Some Influences on Future Modeling

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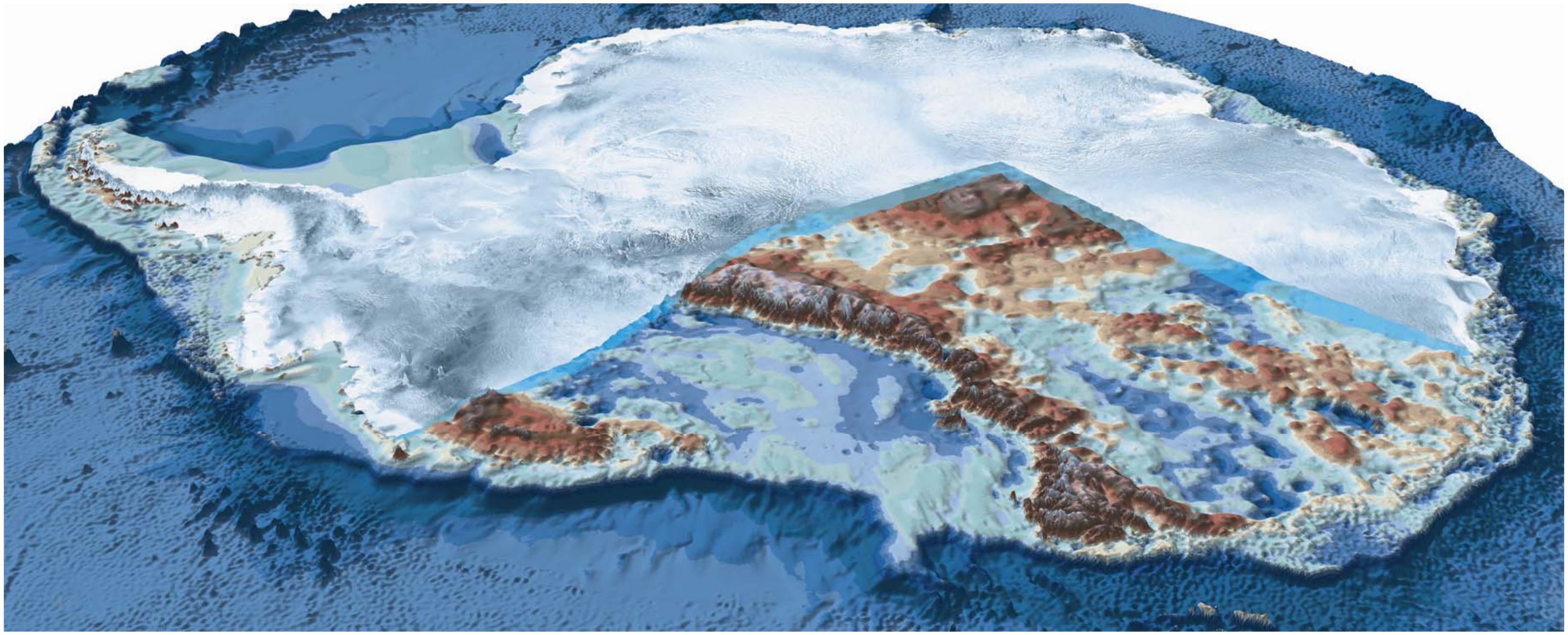
Emergence of new observations – 3 dimensional Imaging (GISMO)

Incorporation of more physics (acceleration)

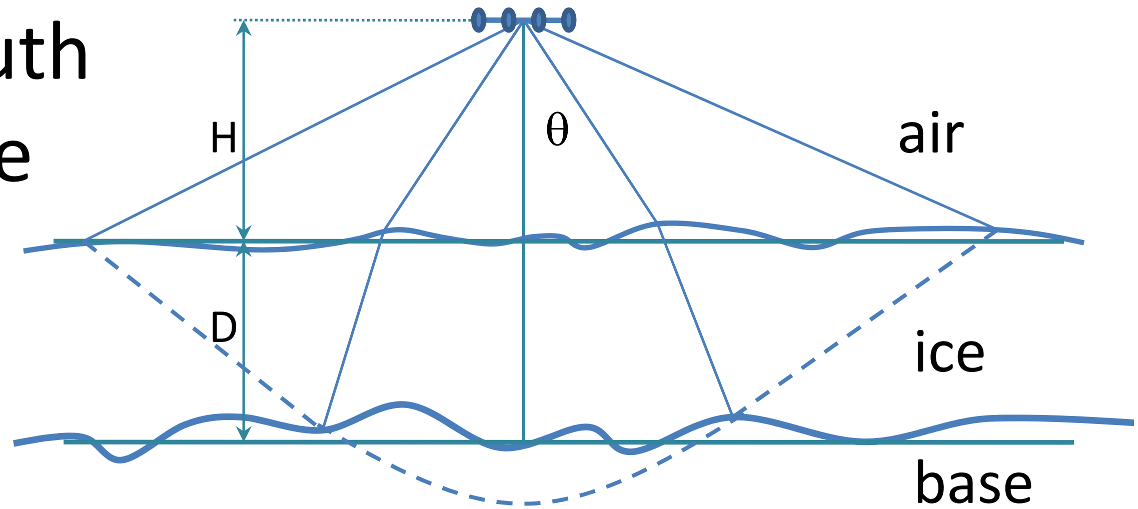
Better controls on model believability and predictability (Bayes)

Basal Ice Imaging Radar

Create 3-dimensional image maps of Greenland and Antarctica as they would appear were the ice sheets stripped away. Use these first ever maps to constrain estimates of present ice sheet mass balance. Use the maps to feed numerical models that describe the past and predict the future behavior of the ice sheets and their contribution to global sea level rise.



Tomography formulation for range and azimuth compressed image



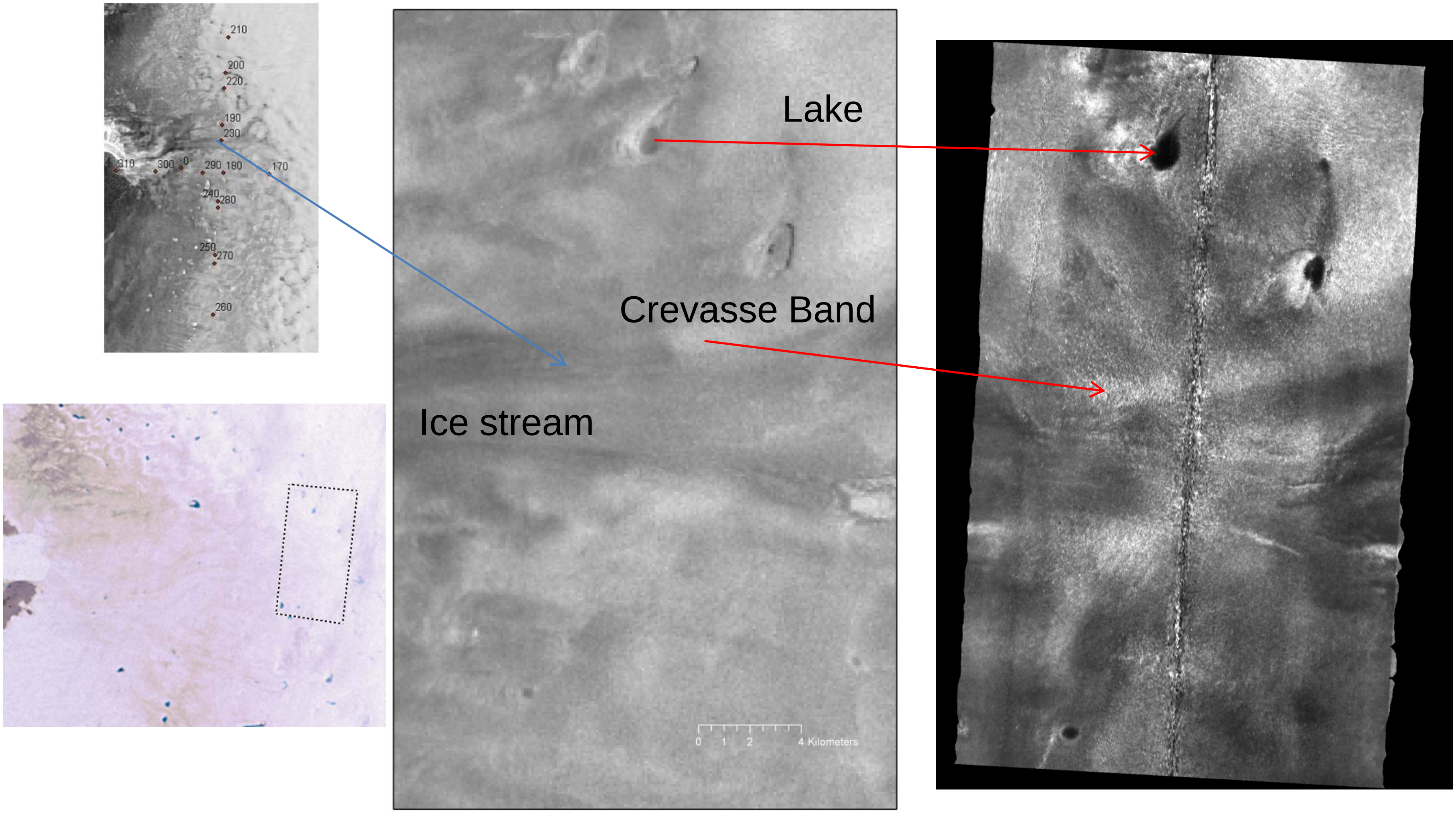
- Received signals at each sensor:

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} = \begin{bmatrix} e^{jkd_1 \sin\theta_1} & e^{jkd_1 \sin\theta_2} & \dots & e^{jkd_1 \sin\theta_q} \\ e^{jkd_2 \sin\theta_1} & e^{jkd_2 \sin\theta_2} & \dots & e^{jkd_2 \sin\theta_q} \\ \dots & \dots & \dots & \dots \\ e^{jkd_p \sin\theta_1} & e^{jkd_p \sin\theta_2} & \dots & e^{jkd_p \sin\theta_q} \end{bmatrix} \begin{bmatrix} s_1 \\ s_1 \\ \vdots \\ s_q \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_p \end{bmatrix}$$

$$\mathbf{x} = \mathbf{A}(\Theta) \cdot \mathbf{s} + \mathbf{n}$$

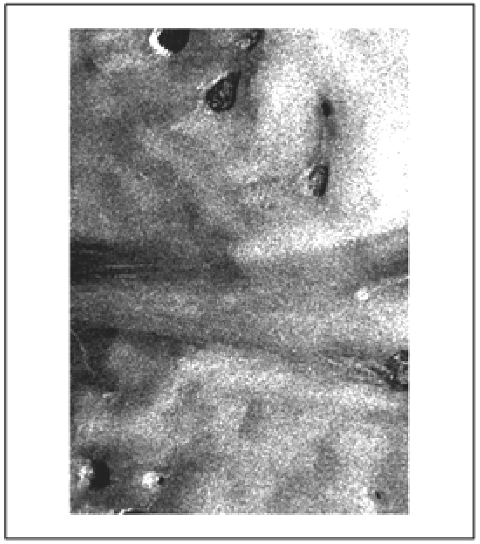
x_i: received signal of sensor i; k: $4\pi/\lambda$;
d_i: distance of sensor i; θ_i: arrival angle;
p: number of sensors; s_i: signal;
q: number of signals; n_i: noise;

