. Initialize the velocity field (use surface observations and balance velocities where obs not available, develop an obs-consistent scheme for guessing the vertical velocity distribution if you have a 3D model) (image courtesy of Dr. Ian Joughin, APL-UW)



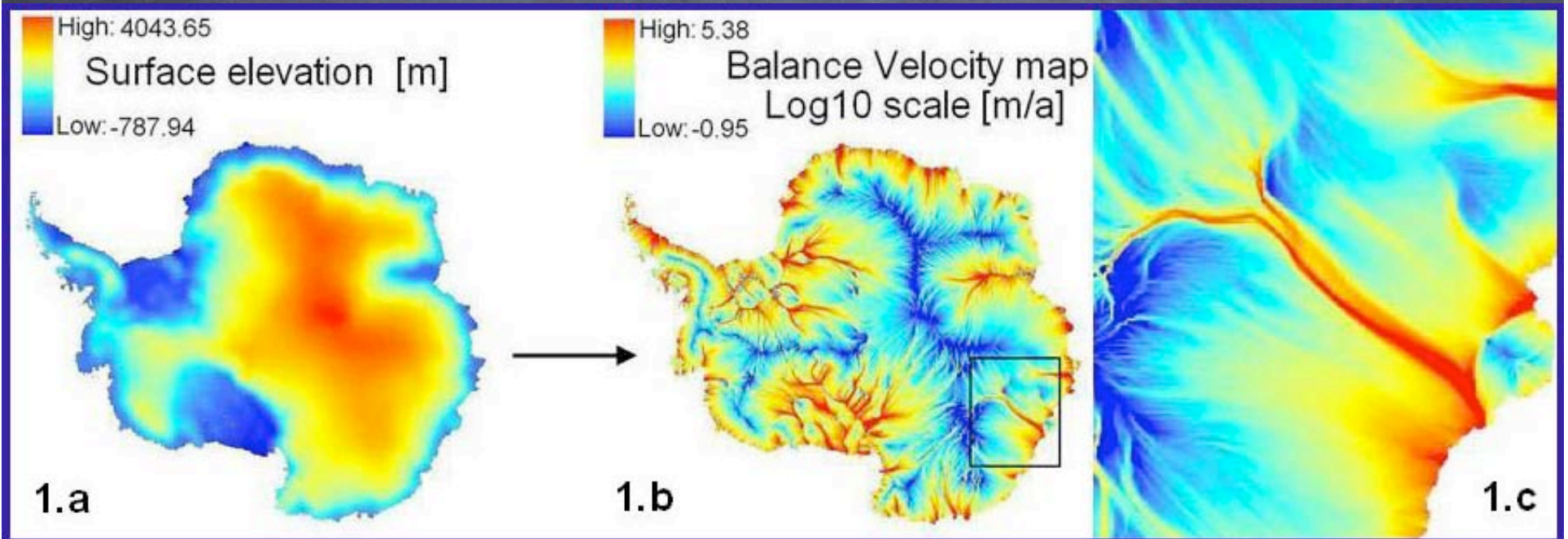
When you don't have measured velocities, calculate balance velocities (you just need to know distribution of accumulation rates and thickness and assume that modern vels are in balance)



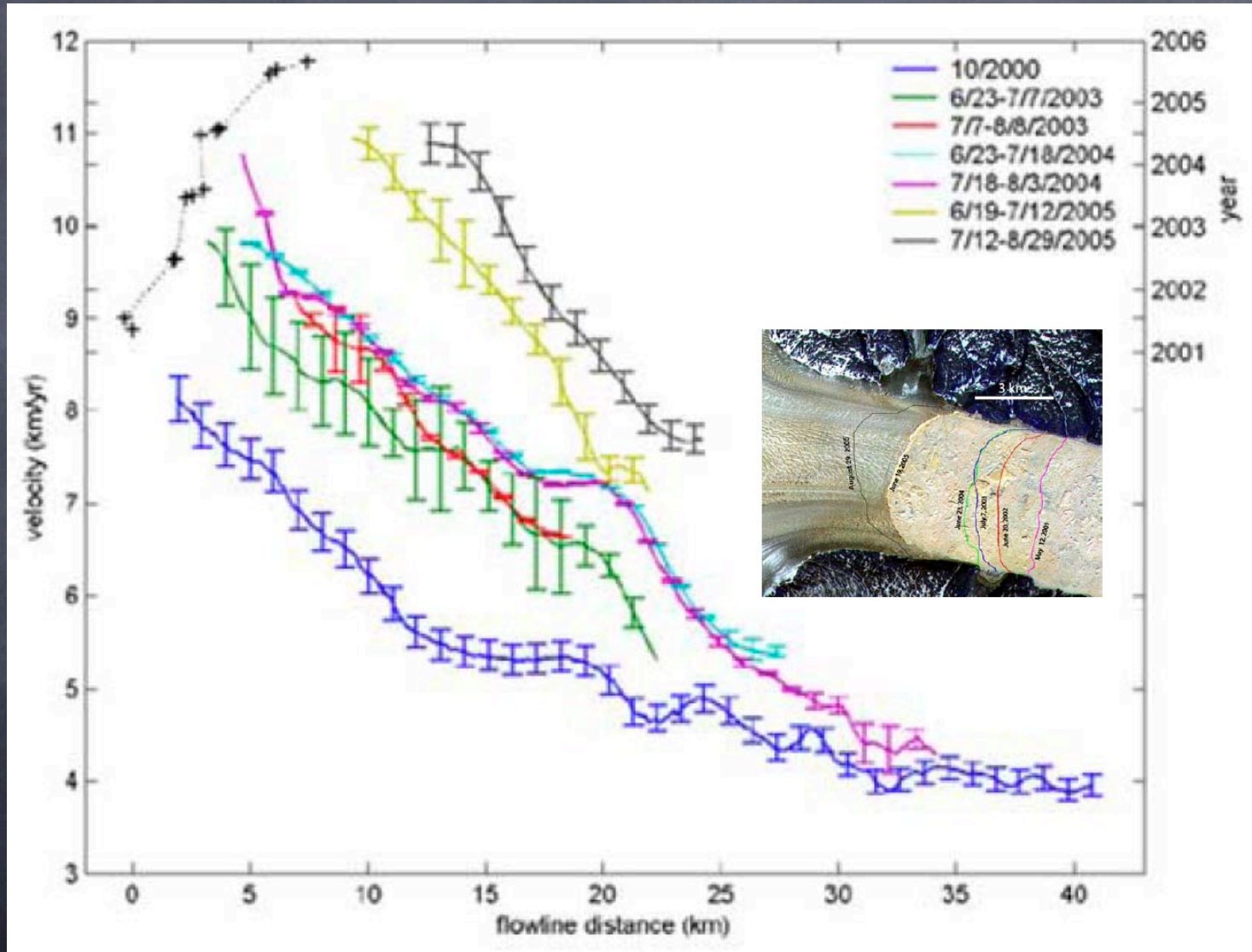
$$\bar{u} = \frac{1}{h(x)} \int_0^x b_n(x) dx$$

Figure 5.1 Schematic diagram illustrating dependence of horizontal velocity on accumulation rate on an ice sheet.

Results from Dr. Mansell, Swansea U.



... and then velocity can change quickly in some dynamic (i.e. interesting) parts of ice sheets (e.g. Howat et al., 2005)

