A brief history of grounding line treatments

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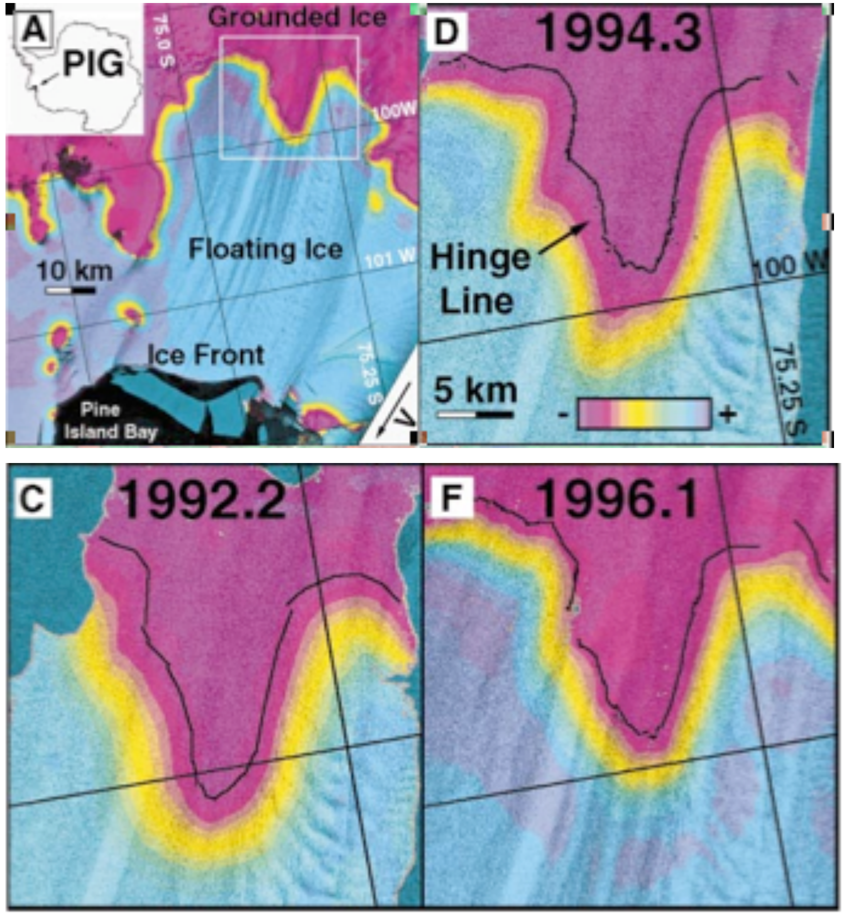
And many conversations with

Duncan Wingham, Andrew Fowler,

Christian Schoof, Richard Hindmarsh…

To float or not to float, that is the question…

- Marine ice sheets and instability
- Comparison of grounding line treatments
- Detailed studies of grounding lines
- Challenges



(Rignot, 1998)

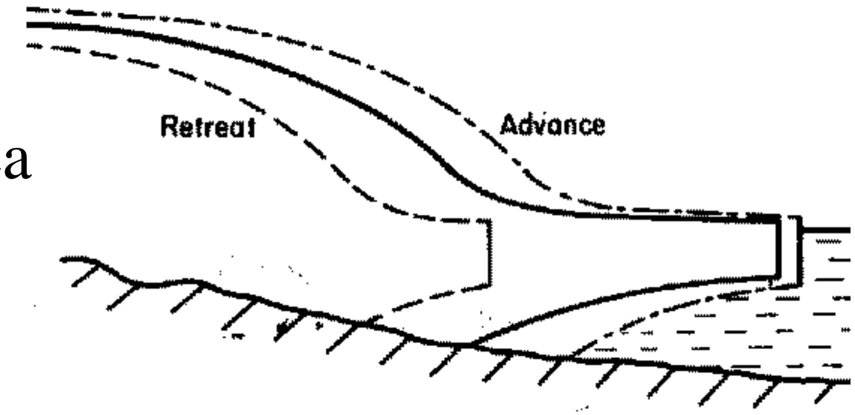
Marine ice sheet instability

The key to the marine ice sheet instability argument is that the mass flux at the grounding line is controlled by ice shelf dynamics, hence a unique function of sea level (Weertman, 1974).

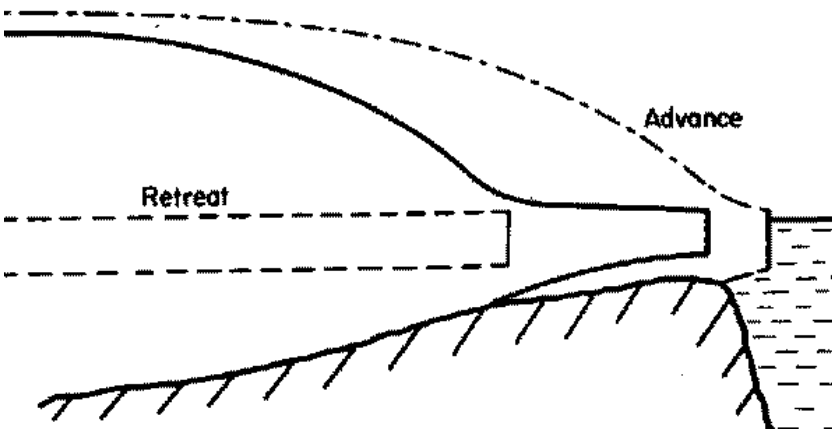
A retreat of grounding line into a deeper bedrock would lead to an increased discharge and further retreat.

In contrast, should the mass flux at the grounding line be controlled by ice sheet dynamics, then the uniquess argument might not hold (Hindmarsh, 1993).

Stable marine ice sheet



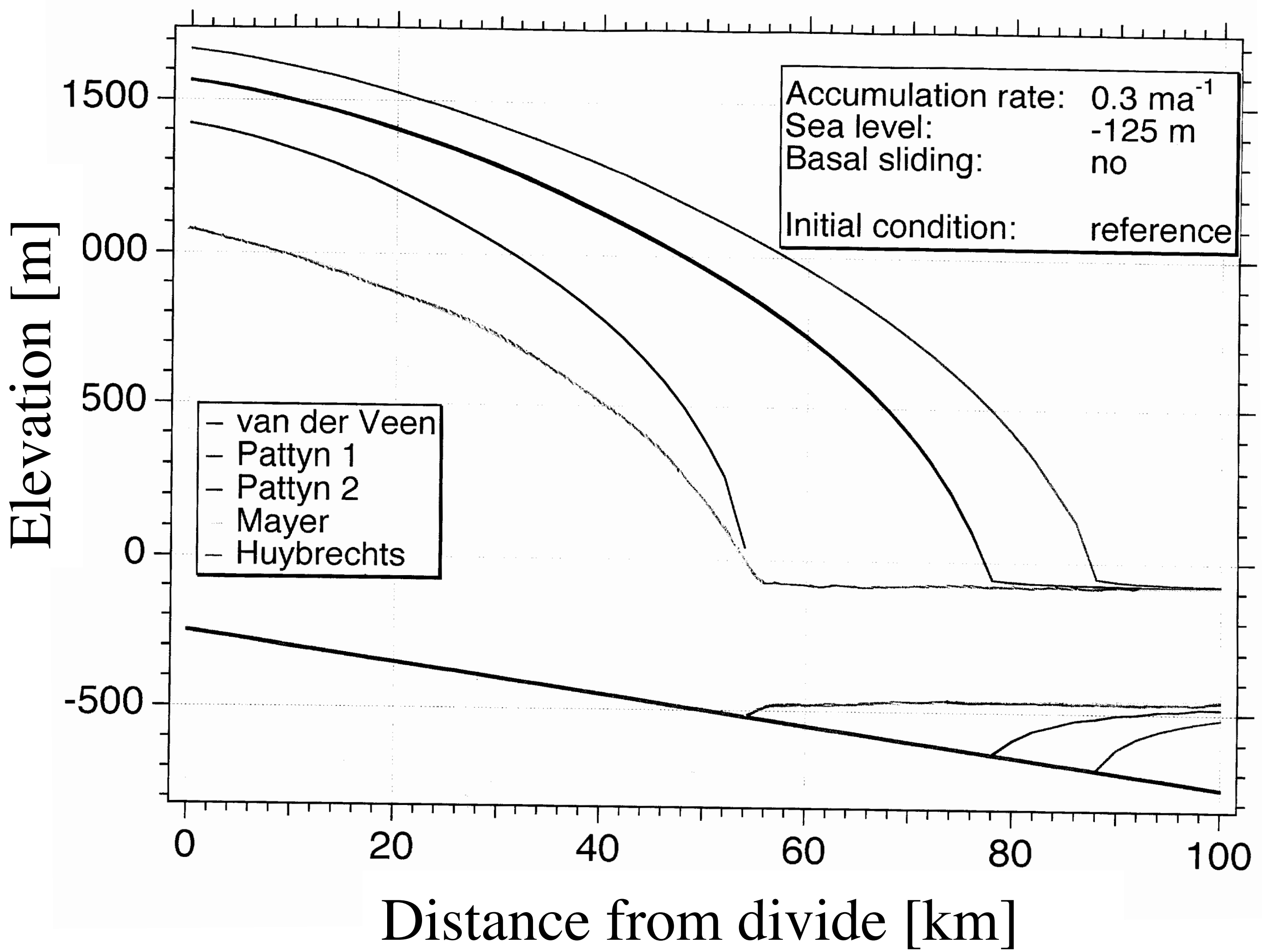
Unstable marine ice sheet



(Thomas, 1979).

