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Response of subglacial sediments to basal freeze-on 2. Application in numerical modeling of the recent stoppage of Ice Stream C, West Antarctica

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[1] Ross ice streams supply over 90% of the ice volume flowing out of the Ross sector of the West Antarctic ice sheet (WAIS). Stoppage of Ice Stream C (ISC) ca. 150 years ago appears to have pushed this sector of WAIS from negative into positive mass balance [*Joughin and Tulaczyk*, 2002]. We propose an explanation for the unsteady behavior of ISC using a new numerical ice-stream model, which includes an explicit treatment of a subglacial till layer. When constrained by initial conditions emulating prestoppage geometry, dynamics, and mass balance of ISC, the model yields a rapid (∼100 years) stoppage of the main ice-stream trunk. The stoppage is triggered by basal freeze-on, which consolidates and strengthens the subglacial till. Our numerical simulations produce results consistent with a number of existing observations, for example, continuing activity of the two tributaries of ISC. The model always yields rapid stoppage unless we specify ice-stream width that is smaller than its prestoppage values (maximum of ∼80 km). We conjecture that if ISC was active for at least a few thousand years before slowdown, its width was significantly smaller than today to sustain the long active phase. Ice-stream width is a key control that helps determine whether ice-stream flow is sustainable over a long term. Our work indicates that the recent stoppage of Ice Stream C could have been part of inherent ice-stream cyclicity, and it leaves open the possibility that other active ice streams may evolve in the future toward rapid shutdowns. *INDEX TERMS:* 1823 Hydrology: Frozen ground; 3210 Mathematical Geophysics: Modeling; 3220 Mathematical Geophysics: Nonlinear dynamics; 4556 Oceanography: Physical: Sea level variations; *KEYWORDS:* ice stream, ice dynamics, till, stoppage, ice modeling

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1. Introduction

[2] The West Antarctic Ice Sheet (henceforth WAIS) is a marine ice sheet grounded largely below sea level. Geological and geophysical evidence indicates that this ice sheet decreased significantly in extent and volume in response to the last glacial termination [*Anderson and Andrews*, 1999; *Domack et al.*, 1999; *Hall and Denton*, 1999; *Shipp et al.*, 1999]. Concerns have been voiced regarding its possible future collapse, which could raise the global sea-level by up to 6 m [*Bindschadler*, 1997, 1998; *Hughes*, 1975; *Oppenheimer*, 1998]. The possibility of such a collapse during the current and the past interglacials, however, is still controversial [*Bentley*, 1997, 1998; *Cuffey and Marshall*, 2000; *Joughin and Tulaczyk*, 2002; *Scherer et al.*, 1998].

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[3] One of the least understood elements of the WAIS system are the fast-flowing ice streams (Figure 1) that drain the interior ice reservoir and transport ice to the surrounding ice shelves [*Oppenheimer*, 1998]. These ice streams account for most of ice discharge from this ice sheet [*Alley and Whillans*, 1991; *Bentley*, 1987]. Extensive geophysical and glaciological research on the Ross ice streams has shown that their fast flow can be attributed to the presence of a weak subglacial till layer [*Alley et al.*, 1986, 1987; *Anandakrishnan et al.*, 1998; *Blankenship et al.*, 1987; *Engelhardt et al.*, 1990; *Kamb*, 1991; *Tulaczyk et al.*, 2000b]. In the context of the current concerns regarding the near-future evolution of the WAIS, transient behavior of ice streams on relatively short time scales (10's to 100's of years) is particularly important [*Bindschadler and Vornberger*, 1998; *Jacobel et al.*, 1996].

[4] There are many pieces of evidence showing that ice streams behave in an unstable manner on timescales of 10's and 100's of years [*Fahnestock et al.*, 2000], for instance,

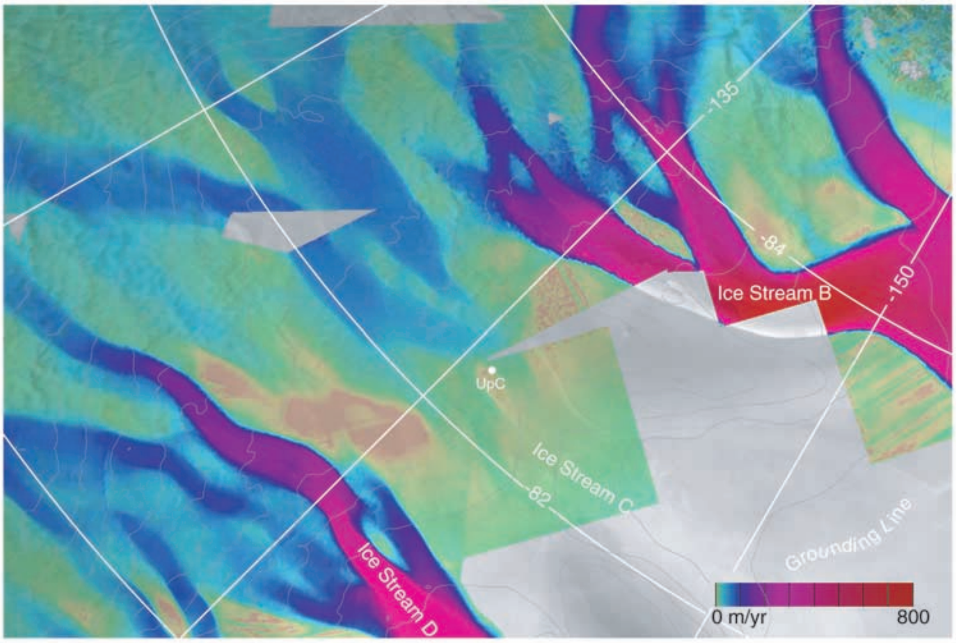

Figure 1. Location map showing ice velocity distribution (colors) overlain on top of a satellite image of ice sheet surface [*Joughin et al.*, 2001]. The area shown is focused on ISC and WIS.

the slow down of Whillans Ice Stream (henceforth WIS) [*Joughin and Tulaczyk*, 2002] and changes in ice-stream boundaries revealed by radar and recorded in the Ross ice shelf [*Fahnestock et al.*, 2000; *Jacobel et al.*, 2000]. The most significant example of recent transient ice-stream behavior is the stoppage of Ice Stream C (henceforth ISC) ca. 150 years ago [*Retzlaff and Bentley*, 1993; *Rose*, 1979]. *Anandakrishan et al.* [2001] reviewed previously proposed conceptual models for this shutdown. They identify the water-piracy hypothesis [*Alley et al.*, 1994; *Anandakrishnan and Alley*, 1997a] as the most likely explanation for the stoppage. According to this model, ISC stopped because of a diversion of subglacial water from its upper part to WIS. This diversion is thought to have happened due to a particular combination of surface and bed slopes in the upper part of ISC that generate water pressure gradients that promote basal water flow toward WIS rather than along the trunk of ISC. *Price et al.* [2001], however, argue that the current ice-surface configuration of the upper part of ISC is the result of the rapid growth of an ice bulge that formed after the stagnation of ISC [*Joughin et al.*, 1999]. Thus, the unusual water pressure gradients pushing basal water from beneath ISC to WIS may be a consequence and not the cause of the stoppage [*Alley et al.*, 1994; *Price et al.*, 2001].

[5] An alternative model for stoppage of an ice stream invokes the removal of basal water lubrication due to basal freeze-on [*MacAyeal*, 1993a, 1993b; *Payne*, 1995; *Payne and Dongelmans*, 1997; *Tulaczyk et al.*, 1998, 2000b]. The basic idea behind this model is that the fast flow of ice streams may lead to advection of cold ice from upstream and/or to thinning of an ice stream. As a result, the basal temperature gradient may steepen sufficiently to switch the basal thermal regimen from melting to freezing. Once basal freezing is initiated, removal of water from the subglacial system decreases basal lubrication and may cause a complete shutdown. This thermodynamics-based model of ice-stream stoppage has fundamentally different implications for the future behavior of Ross ice streams than the water-piracy model. In the latter, stoppage of ISC is an unusual event that took place because of a specific coincidence of surface and bed geometry in the upper part of this ice stream. In the thermodynamics-based model, ice streams are inherently oscillatory features, whose lifecycle includes alternating periods of activity and quiescence [*MacAyeal*, 1993a, 1993b; *Payne*, 1995; *Payne and Dongelmans*, 1997]. If the concept of ice streams as thermodynamic oscillators is correct, future behavior of the Ross ice streams could be predicted by constraining thermodynamic ice-stream models with observations of present-day ice-stream temperature field.

[6] Here, we model the stoppage of ISC using the recently developed Undrained Plastic Bed (henceforth UPB) model for soft-bedded ice-stream thermodynamics [*Tulaczyk et al.*, 2000b]. The salient feature of the UPB model is its ability to produce two equilibrium states, one with strong till ('ice-sheet mode') and slow ice velocities and one with weak till and fast ice velocities ('ice-stream mode'). Our goal is to test whether the stoppage of ISC may have been caused by the internal feedbacks between ice thermodynamics and till mechanics encapsulated in the UPB formulation. We will judge model performance by comparing results of simulations to geophysical and glaciological observations made previously on ISC [*Anandakrishnan and Alley*, 1997b; *Anandakrishnan and Bentley*, 1993; *Bentley et al.*, 1998; *Gades et al.*, 2000; *Jacobel et al.*, 2000; *Price et al.*, 2001; *Retzlaff and Bentley*, 1993; *Retzlaff et al.*, 1993; *B. E. Smith et al.*, 2001].