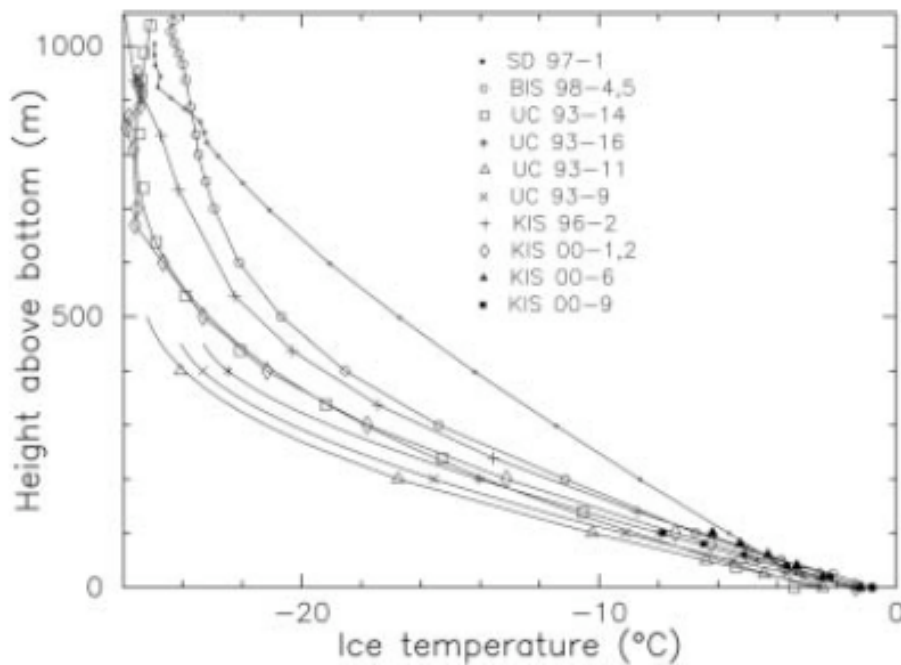


## A. Initial conditions continued:

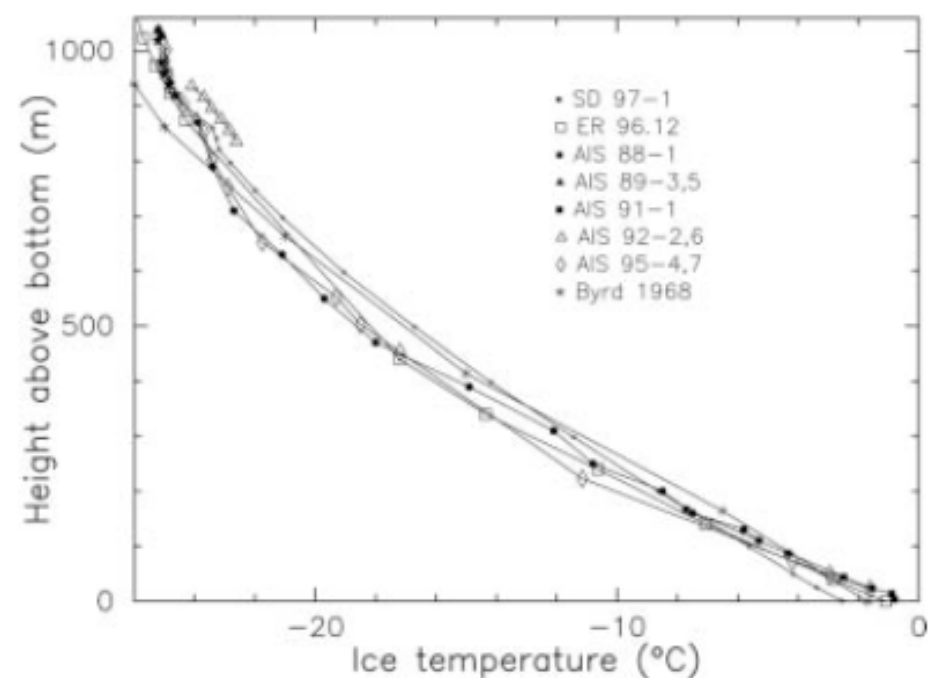
3. Prescribe the ice temperature distribution within the ice sheet (remember the part about the ice viscosity being T-dependent? And we need to calculate distribution of basal melting typically used to determine where basal sliding occurs)

There are some borehole measurements of ice temperature profiles and estimates from temperature-sensitive radar signal attenuation may work in the future. For the most part, one has to either run a spin-up model (say, since last interglacial 125kys ago) or do a steady-state estimate (e.g. Joughin et al., 2004). Borehole data should be used to check how good such estimates are.

### Advection-dominated profiles



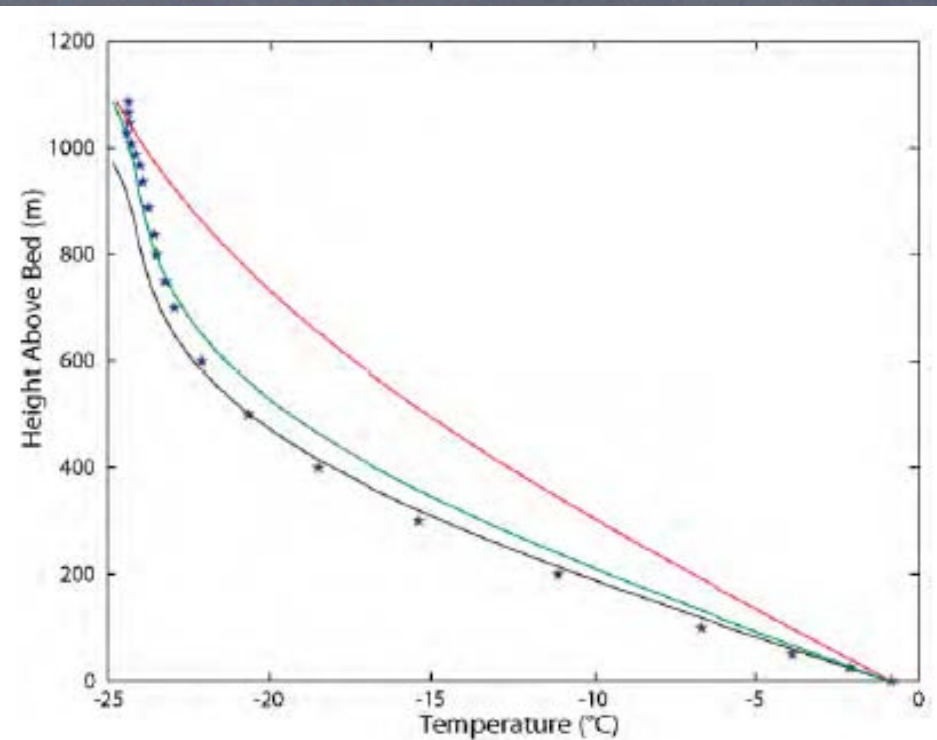
### Diffusion-dominated profiles



$$\frac{\partial \theta}{\partial t} = \kappa \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right] - u \frac{\partial \theta}{\partial x} - v \frac{\partial \theta}{\partial y} - w \frac{\partial \theta}{\partial z} + \frac{Q}{\rho C}.$$

Joughin et al., 2004

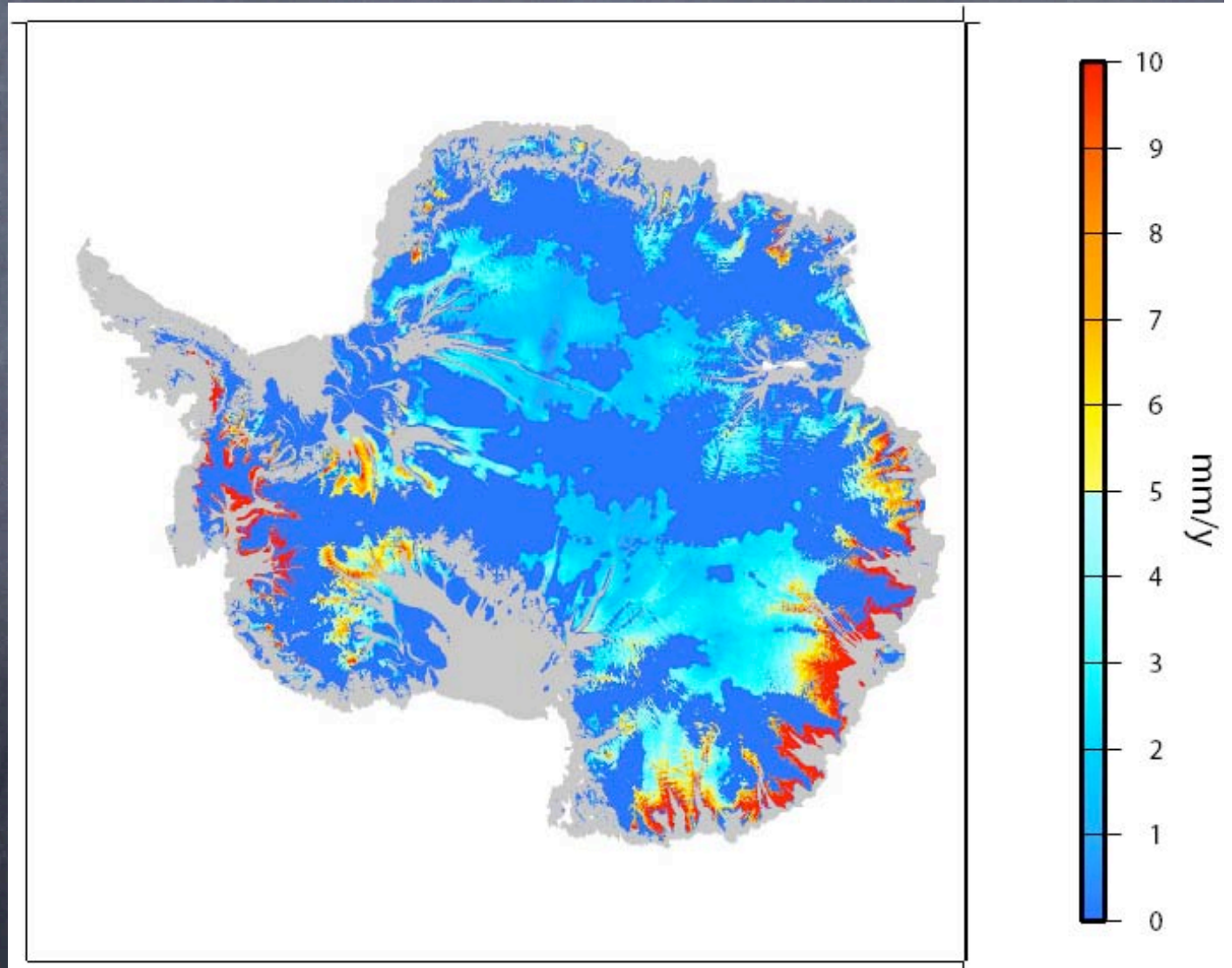
The problem is that the ice sheet does not forget the past very readily; in particular, the thermal field may remember climate and velocity changes that took place thousands of years ago.