Comparison of grounding line treatments

(Huybrechts, 1997)



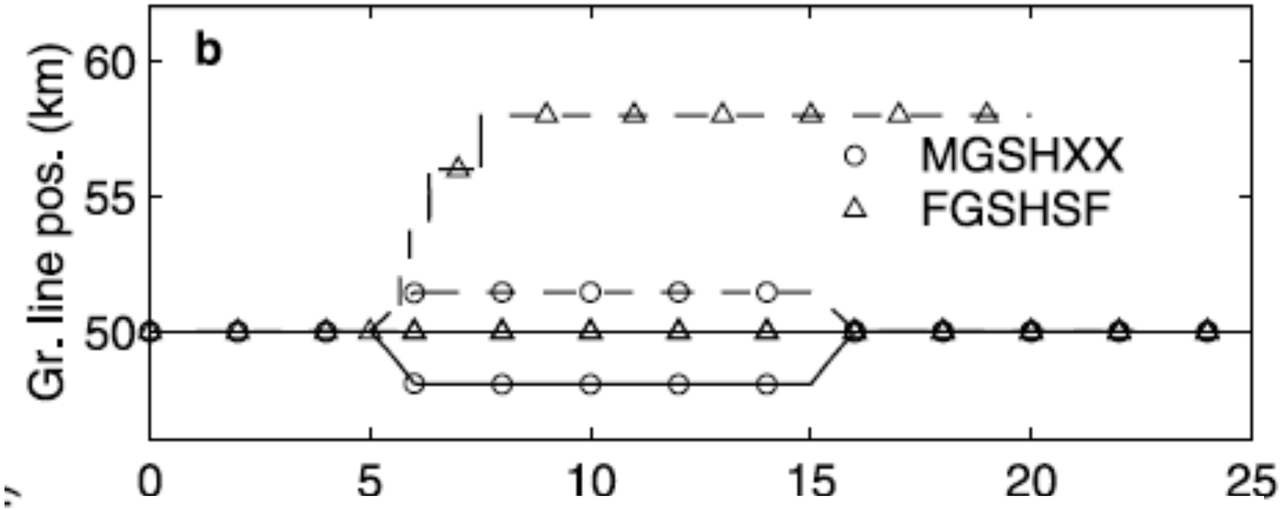
Result of EISMINT: No consensus on how to model MIS.

Comparison of grounding line treatments
(Vieli & Payne, 2005)

Table 1. Summary of Used Numerical Models for Grounding Line (GL) Migration, With Citations Where Appropriate

Name	Short Description	Type of Grid	Coupling at GL	GL Treatment
		Fixed Grid Models		
FGSHSF	sheet shelf, *Huybrechts* [1990], *Ritz et al.* [2001]	fixed grid	no mechanical coupling, coupling only through flux and thickness evolution	flotation condition
FGSTSF	stream shelf	fixed grid	full mechanical coupling	flotation condition
		Moving Grid Models		
MGSHXX	sheet, *Hindmarsh and LeMeur* [2001]	moving grid	no coupling (no shelf)	GL migration equation
MGSHSF	sheet shelf	moving grid	no mechanical coupling, only through flux at GL	GL migration equation
MGSTSF	stream shelf	moving grid	full mechanical coupling	GL migration equation

Grounding line response to change in sea level



Time (ka)

Major conclusions:

- Grid size and grounding line physics matter.

- No reliable model of GL migration.

Trade off between flotation condition and GL migration

(Pattyn, 2006)

- Allow the determination of grounding line to subgrid precision.

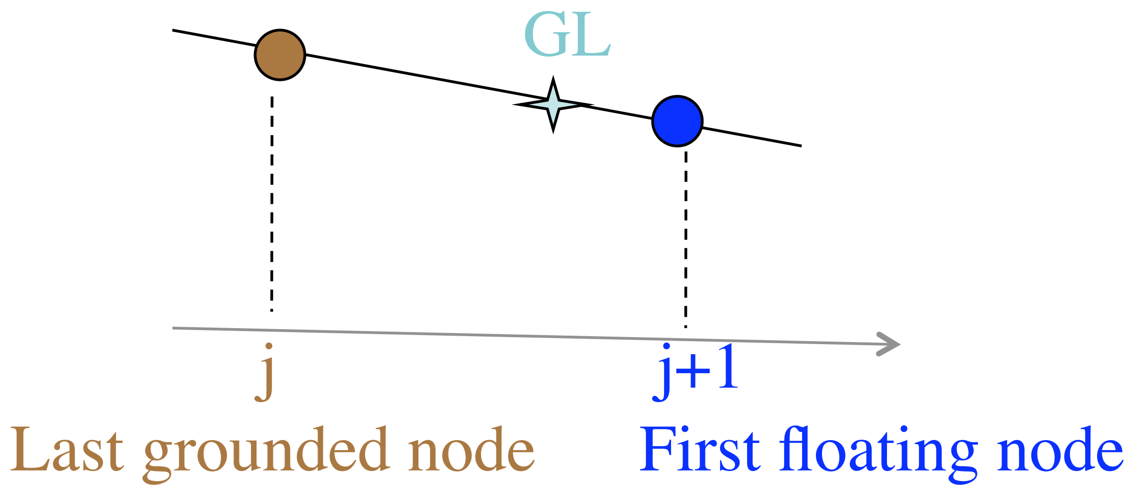
$$f = \frac{(l-b)\rho_w}{\rho_i h} \quad \text{where} \begin{cases} f < 1 & \text{for grounded sheet} \\ f = 1 & \text{at grounding line} \\ f > 1 & \text{for ice shelf} \end{cases}$$

- Grounding line position:

$$x_g = \frac{1 - f_j + x_j \nabla f}{\nabla f}$$

where

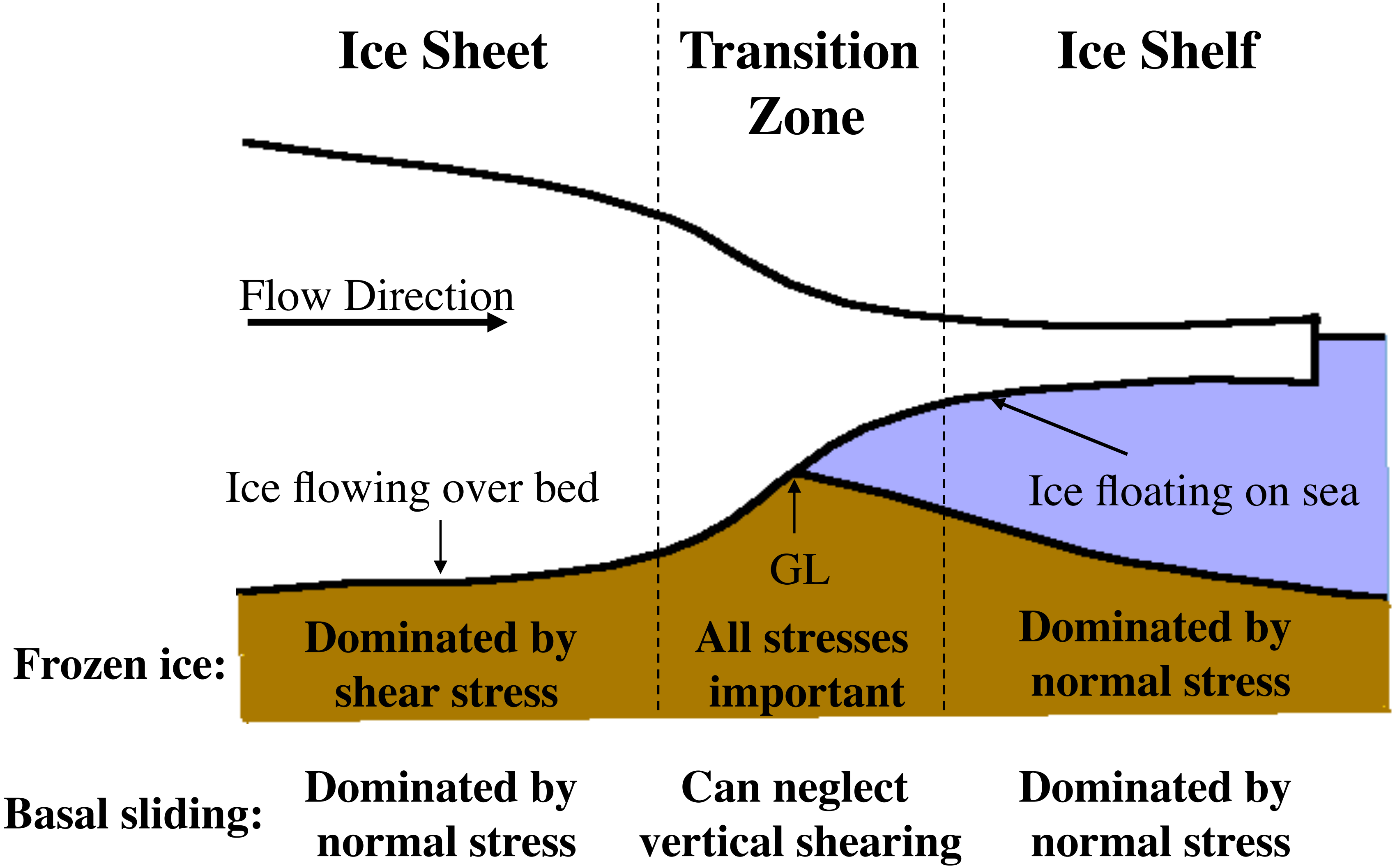
$$\nabla f = (f_{j+1} - f_j)/\Delta x$$



- Procedure still affected by the grid size since requires interpolation of variables near the grounding line.