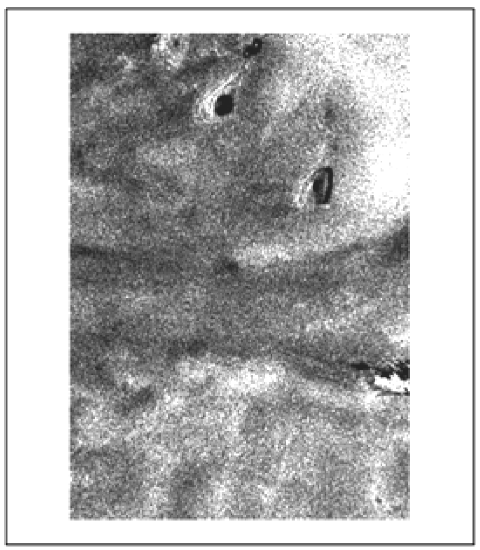
Multi-frequency Images of Ice Sheet Surface

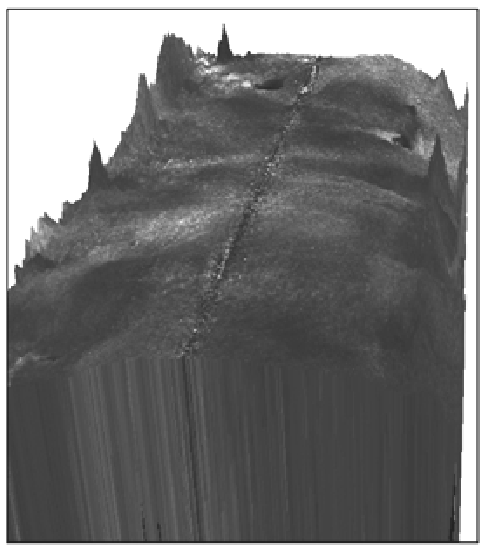


July 20, 2008, 17 km wide, 150 MHz radar tomography GISMO image (geocoded) of the upper surface of the ice sheet across Jacobshavn Glacier (right). 2000 Radarsat C-band image (center). Inset map from Radarsat mosaic (left). July 15, 2008, MERIS optical image (lower left). GISMO image located at about 69.3N, 48.3 W

GISMO Lakes Result







Upper Left: Radarsat winter scene.

Upper Right: 150 MHz GISMO surface image

Lower left: Radarsat summer scene

Lower Right: GISMO image on top of Gismo topography (preliminary result in need of a tilt correction)

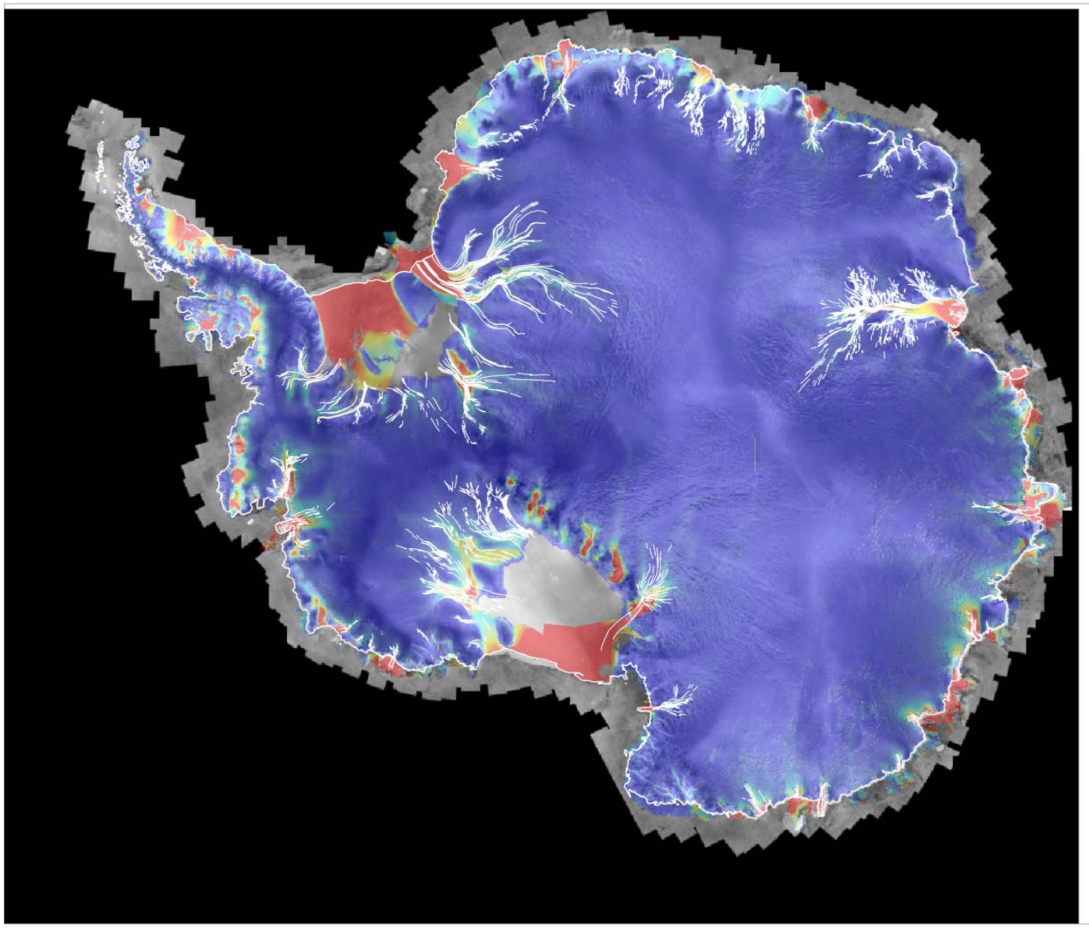
GISMO Basal Imagery

5x20 Km 3-d image of the base of the ice sheet. Scene is an orthorectified mosaic located just south of the main Jacobshavn Drainage Channel (to be corrected for cross track bias between mosaic swaths)

BIIR Drivers on Future Models

1. To what extent does bed topography (valleys and roughness) control the present day locations of ice stream networks and do any such subglacial valleys anchor the ice streams in place, providing resistance to any flips in routing to the margin?
   - *Compare ice stream networks to BIIR subglacial topography, in trunk zones and upstream of onset zones. Compare known fluctuations of ice stream width and shutdowns to underlying topography.*
2. Does ice stream location and vigor depend on the ability of the upstream bed topography to capture and route meltwater?
   - *Given coincident BIIR data on bed and ice surfaces, compute hydropotential surfaces for basal meltwater to see where it should flow and the extent to which it feeds ice stream lubrication.*
3. Is there a history of ice stream network evolution recorded in the bed geomorphology, and how stable are the existing flow paths?
   - *Use BIIR data to investigate whether today's ice stream configuration is just one of a possible set of network configurations. Learn about longer term variability in ice streams than current observations permit.*
4. Subglacial bedforms (drumlins etc) are the physical manifestation of processes operating at the ice-bed interface and which facilitate fast ice flow. Are the existing numerical models on the formation of these bedforms correct?
   - *Current work uses predictions from such models tested against the scale and shape properties of now-exposed landforms, but in the absence of the glaciological parameters to feed the model. Having BIIR data on bedforming in action along with ice thickness and slope data provide the first chance to make better constrained tests of these models.*
5. How well matched is basal and ice surface roughness?
   - *BIIR will provide the first 3-d information to test transfer function models that describe the between surface and basal topography. BIIR information on the 3-d slope of internal layers can be incorporated into the models.*

Accelerating Ice Flow Fields



Ice flow models usually assume a quasi-equilibrium ice sheet. Gradients in velocity are used to compute stresses

Equations of Motion

- Instead of making usual assumption allow

$$\rho \frac{D\bar{u}}{Dt} = -\nabla \bullet \bar{\bar{\sigma}} - \rho \bar{g}$$

$$\frac{D\bar{u}}{Dt} = \frac{\partial \bar{u}}{\partial t} + (\bar{u} \bullet \nabla)\bar{u}$$

Given the detailed spatial information about the velocity field, we will make some estimates of the left side of the equations

Now $\frac{\partial \bar{u}}{\partial t}$ poorly known. $\frac{\partial}{\partial z}$ requires assumptions

about depth dependence. So here will simply assume depth independence and evaluate

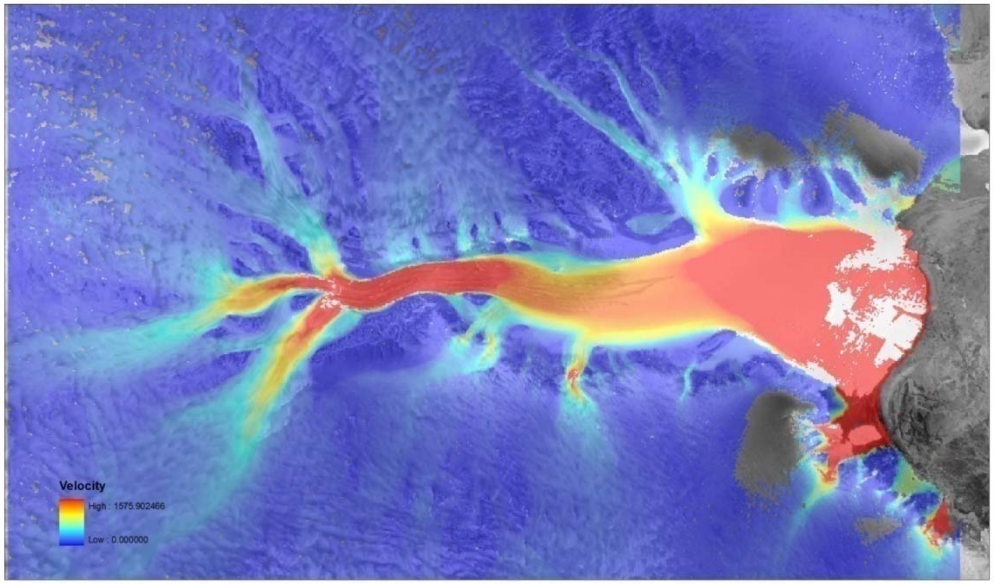
Analysis Method

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = -\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right)$$