[7] As we will show later, our model reproduces many observed aspects of the stoppage of ISC reasonably well. However, the model is based on an end-member assumption of undrained bed. By making this assumption, we neglect

the possibility of significant long-distance basal water transport beneath ISC. The consequences and the scientific context of the undrained-bed model are discussed in more detail in the companion paper [*Christoffersen and Tulaczyk*, 2003, section 2]. In our opinion, the undrained-bed model is the best existing description of ice-stream till beds but we have no relevant conclusive evidence. Hence, the reader should remember that our work is concerned with a plausible end-member of ice-stream behavior. Our model builds upon previous efforts to simulate time-dependent ice-stream behavior [*Fastook*, 1987; *Van der Veen and Whillans*, 1996]. Its distinguishing characteristic is the explicit inclusion of the UPB treatment for the subglacial till. We allow the coupled ice-till system to evolve with time and calculate changes in till properties with depth. Inclusion of the subglacial till layer into our model is not only important in closing the feedback loops between ice dynamics and subglacial dynamics but permits also comparisons of simulated basal conditions with existing subglacial observations [*Engelhardt et al.*, 1990; *Kamb*, 1991, 2001]. Our results indicate that the shutdown of ISC may have been triggered by a switch to basal freezing following ice-stream thinning. The model is able to reproduce the important characteristics of the current ice velocity field in which the main trunk of ISC is completely shutdown but its tributaries are still moving relatively fast. In addition, the model output compares favorably with several ice and till properties measured in the field (e.g., ice temperature distribution, till porosity and till porosity gradients). Model runs with a range of different boundary and initial conditions consistently produce an ice-stream stoppage. In fact, given the assumption behind the UPB model of subglacial processes [*Tulaczyk et al.*, 2000b] our model does not produce long-term (1000's of years) stability without assuming that ISC was significantly narrower than at the time of the shut down. Based on the model results we propose that further observational verification of the mechanism of ISC stoppage could be obtained by geophysical and borehole measurements in the lower parts of the stagnant ice stream where we predict that the shutdown has initiated.

2. Physical Basis of the Model

[8] The main objective of our modeling is to test the hypothesis that the shut down of ISC may have been caused by freeze-on-driven consolidation of the till layer present beneath this ice stream. Our treatment of the coupled ice-till flow expands upon the analytical UPB model developed previously by *Tulaczyk et al.* [2000a, 2000b]. More detailed analysis of processes taking place in a subglacial till experiencing freeze-on is presented in the companion paper [*Christoffersen and Tulaczyk*, 2003]. The formulation of ice continuity and thermodynamics is patterned after the flowline model of an ice stream built previously by *Van der Veen and Whillans* [1996]. We chose to emulate their simple flowline approach rather than to construct a vertically integrated map-view model [e.g., *Hulbe and MacAyeal*, 1999] because the flowline model facilitates treatment of time-dependent ice-stream width [*Van der Veen and Whillans*, 1996]. Time variability of ice-stream width is not only well documented [*Clarke et al.*, 2000; *Echelmeyer and Harrison*, 1999; *Harrison et al.*, 1998; *Jacobel et al.*, 2000] but, as we will later show, may be the primary process controlling ice-stream evolution. Figure 2a illustrates schematically our model domain and the axis convention. We minimize the complexity of our ice-flow module by running

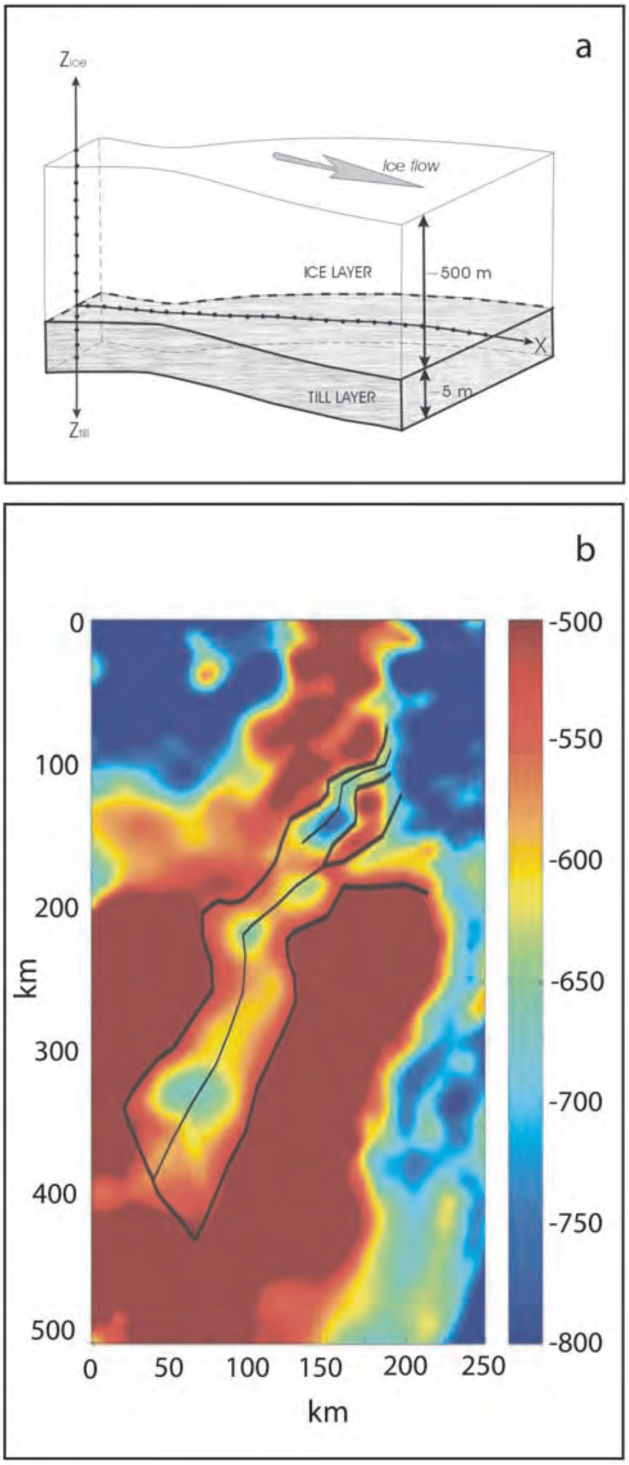

Figure 2. Schematic diagram of our model domain (a), with axis directions used in the model. ISC bed topography map (b) with outline of the ice-stream boundaries. We defined the bed topography of our model by averaging cross-sectional values perpendicular to the displayed flow line.

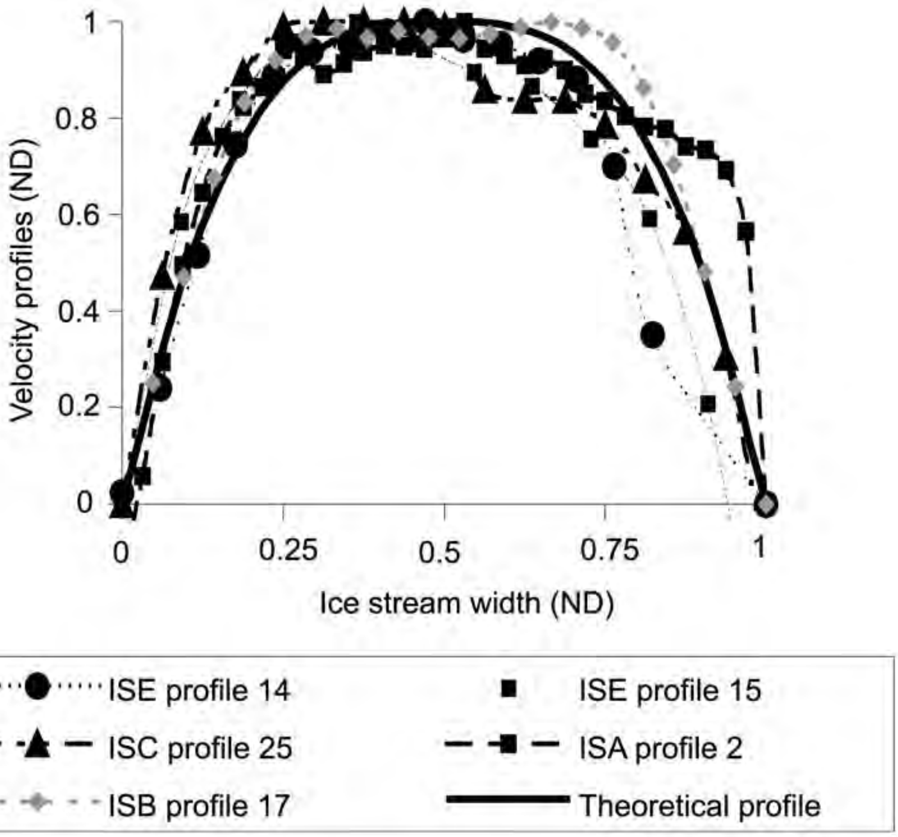

Figure 3. Comparison between the theoretical equation for the velocity profile across an ice stream (equation (2)) and velocity data for different ice-stream widths [*Joughin et al.*, 1999]. The velocities and distances have been non-dimensionalized using the maximum velocity in each cross-section. Ice-stream margins are assumed to lie where the velocity gradient across lateral boundaries peaks [*Raymond et al.*, 2001]. The theoretical equation reproduces very well the general pattern of the data.

ice-stream simulations in a flowline mode (Figure 2b) with an implicit inclusion of several important transverse effects. This approach reduces the computational requirements of our ice-flow module while allowing for a detailed treatment of these aspects of ice-stream dynamics that are in the focus of our numerical experiments: (1) ice thermodynamics, (2) basal conditions, and (3) ice-stream widening. Following *Tulaczyk et al.* [2000b, equation (15)] (modified from *Raymond* [1996, equation (38)]), we calculate the centerline ice velocities, U_s, from:

$$U_s = U_b + U_{def} = U_d\left[\left(1 - \frac{\tau_b}{\tau_d}\right)^n \left(\frac{W}{2H}\right)^{n+1} + \left(\frac{\tau_b}{\tau_d}\right)^n\right] \quad (1)$$

where W is the width of the ice stream (m), H is the ice thickness (m), τ_d is the driving stress (Pa)($\tau_d = \rho_i g H \alpha$, where ρ_i is the ice density (kg m^{-3}), g is the constant of acceleration due to gravity (m s^{-2}), H is the ice thickness (m), and α is the surface slope). The surface velocity includes a basal sliding component U_b and an internal deformation component U_d ($U_d = 2^{1-n}\tau_d^{\,n} H (n+1)^{-1} B^{-n}$). In the latter, B is the stiffness parameter (Pa yr$^{1/3}$), and we use the Arrhenius relationship [*Paterson*, 1996] to compute a column-averaged value of B from vertical ice temperature profiles calculated in the model.

