Finite Differencing: Introduction

- To do ice sheet modelling (and lots of other things), we have to solve differential equations
- ***BUT***, these deal with the gradients of continuous functions in the limit of infinitessimal changes
- A potentially ***infinite*** amount of information...

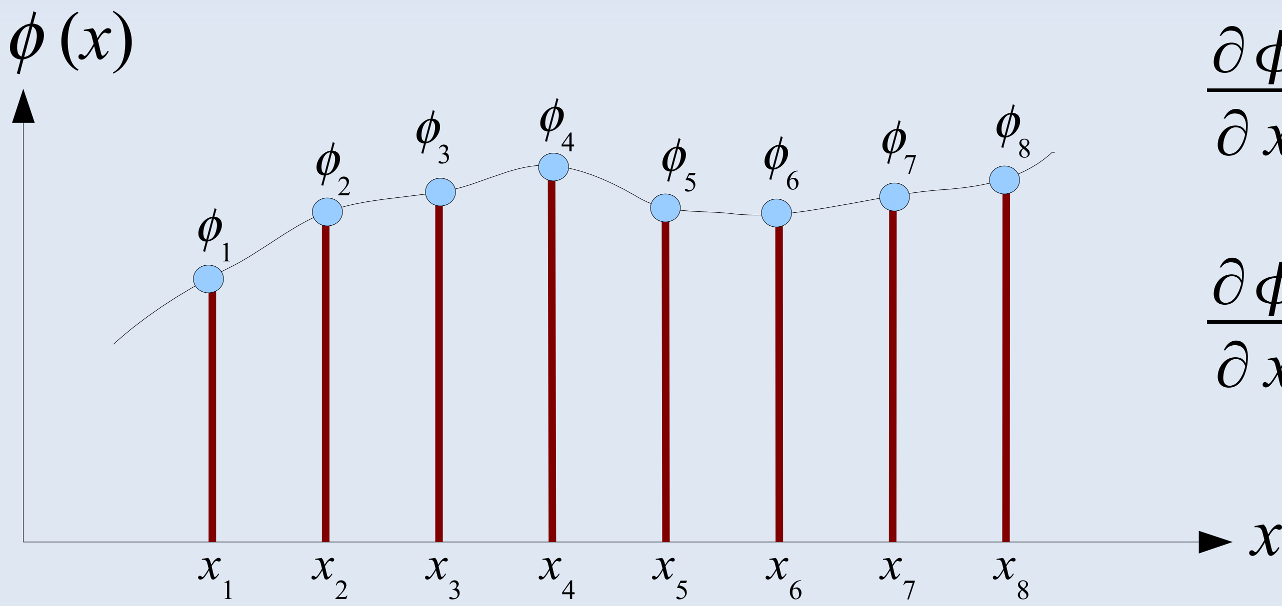
Tricky...

So... discretize

- Discretization is any technique for representing continuous functions as a set of discrete values
- Many common techniques:
  - Finite difference
  - Finite volume
  - Finite element
  - Spectral decomposition
- Finite difference is probably the simplest and most intuitive

Intuitive explanation

- Easy part is to divide a continuous function into a set of discrete points



$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i}$$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

But...

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

- How accurate is this expression?
- *Where* is it valid?
- What about higher-order derivatives?

More formal: Taylor's Theorem

- Taylor's theorem can be used to construct FD expressions:

$$\phi_{i+1} = \phi_i + \frac{\Delta x}{1!} \left. \frac{d\phi}{dx} \right|_i + \frac{\Delta x^2}{2!} \left. \frac{d^2\phi}{dx^2} \right|_i + O(\Delta x^3)$$

$$\phi_{i-1} = \phi_i - \frac{\Delta x}{1!} \left. \frac{d\phi}{dx} \right|_i + \frac{\Delta x^2}{2!} \left. \frac{d^2\phi}{dx^2} \right|_i + O(\Delta x^3)$$

$$\left. \frac{d\phi}{dx} \right|_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} + O(\Delta x^3)$$