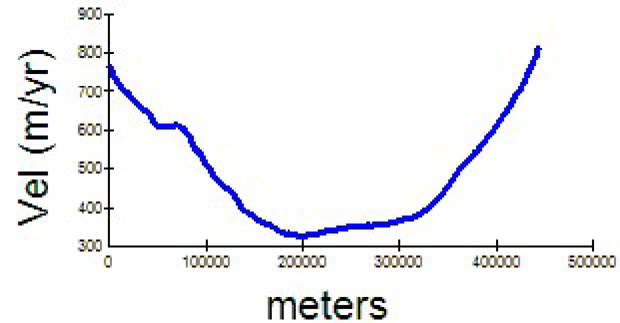
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = -\rho g - \rho\left(v_y \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y}\right)$$

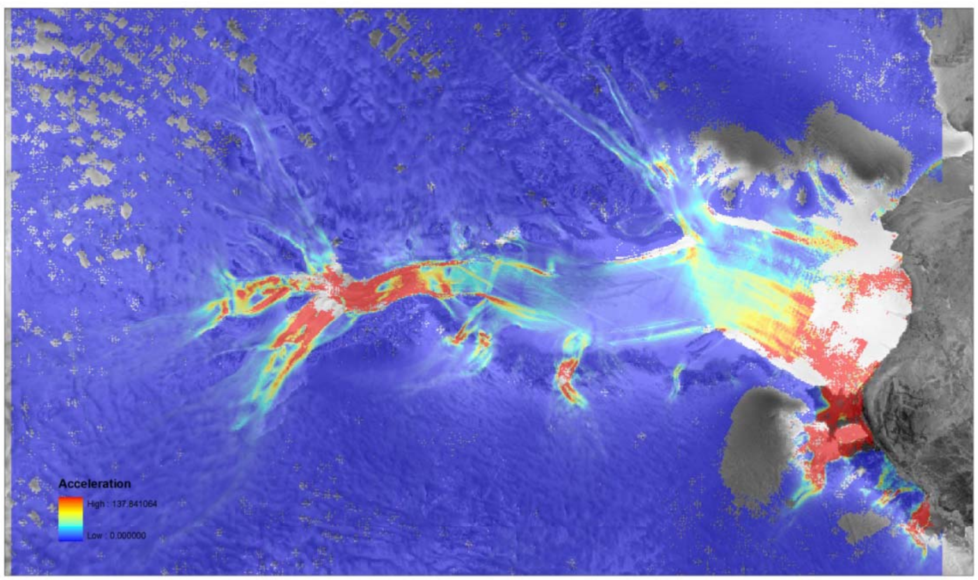
- Evaluation Method

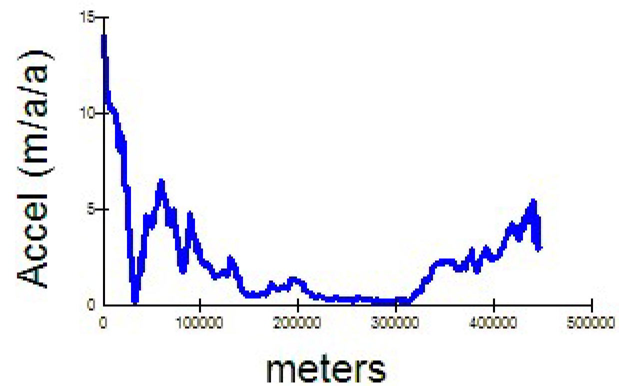
$$a_i(m,n) = \sum_{k=i}^{j} \frac{dV_i}{dx_k} V_k \approx \frac{V_i(m+1,n) - V_i(m-1,n)}{2\Delta x_i} V_i + \frac{V_i(m,n+1) - V_i(m,n-1)}{2\Delta x_j} V_j$$