[9] The velocity equation (1) is strictly applicable only to an ice stream flowing in a rectilinear channel with rigid walls. It assumes also that the gravitational driving stress is balanced fully by a combination of the basal shear stress and side shear [*Raymond*, 1996, equation (3)]. All other terms of the general force balance equations [e.g., *Van der Veen and Whillans*, 1989a] are ignored. Previous studies of West Antarctic ice streams indicate that basal resistance and marginal shear stress support typically almost all of the gravitational driving stress [*Echelmeyer et al.*, 1994; *Harrison et al.*, 1998; *Jackson and Kamb*, 1997; *Van der Veen and Whillans*, 1989b; *Whillans et al.*, 1989]. Our treatment also neglects transverse variations in ice thickness and driving stress. The convenient result of this approach, however, is that while we concentrate on calculating explicitly only the centerline ice velocities, the distribution of velocity across the whole ice stream can be obtained from a generalization of equation (1) in the transverse direction (modified from *Raymond*, 1996, equation (39)):

$$U_s(y) = U_b\left[1 - \left(1 - \frac{y}{W}\right)^{n+1}\right] + U_{def} \quad (2)$$

where y is the transverse direction, and all other parameters have been defined earlier. Figure 3 illustrates that this equation reproduces well the observed transverse velocity distribution on Ross ice streams.

[10] Given the centerline ice-stream velocity from equation (1) and the known along-flow distribution of ice-stream width and thickness, we can calculate the time rate of thickness change by imposing the flowline form of the mass continuity equation [*Nye*, 1959; *Paterson*, 1996, pp. 256–257]:

$$\frac{\partial H}{\partial t} = \frac{2Hv}{W} + \dot{a} - \dot{m} - \bar{u}\left(\frac{\partial H}{\partial x} + \frac{H}{W}\frac{\partial W}{\partial x}\right) - H\frac{\partial \bar{u}}{\partial x} \tag{3}$$

where t denotes the time (yr), the first right hand side term is the transverse flow rate (m yr^{-1}), v is the transverse velocity (m yr^{-1}), $\dot{a}$ is the accumulation rate ($\sim$0.1 m yr^{-1}) [*Giovinetto et al.*, 1990; *Vaughan et al.*, 1999], $\dot{m}$ is the basal melting/freezing rate (m yr^{-1}), $\bar{u} = 0.8U_s$ is the ice velocity in the x-direction (Figure 2a) averaged over ice-stream width (obtained by integration of equation (2)).

[11] The basal resistance to ice-stream flow, τ_b, is assumed to be controlled everywhere by the till strength, τ_f, which in turn is assumed to be a function of till void ratio, following an experimentally-determined relationship of the form [*Tulaczyk et al.*, 2000a, equation (3d)]:

$$\tau_b = \tau_f = a\exp(-be) \tag{4}$$

where a and b are two empirical constants, and e is the void ratio in the till. The relation $\varphi = e/(1+e)$ links the engineering void ratio to porosity, φ, which is a measure commonly used in geosciences. Our treatment makes the additional assumption that basal resistance (i.e. till strength) does not change in the direction transverse to flow. This is consistent with borehole observations on WIS and ISC where no consistent changes in till properties were observed on length scales of up to $\sim$10 km, both transverse and longitudinal to ice-stream flow [*Kamb*, 2001; *Tulaczyk et al.*, 2001, 1998]. Our model does not include transverse variability in basal stickiness that may exist beneath ice streams resulting from local till thinning or from sticky spots [*Alley*, 1993; *Anandakrishnan and Alley*, 1997a; *Hulbe and Whillans*, 1997; *MacAyeal et al.*, 1995]. Rather, we assume that our simulated ice stream is underlain everywhere by a continuous, several-meter-thick layer of till. This assumption is consistent with existing geophysical and borehole observations that have shown widespread presence of several-meter-thick till beneath active and stopped Ross ice streams [*Anandakrishnan*, 2003; *Blankenship et al.*, 1986, 1987; *Engelhardt et al.*, 1990; *Kamb*, 1991, 2001; *Rooney et al.*, 1987]. In the model, we do not treat till mass balance explicitly [*Alley et al.*, 1997] but assume that a constant amount of till solids is present everywhere beneath the simulated ice stream. Using a simplified version of the same numerical model we conducted investigations of the rates of subglacial erosion, transport, and deposition necessary to sustain such steady state till thickness. The erosion/deposition rates lie within a reasonable range of values of the order of 1 mm/yr or less [*Bougamont and Tulaczyk*, 2000, 2003].

[12] Through the till void ratio, e, the basal shear stress, τ_b, used in ice-velocity calculations is coupled to the basal melting/freezing rate, $\dot{m}$, which, in turn, is determined by the basal thermal energy balance:

$$\dot{m} = \frac{\tau_b U_s + G - K_i\theta_b}{L\rho_i} \tag{5a}$$

$$\frac{de}{dt} = \dot{e} = \frac{\dot{m}}{Z_s} \tag{5b}$$

where Z_s is the thickness of till solids (m), G is the geothermal flux (W m^{-2}), K_i is the thermal conductivity of ice (J m^{-1} s^{-1} K^{-1}), θ_b is the basal temperature gradient (°C m^{-1}) (negative quantity in our coordinate system, Figure 2a), L is the latent heat (J kg^{-1}), and ρ_i is the ice density (kg m^{-3}). As discussed by *Tulaczyk et al.* [2000b], in the above equations basal melting/freezing (5a) is assumed to be balanced locally by increased/decreased water storage in till (5b).

[13] From the existing measurements of void ratio in sub-ice stream till [*Tulaczyk et al.*, 2001], we infer that distribution of water in till is determined by a combination of vertical diffusion and advection. Void ratio in till cores from beneath the active WIS is relatively constant with depth, indicating that the layer is well mixed, presumably due to till deformation associated with the fast basal ice motion [*Tulaczyk et al.*, 2001]. On the other hand, measurements on cores recovered from beneath the stopped ISC show a strong increase in till void ratio with depth, consistent with diffusion of till pore water toward a freezing ice-base [*Christoffersen and Tulaczyk*, 2003]. We expect that in Nature there is a gradual transition between mixing-dominated till void ratio distribution under high sliding velocities and diffusion-dominated distribution under low velocities. In quantitative treatments, smooth scaling of such transition is achievable using, for instance, the concept of a Peclet number. To do so, however, we would need some constraints on the rates of vertical till mixing. Pending further pertinent studies, we apply in the present model a sharp velocity threshold (set to 30 m yr^{-1}). The perfectly-mixed approximation (i.e., void ratio constant with depth) is used when local sliding velocity is greater than the threshold. Below this threshold, the distribution of excess pore water pressure with depth is calculated from:

$$\frac{\partial u}{\partial t} = c_v\frac{\partial^2 u}{\partial z^2} + U_s\left(\frac{z}{H}\right)^r\frac{\partial u}{\partial x} \tag{6}$$

where u is the excess pore pressure (Pa) ($u = P_w - P_h$, where P_w is the total pore pressure and P_h is the hydrostatic pressure), c_v is the hydraulic diffusivity of till (m^2 yr^{-1}), z is the vertical coordinate. The last term of equation (6) represents the horizontal advection in the till and r is an exponent describing the non-linearity of the till deformation [*Cuffey and Alley*, 1996]. We set $r = 1$ to maximize the effect of till advection, with the additional assumptions that the till is not sliding on its substratum, and that the velocity at the top of the layer equals the ice velocity [*Alley*, 1989]. We included horizontal till advection only in some numerical experiments, because it turns out to have a negligible effect on the fundamental results of the simulations but increases the computational effort. Once excess pore pressure distributions are obtained from equation (6),

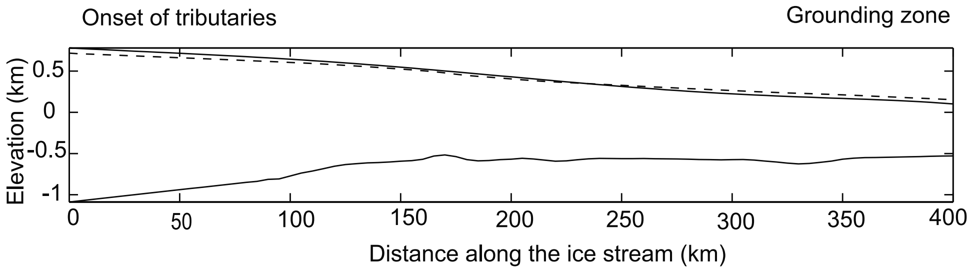

Figure 4. Geometry of Ice Stream C used in the model. The bed topography profile has been obtained with the map given in Figure 2b, and the initial surface profile (solid line) is from *Shabtaie and Bentley* [1987]. The dashed line represents the modeled surface profile 150 years after the complete stoppage of ISC. It is within <100 m the present observed surface profile.

vertical water fluxes v_w can be calculated from the Darcy equation [*Scott*, 1963, pp. 5–49]:

$$v_w = K_h \frac{\partial u}{\partial z} \frac{1}{\rho_w g} \tag{7}$$

where K_h is the hydraulic conductivity in the till (m s^{-1}), $\frac{\partial u}{\partial z}$ is the excess pore pressure gradient (Pa m^{-1}), ρ_w is the water density (kg m^{-3}).