*Fig. 7. Measured and modeled temperature profiles at the UpD camp. Blue stars show the measured temperature profile (Engelhardt, unpublished information). The black curve shows the result from the model with horizontal advection. The green curve shows the same model result as the black curve, but with the vertical coordinate rescaled so that the thickness matches the borehole thickness. The red curve shows the rescaled vertical-advection-only solution.*

Sliding is coupled to basal hydrology and in IS models this is captured as a binary (0-1) switch, where basal melting is calculated, sliding is turned on (1), when basal freezing is calculated, sliding is turned off (0)  
Melting/Freezing rates are calculated but basal water is not conserved or routed. There is no feedback between hydrology and sliding (notable exception is Johnson & Fastook 2002)

Llubes et al. 2006



Let's step back for a moment into theory (collective sigh of relief now, please!)

Why should there be water generation beneath ice sheets?

Melting = Potential + Geothermal – Warming

$$E_m = E_p + E_g - E_w$$

$$mAL = Mg\Delta h + AG - MC_p\Delta T$$

$$m \approx 100 + 50 - 50 \text{ [Gt/yr]}$$

$m$  = melted ice mass

$A$  = basal area (~10 mln sq. km in Antarctica)

$L$  = latent heat of ice fusion (335 kJ/kg)

$M$  = ice mass discharged to the ocean (~2,000 Gt/yr)

$g$  = gravitational acceleration

$\Delta h$  = average elevation of snow deposition (~1000 m)

$G$  = geothermal flux rate (~1.5 MJ/yr)

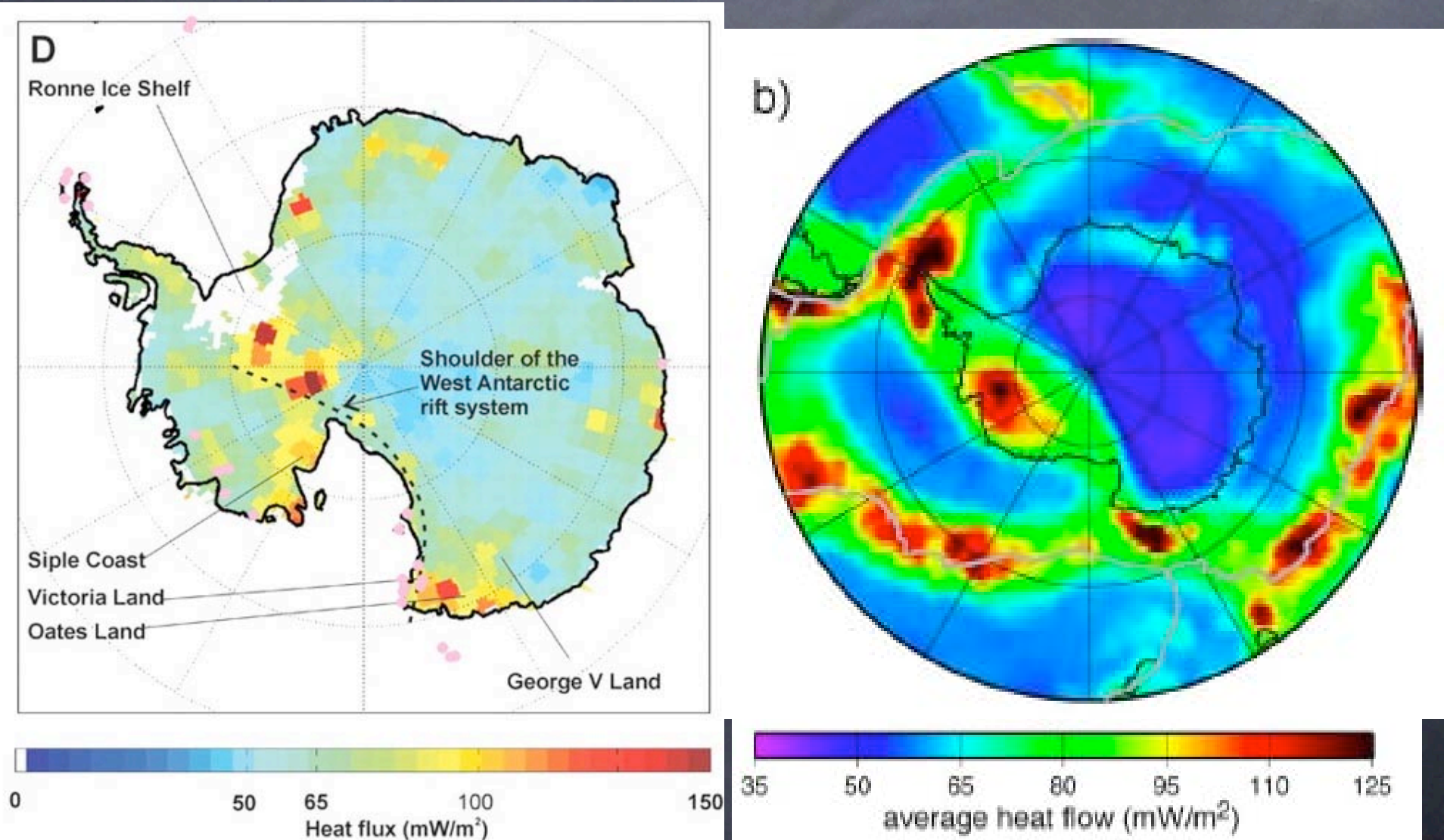
$C_p$  = heat capacity of ice (4 kJ/kg/deg.)

$\Delta T$  = average ice warming (~10 K)





And so we have stumbled upon the least known, and the hardest to measure, initial/boundary condition, geothermal flux (estimates from magnetic crust thickness and seismic data: Maule et al. 2005; Shapiro and Ritzwoller, 2004)

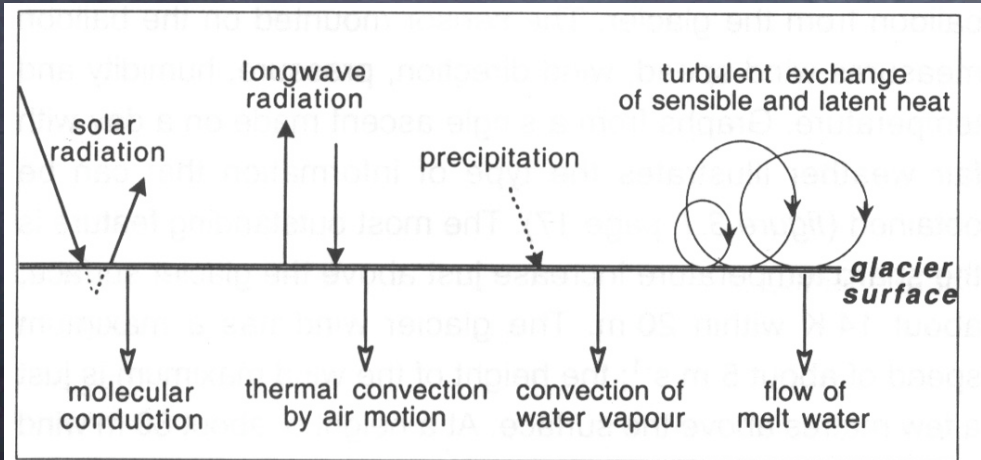


B. Boundary conditions => constraints on ice-atmosphere and ice-ocean interactions (time-dependent under changing climate)