Modeling marine ice sheets

$$\nabla \cdot \underset{\rightarrow}{V} = 0, \quad \nabla \cdot \underset{\rightarrow}{T} + \rho \underset{\rightarrow}{g} = 0, \quad \varepsilon_{\mathrm{ij}} = A T_{eff}^{n-1} T_{ij}^{'}$$

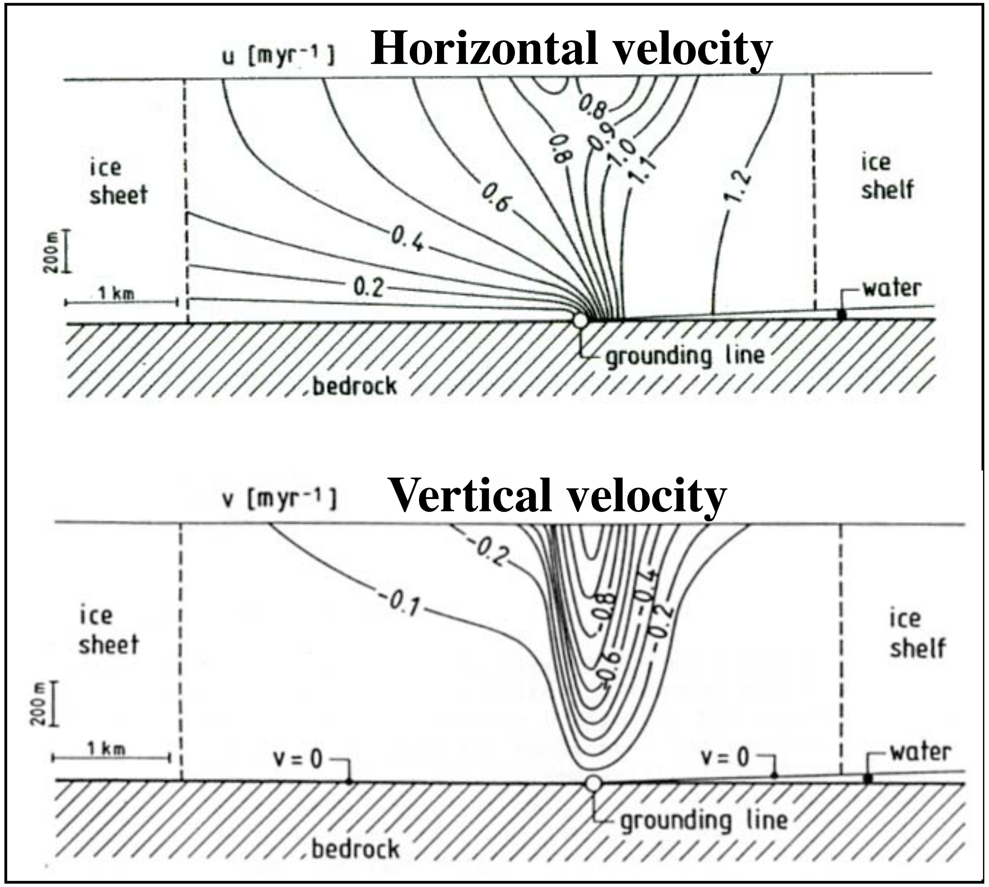


First detailed analysis of flow in vicinity of grounding line (Herterich, 1987)

- Transition zone: fixed shape.
- Not full steady state Stokes equations (ignores horizontal variations in shear stress):

$$-\rho g \frac{\partial h}{\partial x} + 2\frac{\partial T'_{XX}}{\partial x} + \frac{\partial T'_{XZ}}{\partial z} = 0$$

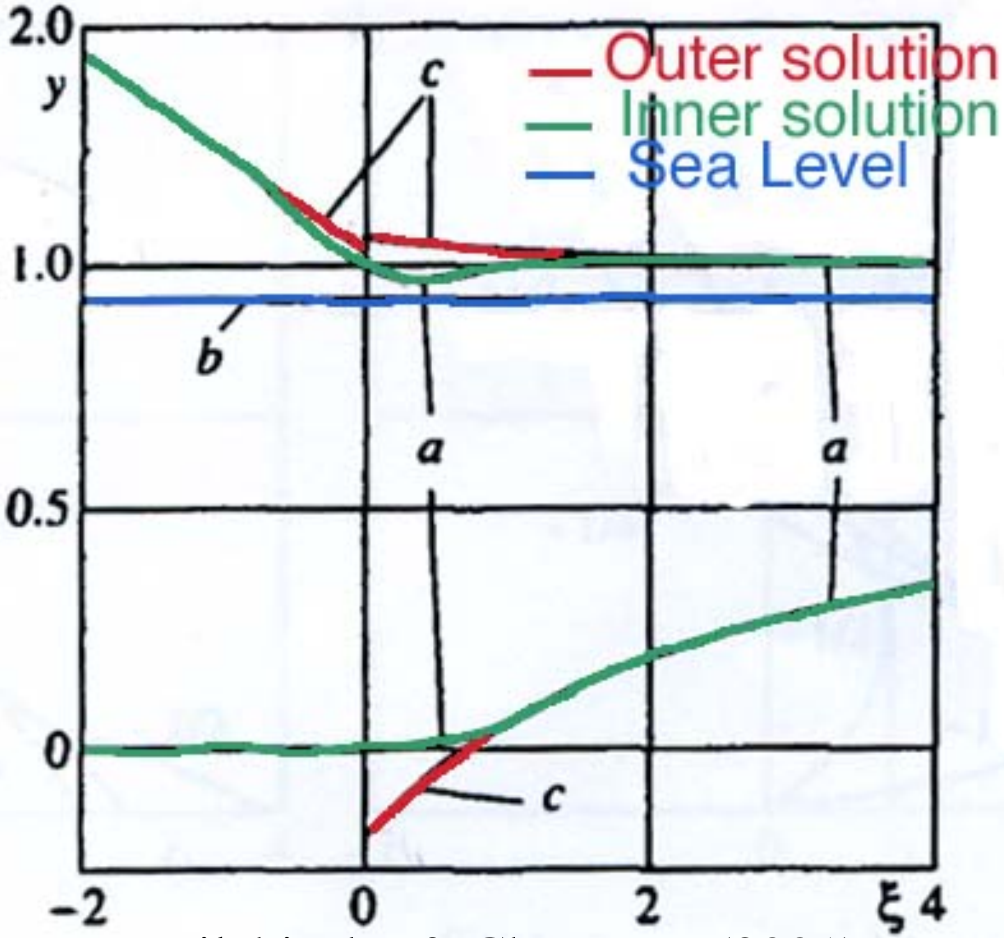
- Linear rheology.
- Constant temperature.
- **Size of TZ ~ ice thickness.**
- **Gradual transition from sheet to shelf flow.**



No basal sliding

Chugunov & Wilchinsky (1996)

Need to solve the full Stokes, free surface problem.



Wilchinsky & Chugunov (2001)

Two non-dimensional parameters

$$\varepsilon \sim \frac{\tau_{xz}}{\rho g H_{st}} \sim \frac{\partial s}{\partial x} \sim 10^{-3}$$

$$\delta \sim \frac{\tau_{xx}}{\rho g H_{sf}} \sim \left(\frac{\rho_w}{\rho} - 1\right) \sim 10^{-1}$$

$$\frac{H_{sf}}{H_{st}} \sim \left(\frac{\varepsilon}{\delta}\right)^{1/3}$$

Assuming that

$$\frac{\partial b}{\partial x}\left(x_g\right) = 0 \quad \frac{\partial^2 b}{\partial x^2}\left(x_g\right) = 0$$