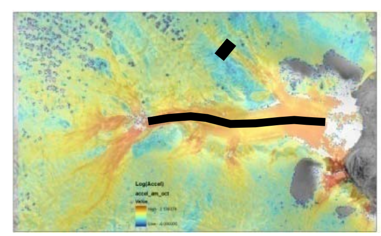


Lambert Glacier



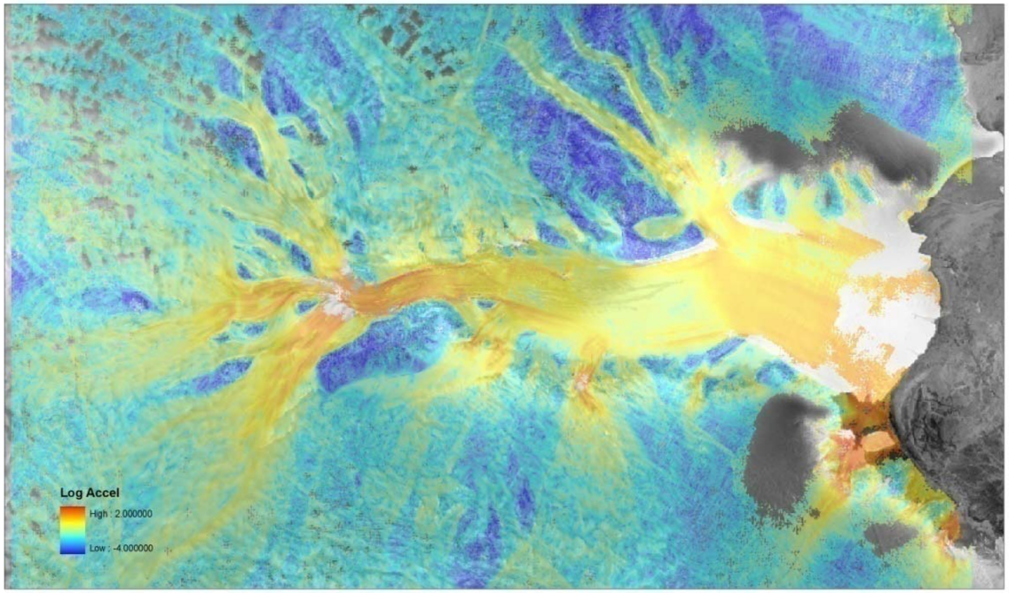


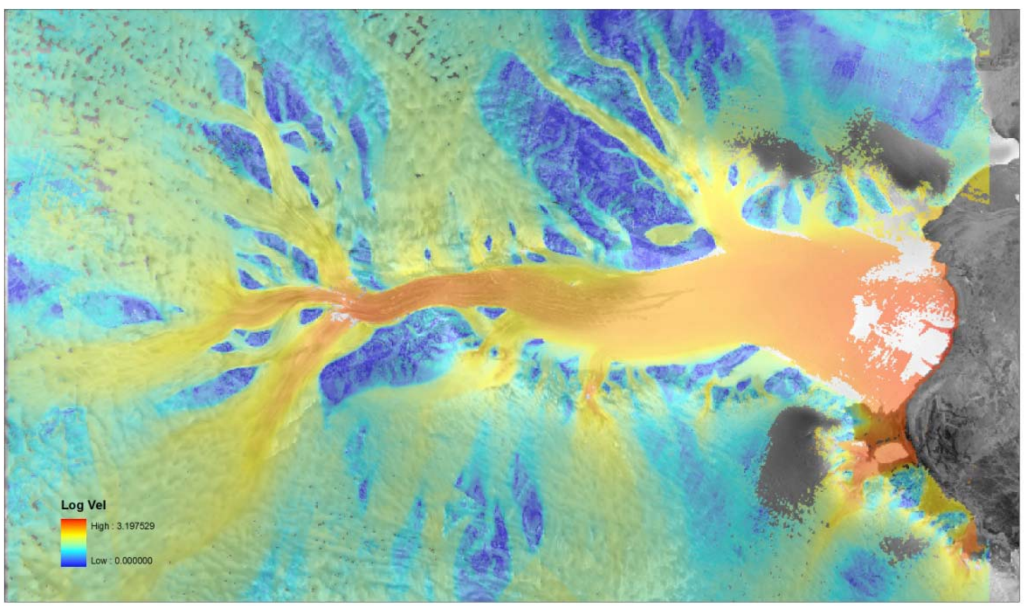




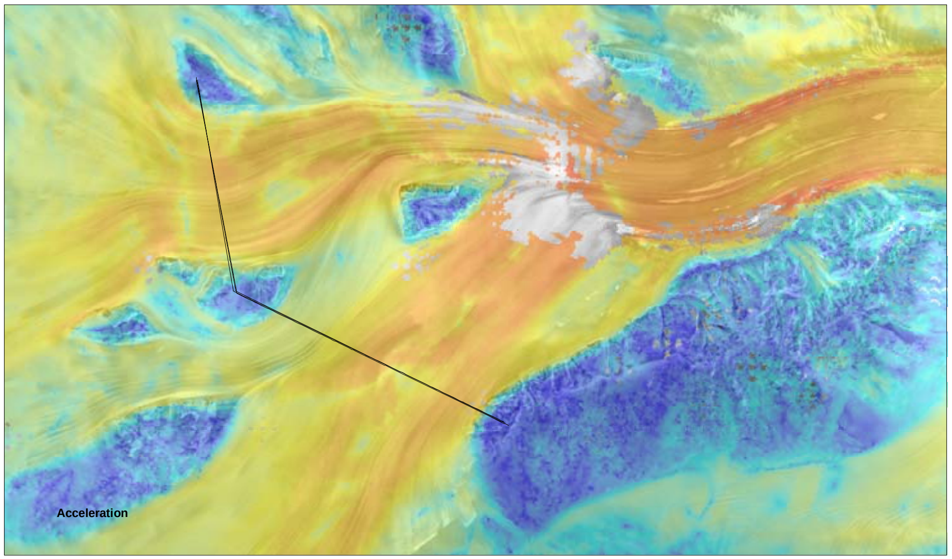
Lambert Glacier

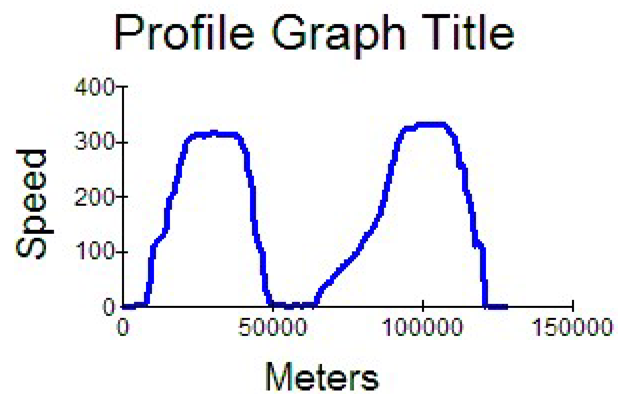
- Log Scales

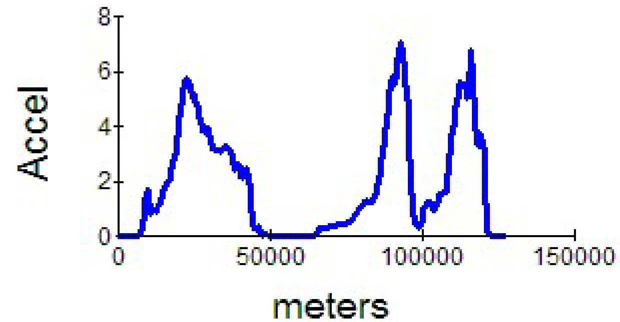




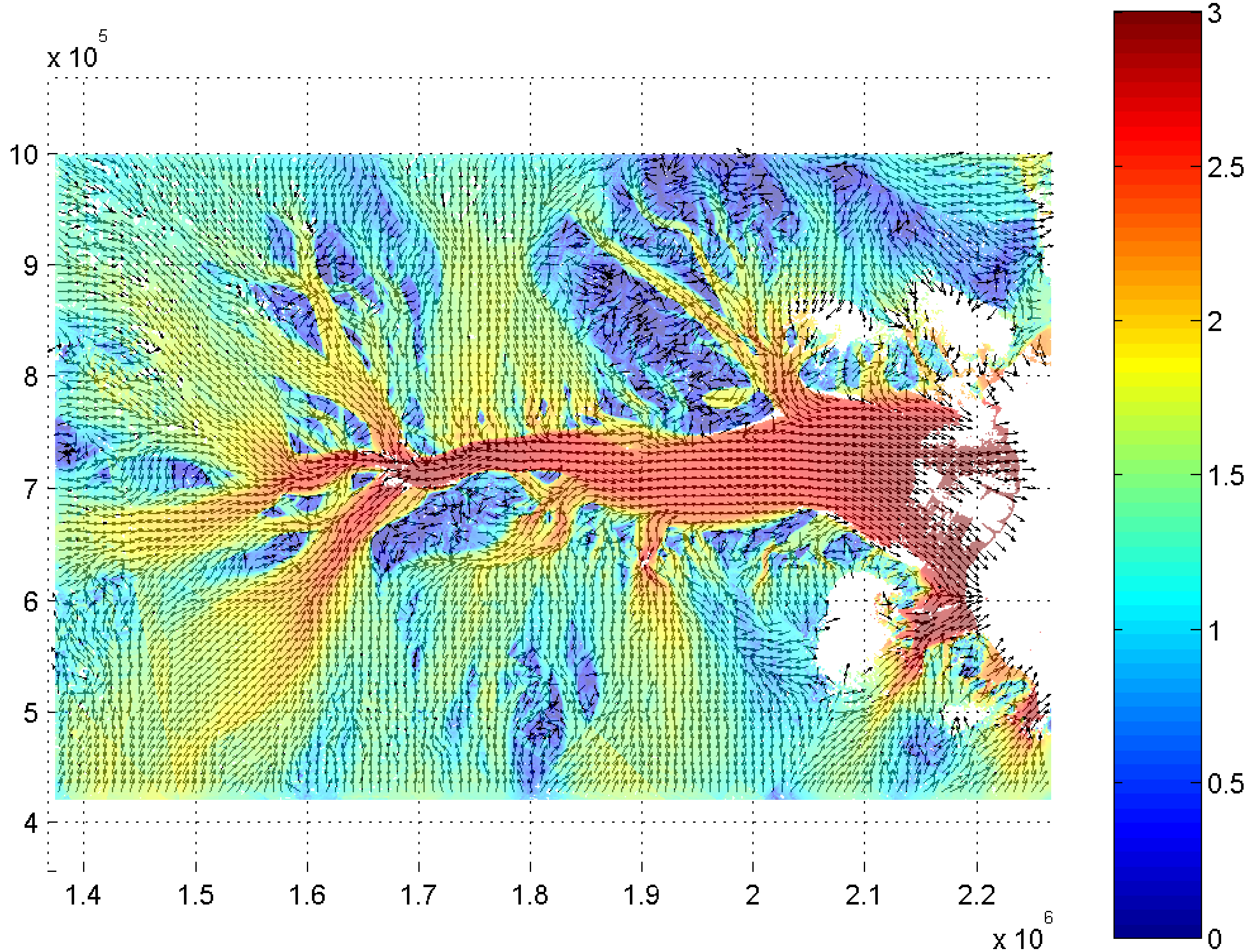
Lambert Glacier



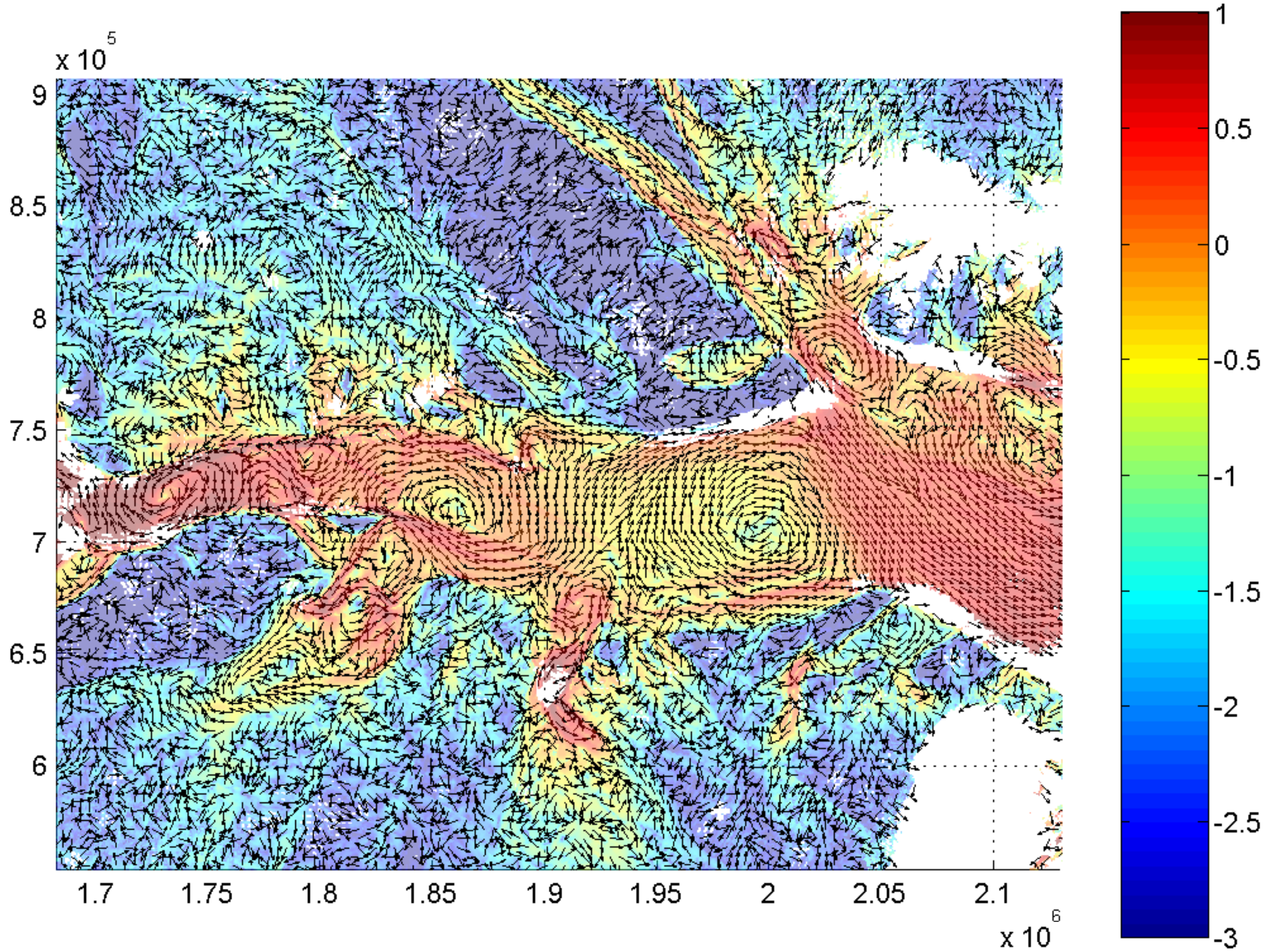




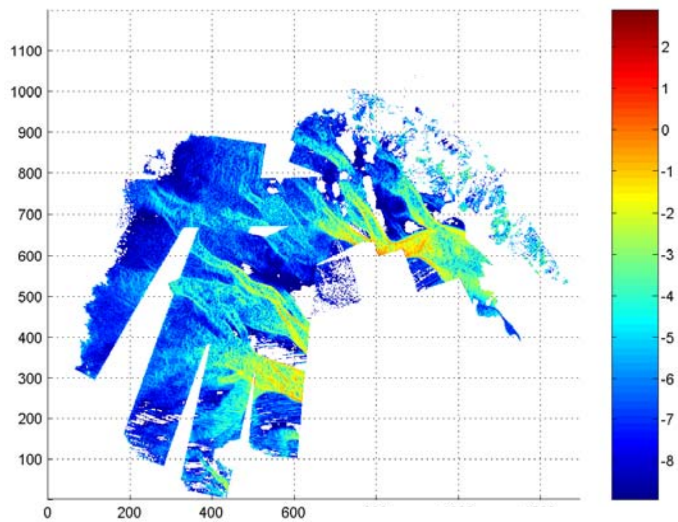
Velocity Field



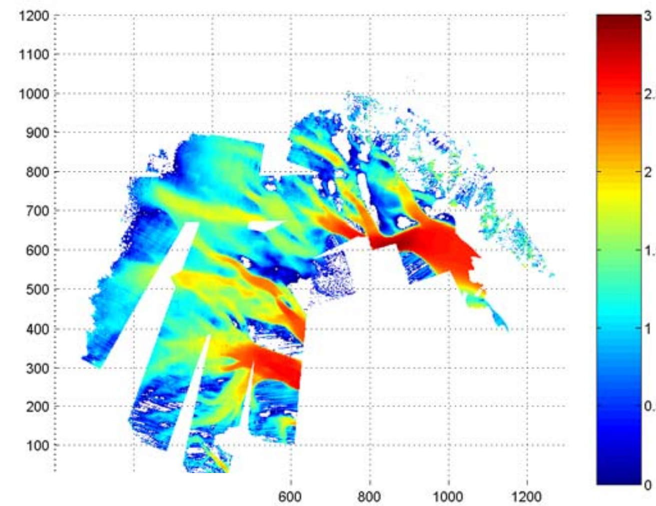
Amery Ice Shelf



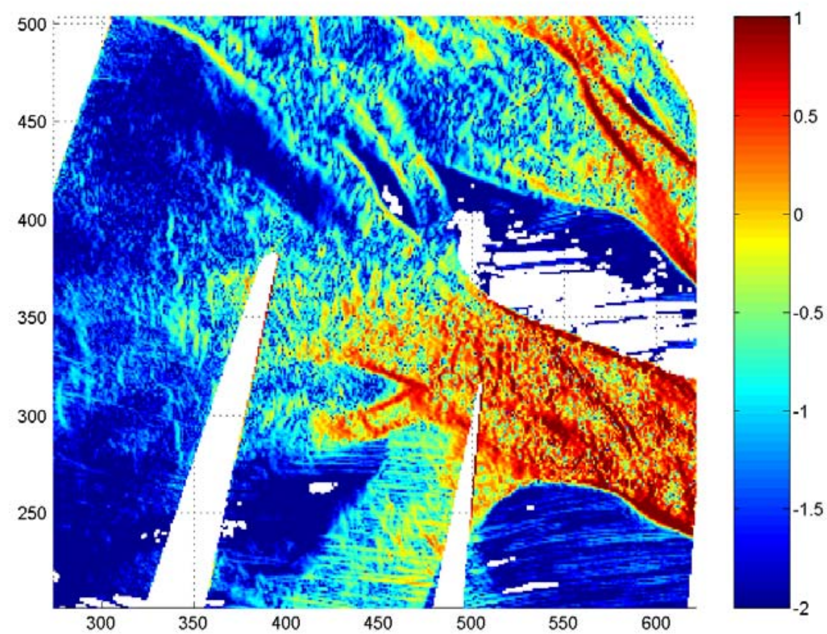
WAIS



1100
1000
900
800
700
600
500
400
300
200
100
0
200
400
600
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8

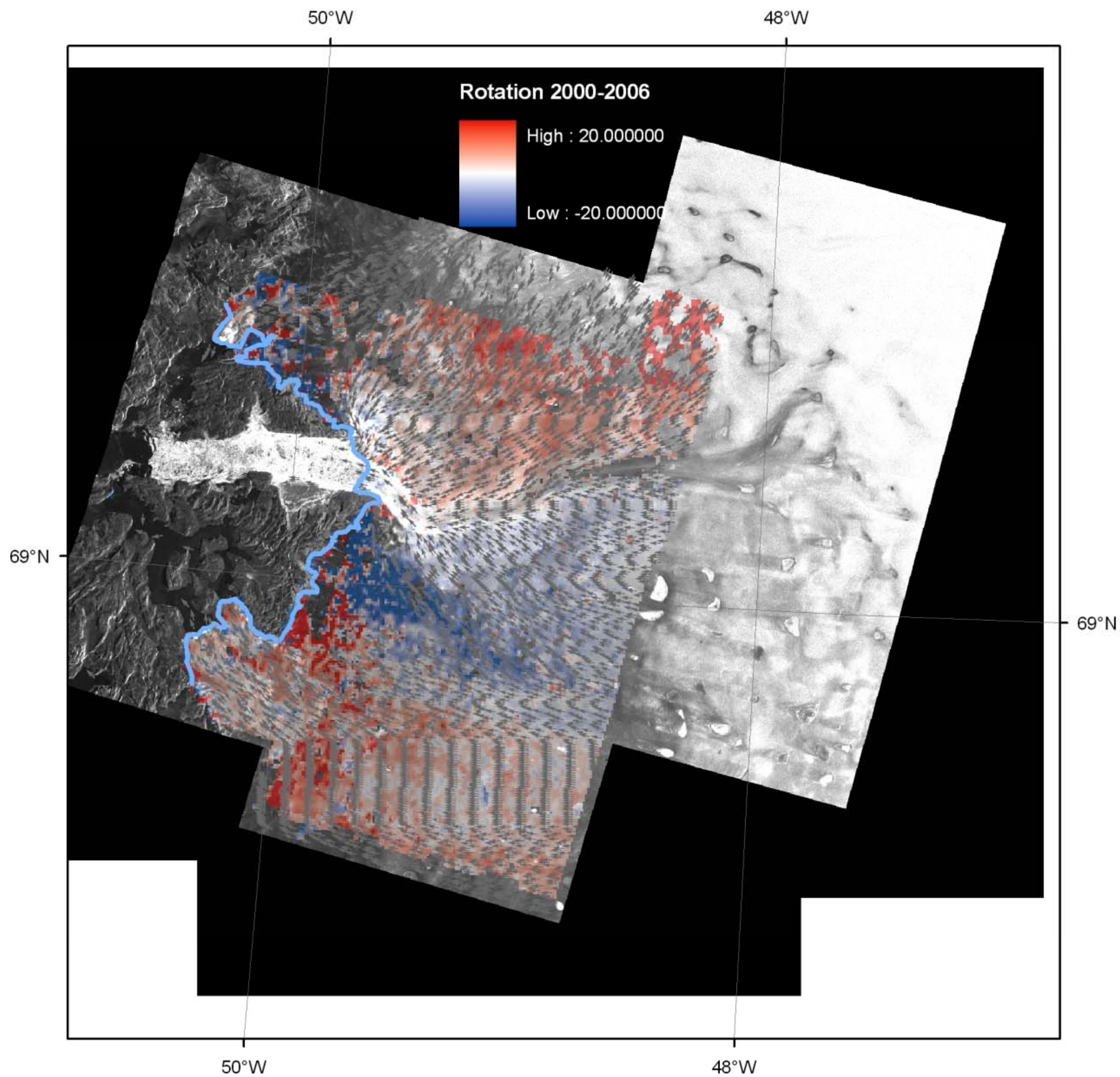


1200
1100
1000
900
800
700
600
500
400
300
200
100
600
800
1000
1200
3
2.5
2
1.5
1
0.5
0



500
450
400
350
300
250
300
350
400
450
500
550
600
1
0.5
0
-0.5
-1
-1.5
-2

Acceleration can be a flow direction change



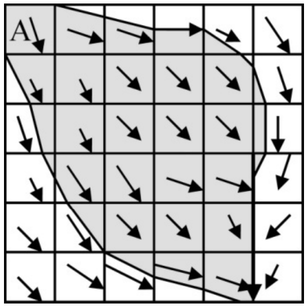
Accelerations – what next

- Accelerations are small, but measurable. How do we model these small observables?
- Acceleration velocity fields are organized on larger scale but edge effects complicate small scale investigations