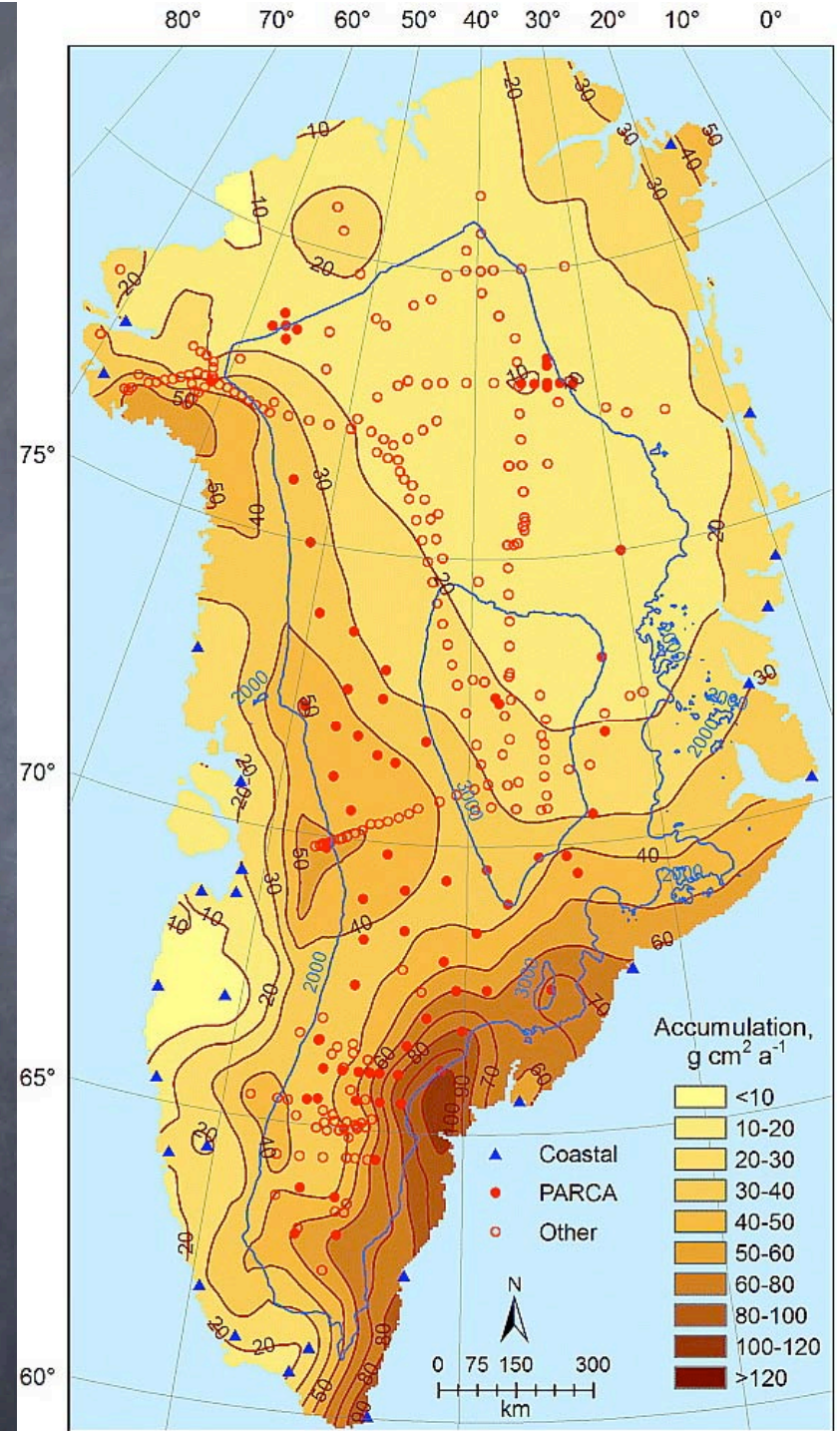
1. Atmospheric boundary conditions - surface mass balance

e.g. recent surface accumulation rates from AWS, ice cores, shallow radar and temperature for PDD or EBM parameters for melt calculations

(Bales et al., 2009)



**Figure 3.2.** The most important processes determining the energy flux at the glacier - atmosphere interface and the thermal structure in the upper layer of the glacier.



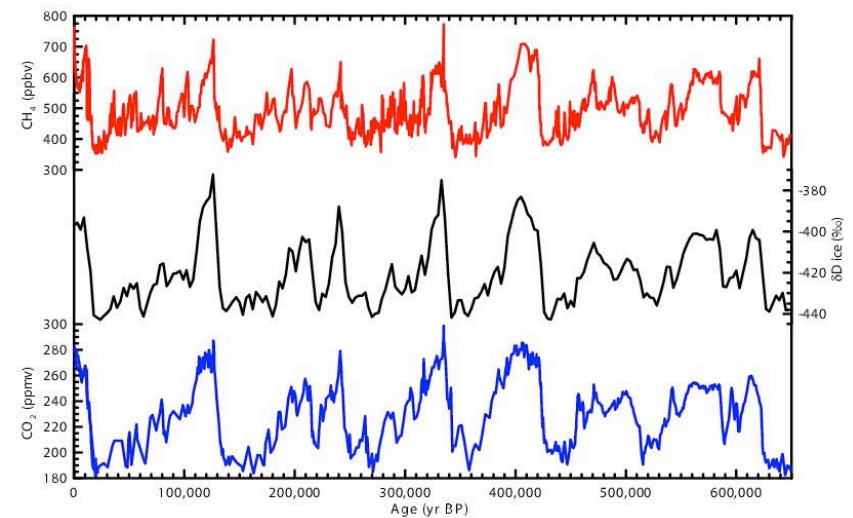
B. Boundary conditions => constraints on ice-atmosphere and ice-ocean interactions (time-dependent under changing climate)

1. Atmospheric boundary conditions - surface mass balance

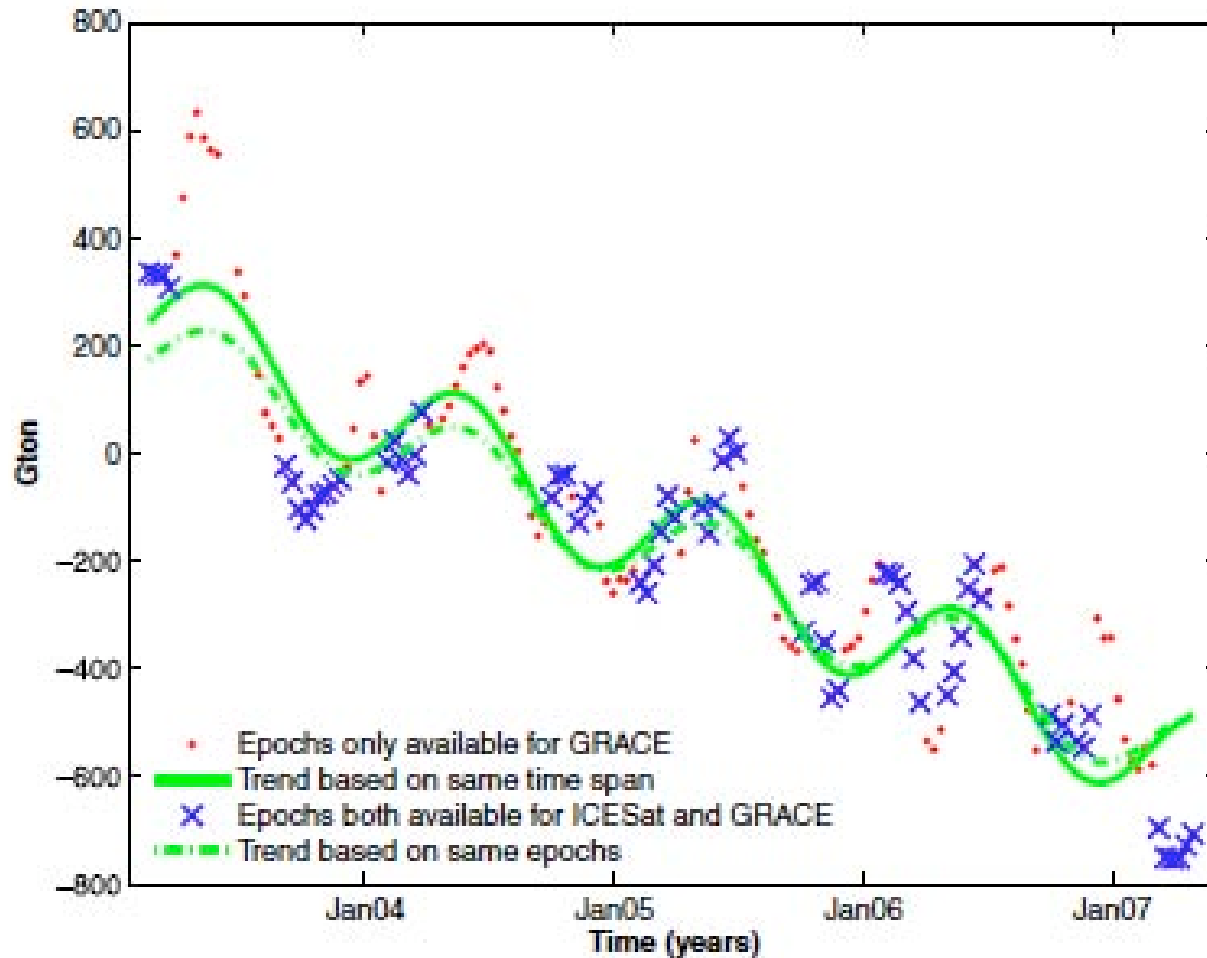
Time-variable surface conditions for future climate changes will come from climate model output e.g. (EBM parameters from CCSM). GCMs often do poorly at getting precip right.

Simulations of paleo-ice sheets require either paleo-climate simulations or parametrization of precip and temperature changes based on climate records (e.g. ice cores, deep-sea records)

Mismatch of horizontal resolutions between GCMs and ISM (10:1). Use RCM and/or downscaling schemes.



Space-borne measurements may make it possible to calculate your ice sheet mass balance scheme using constraints on annual ice sheet elevation and mass changes (e.g. Slobbe et al., 2009)



**Figure 4.** Time-series and estimated trends for CNES solutions using only the months for which ICESat data are available and over the same time span as we have for ICESat data.