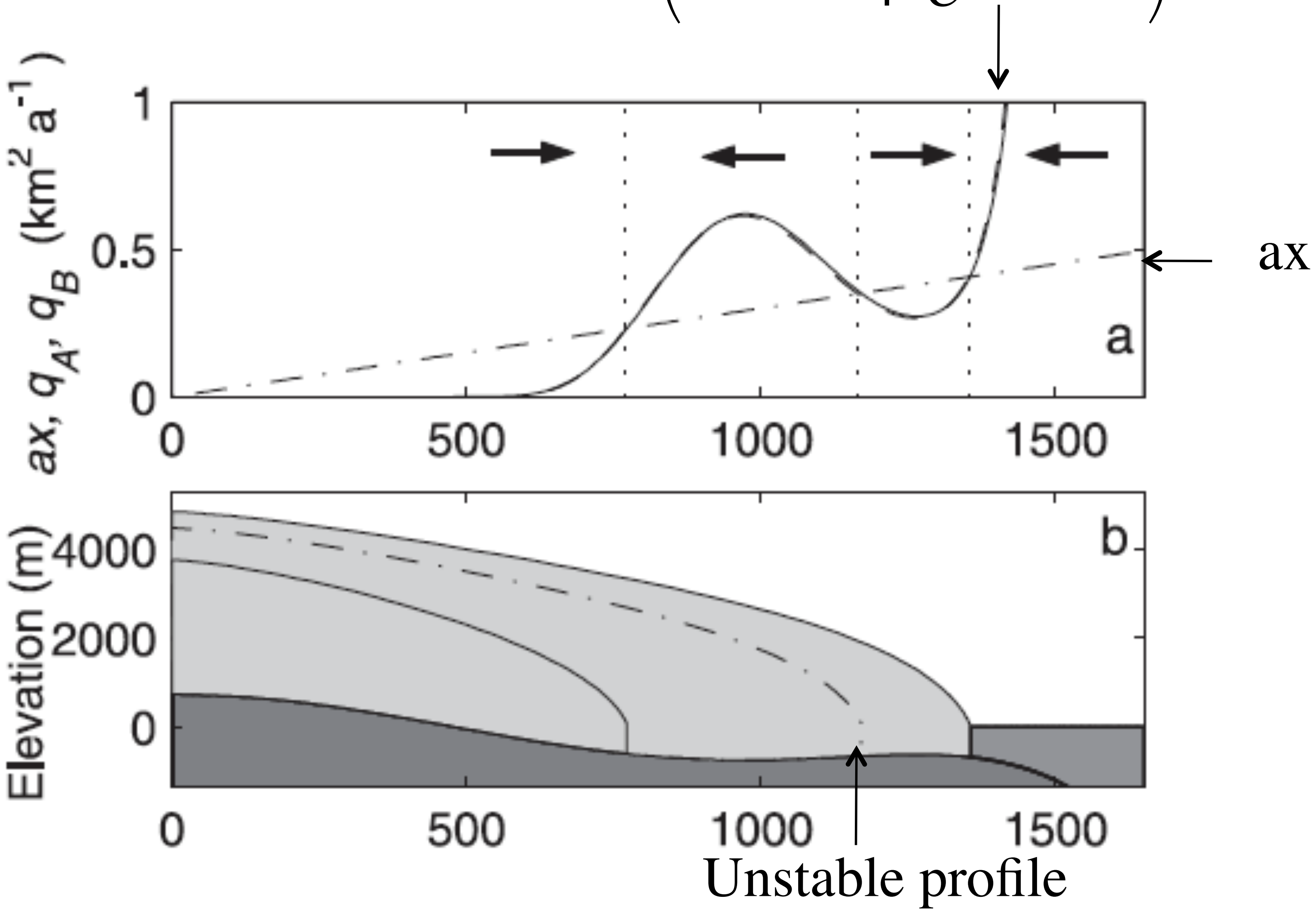
then $\quad q = q(l)$

Rapid basal sliding

Schoof (2007)

The mass flux at the grounding line is an unique function of sea level

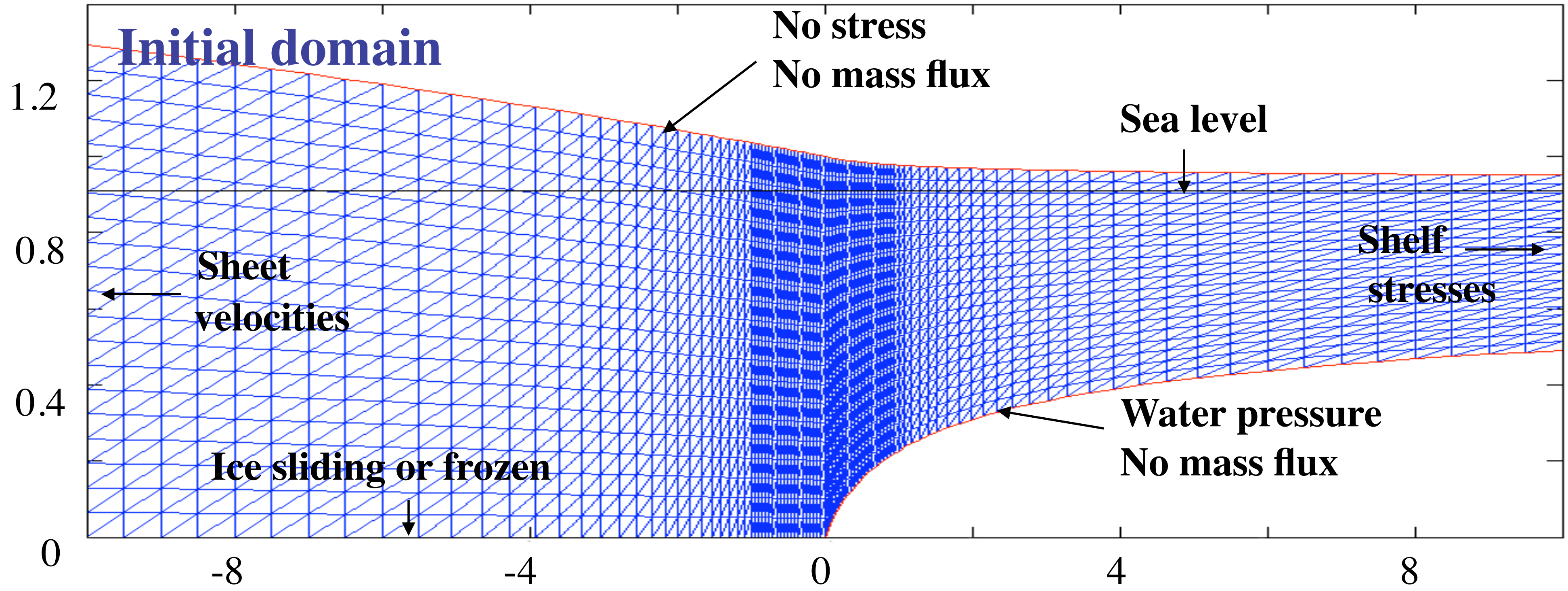
$$q = \left(\frac{A(\rho g)^{n+1}(1-\rho/\rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h^{\frac{m+n+3}{m+1}}$$



Contact inequalities for the grounding line

Nowicki & Wingham (2008)

- Full Stokes finite element model.
- Glen's flow law.
- Grounding line is fixed.
- Free surfaces to be determined.



Once a steady state solution is found check that:

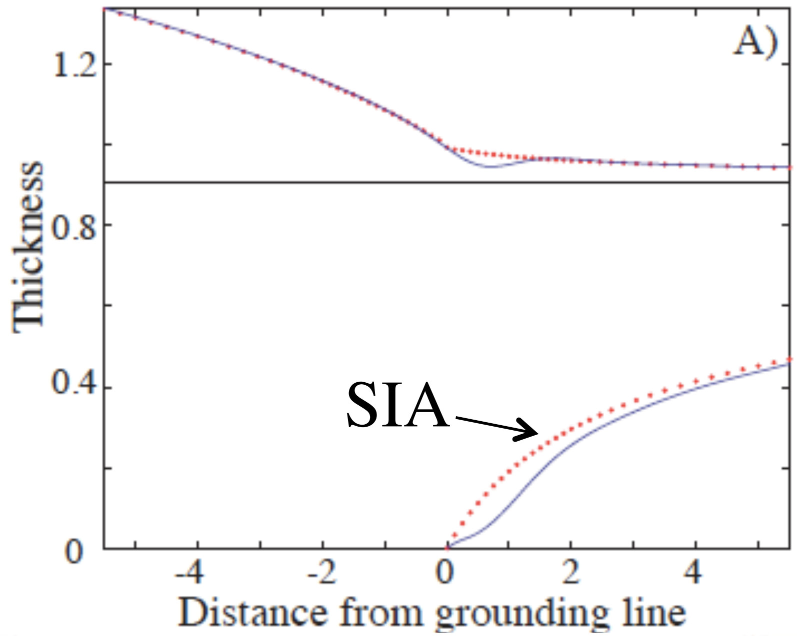
- Upstream from the grounding line: $\tau_{zz} + p_w \leq 0$
- Downstream from the grounding line: $b > b_r$

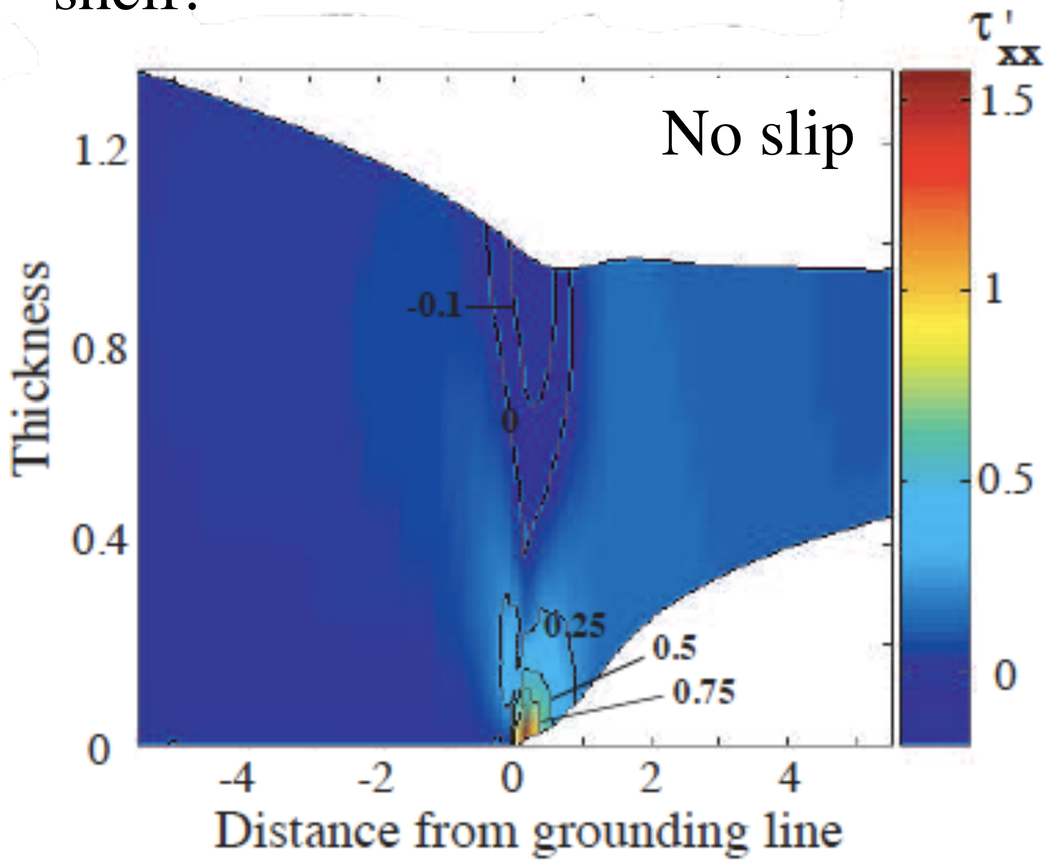
Cf: subglacial cavities (Lliboutry, 1968, Iken, 1981; Schoof, 2005, Gagliardini et al., 2007)