- Next steps are to investigate depth integra $\frac{\partial \overline{u}}{\partial t}$ force field, the possibility of extracting

Models, Predictions, Confidence

- Model uncertainties can be computed using <u>judicious</u> application of standard error propagation methods and useful shortcuts.

- Bayes Theorum provides a more direct and robust approach for estimating the probability density function of a parameter – but the apparatus can be cumbersome.



Balance Velocity and Error Propagation

- The balance velocity (V) is estimated as

$$|V_s| = \frac{|F|}{H \cdot W \cdot \left(|\sin\theta| + |\cos\theta|\right) \cdot r}$$

- where the ratio (r) between balance velocity and the ice surface balance velocity takes into account the fact that the velocity decreases from the surface towards the bed.
- Errors are estiamted using the total derivative short cut, which assumes normally distributed variables

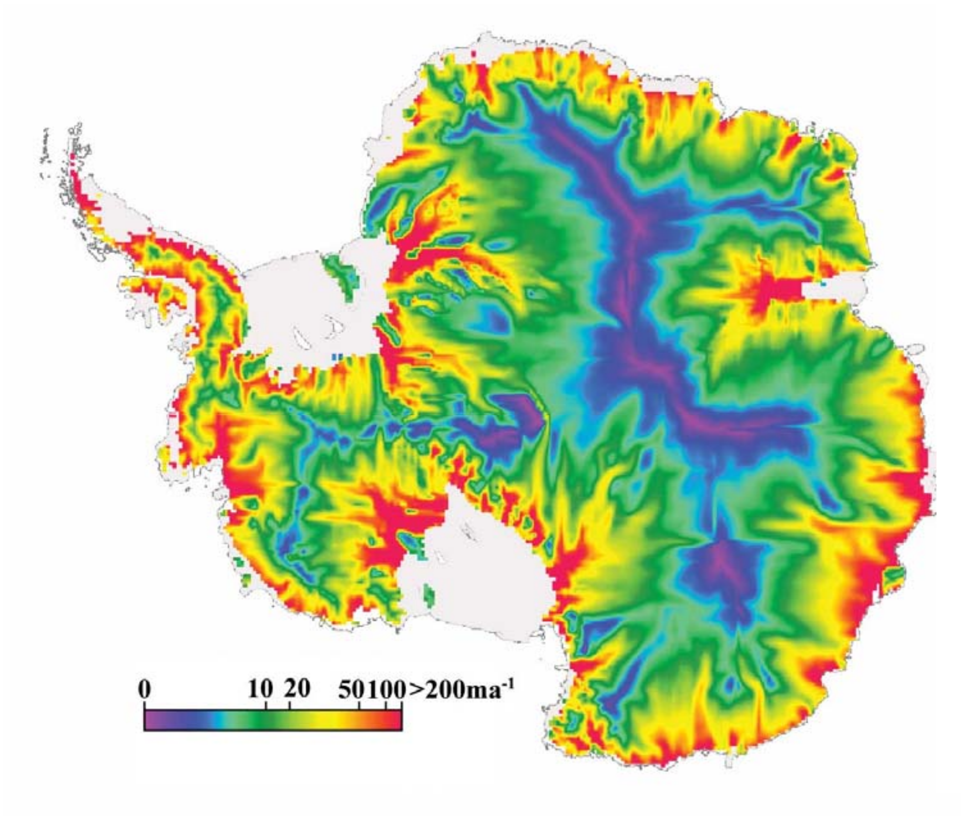
$$\Delta_y = \left|\frac{\partial f}{\partial x_1}\right|\Delta_{x_1} + \left|\frac{\partial f}{\partial x_2}\right|\Delta_{x_2} + \ldots + \left|\frac{\partial f}{\partial x_m}\right|\Delta_{x_m}$$