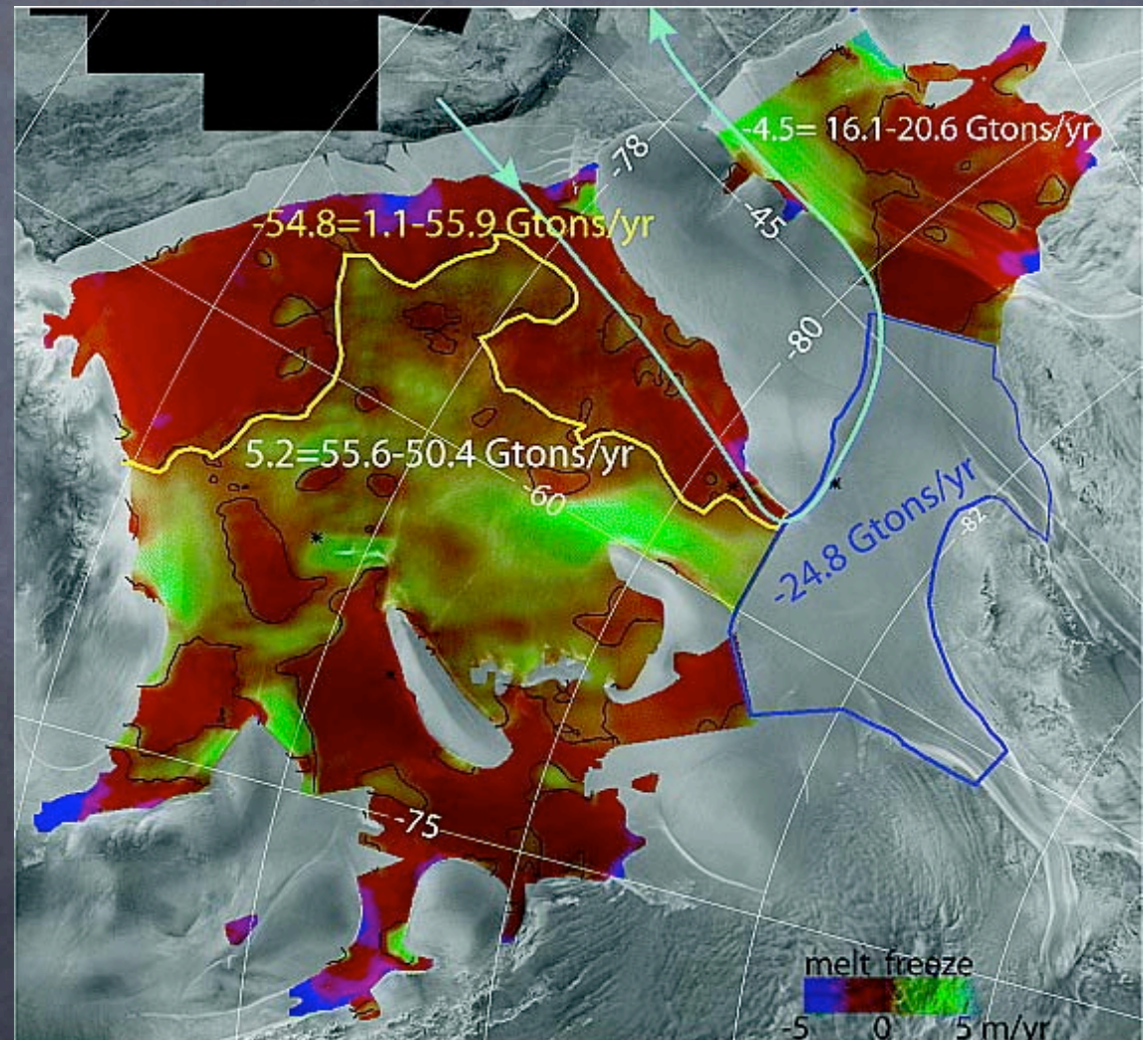
B. Boundary conditions => constraints on ice-atmosphere and ice-ocean interactions (time-dependent under changing climate)

2. Oceanographic boundary conditions - thermal ocean forcing of basal melt rates (relatively new)

Parametrizations from measurements of modern conditions and ocean temperatures from measurements or climate models.

How to deal with cavities?

(Joughin and Padman, 2003)



## C. Data for parametrizations of physical processes:

- Basal sliding velocity and basal tractions – in 1970s and 1980s empirical 'sliding laws' multiplied like bunnies (below is a review table from Bindschadler, 1987). And these are just for the case of ice motion over hard beds. For ice motion over sediments, stress exponent tends towards infinity → plastic behavior.

TABLE 1. Calculated Values of Sliding Speed for Ice Stream B According to Various Models Using the Surface Slope and Ice Thickness Measured at UpB Camp

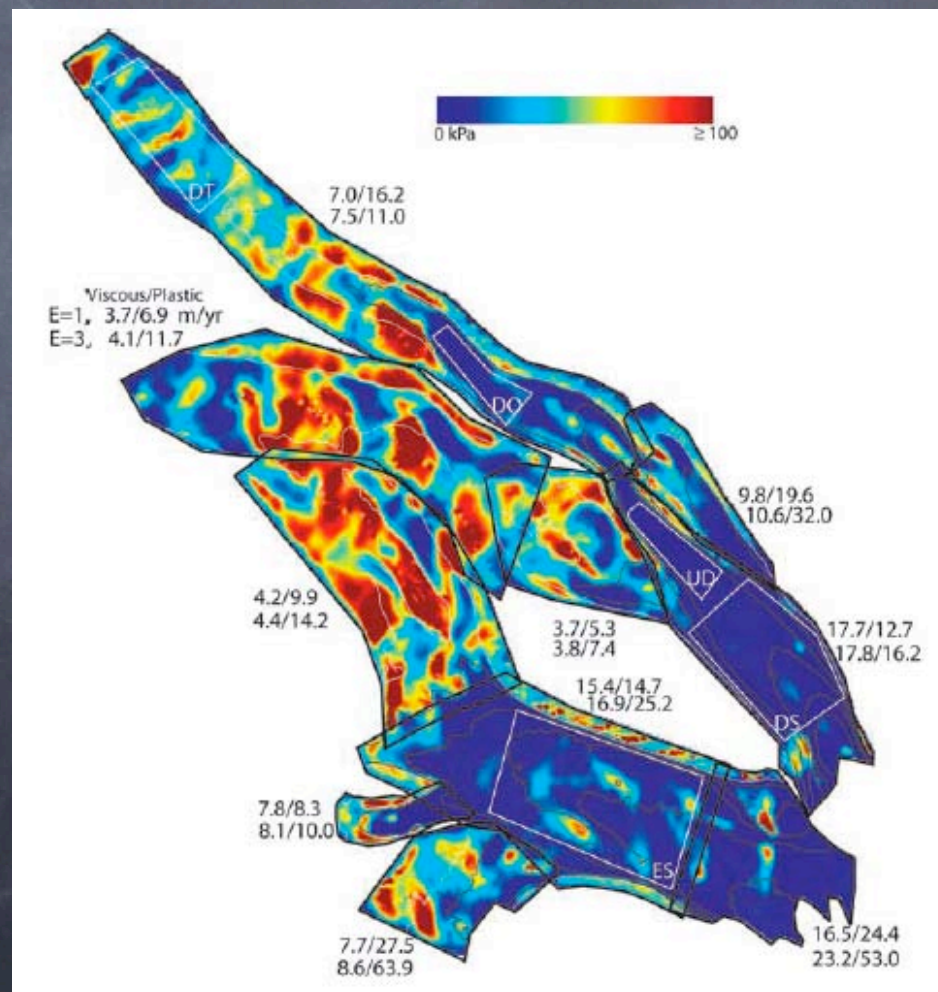
Equation	Authors' Parameter Values		$u_b$ for UpB, m yr <sup>-1</sup>		Reference	
	k or $k_1$	$k_2$ , Pa <sup>-1</sup>	$N_e$ , 10 <sup>5</sup> Pa	Using Authors' Value of $N_e$		Using Measured Value of $N_e$
$u_b = k\tau_b^2$	$1.25 \times 10^{-6} \text{ m yr}^{-1} \text{ Pa}^{-2}$			280		Thomas and Bentley [1978]
$u_b = k\tau_b^3$	$8 \times 10^{-11} \text{ m yr}^{-1} \text{ Pa}^{-3}$ $8 \times 10^{-10} \text{ m yr}^{-1} \text{ Pa}^{-3}$			270 2,700		Thomas and Bentley [1978] Thomas et al. [1979]
$u_b = k \frac{\tau_b^3}{N_e}$	$8.4 \times 10^{-9} \text{ m yr}^{-1} \text{ Pa}^{-2}$		5	0.06	0.6	Bindschadler [1983]
$u_b = k \frac{\tau_b}{N_e^2}$	$5 \times 10^9 \text{ m yr}^{-1} \text{ Pa}$		25	12	32,000	Budd et al. [1984], also van der Veen [1985]
$u_b = k \frac{\tau_b}{N_e}$	$4,670 \text{ m yr}^{-1}$		3.5	200	1,400	Lingle and Brown [1987]
$u_b = \frac{k_1 \tau_b}{N_e + k_2 N_e^2}$	$1.5 \times 10^4 \text{ m yr}^{-1}$	$2.5 \times 10^{-7}$	25	55	4,400	McInnes and Budd [1984]
$u_b = \frac{k_1 \tau_b}{(N_e + k_2 N_e^2)^2}$	$1.5 \times 10^{11} \text{ m yr}^{-1} \text{ Pa}$	$3.5 \times 10^{-7}$	25	100	870,000	Jenssen et al. [1985]

Two calculations are presented where relevant, one using the definition of  $N_e$  preferred by the author(s) of the model, and one with the seismically determined value  $N_e = 0.5 \times 10^5 \text{ Pa}$ . Other numerical values used are  $\tau_b = 1.5 \times 10^4 \text{ Pa}$  and  $h^* = 250 \text{ m}$  (cf. (7)). The measured value of  $u_b$  is  $443 \text{ m yr}^{-1}$ .

