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Solving Trapezoidal Fuzzy Linear Programming Problem using Modified Big-M

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ABSTRACT
The fuzzy logic and fuzzy numbers have been applied in many areas of Mathematical Programming. Optimization under uncertainty is one of the most important problem in Mathematical Programming. This paper proposes a Modified Big M method to solve fully fuzzy Trapezoidal linear programming problem with fuzzy decision variables and fuzzy parameters.

Keywords: *Linear programming problem, fuzzy numbers*

INTRODUCTION

Decision making is most important phenomenon for every person. In our daily life, there arise several problems where we face problems of finding optimal solution irrespective of limited constraints. To find the optimal solution we use mathematical programming technique called linear programming.

Linear programming is a best technology to find the optimal solution. Fuzziness in the data is described by the fuzzy membership function. Trapezoidal fuzzy membership function is defined by four parameters. Here in this paper we are solving fully fuzzy trapezoidal Linear programming problem with constraints having ≥ sign using Modified Big-M method.

George Dantzig in 1947 first introduced Linear programming problems. Fuzzy decision making in a Fuzzy environment was first introduced by Bellman and Zaheh [1], further Zimmermann [10] illustrated Fuzzy programming and Linear programming, Since then it has been used to solve a variety of problems. Deldago, Verdegay, Vila [2] introduced A General Model for solving Fuzzy Linear Programming. Fang, Hu, Wu & Wang [3] proposed Linear Programming with Fuzzy Coefficients in Constraint. Maleki, Tata and Mashinchi [5] proposed a new method to solve Linear Programming with fuzzy variables using ranking function. Nasseri and Ardil [6] developed a Simplex Method for Fuzzy Variables Linear Programming Problems. Safi, Maleki and Zaeimazad [9] proposed A geometric approach for solving fuzzy linear programming problems. Nasseri, Alizadeh, and Khabiri [7] solved fuzzy linear Algorithm Using a Two Phase Method. Ghanbari, Ghorbani-Moghadam, Mahdavi-Amiri, Baets [8] investigated various types of fuzzy linear programming problems based on models and solution methods. Recently Figueroa–García, Hernández and Franco [4] reviewed on history, trends and perspectives of fuzzy linear programming models and methods.

PROPERTIES OF FUZZY NUMBERS

1. If a fuzzy number $X_1 = (x_1, y_1, z_1, \gamma_1)$ and another fuzzy number $X_2 = (x_2, y_2, z_2, \gamma_2)$, then a fuzzy number $X_1 \oplus X_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2, \gamma_1 + \gamma_2)$
2. If a fuzzy number $X_1 = (x_1, y_1, z_1, \gamma_1)$ and another fuzzy number $X_2 = (x_2, y_2, z_2, \gamma_2)$, then a fuzzy number $X_1 \Theta X_2 = (x_1 - x_2, y_1 - y_2, z_1 - z_2, \gamma_1 - \gamma_2)$
3. If a fuzzy number $X_1 = (x_1, y_1, z_1, \gamma_1)$ and another fuzzy number $X_2 = (x_2, y_2, z_2, \gamma_2)$, then a fuzzy number $X_1 \otimes X_2 = (x_1 x_2, y_1 y_2, z_1 z_2, \gamma_1 \gamma_2)$
4. If a fuzzy number $X_1 = (x_1, y_1, z_1, \gamma_1)$ and $k > 0$ then $X_2 = (kx_1, ky_1, kz_1, k\gamma_1)$ is also a fuzzy number.
5. If a fuzzy number $X_1 = (x_1, y_1, z_1, \gamma_1)$ and another fuzzy number $X_2 = (x_2, y_2, z_2, \gamma_2)$, then a fuzzy number $X_1 / X_2 = (x_1 / x_2, y_1 / y_2, z_1 / z_2, \gamma_1 / \gamma_2)$

Steps to Solve Fuzzy LPP

1. Convert the given problem into maximization form. If the given problem is in Minimization form then first convert it into Maximization form.
2. Convert the Constraints into equations. Add surplus and slack variables in the constraints to convert the inequalities into equations, Therefore express the constraints in the form Ax = b.
3. Add artificial variables if needed

Add artificial variables in constraints. Corresponding to these artificial variables add these in the objective function with cost –M.

If the inequality type is ≤ , add slack variable.

If the inequality type is = , add artificial variable.

If the inequality type is ≥ , subtract surplus variable and artificial variable.

4. Preparing Initial Fuzzy Simplex table

The initial table of Simplex method consists of all the coefficients of the decision variables of the original problem and the slack and artificial variables. The C_B column contains the variables that are in the base. The C_j column contains the coefficients of the variables that are in the base. The first row consists of the objective function coefficients, while the last row contains the objective function value and reduced costs

The last row is calculated as follows:

$$Z_j - C_j : \min \left\{ \left(\frac{x_1 + y_1 + z_1 + \gamma_1}{4} \right), \left(\frac{x_2 + y_2 + z_2 + \gamma_2}{4} \right), \ldots, \left(\frac{x_n + y_n + z_n + \gamma_n}{4} \right) \right\}$$

5. Stopping Condition

If $\forall Z_j - C_j \geq 0$ that is there is no negative value. This implies that the stop condition has been reached. Then the method reaches at end and we get the optimal value of the objective function. Another conceivable situation is all values are negative or zero in the input variable column of the base. This demonstrates that the problem isn't restricted and the arrangement will constantly be improved along.