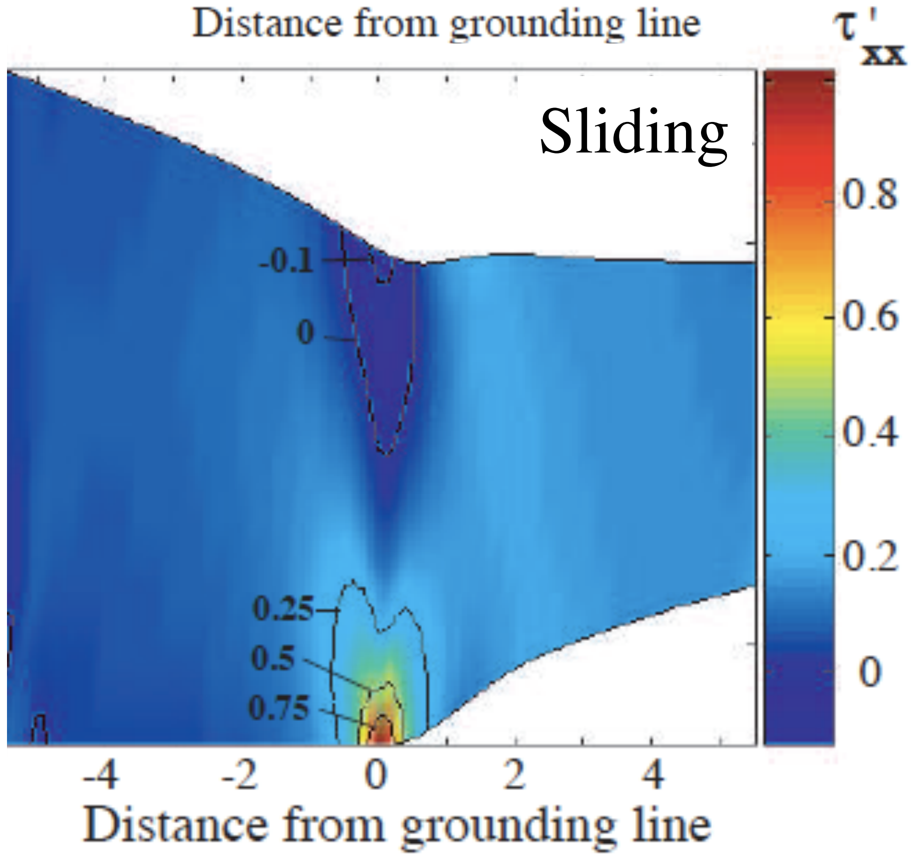
Why use full Stokes?

(Nowicki, 2007)

- Surface dip over the grounding line.
- Takes a few ice thickness for shelf to adjust to SIA.
- Stress transfer between sheet and shelf.

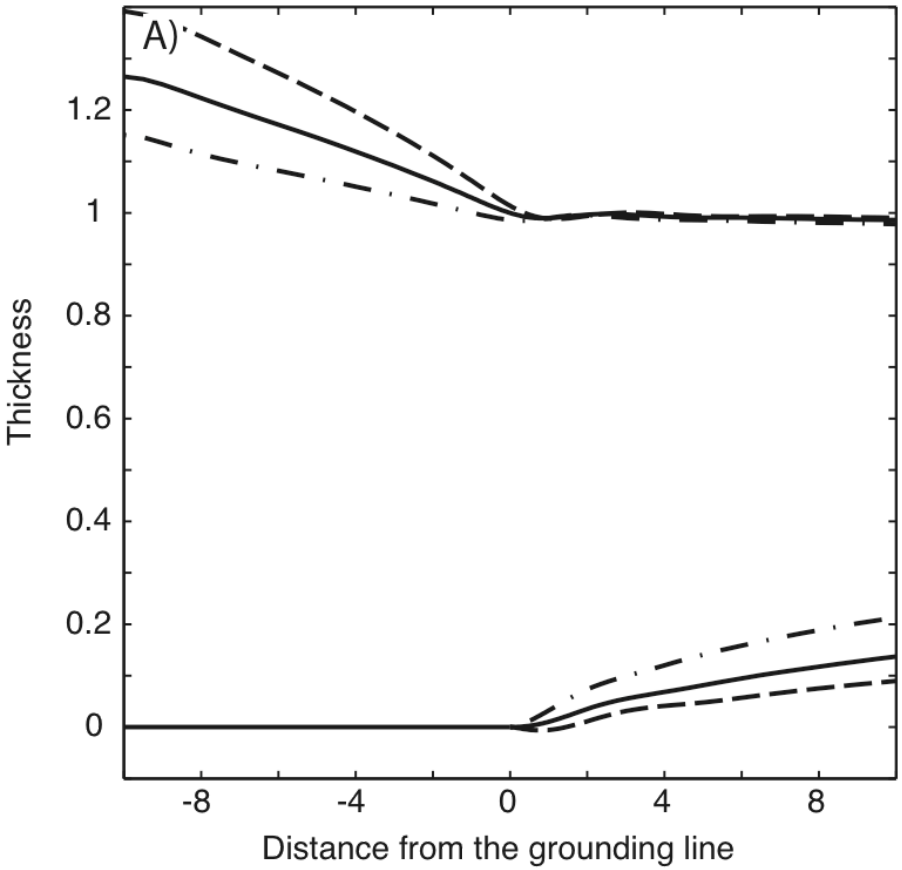




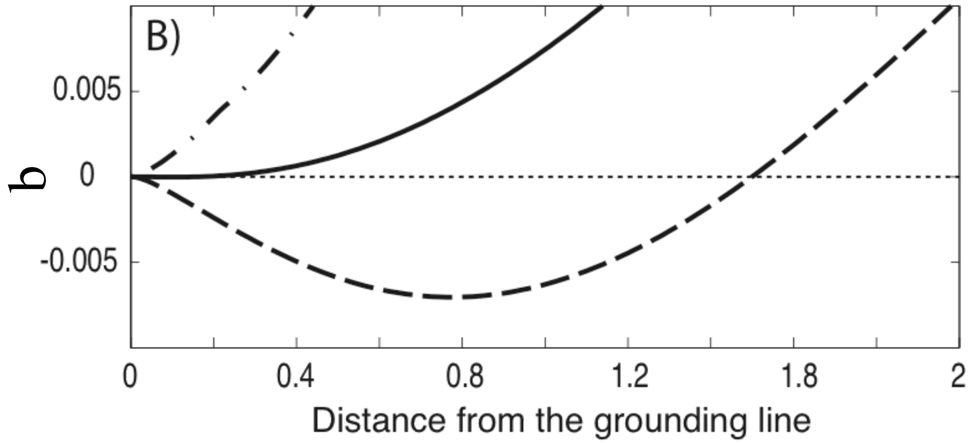


Contact problem for fixed basal slipperiness, varying mass flux.