[14] Compared to the detailed treatment of water, solute, and heat transport in till during freezing investigated in a companion paper [*Christoffersen and Tulaczyk*, 2003], we neglect here several of the secondary effects that appear not to be crucial to modeling shutdown of the coupled ice-stream-till system. Inclusion of the non-linear processes and threshold conditions investigated by *Christoffersen and Tulaczyk* [2003] would make our ice-stream model computationally impractical by increasing the time needed for a single run on a double-processor SUN Ultra 80 to about a year.

[15] To complete the coupling of the ice and till components of our model, we need an expression for the temperature gradient at the base of the simulated ice stream. This temperature gradient will permit us to close the feedback loops by determining the basal melting rate (5a) and making our system of equations self-contained. Inclusion of the temperature calculation into our model also makes it possible to verify the model performance by comparing the theoretical temperature profile with the measured profile at the UpC camp (H. Engelhardt, Caltech, personal communication, 2000). Previous studies have shown that it is challenging to get satisfactory numerical solutions for temperature distribution in ice streams [*Hulbe*, 1998] where frictional heat dissipation may be variably partitioned between the bed and the shear margins [*Jacobson and Raymond*, 1998; *Raymond*, 2000]. In line with our simplification of the ice-till system to a centerline model, we calculate ice temperatures from the ice-column formulation of a transient, 2-D thermal diffusion-advection equation [*Budd et al.*, 1971; *Hooke*, 1998, pp. 79–80]:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} - \frac{(w_s - w_b) z}{H} \frac{\partial T}{\partial z} + U_s \frac{\partial T}{\partial x} \tag{8}$$

where κ is the thermal diffusivity in the ice (m^2 yr^{-1}), T is the ice temperature (°C), w_s and w_b are respectively the vertical velocity at the surface and the base of the ice column (m yr^{-1}). As is typically done, the horizontal diffusion term has been omitted [*Hooke*, 1998 p.79]

3. Numerical Treatment: Fixed Margin (FM) Model

3.1. Initial and Boundary Conditions

[16] Our initial approach to modeling of the stoppage of ISC is to use modern observations to constrain the geometry of the model domain (surface elevation [*Shabtaie and Bentley*, 1987], bedrock topography and ice-stream width (Figure 2a and Figure 4)). This is the simplest approach given that there are few data on prestoppage geometry of ISC, except for some constraints on former positions of ice-stream margins near the grounding line [*Bindschadler and Vornberger*, 1998; *Fahnestock et al.*, 2000; *Gades et al.*, 2000; *Jacobel et al.*, 2000, 1996]. Under these assumptions, our model is effectively used to test whether the combination of the selected treatment of ice-till thermodynamics with the modern ice-stream geometry may satisfactorily reproduce the shutdown of ISC. The ice dynamics and mass-balance component of our model has only one important boundary condition, the one at the upstream end of the tributaries. We have designed the numerical treatment of the model physics in such a way as to treat the downstream end of the model domain (i.e. the area of the modern grounding zone) in the same way as the interior of the model.

[17] The upstream end of the model domain is a convenient place to set boundary conditions because the two ice-stream tributaries of ISC appear to be more stable than ice-stream trunks [*Joughin et al.*, 1999; *Price et al.*, 2001]. We have made the simulated tributaries quite long to put the upstream ('onset') boundary condition far away from the main trunk of ISC, which is the part of the ice-stream system that is of main interest to us because it has experienced near-stoppage. Consequently, we drive the ice-dynamics component of our model with the assumption that ice input at the onset of the simulated tributaries remains constant throughout each model run. We also keep the ice surface slope at the onset constant, because this approach permits ice thickness changes. Experiments with constant thickness at the onset lead typically to a numerical instability caused by development of inverse ice surface slopes. Since the ice thickness at the onset is allowed to change, the ice velocity has to adjust itself to keep in line with the requirement of constant influx.

[18] As part of our strategy to concentrate boundary conditions in the 'onset' area of the tributaries, we use standard Euler backward-difference approximations with second-order error terms to approximate horizontal gradients of different variables in our equations (e.g., equation (3)). Because of a persistent problem with numerical instability, we had to use the forward-difference approximation in calculations of surface slope (e.g., equations (1) and (2)). Therefore we had to set one boundary condition at the downstream end of the model (i.e., the grounding zone area); the curvature of ice surface elevation is assumed there to be zero. This should be a reasonable assumption because ice surface slopes and curvatures vary very little in the downstream sections of ice streams [*Bindschadler et al.*, 1993, 1987]. The initial distribution of basal stress (4) was computed by assigning a spatially homogeneous initial value for till void ratio of 66% (ca. 40% porosity). This value is consistent with the existing measurements and estimates of till porosity beneath modern active ice streams [*Blankenship et al.*, 1987; *Engelhardt et al.*, 1990; *Kamb*, 1991; *Tulaczyk et al.*, 2001, Figure 7]. In the cases in which the horizontal till advection was included, we used the Euler backward-difference approximation to obtain the horizontal derivative of excess pore pressure. This approach allows us to set a single boundary condition at the onset, where the till porosity is assumed to be always at 40%. When ice velocity drops at any location along the horizontal flowline below the arbitrary threshold value of 30 m/yr, we turn on also the vertical pore pressure diffusion term in equation (6). At this stage, the initial excess pore pressure distribution is calculated using the porosity of the previously well-mixed layer:

$$u = (\rho_s - \rho_w)(1 - \varphi)gz - p' \tag{9}$$

where ρ_s is the solid till density (kg m^{-3}), φ is the porosity, and p' is the effective stress (Pa). The lower boundary is defined by no-flow condition and the upper boundary condition consists of prescribed flux, set to the melting rate $v_w = \dot{m}$, (equation (7)).

[19] Finally, we initialize the ice temperature field using the analytical solution to a vertical diffusion-advection heat equation [*Hughes*, 1998, pp. 294–295; *Zotikov*, 1986]:

$$T = T_{pmp} + \left(T_s - T_{pmp}\right)\frac{erf\left(\sqrt{0.5Pe_{ice}}z/H\right)}{erf\sqrt{0.5Pe_{ice}}} \tag{10}$$

where $Pe_{ice} = \dot{a}H/\kappa$ is the non-dimensional Peclet number expressing the relative importance of vertical heat conduction and advection, T_s is the surface temperature and T_{pmp} is the basal temperature assumed to be everywhere at the pressure melting point (°C). Compared to (8), this equation does not include the influence of horizontal advection on temperature distribution. However, the numerical solution to (8) initialized with (10) converges relatively quickly to a steady-state case. We use the pressure melting point as the basal boundary condition [*Hooke*, 1998, p.74], thereby neglecting any supercooling that may develop during basal freeze-on [*Christoffersen and Tulaczyk*, 2003]:

$$T_{pmp} = \rho_i gH\left(-0.098.10^{-6}\right) \tag{11}$$

The upper boundary condition is provided by a parameterization of surface temperatures. At UpC, the temperature is defined from field measurement ($T_s(UpC) = -27$°C), and the other surface temperatures are calculated from surface elevations and an atmospheric lapse rate, λ:

$$T_s = T_s(UpC) + \Delta S\lambda \tag{12}$$

where T_s is the temperature at a given position along the profile (°C), ΔS is the elevation of any given point on the ice surface above UpC (m), and λ is the regional atmospheric lapse rate estimated to be -0.004 °C m^{-1}. The values range from -25°C at the grounding zone area, to almost -28°C at the onset, which is a good approximation when compared to a regional map of surface temperatures (http://uwamrc.ssec.wisc.edu/aws/databook/1995/db8.html). The ice temperature profile at the onset is recomputed with the equation (10) to provide the boundary condition needed to solve (8) over the rest of the temperature field.

[20] Before starting each numerical experiment, we spin up the model without permitting the till porosity and strength to change from their initial values. The post spin up, experimental part of each model run begins when the feedback between the ice thermodynamics portion of the code and the till portion is turned on. The model spin up stage is ended when the simulated ice velocity is not changing anywhere in the domain at a rate faster than $\sim$1 m yr^{-2}. Initial rates of velocity changes are much greater than that. Individual numerical experiments last between ca. 500 and ca. 10,000 model years. Due to the non-linearity of equations (1) and (4) and the employed Euler backward approximations, stability can be achieved only with short time steps of the order of a fraction of a day. A 500 years model run takes about 10 hours on a 2 × 500 MHz machine with 1 GB RAM.

3.2. FM Model: Results and Discussion

[21] The most important result of our model runs with fixed margins is that the simulated ISC at first moves at high ice-stream-like velocities and then stagnates (Figure 5) because the basal thermal regime switches to freezing ($\sim$1 to 6 mm yr^{-1}) which strengthens the underlying till. With the current geometry of ISC, no other mechanism is necessary in our model to trigger a near shut-down of the ice stream. These results demonstrate that the UPB model applied to ISC provides a plausible explanation for the stoppage of ISC that occurred 150 years ago. A major implication is that we might see other ice streams behaving like ISC in the near future, because this stoppage is inherent to the UPB model physics that invokes thermodynamically-driven oscillatory behavior [*Tulaczyk et al.*, 2000b]. Gradual loss of volume and retreat of the West Antarctic ice streams that has been triggered by the last glacial-to-interglacial transition may have made them sufficiently thin to start switching from basal melting to basal freezing. In addition to the stoppage of ISC, the Siple Ice Stream has shut down ca. 500 years ago as well [*Nereson*, 2000; *Nereson et al.*, 1998]. WIS may be experiencing shutdown at the present time. In the last several decades, the ice plain of WIS slowed down by $\sim$200 m/yr and is currently decelerating at a rate of about 5.5 m yr^{-2} [*Joughin and Tulaczyk*, 2002].