APPLICATION

In this paper we are going to solve a linear programming problem by trapezoidal fuzzy number using simplex algorithm. Our problem is described below:

$$\max z = (-1,1,-5,-2)x_1 + (-1,5,-2,-1)x_2$$
s.t.
$$(2,3,3,3)x_1 + (1,4,1,1)x_2 \geq (4,6,4,3)$$
$$(1,1,2,4)x_1 + (7,3,1,3)x_2 \geq (7,2,3,6)$$

Introducing surplus and artificial variables, Modified LPP will be

$$\max z = (-1,1,-5,-2)x_1 + (-1,5,-2,-1)x_2 + (0,0,0,0)x_3 + (0,0,0,0)x_4 + (-M,-M,-M,-M)x_5 + (-M,-M,-M,-M)x_6$$
$$s.t.\ (2,3,3,3)x_1 + (1,4,1,1)x_2 + (-1,-1,-1,-1)x_3 + (1,1,,1,1)x_5 = (4,6,4,3)$$
$$(1,1,2,4)x_1 + (7,3,1,3)x_2 + (-1,-1,-1,-1)x_4 + (1,1,,1,1)x_6 = (7,2,3,6)$$

	c_j	(-1,1,-5,-2)	(-1,5,-2,-1)	(0,0,0,0)	(0,0,0,0)	(-M,-M,-M,-M)	(-M,-M,-M,-M)
BV	X_B	x_1	x_2	x_3	x_4	x_5	x_6
x_5	(4,6,4,3)	(2,3,3,3)	(1,4,1,1)	(-1,-1,-1,-1)	(0,0,0,0)	(1,1,1,1)	(0,0,0,0)
x_6	(7,2,3,6)	(1,1,2,4)	(7,3,1,3)	(0,0,0,0)	(-1,-1,-1,-1)	(0,0,0,0)	(1,1,1,1)
z_j		(-3M,-4M,-5M,-7M)	(-8M,-7M,-2M,-4M)	(M,M,M,M)	(M,M,M,M)	(-M,-M,-M,-M)	(-M,-M,-M,-M)

First Big-M table

From the above table $\frac{z_j - c_j}{4} = \left(\frac{-8M+1-7M-5-2M+2-4M+1}{4}\right)$ is minimum. Therefore x_2 is entering vector.

For departing vector find: $\min\left(\left\{\frac{4+\frac{3}{2}+4+3}{4}\right\},\left\{\frac{1+\frac{2}{3}+3+2}{4}\right\}\right)$, Therefore x_6 is departing vector. Now, the next table will be:

	c_j	(-1,1,-5,-2)	(-1,5,-2,-1)	(0,0,0,0)	(0,0,0,0)	(-M,-M,-M,-M)
BV	X_B	x_1	x_2	x_3	x_4	x_5
x_5	(3,10/3,1,1)	(13/7,5/3,1,-1)	(0,0,0,0)	(-1,-1,-1,-1)	(1/7,4/3,1,1/3)	(1,1,1,1)
x_2	(1,2/3,3,2)	(1/7,1/3,2,4/3)	(1,1,1,1)	(0,0,0,0)	(-1/7,-1/3,-1,-1/3)	(0,0,0,0)
z_j		(-13/7M-1/7,-5/3M+5/3,-M-4,M-4/3)	(-1,5,-2,-1)	(M,M,M,M)	(-M/7+1/7,-4/3M-5/3,-M+2,-M/3+1/3)	(-M,-M,-M,-M)

Second Big-M table

From the above table $\frac{z_j - c_j}{4} = \left(\frac{-\frac{13}{7}M + \frac{6}{7} - \frac{5}{3}M + \frac{2}{3} - M + 1 + M + \frac{2}{3}}{4} \right)$ is minimum.

Therefore x_1 is entering vector. For departing vector find: $\min\left(\left\{ \frac{\frac{21}{13} + 2 + 1 - 1}{4} \right\}, \left\{ \frac{7 + 2 + \frac{3}{2} + \frac{3}{2}}{4} \right\} \right)$, Therefore x_5 is departing vector. Now, the next table will be:

	c_j	(-1,1,-5,-2)	(-1,5,-2,-1)	(0,0,0,0)	(0,0,0,0)
BV	X_B	x_1	x_2	x_3	x_4
x_1	(21/13,2,1,-1)	(1,1,1,1)	(0,0,0,0)	(-7/13,-3/5,-1,1)	(1/13,4/5,1,-1/3)
x_2	(10/13,0,1,10/3)	(0,0,0,0)	(1,1,1,1)	(1/13,-1/5,-2,4/3)	(-2/13,-3/5,-3,1/9)
z_j		(-1,1,-5,-2)	(-1,5,-2,-1)	(7/13,-3/5,5,-2)	(-1/13,4/5,-5,2/3)

Third Big-M table

Now since all $\frac{z_j - c_j}{4}$ are positive. Therefore the above solution is optimal. Hence the required solution is x_1 = (21/13, 2, 1, -1) and x_2 = (10/13, 0, 1, 10/3)

CONCLUSION

In this paper the arithmetic operations of fuzzy numbers described, we have also solved fully fuzzy Trapezoidal linear programming problem with fuzzy decision variables and fuzzy parameters. The procedure of solving fully fuzzy linear programming problem using modified Big-M method may help us to solve many optimization problems.

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