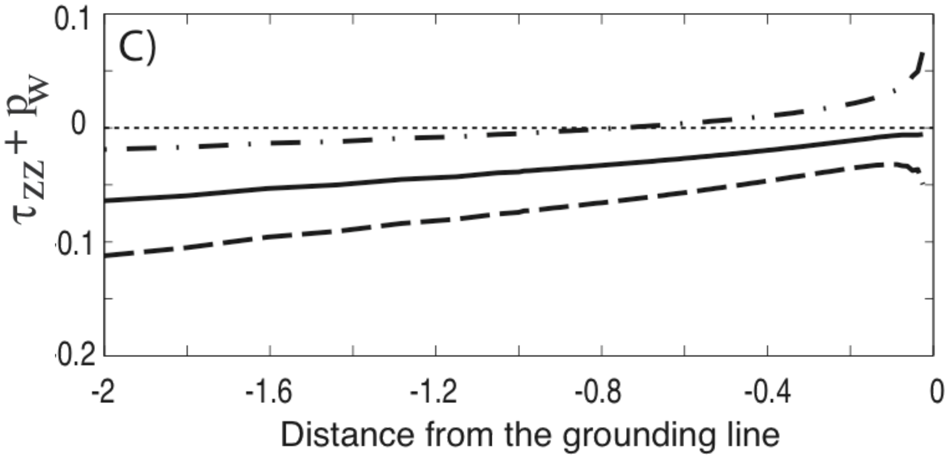
Final surface profiles



Shelf lower surface

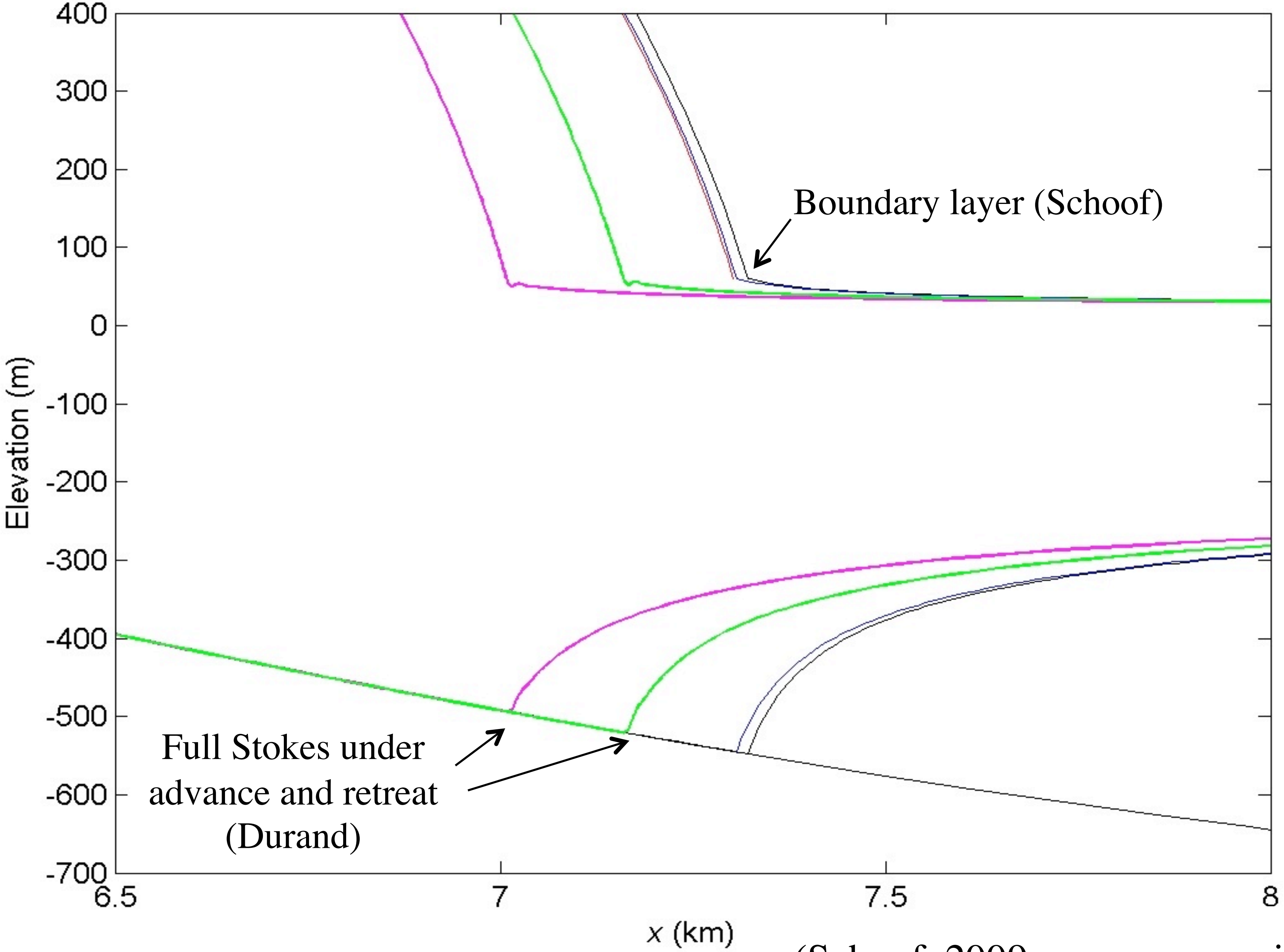


Upstream stresses



For grounding line migration based on the contact conditions, see Durand et al (2009).

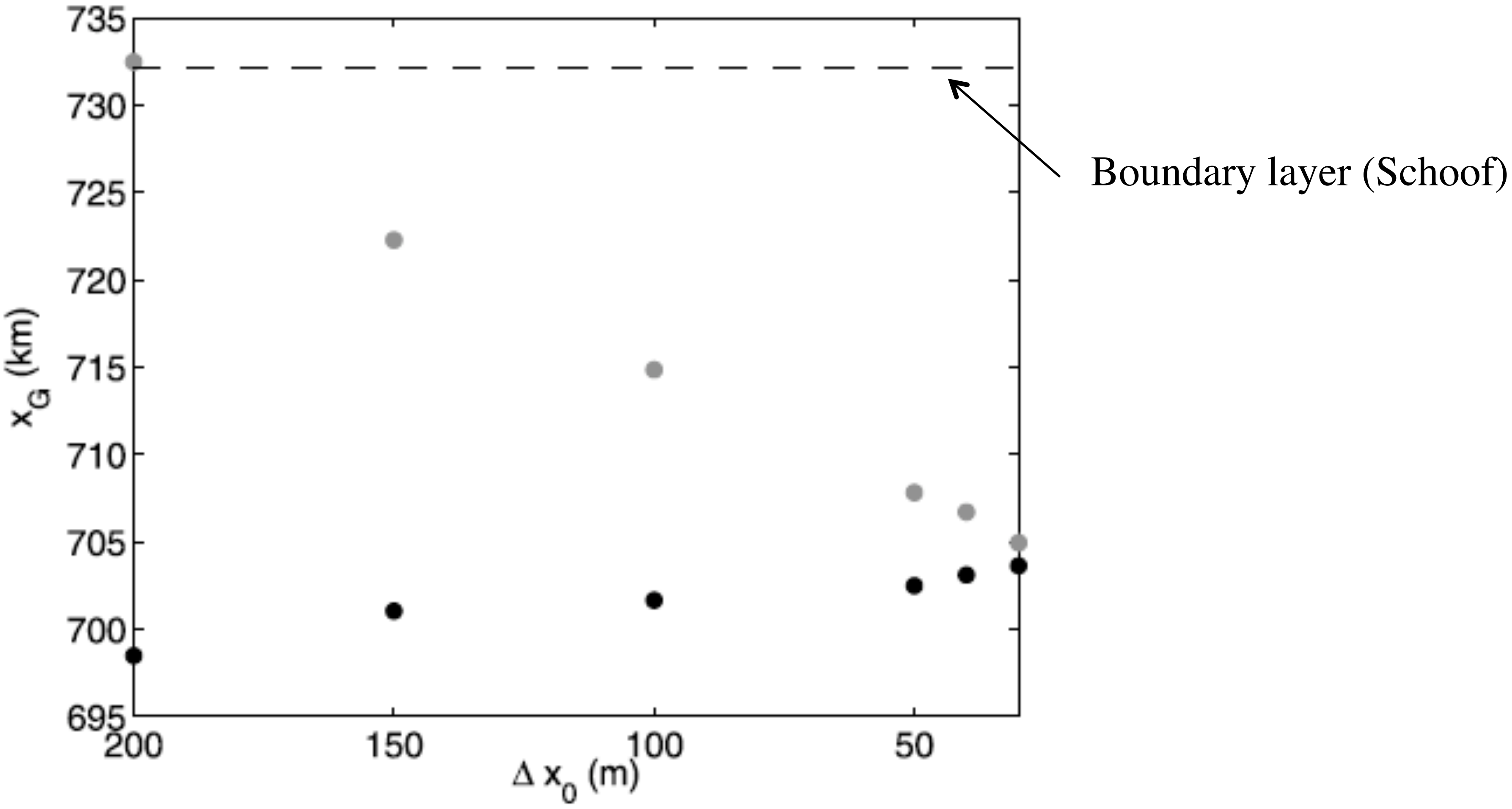
Rapid basal sliding



(Schoof, 2009, pers communication)

Effect of grid size on grounding line position determined from full Stokes solution

Durand et al (2009)



Using boundary layer flux as a constraint on grounding line flux in coarse grids (10-40km)

Pollard & DeConto (2009)

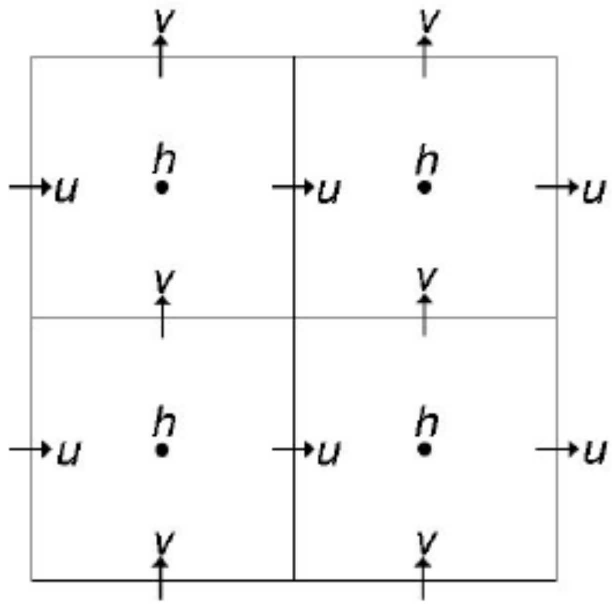
- Grounding line position from floatation and interpolation (Pattyn, 2006).