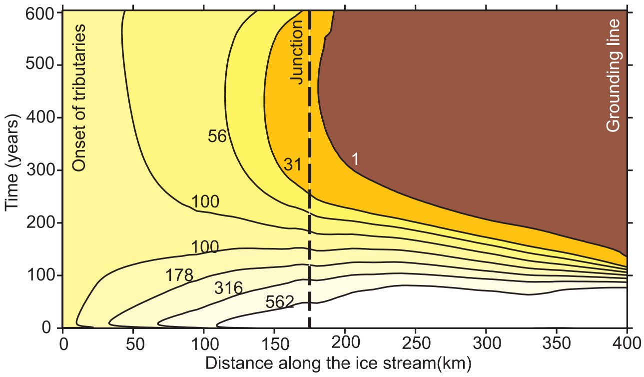

Figure 5. Fixed Margin model velocity results: x = 0 km corresponds to the tributaries onset, and x = 400 km corresponds to the grounding line position. The junction between tributaries and main trunk is at x = 175 km. Time is displayed on the vertical axis. Contour lines represent ice flow velocity values (in m/yr), plotted in a log scale. For this experiment, the ice-stream width corresponds to the modern ISC width; the initial velocity field in the main trunk is high (565 m/yr), and drops rapidly to 1m/yr or less at the grounding zone. The ice velocity in the tributaries is much more stable.

[22] One of the most important known characteristics of the shutdown of ISC was its abrupt nature. Analysis of crevasse burial depths has shown that the shutdown of ISC was rapid enough to cause cessation of crevasse opening along the whole length of the main ice-stream trunk within less than 100 years [*Retzlaff and Bentley*, 1993; *I. J. Smith et al.*, 2001]. To compare the performance of our model to this recognized fact, we have computed from force balance the evolution of marginal shear stress, τ_s, along our simulated ice stream:

$$\tau_s = \frac{W}{2H}(\tau_d - \tau_b) \tag{13}$$

[23] *Vaughan* [1993] reviewed existing data pertinent to the tensile stress magnitude that causes opening of crevasses and determined that in the Ross Embayment active crevasses occur where the tensile stress is between 190 and 240 kPa. In ice-stream margins, crevasses open at an angle of ca. 45 degrees from the direction of marginal shear [*Vornberger and Whillans*, 1990]. From Mohr circle construction we conclude that the relevant value of tensile stress in the margins is twice the marginal shear stress [see also *Jackson and Kamb*, 1997, equations (1)–(4)]. Thus, we assume that in our simulated ice-stream crevasses stay active as long as the marginal shear stress remains at or above the range of 95 to 120 kPa. Figure 6 shows the evolution of marginal shear stress along the simulated ice stream. These results are consistent with the crevasse-burial data because the modeled stoppage deactivates crevasses within a time period of <100 years along the whole length of the main ice-stream trunk. This deactivation propagates also somewhat into the two ice-stream tributaries but their upper parts experience continuous crevassing. The model also predicts a progression in crevasse-burial ages from the oldest near the grounding line toward the junction of the tributaries where crevasses are several decades younger. The existing crevasse-burial data show some indication of similar progression in crevasse-burial ages [*I. J. Smith et al.*, 2001]. Unfortunately, the error bars associated with the crevasse-burial dating technique are large (±20–30 years) compared to the total crevasse-deactivation period (<100 years). This fact makes it difficult to ascertain the temporal progression of crevasse deactivation along the ISC [*I. J. Smith et al.*, 2001].

[24] Another very satisfactory result of our model of ISC shutdown is that it shows the narrow tributaries moving at a relatively constant velocity (~80 m yr^{-1}), even when the main ice-stream trunk is completely shut down. This is consistent with the current pattern of velocity observed on ISC [*Joughin et al.*, 1999; *Price et al.*, 2001]. The tributaries do not shut down, although they do gradually slow down, even when we keep the numerical model running for much longer (~1,000–2,000 years) after the stoppage of the main trunk is completed (see Figure 6). In considering the reason for the distinctly different behavior of the simulated tributaries and the ice-stream trunk, it is important to note that the tributaries and the trunk of the ice stream are treated in our model with exactly the same equations and numerical methods. We attribute this difference in behavior solely to the difference in width of the tributaries and the trunk.

[25] As illustrated by equation (1), ice-stream/tributary velocity is very sensitive to channel width. The velocity in wide ice streams can easily reach values that are above the balance velocity (Figure 7). This leads to gradual ice thinning, which steepens the basal temperature gradient, and triggers basal freezing. On the other hand, the tributaries are much more narrow than the main trunk of ISC (W_{trib} = 25 km, 30 km < W_{trunk} < 75 km) and can adjust their velocity to the balance velocity (Figure 7). With narrower widths, the simulated ice-stream tributaries are able to adjust their driving stress to keep themselves flowing at the balance velocity. This prevents ice thinning and

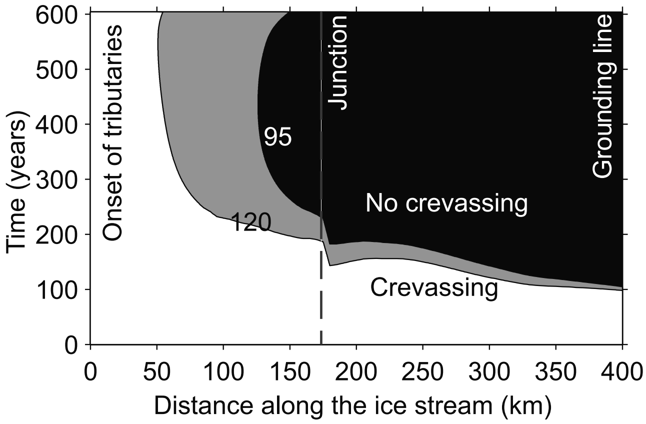

Figure 6. Marginal stress along the ice stream (horizontal axis), and through time (vertical axis). The axes arrangement is similar to that shown in Figure 5. The values represent marginal shear stress (in kPa). Crevasses are considered active if the marginal stress exceeds 95–120 kPa and inactive if it is less than 95–120 kPa [*Vaughan*, 1993].

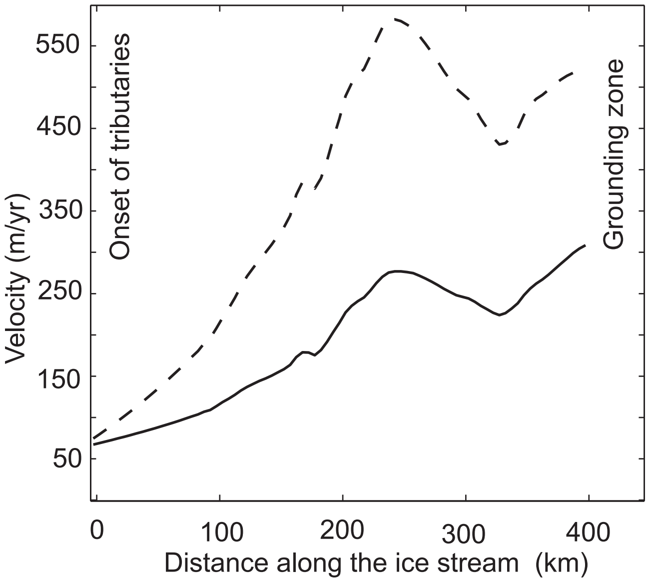

Figure 7. Balance velocity computed for ISC (solid line), compared with the prestoppage velocity profile in our model (dashed line).

development of basal freezing. Tributaries and the trunk of ISC show also distinctly different patterns of till porosity (Figure 8a). The till layer beneath the tributaries remains very wet and weak because of negligible water loss to basal freezing. Basal temperature gradients in the tributaries range between $-0.05°C\ m^{-1}$ and $-0.03°C\ m^{-1}$ at the time of the stoppage. They range between $-0.05°C\ m^{-1}$ and $-0.07°C\ m^{-1}$ in the main trunk. This difference is mainly due to the contrast in ice thickness and due to greater importance of horizontal advection of cold ice from upstream in the fast-moving trunk. Admittedly, our model incorporates ice thermodynamics without taking into account all possible complications (e.g., influence of past climate changes on ice temperatures). However, we are encouraged by the relatively good agreement between model results and the temperature profile measured at UpC by Engelhardt and Kamb (Figure 9) (Caltech, personal communication, 2000). This agreement is at least as satisfying as that produced previously with 2.5D, finite-element models of ice-stream thermodynamics that used variable climate [*Hulbe*, 1998; *Hulbe and MacAyeal*, 1999].