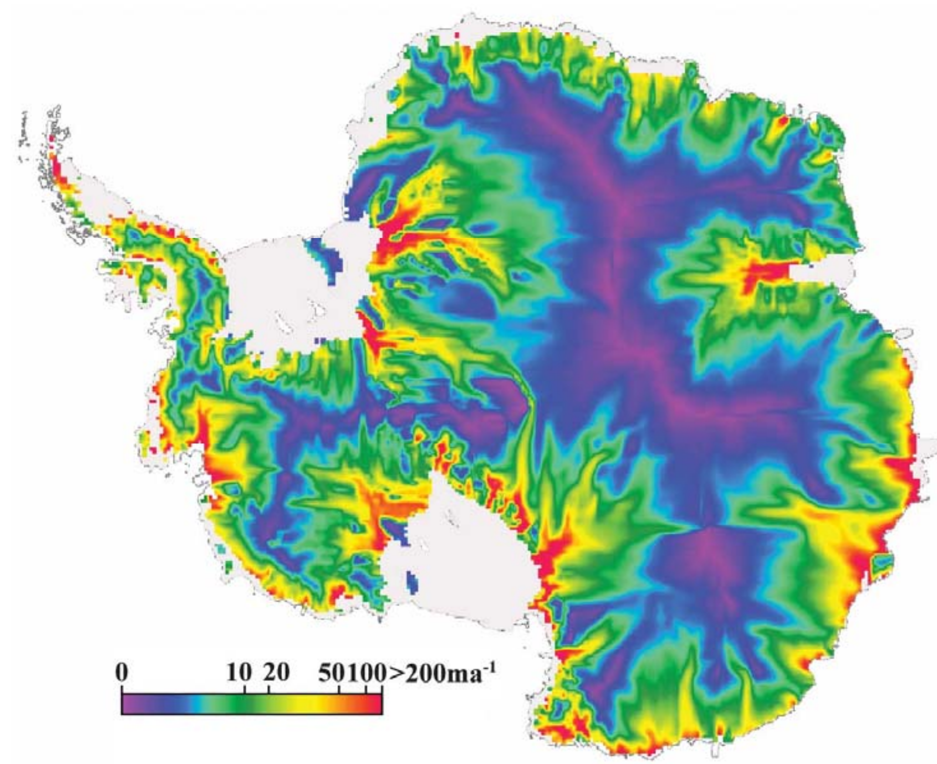
- and

$$\Delta V_s = \frac{1}{H \cdot (\sin\theta + \cos\theta) \cdot W \cdot r}\Delta\hat{F}_{i,j} + \frac{\hat{F}_{i,j}}{(\sin\theta + \cos\theta) \cdot W \cdot r \cdot H^2}\Delta H + \frac{\hat{F}_{i,j} \cdot (\cos\theta - \sin\theta)}{H \cdot W \cdot r \cdot (\sin\theta + \cos\theta)^2}\Delta\theta$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{\hat{F}_{i,j}}{(\sin\theta + \cos\theta) \cdot W \cdot H \cdot r^2}\Delta r \qquad (26)$$

Model and Error Estimate





- Balance Velocity Model Estimated Error

Comments on Error Propagation

- The total derivative short cut is useful if the variables are normally distributed
- This allows for an expansion about the local maximum of the PDF resulting in typical least squares type estimates.
- A curious question: The angle Theta is a normally distributed random variable with mean Theta 0. Compute the PDF(y) given

$$y = \cos(\theta)$$

Bayesian Inference

- Wikle and Berliner, 2007:

  DA is an approach for fusing data (observations) with prior knowledge (e.g., mathematical representations of physical laws; model output) to obtain an estimate of the distribution of the true state of a process. From this perspective, one needs the following components to perform DA: a statistical model for observations (i.e., a data or measurement model), and an a priori statistical model for the state process (i.e., a state or process model).

- Gelman et al. [20, p. 2] define Bayesian inference as
- "... the process of fitting a probability model to a set of data and summarizing the result by a probability distribution on the parameters of the model and on unobserved quantities such as predictions for new observations".

$$P(H \mid R,I) = \frac{P(R \mid H,I)P(H)}{constant}$$

Bayesian Hierarchical Modeling (BHM)

- BHM: sequence of conditional probability models
- Quintessential BHM: Data Y; Process of interest X
  1. <u>Data Model</u> $[\ Y \mid X, \theta\ ]$
  2. <u>Process Model</u> $[\ X \mid \theta]$
  3. <u>Parameter Model</u> $[\ \theta\ ]$
- Bayes' Theorem: $[\ X, \theta \mid Y\ ]$

Compare

- "Statistics": $[\ Y \mid \theta\ ]$ (& $[\ \theta\ ]$ for Bayesians)
- "Physics": $[\ X \mid \tilde{\theta}(Y)\ ]$

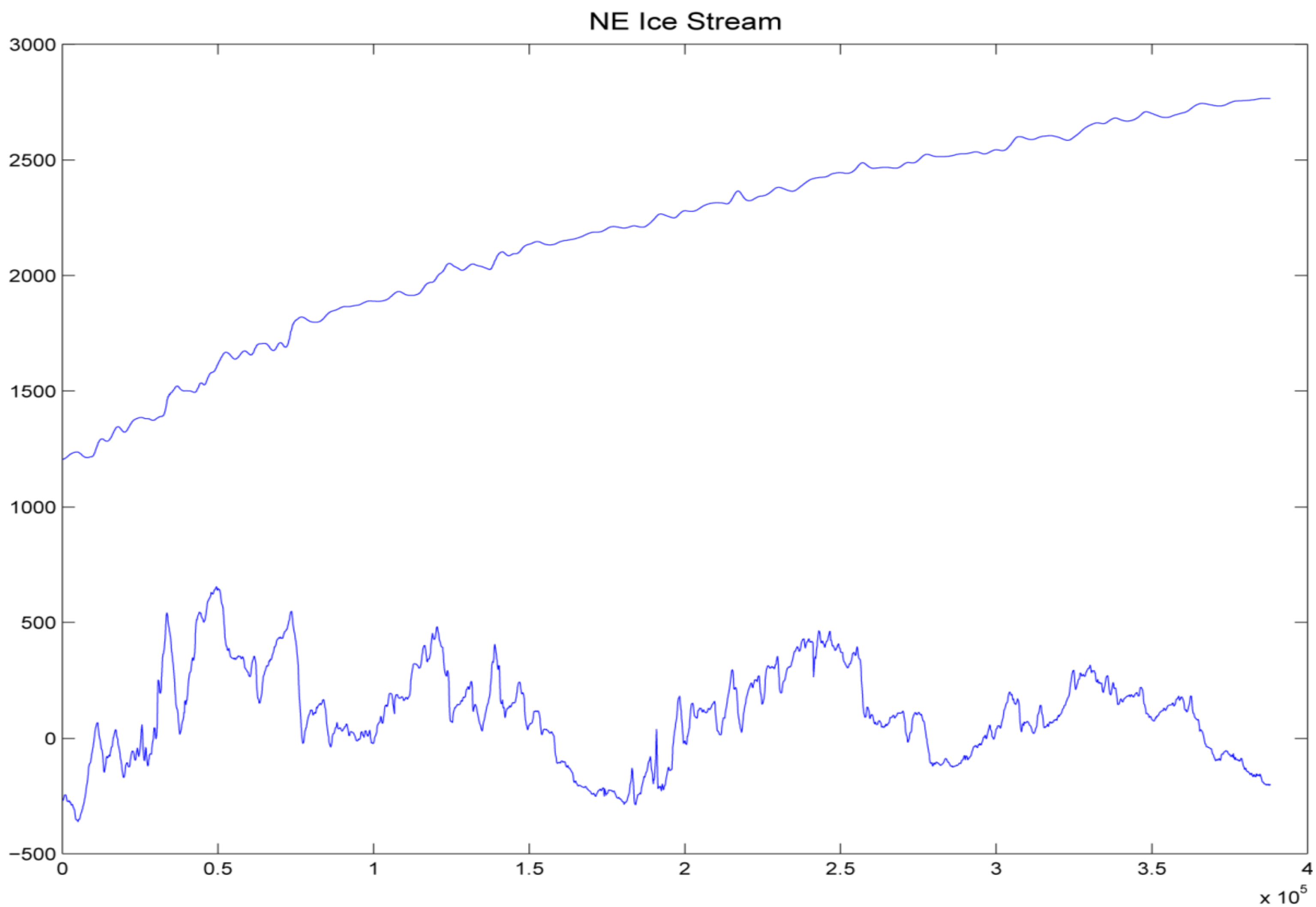
Glacial Dynamics *Berliner et al. 2008 J. Glaciol.*

- Flow: gravity moderated by drag (base and sides) & ....
- Simple flow models: flow from geometry.

Data

Program for Arctic Climate Regional Assessments (PARCA)
Radarsat Antarctic Mapping Project (RAMP)