- The flux

$$q_g = \left(\frac{A(\rho g)^{n+1}(1-\rho/\rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h_g^{\frac{m+n+3}{m+1}} \left(\frac{\tau_{xx}}{\tau_f} \right)$$

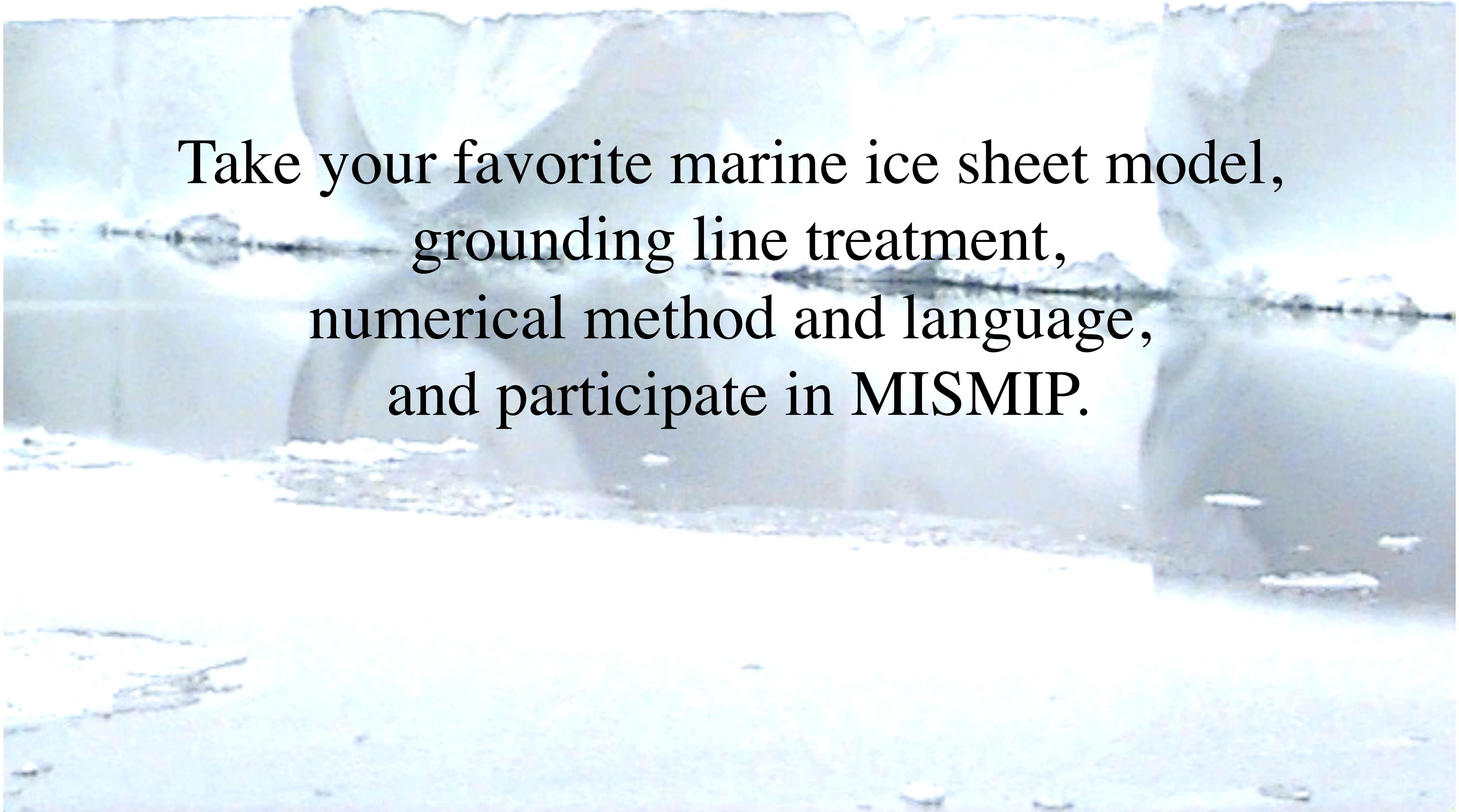
gives the average velocity, $u_g = q_g/h_g$, used as boundary condition for the shelf velocity at the grounding line:

-If $q_g > q_m$: impose u_g at u-grid gl point
-If $q_g < q_m$: impose u_g at u-grid downstream from gl