[26] The only direct observations of till porosity come from the area of the UpC camp where boreholes were drilled into the transitional zone between the active ice-stream tributaries and the inactive main trunk of ISC [*Kamb*, 2001]. Hence, we cannot verify whether the tributary-trunk contrast in porosity predicted by the model exists in reality (Figure 8a). Even the limited UpC porosity data do, however, indicate that the process of basal freezing is likely to have affected the subglacial distribution of till porosity at this location. This inference can be drawn by comparing the vertical porosity structure in till cores recovered from beneath UpB and UpC. UpB is located on the active WIS and core data show relatively uniform distribution of porosity with depth (Figure 8b). At UpC, five out of six examined till cores have demonstrated an increase in porosity with depth (Figure 8b). As shown in Figure 8a

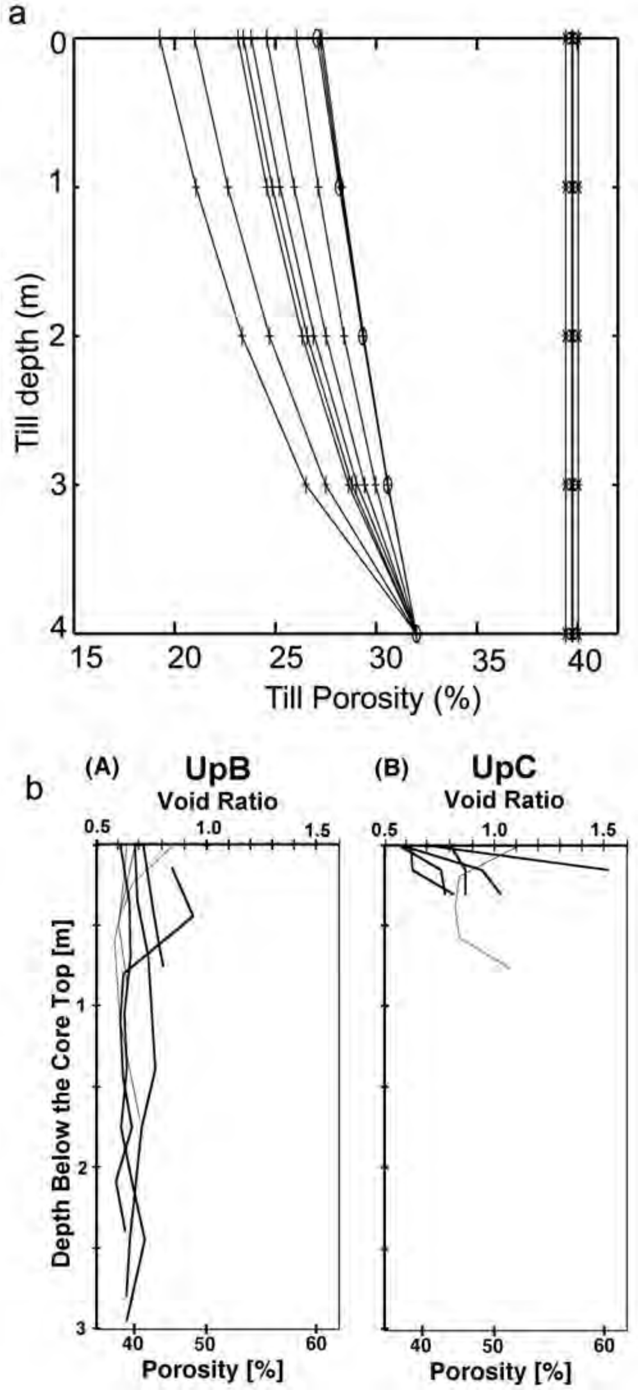

Figure 8. Simulated distribution of till porosity at the time of ISC stoppage (a). Beneath active tributaries (star symbols) porosity shows no vertical gradient but it increases with depth beneath the stopped ice-stream trunk (crosses). Porosity profile from the transition zone beneath the trunk and the tributaries is also shown (circles). (b) For comparison, we show porosity data from UpB and UpC till samples collected on WIS and ISC, respectively (UpB data from *Tulaczyk et al.* [2001]; UpC data have not been published before).

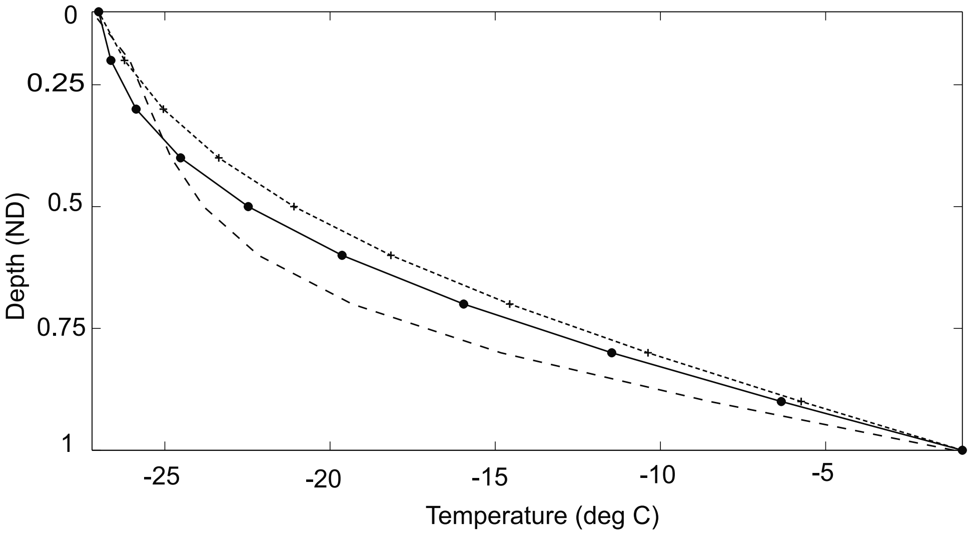

Figure 9. Temperature profiles: from field measurements at UpC (long dash) (H. Engelhardt, personal communication, 2000); modeled profile at x = 230 (solid line); comparison with an analytical solution (short dash) [*Zotikov*, 1986]. The temperature in °C is indicated on the horizontal axis, and the depth in the ice is indicated on the vertical axis, which has been nondimensionalized using ice thickness.

and in more details by *Christoffersen and Tulaczyk* [2003], freezing at the ice/till interface beneath a stopped/stopping ice stream can produce such an increase of till porosity with depth. This vertical porosity structure may then develop because the process of basal freezing withdraws pore water from the top layers of the till leading to a gradual progression of till consolidation into the till layer. The one till core from the UpC area which did not show the same porosity trend as the five other cores was retrieved from a borehole drilled into the southern paleo-margin of the stopped ice stream. We speculate that ice in the paleo-margin has been made warmer by the marginal shear heating that focuses along lateral ice-stream boundaries during active flow [*Echelmeyer et al.*, 1994; *Jacobson and Raymond*, 1998; *Raymond et al.*, 2001]. This warming may have prevented initiation of basal freezing beneath the inactive margin.

[27] The calculated basal freezing rates (of the order of mm yr^{-1}) change the value of till porosity only moderately (from ~40% to ~30%) during the simulated ice-stream shutdown (Figure 8a). In some cases, the magnitude of porosity change does not appear to be as large as the vertical porosity gradients shown in Figure 8b. This mismatch could result from our simplifying assumptions, in particular the assumption of constant till thickness. A thinner till layer would produce faster changes in till void ratio (equation (5b)), and a steeper vertical porosity gradient. However, to first approximation, our modeling results are comparable with available data. In fact, the magnitude of porosity change occurring in our model during ISC stoppage is consistent with previous inferences concerning the porosity of 'weak' and 'strong' till beds that, respectively, can and cannot facilitate fast ice streaming [*Alley et al.*, 1986]. At the same time, this moderate drop-off in porosity during shutdown may help explain the high reflectivity of the ice-till interface beneath ISC in radar and seismic surveys [*Anandakrishnan and Bentley*, 1993; *Bentley et al.*, 1998; *Gades et al.*, 2000]. Past interpretations of the geophysical data assume that the high reflectivity indicates that the till is still as weak as beneath an active ice stream [*Anandakrishnan and Bentley*, 1993]. High sensitivity of till strength to changes in till water content (equation (4)) combined with low driving stresses (typically 1–20 kPa), however, means that it may be possible to strengthen the till sufficiently to stop an ice stream while not changing the till porosity enough to make that change detectable with surface- and aircraft-based geophysics. *Gades et al.* [2000] demonstrated that the bed of the Siple Ice Stream still shows much greater reflectivity in radar surveys than the bed of Siple Dome, which is presumably frozen to its bed [*Bentley et al.*, 1998]. This higher reflectivity persists in spite of the fact that Siple Ice Stream is thought to have stopped ca. 500 years ago [*Nereson*, 2000; *Nereson et al.*, 1998]. The mere presence of unfrozen till beneath an ice stream does not guarantee that the till bed is lubricated enough to facilitate fast ice motion.

[28] The tributary-trunk contrast in the magnitude of till porosity calculated with our model is also consistent with the peculiar spatial distribution of microearthquakes recorded from beneath ISC [*Anandakrishnan*, 1990; *Anandakrishnan and Alley*, 1997b; *Anandakrishnan et al.*, 2001; *Anandakrishnan and Bentley*, 1993; *Anandakrishnan et al.*, 1998]. The most striking feature of this distribution is its inverse relationship with ice velocity [*Anandakrishnan et al.*, 2001, Figure 2]. Frequent microearthquakes (~10 to ~100 events per day) occur on the stopped part of the ice stream that moves with velocity of only ~1 to ~10 m yr^{-1}, but these events are rare beneath the active tributaries which move one to two orders of magnitude faster. In the framework of soil mechanics model of till behavior [*Tulaczyk et al.*, 2000a], if till is sufficiently consolidated it attains brittle rheology and its deformation can produce microearthquakes because it is accommodated through fracturing. When till has high porosity, as in the samples obtained beneath WIS