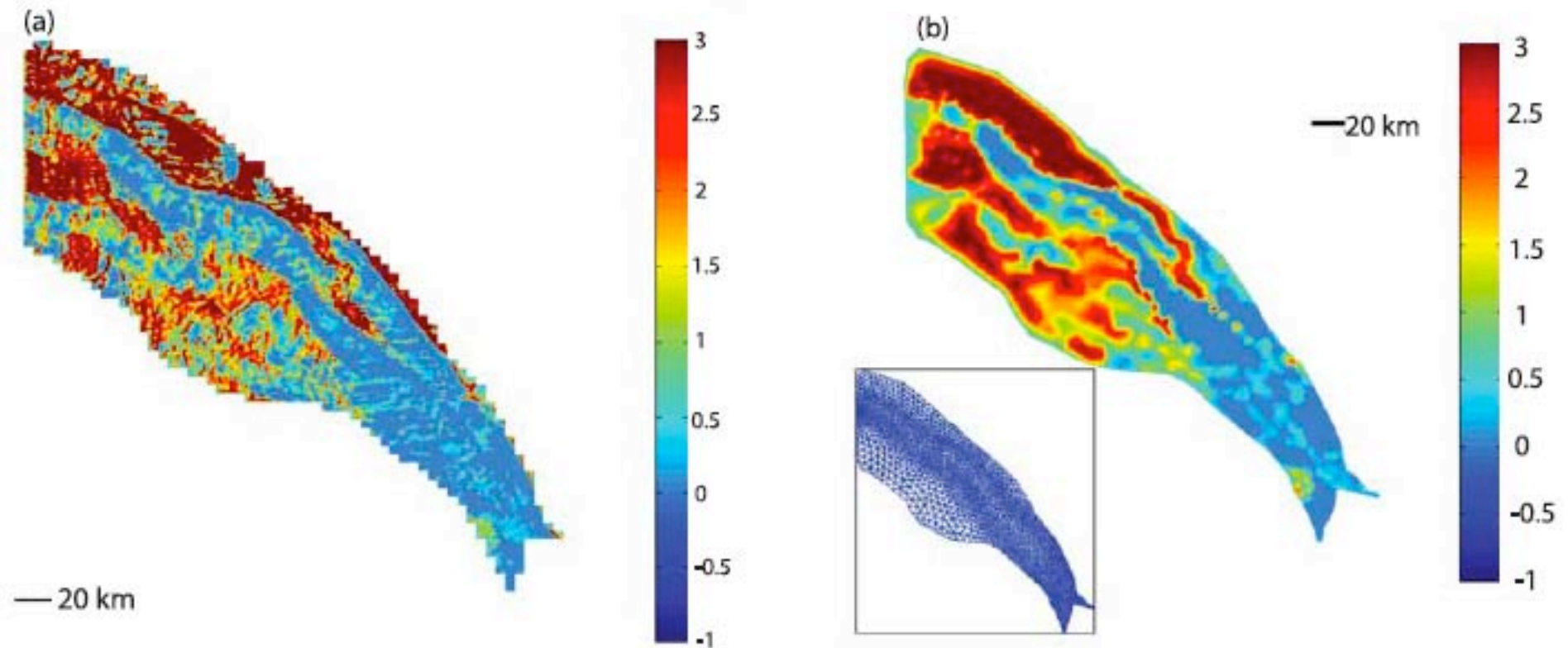
### C. Data for parametrizations of physical processes:

1. Basal sliding velocity and basal tractions – Ice motion over sediments important because it will tend to happen in places showing dynamic behavior (e.g. marine ice sheets overriding marine sedimentary basins – e.g. West Antarctica – and in fjords – Greenland outlet glaciers)

There are some aerogeophysical and surface geophysical data to develop qualitative bed classifications (bedrock vs. sediments). Another approach is to measure the surface velocity field and invert it for basal shear stress. You can call the weak areas 'seds' and the strong ones 'bedrock' (it's a free country) and you also got extra quantitative info you can use to initialize your basal submodel.



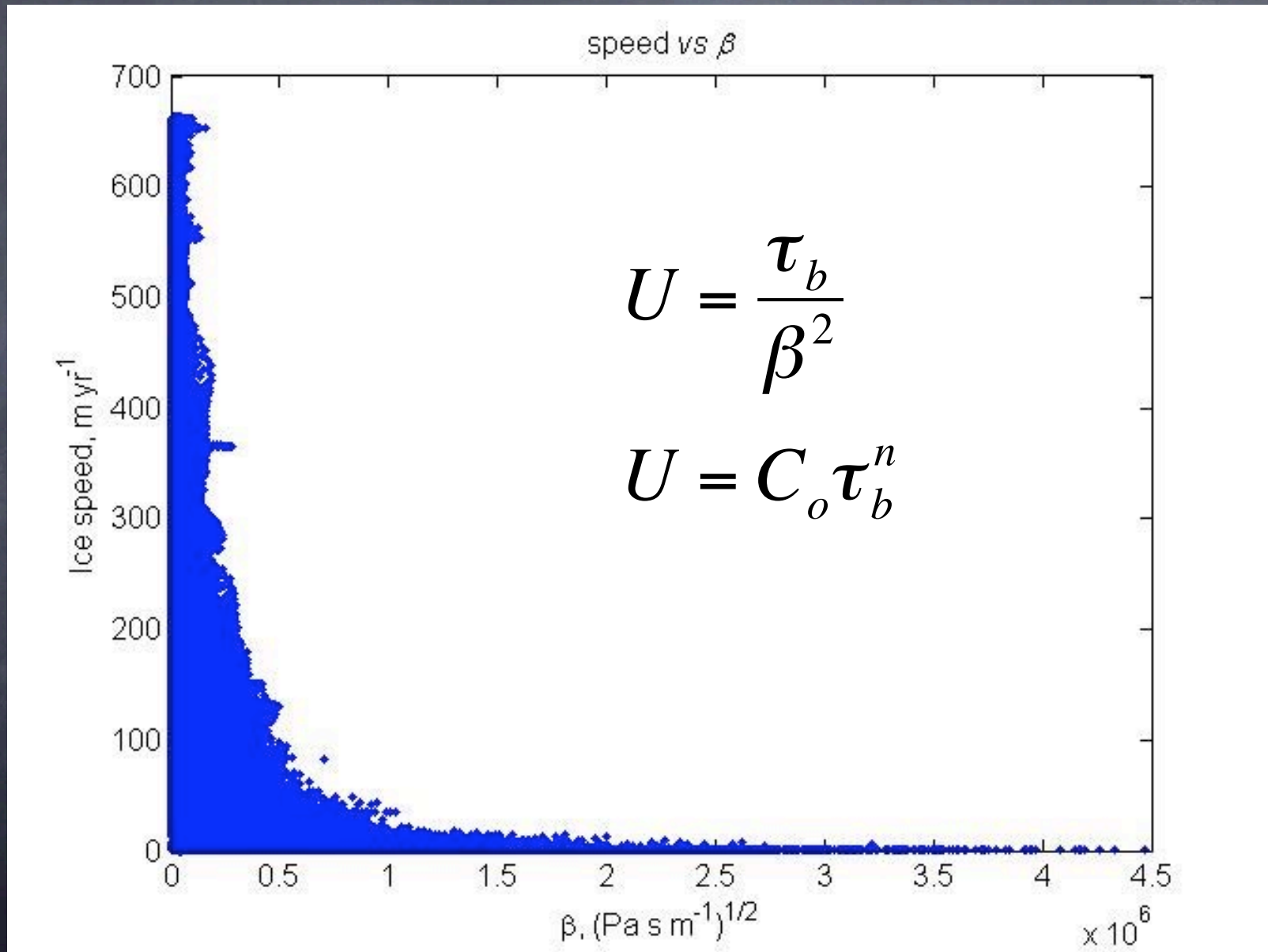
What you are inverting for depends on your assumption about the 'sliding law' - basal sliding parameter (e.g. Sergienko et al., 2008)



**Figure 5.** Inverted basal friction coefficients  $\beta$ ,  $10^5 \text{ (Pa s m}^{-1}\text{)}^{\frac{1}{2}}$ . (a) Block-wise inversion and (b) whole domain inversion, inset shows finite element mesh used for that inversion. Both fields have similar patterns, but block-wise inversion reveals large small-scale variations unable to be resolved by whole domain inversion.