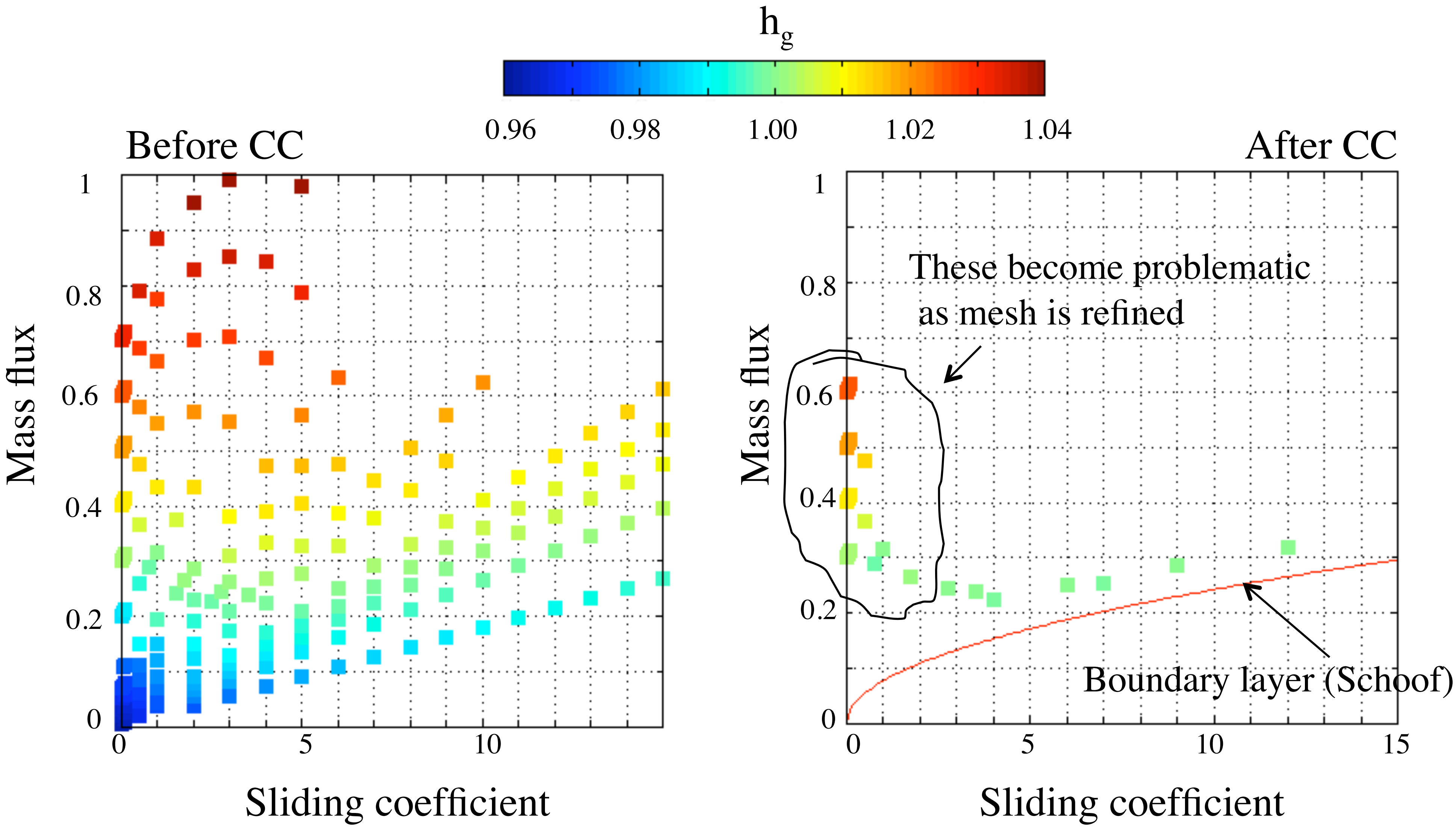
finite-difference Arakawa-C grid

Group exercise



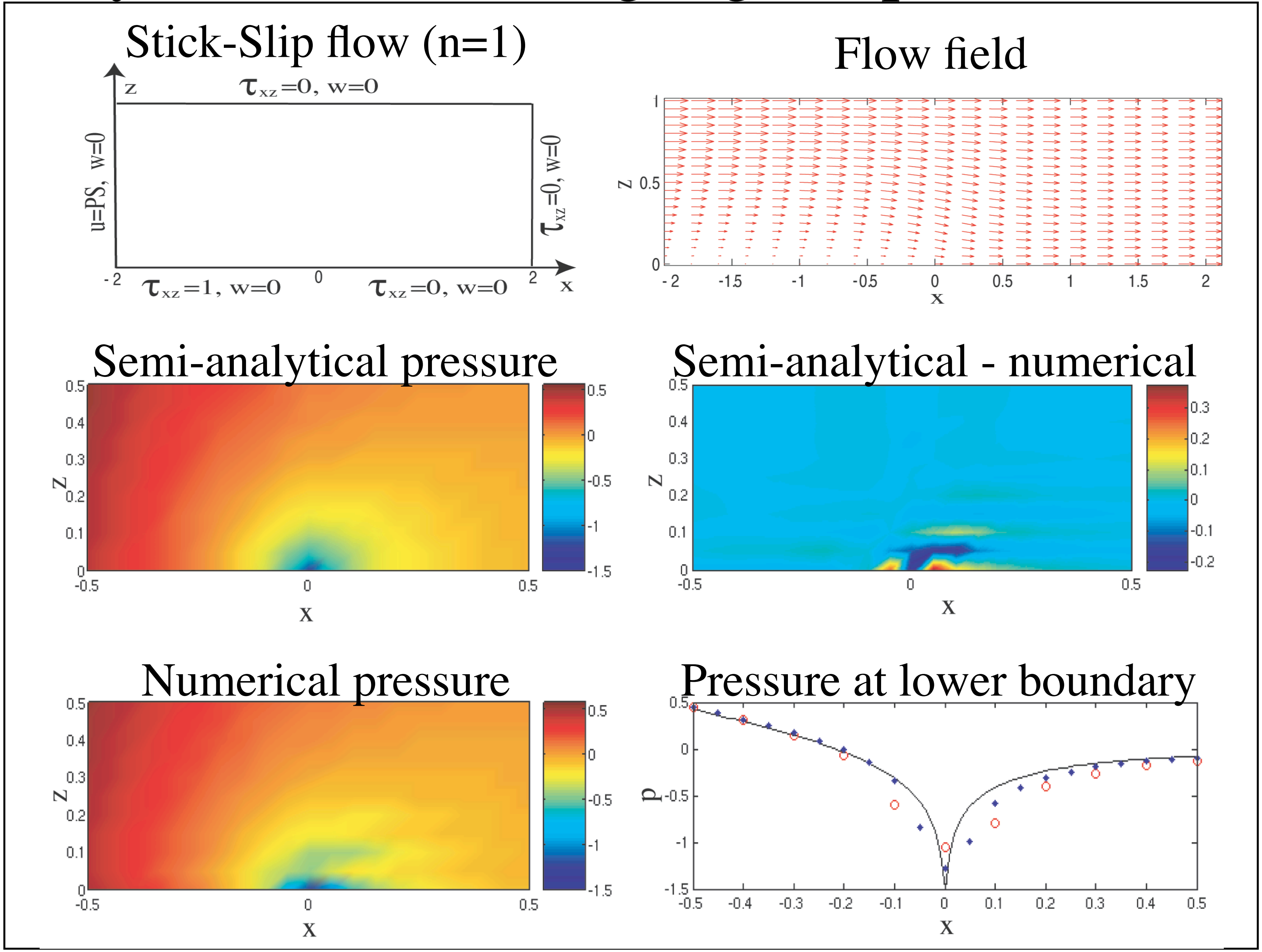
Take your favorite marine ice sheet model,
grounding line treatment,
numerical method and language,
and participate in MISMIP.

Do the contact conditions hold for any sliding?



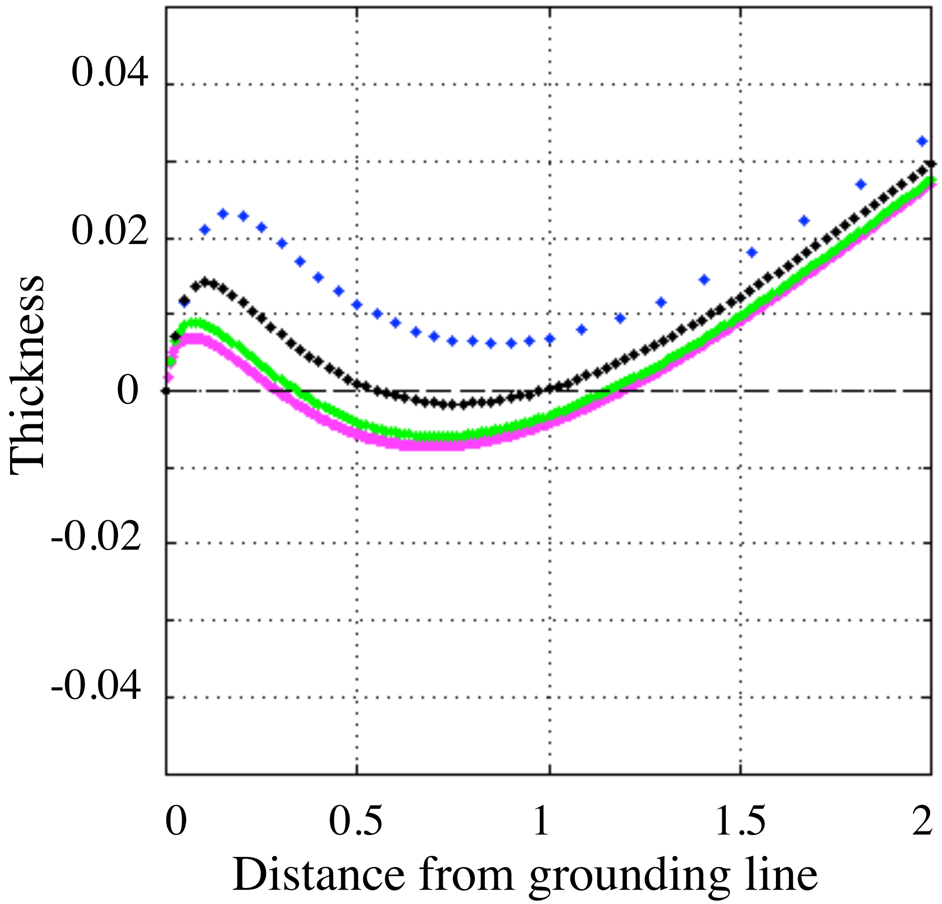
Results for linear sliding law: $w = 0, \quad u - \beta \tau_{xz} = 0$

Why is the low sliding regime problematic?



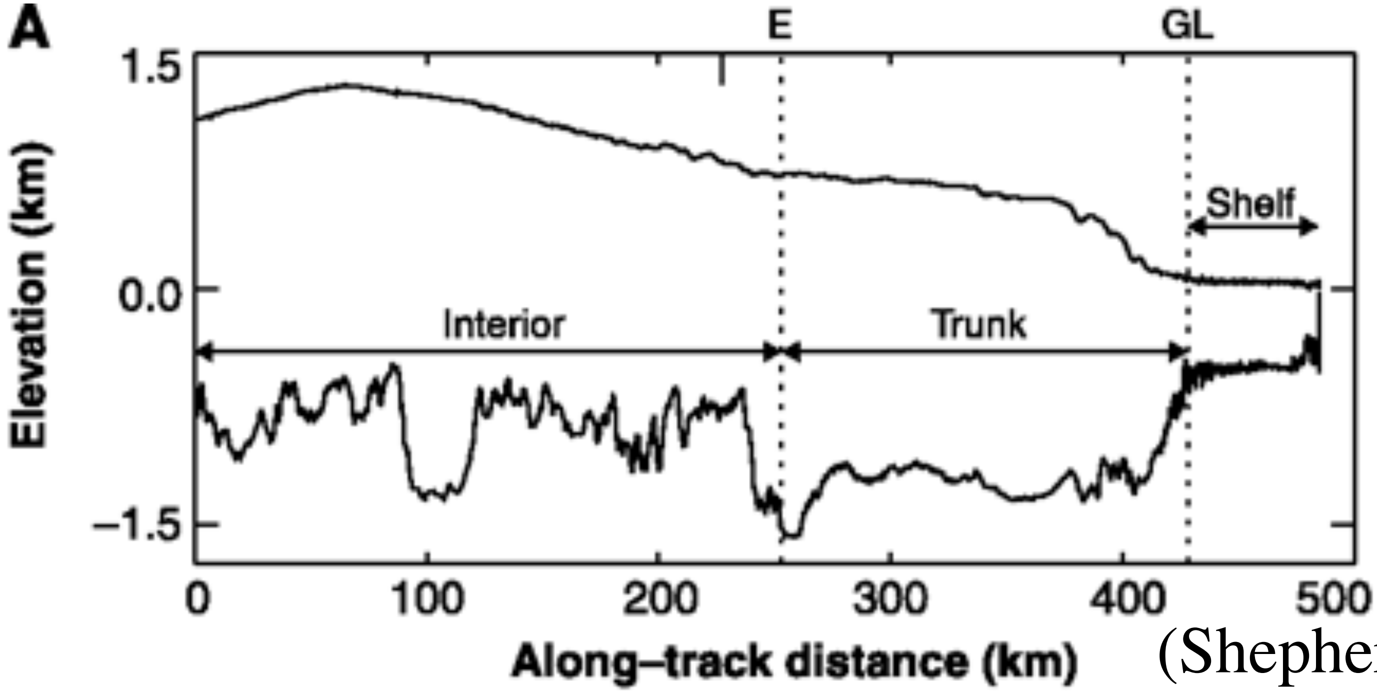
Why is the low sliding regime problematic?

Mesh refinement



- Mesh refinement might be insufficient to capture stress singularity in the vicinity of grounding line.
- A possible solution would be adding a singular trial function (requires knowledge of singularity – for no-slip /free-slip see Barcilon & MacAyeal (1993)).
- Alternative could be using singular elements, where nodes placement are such that derivative of shape function is infinite (Burnett, 1987).

To float or not to float, that is the question…



(Shepherd et al.)

- Grid size matters.
- Grounding line physics matters.
- Non linearity is messy.
- Does full Stokes really matters? Which transition zone do you want to model?

• Keep eyes out on results from MISMIP.
• Do not forget about missing stuff that might turn out to be important (temperature, fracturing, water circulation, shear stresses from the sides…).

Conclusion

- Accurate determination of the position of the grounding line remains one of the challenges faced by current marine ice sheet models.

- We have made progress in understanding that the ‘classical’ hydrostatic hypothesis does not uniquely determine the grounding line position.

- At least one other condition is needed, and it appears that the contact conditions for subglacial cavities are sufficient for ‘fast’ junctions.

- However, the treatment of ‘slow’ junctions requires a careful investigation.