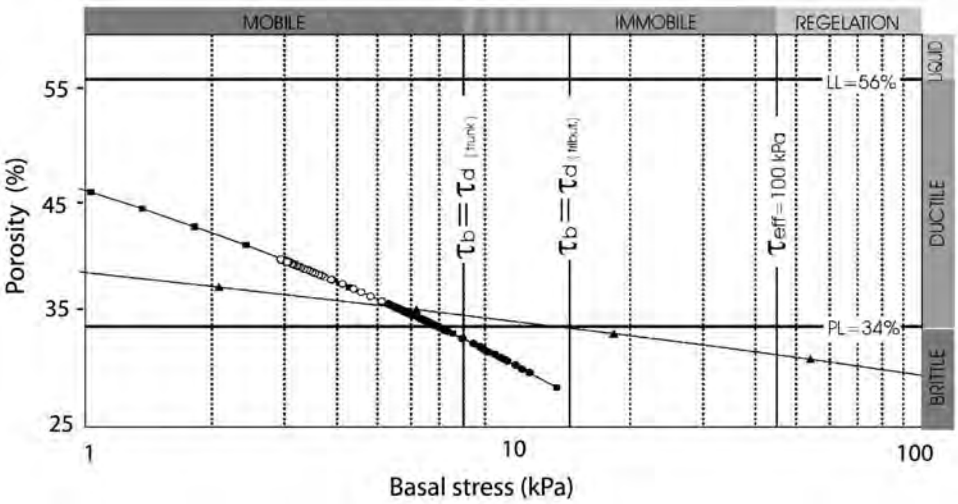

Figure 10. Till porosity (vertical axis) as a function of till strength (horizontal axis on a log scale). Two theoretical relationships are displayed, with different values for *a* and *b* parameters (equation (4)): the profile with black squares is from *Kamb* [2001] and the one with black triangles is from *Tulaczyk et al.* [2000a, 2000b]. The transition between mobile and immobile bed is for each case (tributaries and main trunk) where $\tau_b = \tau_d$. We represent here a range of possible values, bounded by the average value of τ_d beneath the main trunk, and the averaged one beneath the tributaries. The data plotted here represents the till strength beneath the ice stream after stoppage (model year 200). The tributaries data (white circles) stay in a zone of weak and ductile till bed, whereas the main trunk data (black circles) can be in a zone of mobile and ductile bed, mobile and brittle bed, or immobile and brittle bed. After *Christoffersen and Tulaczyk* [2003] we assume that regelation ice invades till when the effective stress τ_{eff} reaches ca. 100 kPa (corresponds to τ_b = 45 kPa).

(~40%, [*Tulaczyk et al.*, 2001]), its motion is accommodated through smooth, ductile deformation that is not able to produce stick-slip events. In soil mechanics literature, the boundary between brittle and ductile sediment behavior is typically called 'the plastic limit' and is expressed in terms of gravimetric water content [*Muir Wood*, 1990]. Using the standard soil mechanics procedure [e.g., *Bowles*, 1992] we have determined the plastic limit on three samples of UpC till provided to us by Kamb and Engelhardt (Caltech, personal communication). The plastic limit is 17% (std. dev. ±1%), which is equivalent to ~55% and ~34% in terms of void ratio and porosity, respectively. Thus, the brittle-ductile boundary for the subglacial till of ISC lies between the porosities associated typically with 'mobile' and 'strong' till beds, as defined originally by *Alley et al.* [1986] and further substantiated by the results of our numerical simulations (Figure 10). If ice-stream stoppage is associated with till consolidation, an inactive ice stream could attain a more 'noisy' bed than an active ice stream, just due to the change in till rheology taking place during till consolidation.

3.3. Failure to Model the Long Phase of Flow: Sensitivity Analysis

[29] As elaborated in the previous chapter, the encouraging result of our FM model is that it can reproduce in a realistic manner the stoppage of ISC. There is evidence, however, that Ross ice streams experienced a period of long (~10,000 years or more) flow before the stoppage [*Anderson*, 1999] whereas our model produces consistently a rapid stoppage over just several hundred years. We reviewed the assumed boundary and initial conditions to identify which of these conditions must be relaxed for the model to yield a long period of steady flow followed by a relatively fast shut down. We have explored through numerical experiments several key parameters that could potentially change the coupled evolution of ice velocity, thickness, and bed strength in a way that delays fast ice-stream shutdown (Table 1). As explained in the introduction, we have deliberately excluded all non-local aspects of subglacial water drainage and balance from our model. The scientific motivation is to test with our model whether inclusion of subglacial drainage is necessary to reproduce the observed stoppage of ISC.

[30] The strategy for modifying our ice-stream model was guided by the observation that the root cause of the simulated rapid ice-stream stoppage was basal freezing triggered by ice overthinning. Firstly, we have examined the sensitivity of model behavior to the volume of ice entering the two simulated tributaries on their upstream

Table 1. FM Model Sensitivity to Control Parameters

Controlled Parameter	Minimum Value Tested	Maximum Value Tested	Number of Experiments	Comments
Q	12 km^3/yr	20 km^3/yr	5	stoppage
$\dot{a}$	0.1 m/yr	0.25 m/yr	4	stoppage
v	0 m/yr	5m/yr	3	stoppage
H_{init}	Modern H	Modern H + 500m	6	stoppage
B	393 kPa $yr^{1/3}$	495 kPa $yr^{1/3}$	3	stoppage
U_{adv}	0 m/yr	Us	2	stoppage

end from outside of the model domain. At the present time ice flux at the location where we have placed the upstream end of the tributaries is estimated to be ca. 12 km^3 [*Joughin and Tulaczyk*, 2002]. Maximum ice surface velocities there are nearly 70 m yr^{-1} but it is unclear how much larger the velocities and the flux may have been before the shutdown of ISC. Examination of tributaries of other ice streams indicates that their maximum velocities are typically closer to 100 m yr^{-1} [*Joughin et al.*, 1999]. We have bracketed the present-day value of ice influx in five numerical experiments that included a maximum flux of ca. 20 km^3. All of the experiments resulted in swift ice-stream stoppage without a prolonged period of ice-stream activity.

[31] Similarly negative results came from several other sensitivity tests in which we have changed the accumulation rate and ice inflow rate across the lateral shear margin. The accumulation rate in our experiments was normally set to 0.1 m yr^{-1}, which is a good approximation of the present day value in the study area [*Vaughan et al.*, 1999]. In the senstitivity experiments we increased it by up to 2.5 times (Table 1). Using data compiled by *Van der Veen and Whillans* [1996] and *Raymond et al.* [2001] we chose to vary the transverse ice influx rate between 0 and 5 m yr^{-1}. In the end, none of the three methods of varying ice flux through the ice-stream system resulted in sufficient thickening of the simulated ice-stream trunk to prevent basal freezing and ice-stream shutdown.

[32] Three additional strategies aimed at modifying the tendency of our model to lead to a rapid ice-stream stoppage included: (1) increasing the initial ice thickness by up to 500 meters, (2) increasing the reference ice stiffness parameter by ca. 25% (393 vs. 495 kPa $yr^{1/3}$ at the reference temperature of $-10°C$), and (3) increasing downstream advection of till. We expected that by increasing the initial thickness of the ice stream, we may allow the ice-stream model to spend some significant time (100's to 1000's of years) in a quasi steady state that would last until ice thinned again enough to turn on basal freezing. Moreover, it is known that in the recent past (10,000–20,000 years BP) this section of the WAIS was indeed thicker by at least several hundred meters [*Conway et al.*, 1999; *Steig et al.*, 2001]. However, the increased initial thickness did not produce the expected longer period of ice-stream activity because it led also to an increase in driving stress, which translated into higher ice-stream velocities (reaching several kilometers per year). These high ice velocities drained the extra ice thickness very quickly (dozens of model years) and the thickened ice stream stopped nearly as rapidly as the ones with modern initial surface geometry. An arbitrary increase in the ice stiffness parameter was aimed at slowing down the ice-stream velocity by increasing the lateral resistance to ice motion. Finally, an increase in downstream advection of till (equivalent at maximum to till plug flow) was aimed at increasing the ability of the till bed to remain weak even in the presence of basal freezing. Both latter strategies have had similarly insignificant influence on the behavior of the simulated ice stream (Table 1).

[33] Although all of the numerical experiments discussed in this section have had a negative result (i.e., no long phase of ice-stream activity was achieved), we believe that it is important to include a discussion of these experiments. Their results demonstrate that the simulated ice-stream shutdown is a robust feature of our model resulting from the physics embedded in the equations used to described the coupled ice-till flow.

4. Migrating Margin (MM) Model

4.1. Model Settings

[34] One important control parameter that has been deliberately ommited from the discussion in the previous section is the ice-stream width. While discussing the phenomenon of stable ice motion in the tributaries we have already pointed out that the narrow width of the tributaries seems to play crucial role in maintaining moderate ice velocities in the tributaries ($\sim$100 m yr^{-1}) and preventing their overthinning. Examination of equation (1) shows that ice-stream velocity is indeed related to width in a sensitive, non-linear manner (with an exponent of $n + 1 \approx 4$). Hence, a long phase of ice-stream activity could be obtained if the main trunk was narrower in the past than it is today. There is only limited direct evidence for the history of ice-stream width changes in West Antarctica. Recent theoretical and observational results suggest that significant margin migration can and has occurred, likely as a result of high heat dissipation in ice-stream margins [*Bindschadler and Vornberger*, 1998; *Clarke and Bentley*, 1995; *Clarke et al.*, 2000; *Echelmeyer and Harrison*, 1999; *Harrison et al.*, 1998; *Jacobel et al.*, 2000; *Jacobson and Raymond*, 1998; *Raymond et al.*, 2001]. Documented cases include both inward and outward margin migration at rates varying between 7 and 100 m yr^{-1} [*Hamilton et al.*, 1998; *Harrison et al.*, 1998; *Raymond et al.*, 2001].

[35] Previously, *Van der Veen and Whillans* [1996] used an ice-stream model with time-dependent width to demonstrate the primary importance of width as a control on ice-stream velocity. They were unable to obtain ice-stream stability for widths greater than ca. 20 km. Their result is consistent with the output of our fixed-margin model, although we are able to obtain stable ice flow in tributaries that are somewhat wider than ca. 20 km. This secondary difference may stem from a different parameterization of basal resistance in the two models. *Van der Veen and Whillans* [1996] have also experimented with two different physically based parameterizations of the rate of lateral shear margin migration. However, the processes that control margin migration appear to be complex [*Jacobson and Raymond*, 1998; *Raymond et al.*, 2001] and more work is needed to develop reliable parameterizations [*Van der Veen and Whillans*, 1996, p. 136]. For the purpose of simulating prestoppage activity of ISC, we have assumed that the ice stream had some arbitrary initial width that can produce millennial-scale ice-stream stability. This width was subsequently increased at a rate consistent with observations from modern migrating margins.