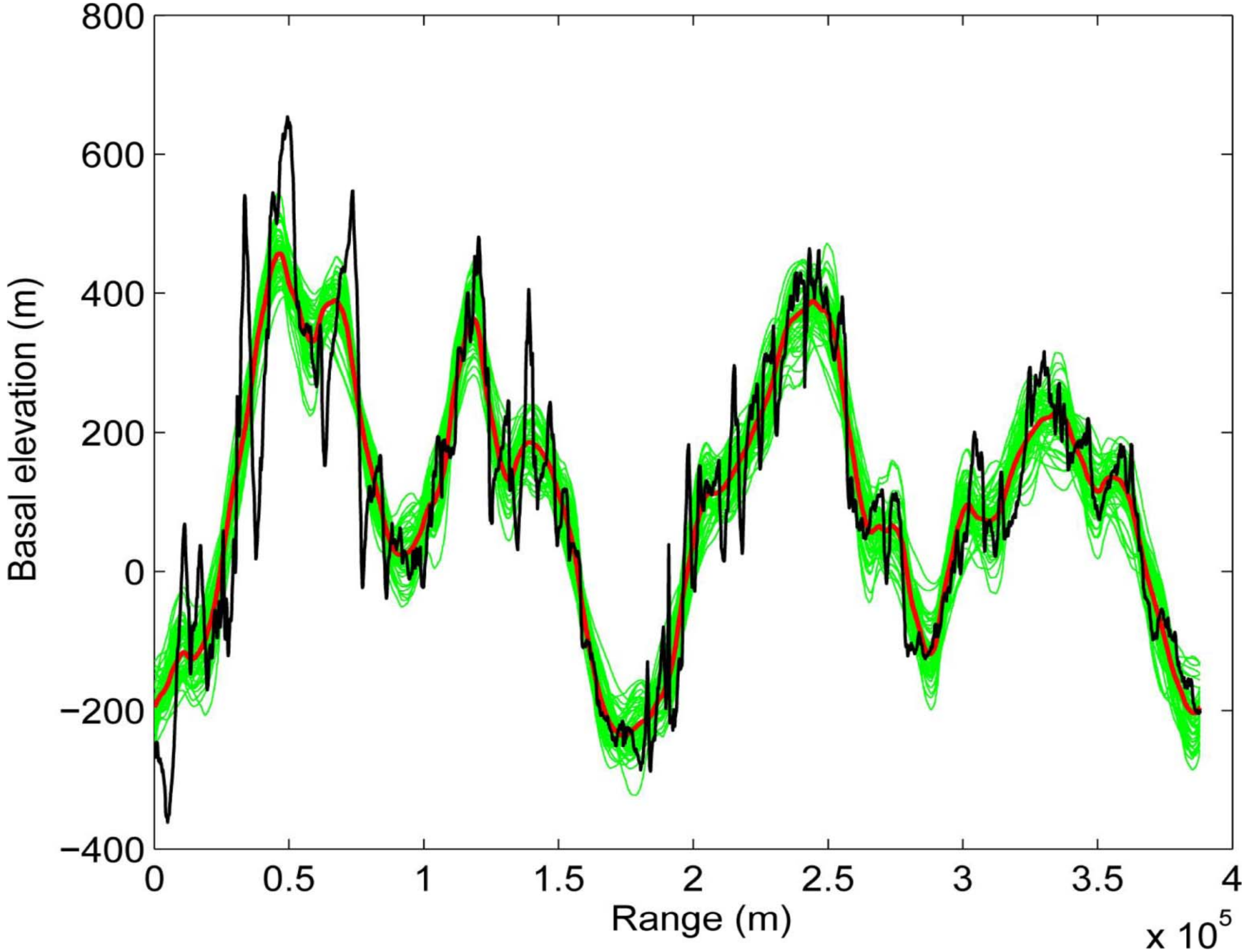
- S: surface topography (Laser altimetry)
- B: basal topography (Radar altimetry)
- U: velocity data (Interferometry)



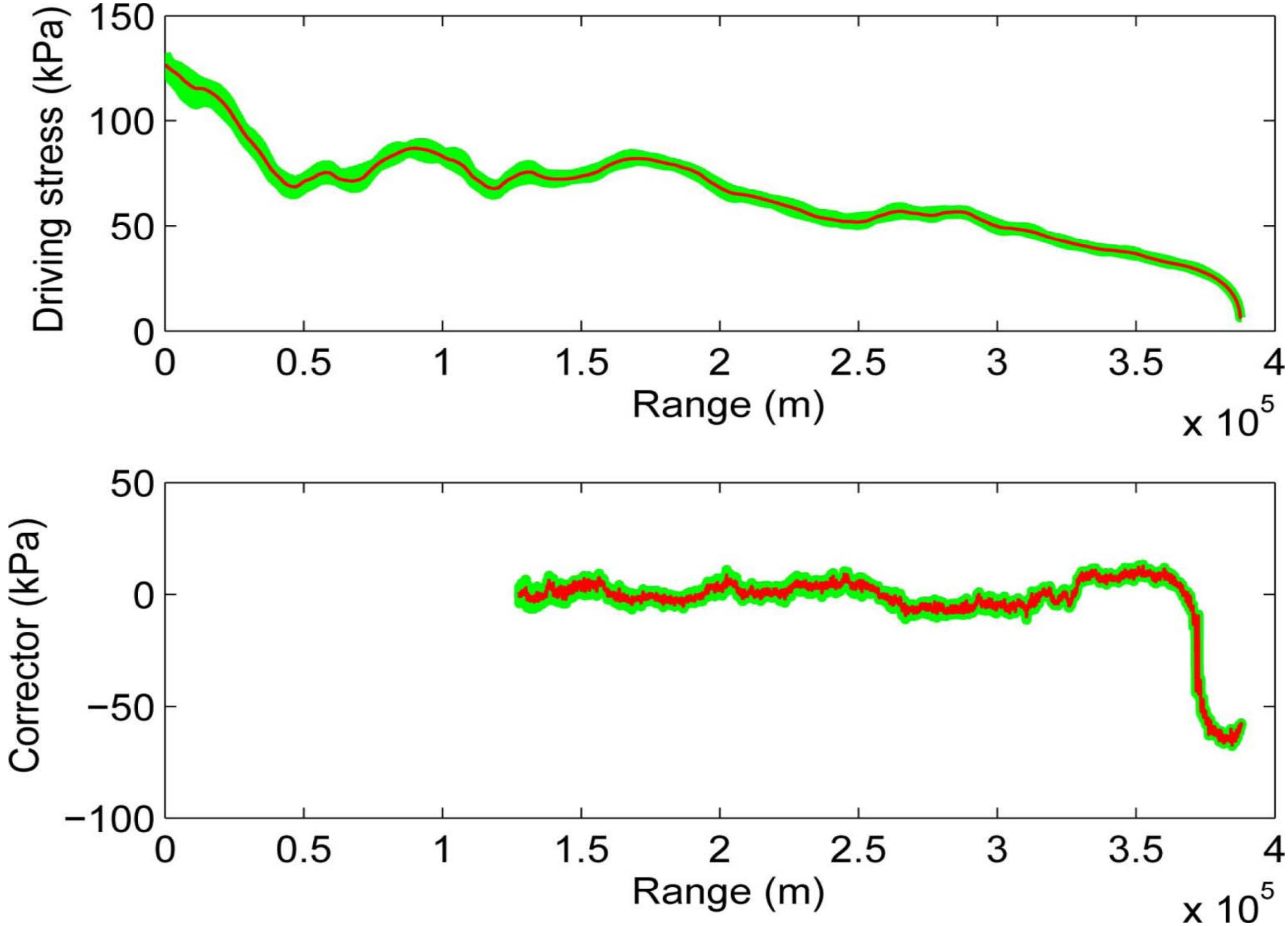
NE Ice Stream
3000
2500
2000
1500
1000
500
0
−500
0
0.5
1
1.5
2
2.5
3
3.5
4
x 10^5

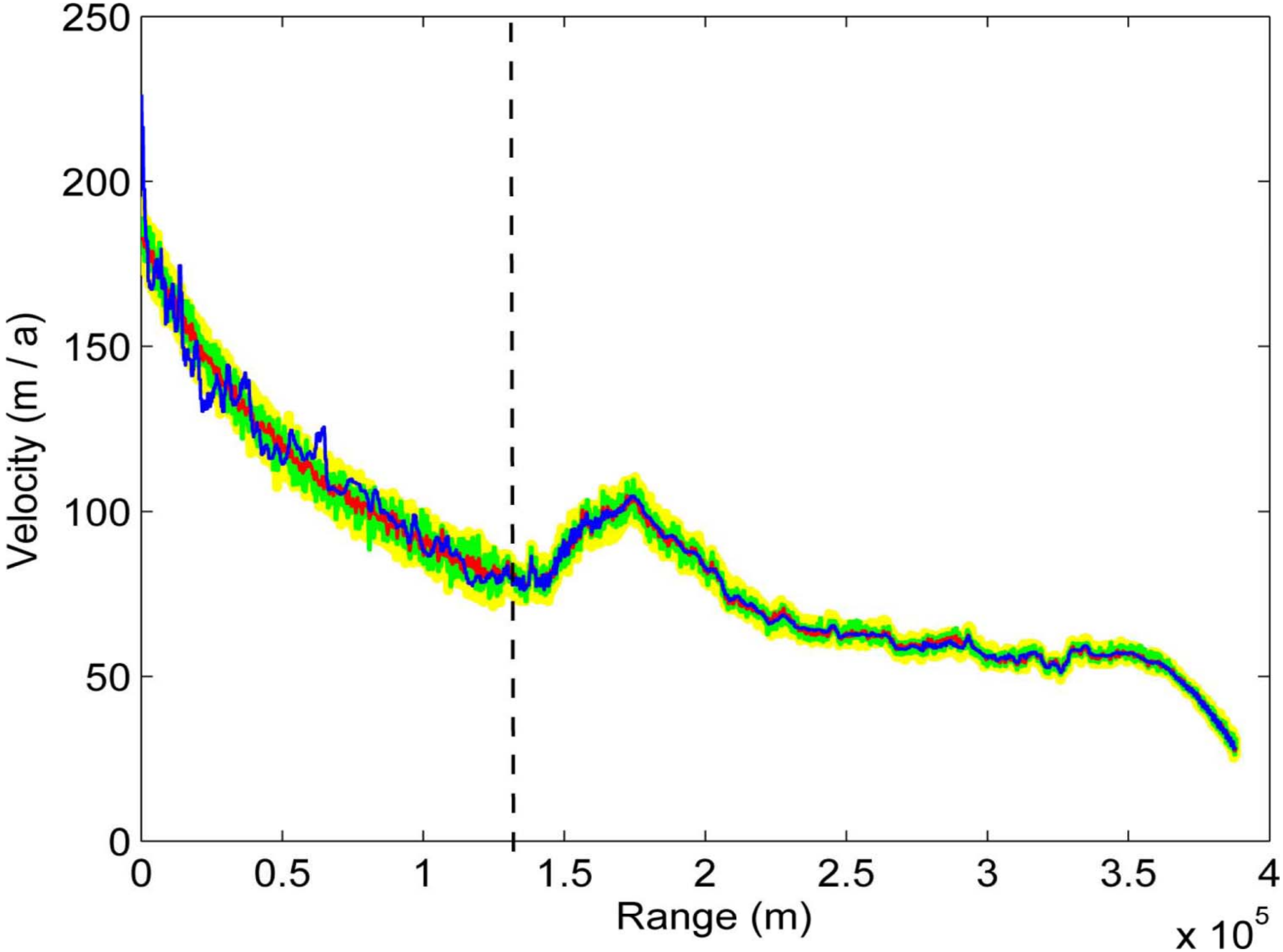
Modelling

- Processes: surface; s: base; H: thickness; u: velocity
- Physical Model
  - Basal Stress $\boldsymbol{\tau} = -\boldsymbol{\rho}\, \mathrm{g}\, \mathrm{H}\, \mathrm{s}' + \mathrm{stuff}$
  - Velocities $\mathrm{u} = \mathrm{u_b} + \boldsymbol{\beta}_0\, \mathrm{H}\, \boldsymbol{\tau}^{\mathrm{n}}$
    where $\mathrm{u_b} = \mathrm{k}\, \boldsymbol{\tau}^{\mathrm{p}}\, (\boldsymbol{\rho}\, \mathrm{g}\, \mathrm{H})^{-\mathrm{q}}$
- Our Model
  - Basal Stress $\tilde{\boldsymbol{\tau}} = -\boldsymbol{\rho}\, \mathrm{g}\, \tilde{\mathrm{H}}\, \tilde{\mathrm{s}}' + \boldsymbol{\eta}$
    where $\boldsymbol{\eta}$ is a "corrector process", $\tilde{\mathrm{H}}, \tilde{\mathrm{s}}$ are unknown
  - Velocities $\mathrm{u} = \tilde{\mathrm{u}}_\mathrm{b} + \boldsymbol{\beta}\, \tilde{\mathrm{H}}\, \tilde{\boldsymbol{\tau}}^{\mathrm{n}} + \mathrm{e}$
    where $\mathrm{u_b} = \mathrm{k}\, \tilde{\boldsymbol{\tau}}^{\mathrm{p}}\, (\boldsymbol{\rho}\, \mathrm{g}\, \tilde{\mathrm{H}})^{-\mathrm{q}}$ or an unknown constant,
    $\boldsymbol{\beta}$ is unknown, e is a noise process.
  - Smoothing