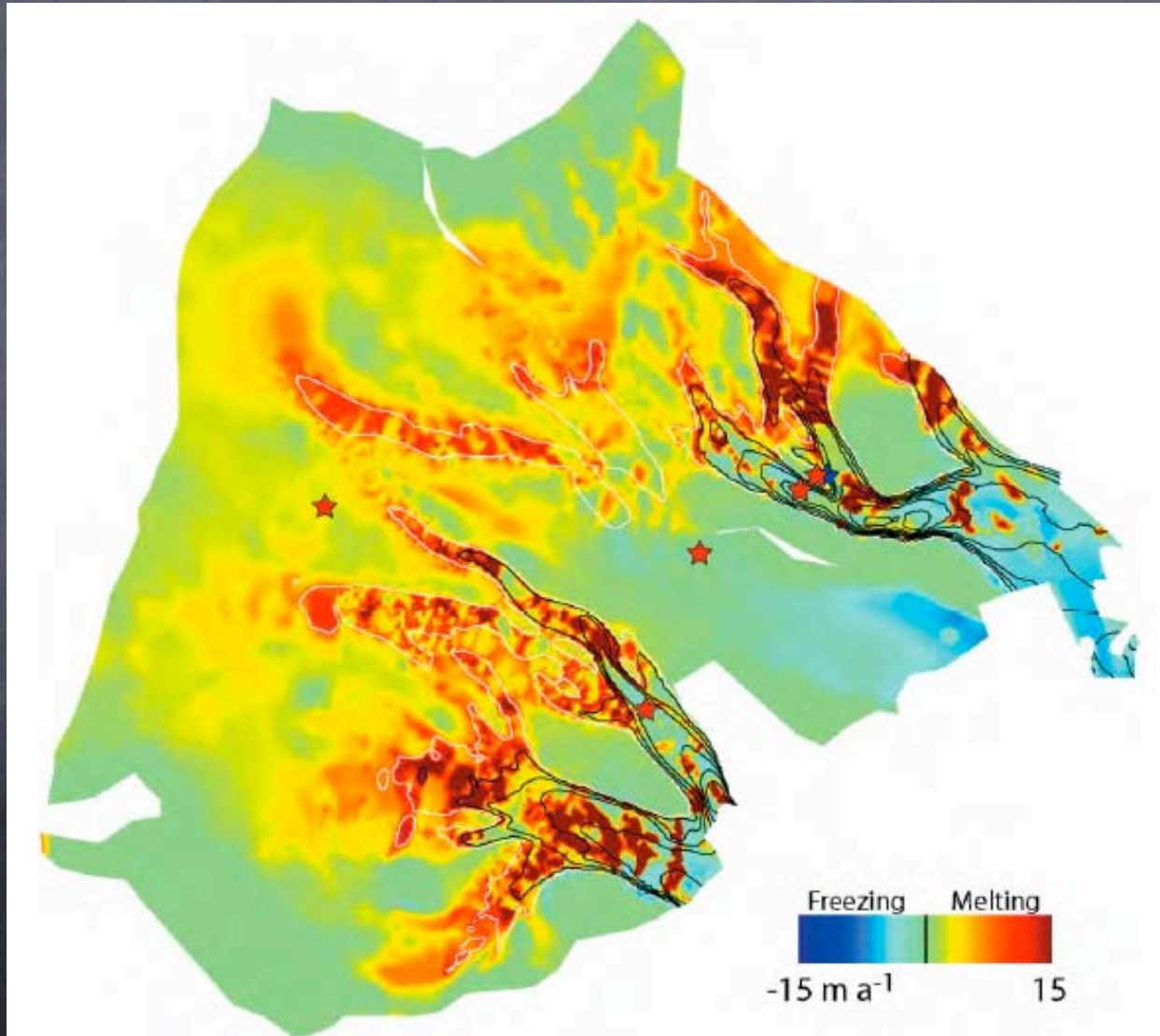


Proof that sliding velocity is not determined locally (bad SIA, bad SIA). Figure from Dr. Olga Sergienko



### C. Data for parametrizations of physical processes:

1. Basal sliding velocity and basal tractions – Once you've inverted for basal shear stress, you can calculate basal shear heating, and if you also calculated basal temperature gradients, and made some assumption about geothermal heat flux, then you can calculate the basal thermal energy balance and basal melting/freezing rates. Again, you can use this to initialize your basal submodel, especially if you're brave enough to include subglacial water flow in it (Johnson and Fastook, 2002).



How is basal sliding represented in IS models now?

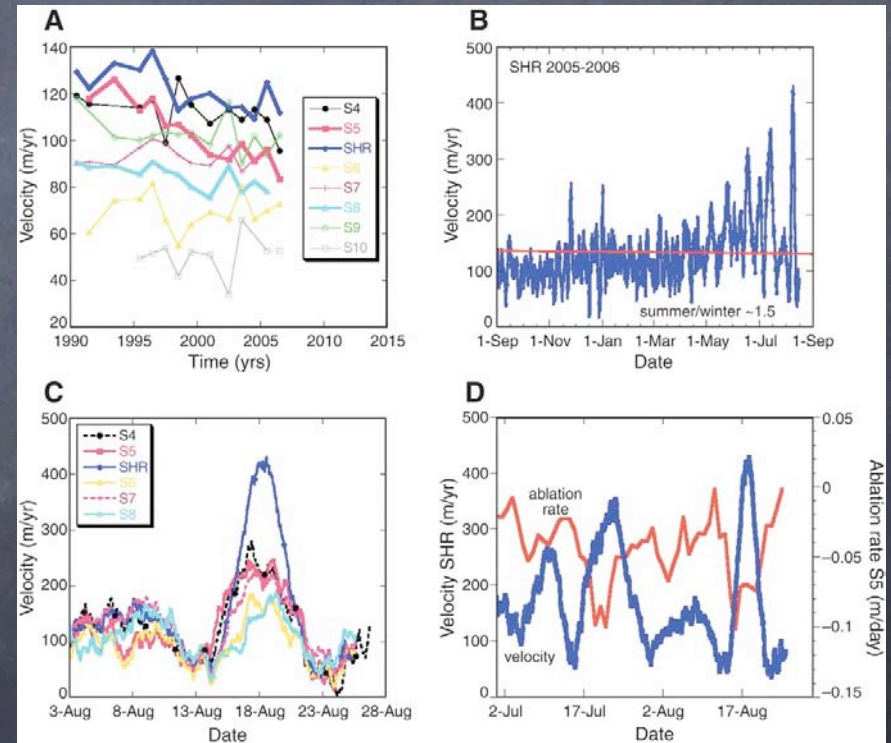
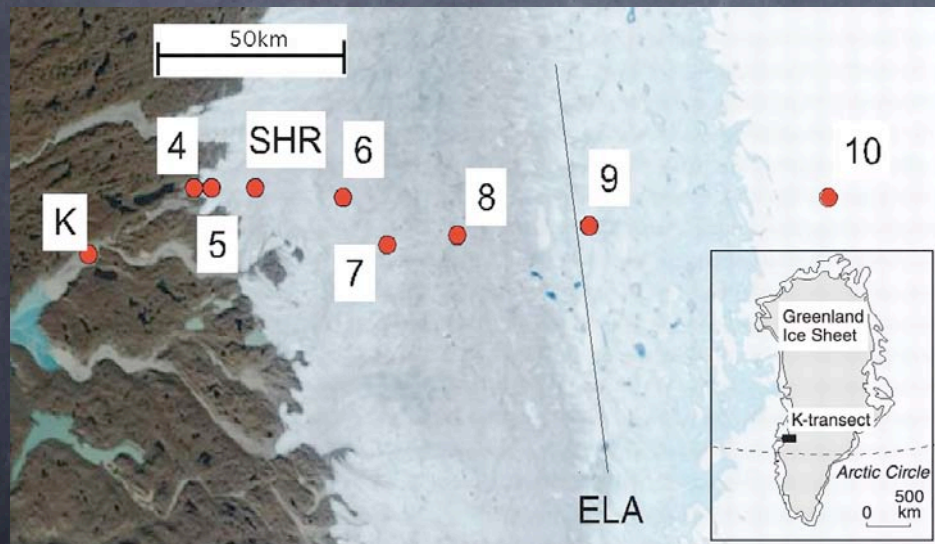
(Nearly)Linear dependence of sliding velocity on stress ( $n > 1$ , e.g. Hebeler et al., 2008)