[36] We have tested the width-dependence of ISC behavior in the following simple model configuration. The model is initialized with tributaries that have width of 33 km and with the main trunk being twice as wide. We then prescribe a widening rate starting at 0 m yr^{-1} at the junction of the tributaries and increasing linearly up to 1.5 m yr^{-1} at the grounding line. We impose this pattern of widening based on the observation that the primary feature

Table 2. MM Model Sensitivity to Different Widening Rates

Maximum Widening Rate	Maximum Width at Instability, km	Period of Steady Flow
1.5 m per year	75.2	6,300–6,800
15 m per year	80	1000–1200
50 m per 50 years	76.5	10,000

of ISC geometry is its gradual downstream widening. However, no attempt is made to reproduce the intricacies of downstream width distribution in the main trunk. The decision to keep ice-stream tributaries at constant width through time is motivated by the fact that the tributaries are located in deep troughs, unlike the main trunk that is less topographically constrained (Figure 2). The rest of the model settings are identical to the FM model, described above.

4.2. MM Model: Results and Discussion

[37] By introducing widening of the simulated ice stream we are able to reproduce both a long period of ice-stream activity and an ice-stream shutdown. Through several numerical experiments we were able to ascertain that the same activity-stoppage scenario arises under different widening rates and modes of widening (Table 2). Moreover, the simulated cessation of ice-stream activity occurs consistently when the maximum ice-stream width (at the grounding line) reaches values similar to the real maximum width of ISC at the time of its shutdown (75–80 km) (Table 2 and Figure 11). An increase in ice-stream width from initial 66 km to 75–80 km at shutdown may seem small. However, the non-linear dependence of ice velocity on width (equation (1)) means that such width change will translate into an increase in ice velocity by 67 to 115%.

[38] The length of the simulated active phase of ice-stream flow depends on the widening rate. For instance, the maximum widening rate of 1.5 m yr^{-1} results in stoppage after almost 7,000 years of activity (Figure 11). A widening rate that was 10 times faster led to a shutdown after <1,000 years (Table 2). As in the FM model, the stoppage phase itself is always short (100–200 years). In Figure 11, we show an example of evolution of the ice-stream-tributary system beyond the stoppage phase. It must be emphasized that the model has not been designed specifically to investigate ice-stream reactivation. However, it is informative to examine the post-stoppage behavior because it again illustrates the high sensitivity of ice-stream evolution to changes in ice-stream width. The lower part of the ice-stream trunk remains shut down for almost 2,000 years. Thereafter, renewed basal melting beneath the whole ice-stream triggers the reactivation. This basal melting is triggered by thickening of ice due to: (1) surface accumulation, and (2) downstream migration of a surge bulge fed by the continuing ice influx from upstream tributaries [*Alley et al.*, 1994; *Price et al.*, 2001]. The reactivated ice stream is at this stage very wide (80 km at the grounding line) and the ice reservoir that was built up during the stoppage phase is drawn down rapidly (ca. 500 years). Ice-stream velocities during this second phase of activity reach over 1,000 m yr^{-1}. As the ice stream thins, it experiences another rapid stoppage.

[39] Results of our MM model demonstrate that ice-stream stability and evolution are highly sensitive to ice-stream width and its changes [see also *Van der Veen and Whillans*, 1996]. In addition, the results suggest existence of a threshold width below which an ice stream can remain active for at least thousands of years and above which ice-stream stoppage commences. Although we have imposed a constant and arbitrary widening rate in our model, in Nature margin migration is likely to be controlled by lateral migration of basal melting-freezing boundary [*Jacobson and Raymond*, 1998; *Raymond et al.*, 2001; *Van der Veen and Whillans*, 1996]. Migration of this boundary is controlled by the basal heat balance on both sides of an ice-

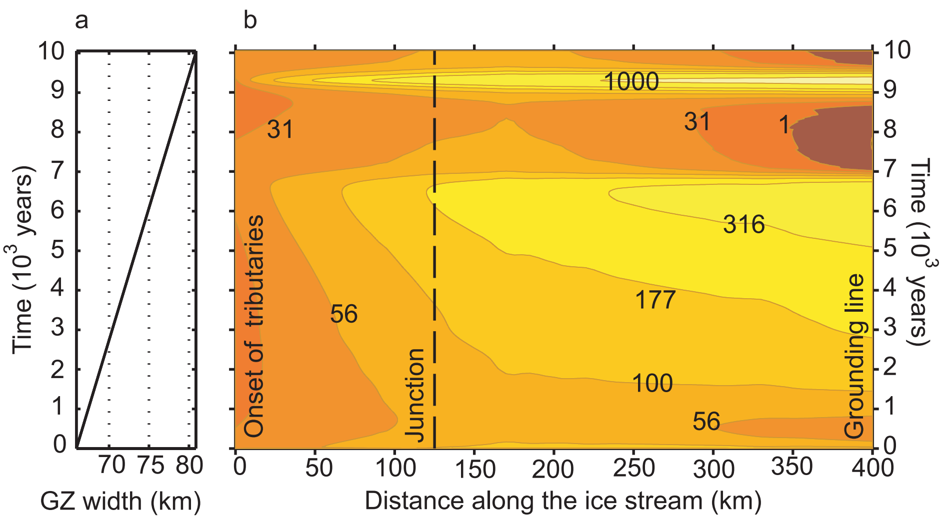

Figure 11. Ice-stream width (a) and velocity (b) evolution calculated from the Migrating Margin model velocity. Time is displayed on the vertical axis. Width evolution shown for the grounding line only. x = 0 km corresponds to the onset of tributaries and x = 400 km corresponds to the assumed grounding line position. The junction between tributaries and main trunk is at x = 175 km. Contour lines and numbers are analogous to those shown in Figure 5.

stream shear margin. Through the basal heat balance, the margin migration rate is then tied to both, climate changes and internal ice thermodynamics.

[40] If ISC was active for at least the last 10,000 years as indicated by geologic evidence [*Anderson*, 1999], our results suggest that the ice stream was narrower than its present-day outlines during this time. Colder conditions during glacial time may have favored more widespread basal freezing and narrowing of West Antarctic ice streams. Gradual propagation of the post-glacial climate warming to the ice base [*Parizek et al.*, 2002; *Whillans*, 1978] may have tipped the basal heat balance in the margins toward gradual widening and thinning of the ice stream. This increase in width may have triggered the recent stoppage of ISC, once the width reached the threshold value of 75–80 km. This qualitative model of widening and stoppage being ultimately driven by post-glacial warming is likely non-unique [e.g., *Van der Veen and Whillans*, 1996]. However, it provides a feasible connection between the stoppage of ISC and the broader problem of the post-glacial retreat and evolution of the WAIS.

5. Conclusions

[41] By incorporating the Undrained-Plastic-Bed treatment of subglacial till into a numerical model of ice thermodynamics, we have generated a new numerical ice-stream model that is capable of reproducing the stoppage of ISC ca. 150 years ago. When the model is driven by initial and boundary conditions that emulate the prestoppage mass balance, dynamics, and geometry of ISC, the stoppage happens within the first few hundred model years after a simulation is started. In our model, the root cause of the stoppage is thinning of the ice-stream trunk, which triggers a switch in basal thermal regime from melting to freezing. Once the freezing starts, a positive feedback between shear heating and basal freezing leads to a runaway increase in basal resistance and decrease in ice velocity. The simulated stoppage of ISC matches several key observations: (1) fast ice motion continues in ice-stream tributaries even after complete shutdown of the main ice-stream trunk, (2) crevasses stop forming in the ice-stream trunk within several decades, and (3) water-saturated till is present beneath the stopped trunk for at least several hundred years. Sensitivity analysis has shown that the model produces stoppage even if we change significantly such important control parameters as the accumulation rate, the rate of ice influx at the onset of ice-stream tributaries, ice input across the lateral margins, initial ice thickness, the rate of downstream advection of till, or the ice stiffness parameter. We found that only when the width of the simulated ice stream was decreased below a threshold value (here 75–80 km), our ice-stream model was able to reproduce a long active phase of flow of ISC. In model runs with initial width of 66 km and a constant widening rate, the length of the active phase was inversely dependent on the value of the widening rate (1.5 to 15 m/yr) and ranged between ~1,000 and ~7,000 model years. Our results indicate that changes in ice-stream width can switch ice-stream behavior between stable and unstable modes. Since they also show that the stoppage of ISC may have been a part of natural ice-stream oscillations, they leave open the possibility that other Ross ice streams (e.g., the slowing down WIS) may stop in the near future.

Notation

Parameters		
a	empirical constant for basal stress determination	131×10^3.
b	empirical constant for basal stress determination	5.7.
c_v	till diffusivity, $m^2\ yr^{-1}$	0.32.
g	constant of gravity, $m\ s^{-2}$	9.8.
G	geothermal flux, $W\ m^{-2}$	0.07.
κ	thermal diffusivity of ice, $m^2\ yr^{-1}$	36.
K_h	hydraulic conductivity of till, $m\ yr^{-1}$	3.15×10^{-3}.
K_i	thermal conductivity of ice, $J\ m^{-1}\ yr^{-1} K^{-1}$	66×10^6.
L	latent heat of ice, $J\ kg^{-1}$	335×10^3.
n	Glen flow law exponent	3.
ρ_i	ice density, $kg\ m^{-3}$	917.
ρ_s	density of till solids, $kg\ m^{-3}$	2600.
ρ_w	water density, $kg\ m^{-3}$	1000.
μ	coefficient of internal friction for till	0.45.
Control variables		
$\dot{a}$	accumulation rate, $m\ yr^{-1}$	0.1.
λ	atmospheric lapse rate, $°C\ m^{-1}$	−0.004.
U_t	transverse velocity, $m\ yr^{-1}$	2.
Z_s	solid till thickness, m	3.

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