800
600
400
200
0
−200
−400
Basal elevation (m)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5


150
100
50
0
Driving stress (kPa)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5
50
0
−50
−100
Corrector (kPa)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5



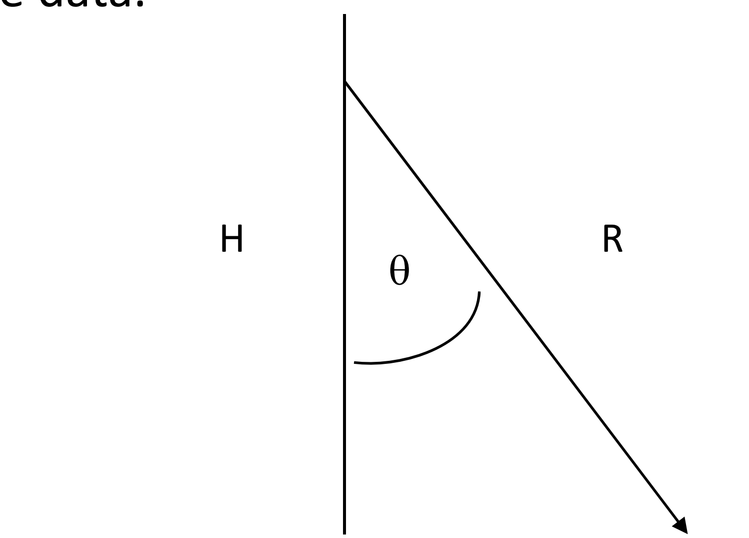
250
200
150
100
50
0
Velocity (m / a)
0
0.5
1
1.5
2
2.5
3
3.5
4
Range (m)
x 10^5

Possible applications of Bayes

- Inference about basal conditions
- Ice Sheet Transfer Functions
- Refine phase estimates in ice sounding radar interferometry and tomography
- Discrimination between surface melt/surface freeze events

A simple example (similar to the "Lighthouse Example")

- Bayesian estimate of Aircraft Elevation
- An altimeter is mounted along the vertical axis of an aircraft. The aircraft rolls in a statistical fashion. Roll is not measured. Range to the surface is measured. Refine the estimate of aircraft height H by updating the prior PDF with range data.



The model is

$$H = R\cos\theta$$

θ is a random variable with probability distribution $P(\theta)$

From change of variables, the distribution of R is

$$P(R) = P(\theta)\frac{d\theta}{dR}$$

$$\frac{H}{R} = \cos\theta$$

$$\frac{d\theta}{dR} = \frac{H}{R^2\sin\theta}$$

$$P(R\,|\,H) = \frac{H}{R^2\sin\theta}P(\theta)$$

$$sin\theta = (1-\cos^2\theta)^{1/2} = (1-\frac{H^2}{R^2})^{1/2}$$

$$P(R\,|\,H) = \frac{H}{R^2(1-\frac{H^2}{R^2})^{1/2}}P(\theta)$$

If $P(\theta)$ is uniformly distributed between angles - c to c

$$P(R\,|\,H) = (\frac{1}{c})\frac{H}{R^2(1-\frac{H^2}{R^2})^{1/2}}$$

If θ is normally distributed with mean θ_0 and standard deviation σ

$$P(\theta) = \frac{1}{2\pi\sigma} \exp - \frac{(\theta - \theta_0)^2}{2\sigma^2}$$

$$\theta = a\cos(\frac{H}{R})$$

$$P(R \mid H) = \frac{H}{R^2(1 - \frac{H^2}{R^2})^{1/2}} \frac{1}{2\pi\sigma} \exp - \frac{(a\cos(\frac{H}{R}) - \theta_0)^2}{2\sigma^2}$$

From Bayes Theorum

$$P(H \mid R,I) = \frac{P(R \mid H,I)P(H)}{constant}$$

here I includes information such as the mean pointing angle and the fact that H must be equal to or less than R.

Assume H is an imprecisely known parameter. Let P(H) be a uniform PDF extending from H_o to H_1 so that $P(H) = 1/b$

- Assume R is measured many times but over a short enough period such that the surface can be taken as flat. The ensemble is $\{R_1, R_2, \ldots R_n\}$. Then

$$P(H \mid R,I) = \prod_{1}^{n} \frac{P(R_n \mid H,I)P(H)}{constant}$$

- So for the case of the uniform distribution on angle

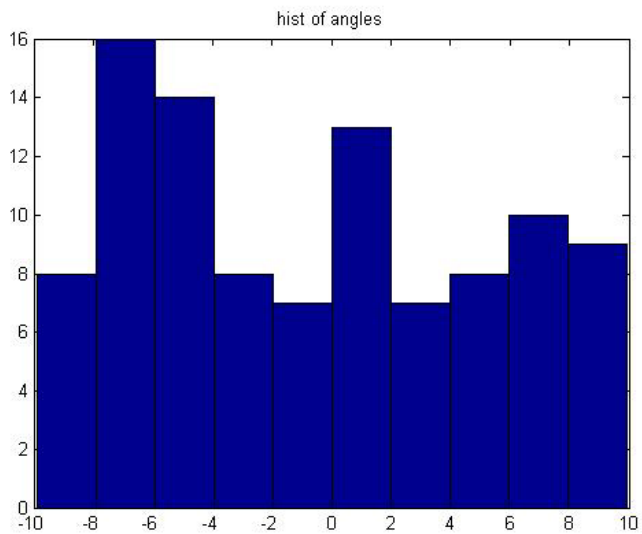
$$P(H \mid R,I) = \prod_{1}^{n} \frac{\dfrac{\boldsymbol{H}}{\boldsymbol{R}^2\left(1-\dfrac{\boldsymbol{H}^2}{\boldsymbol{R}^2}\right)^{1/2}}}{newconstant}$$

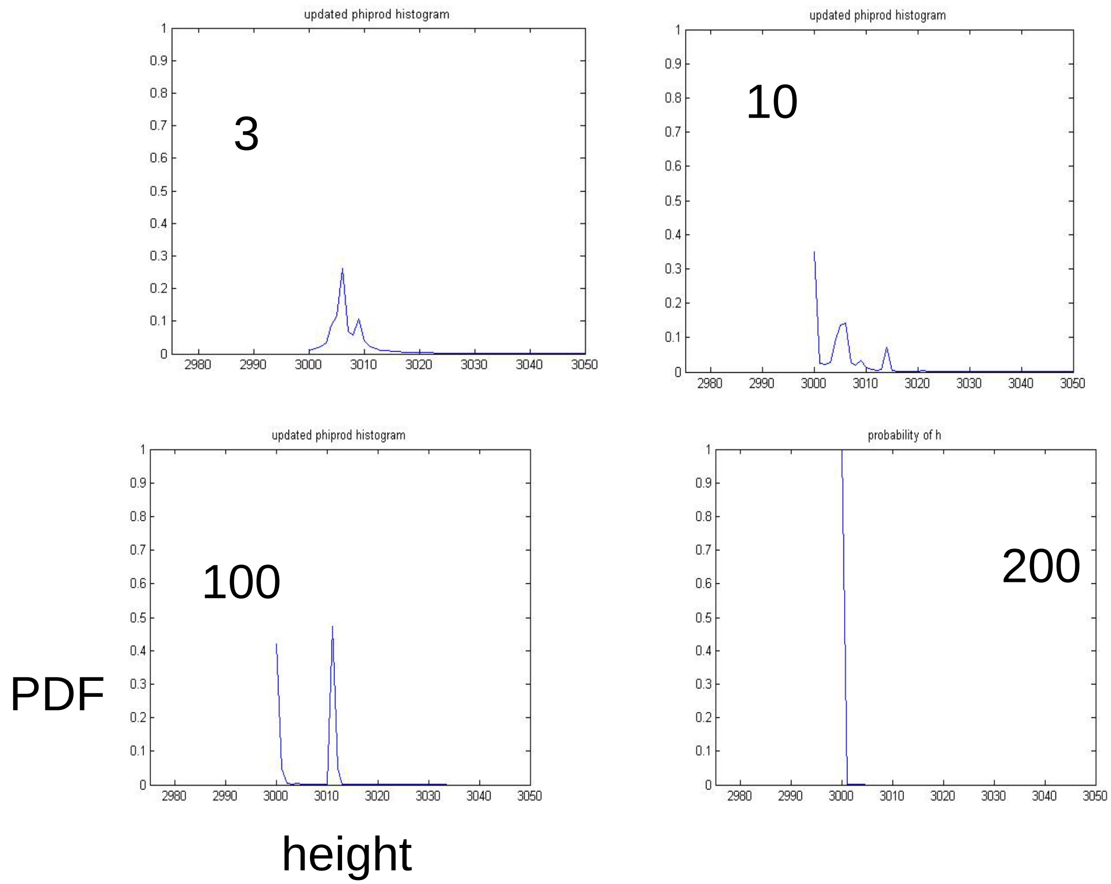
- For the case of the Gaussian PDF on angle.

$$P(H \mid R,I) = \prod_{1}^{n} \frac{\dfrac{\boldsymbol{H}}{\boldsymbol{R}^2\left(1-\dfrac{\boldsymbol{H}^2}{\boldsymbol{R}^2}\right)^{1/2}} \dfrac{1}{2\pi\sigma} \mathbf{exp}- \dfrac{\left(\boldsymbol{a}\cos\left(\dfrac{\boldsymbol{H}}{\boldsymbol{R}}\right)-\theta_0\right)^2}{2\sigma^2}}{newconstant}$$

- Here, the constants in the denominator are determined by ensuring that the integral of P(H|R,I) is unity.

Evolution of Posterior PDF (Uniform distribution)





Evolution of Posterior – Gaussian Distribution