$$v_b = \tau_b^n$$

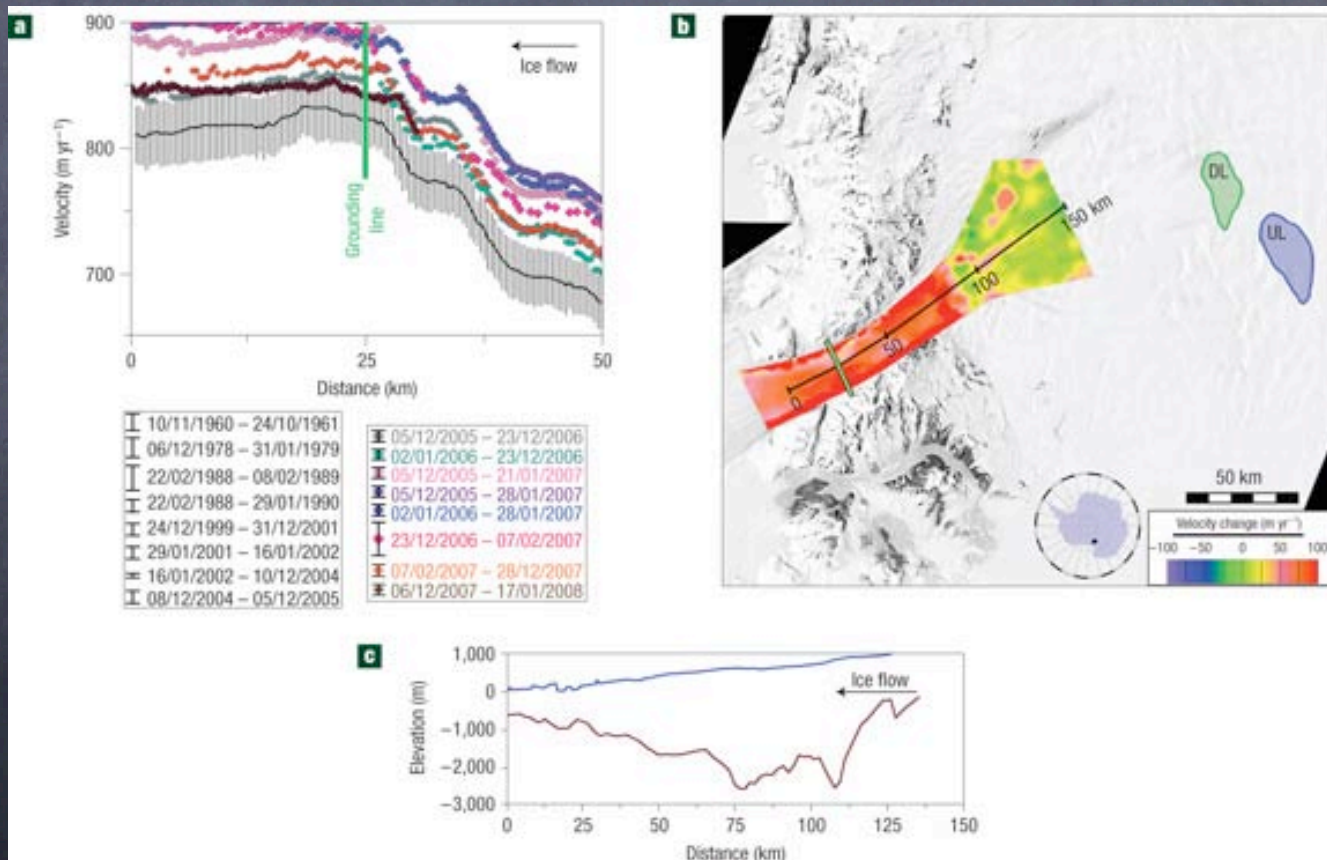
However, observations provide support for non-linear basal sliding law ( $n \rightarrow 2, 3 \rightarrow \text{Infinity}$ , e.g. Tulaczyk, 2006)

Non-linear parametrizations of basal sliding will make the model predictions more sensitive to stress perturbations (e.g., iceberg calving, changes in ice stream geometry, ice shelf back-pressure changes, increased lubrication of the bed by water)

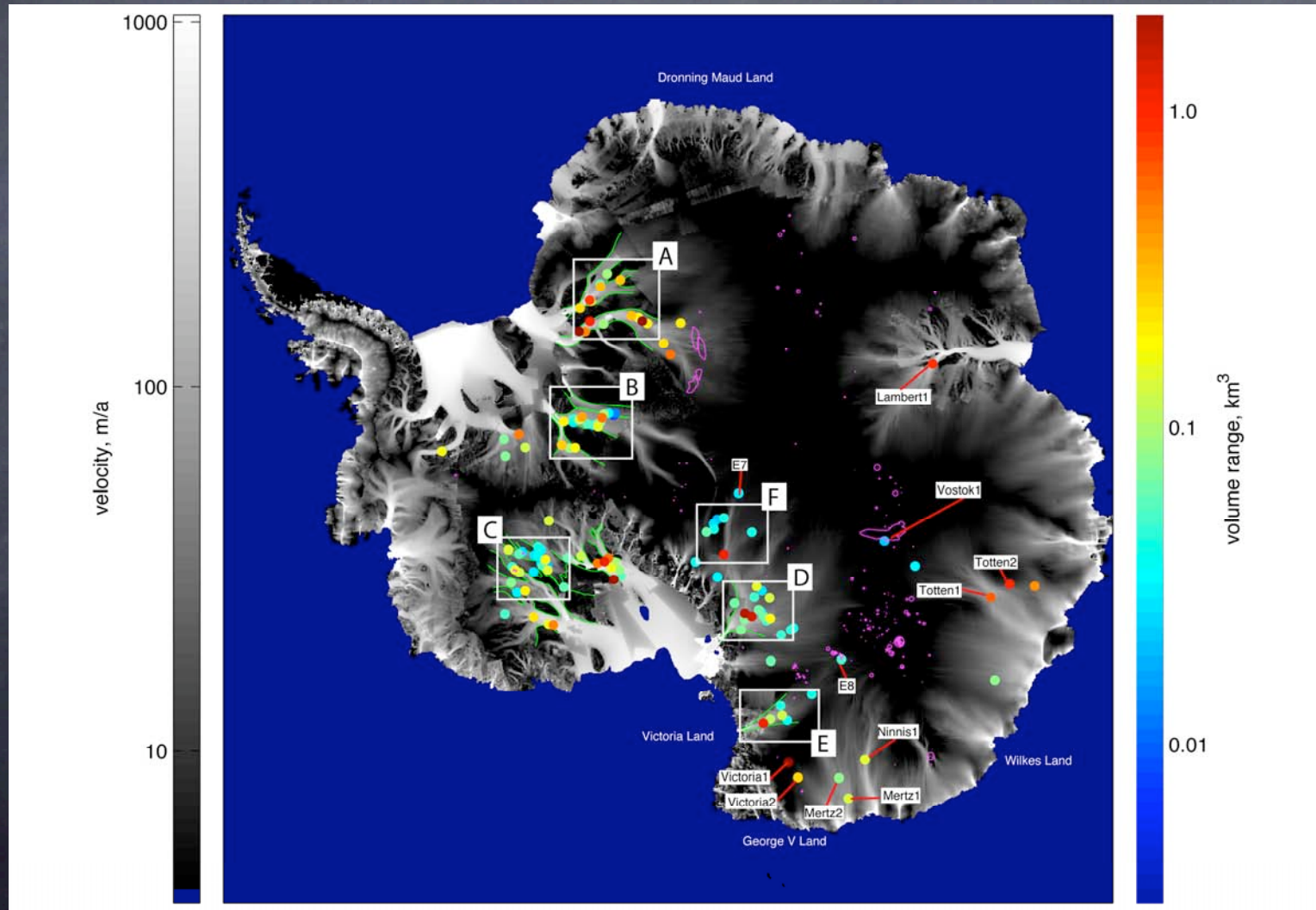
# Velocity of Greenland ice sheet margin fluctuates in response to changes in surface meltwater input to the bed



In Antarctica discharge of water from active subglacial lakes changes ice velocity (Stearns et al., 2008)



More than 120 active lakes discovered by surveying all GLAS laser altimeter data (~5 years, Smith et al., 2009)



## Basal sliding and hydrology challenges:

(1) Getting better data/estimates of spatial distribution of key parameters (e.g., bed topography, basal sliding coefficient, geothermal flux)

(2) Allowing basal boundary conditions to evolve through time (e.g. coupling of hydrology and sliding, perturbations from surface water input, lake discharges, volcanic/geothermal subglacial events?)

### C. Data for parametrizations of physical processes:

2. Iceberg calving – This is the process that is responsible for most ice mass loss from Antarctica. Parametrizations in IS models in the past are very crude (e.g. 100% of ice that gets to the grounding line just disappears due to calving. This is a very restrictive assumption which will make your IS model less interesting than real ice sheets. There are recent developments that will permit making the calving rate dependent on quantities that are observed or calculated in the model (e.g. horizontal ice strain rates) (hopefully Dr. Dupont will speak on that).
3. Grounding line dynamics – What controls horizontal migration of marine ice sheet margins – recent theoretical treatment by Schoof, 2007, more relevant process-oriented studies are ongoing and will start in the near future (e.g. role of basal melting in grounding line stability).



And you thought that solving the 3D mass and momentum conservation equations was the hard part?

