PART I

1. A short history of ice:
   a mechanicists point of view

2. From the discrete to the continuous:
   homogenization & the continuum hypothesis

3. παντα ρει – everything flows: Rheology 1

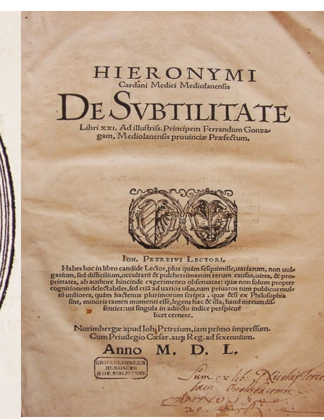
1. A short history of ice: a mechanicists point of view

Ice plays a prominent role in the evolution of material science and crystallography

- according to ancient belief, stone ice is ice frozen in a permanently rocky state

- stone ice (SiO_2) and H_2O ice are referred to as κριсταλλοс (crystals, translating to 'ice')

- Cardano launches the science of ice (1550): stone ice and H_2O ice are distinct as they behave differently under fire



HIERONYMI
Cardani Medici Mediolanensis
DE SVBTILITATE
Libri XXI. Ad illustriss. Principem Ferrandum Gonzagam, Mediolanensis prouinciæ Præfectum.

IOH. PETREIVS LECTORI.
Habes hoc in libro candide Lector, plus quàm sesquimille, uariarum, non uulgarium, sed difficilium, occultarū & pulcherrimarum rerum causas, uires, & proprietates, ab authore hincinde experimento obseruatas: quæ non solum propter cognitionem delectabiles, sed etiā ad uarios usus, tum priuatos tum publicos multò utiliores, quàm hactenus plurimorum scripta, quæ & si ex Philosophia sint, minoris tamen momenti esse, legens hæc & illa, haud mecum dissentiet; uti singula in adiecto indice perspicuè licet cernere.

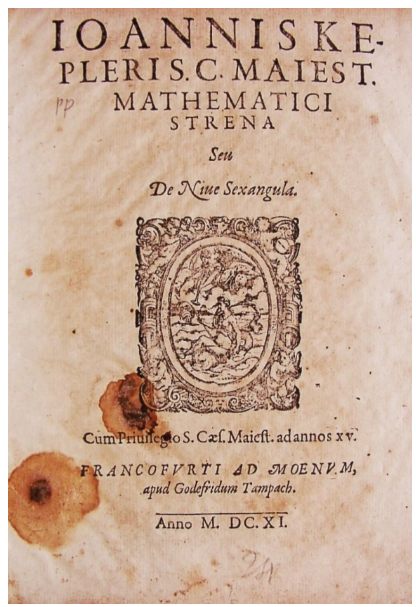
Norimbergæ apud Ioh. Petreium, iam primo impressum.
Cum Priuilegio Cæsar. atq; Reg. ad sexennium.
Anno M. D. L.

GROSSHERZOGLICH HESSISCHE HOF-BIBLIOTHEK

1. A short history of ice: a mechanicists point of view

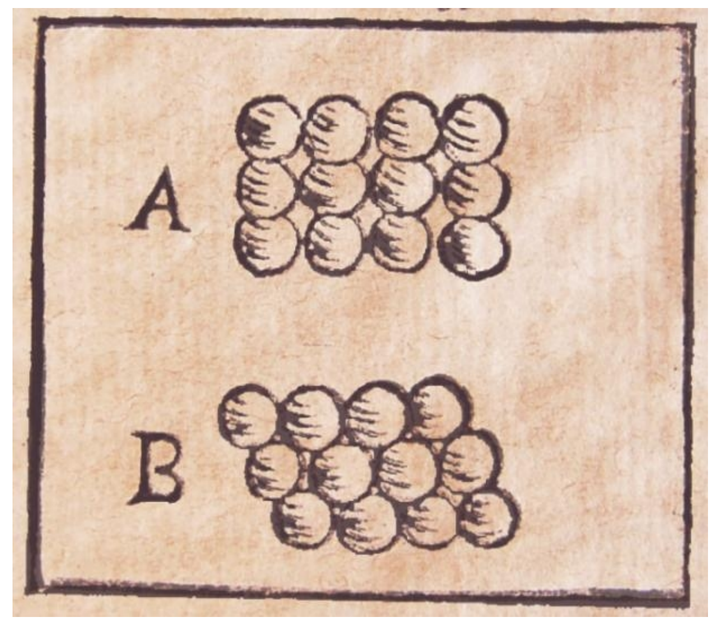
the resolution of the hexagonal structure of ice involved ...

Kepler (1611), explaining the hexagonal structure of ice in terms of cubic and hexagonal packings of spheres



IOANNIS KE-
PLERI S.C. MAIEST.
MATHEMATICI
STRENA
Seu
De Niue Sexangula.

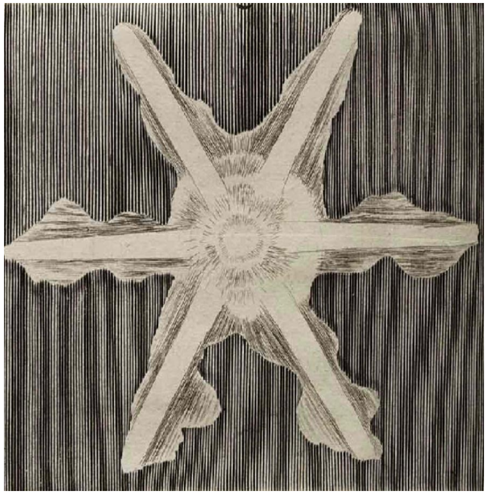
Cum Priuilegio S. Cæs. Maiest. ad annos xv.
FRANCOFVRTI AD MOENVM,
apud Godefridum Tampach.
Anno M. DC. XI.

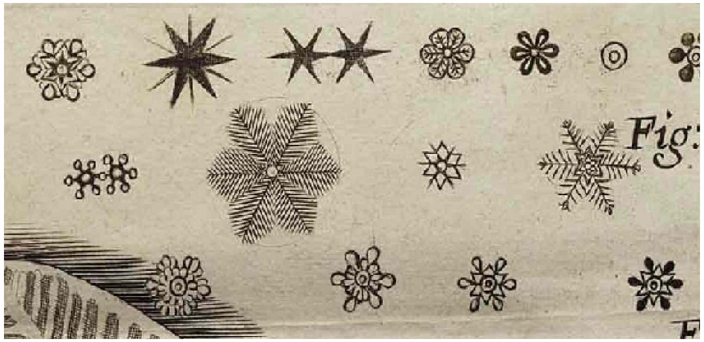


1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

Hooke (1665, „Micrographia"), observing the hexagonal structure of ice under a microscope

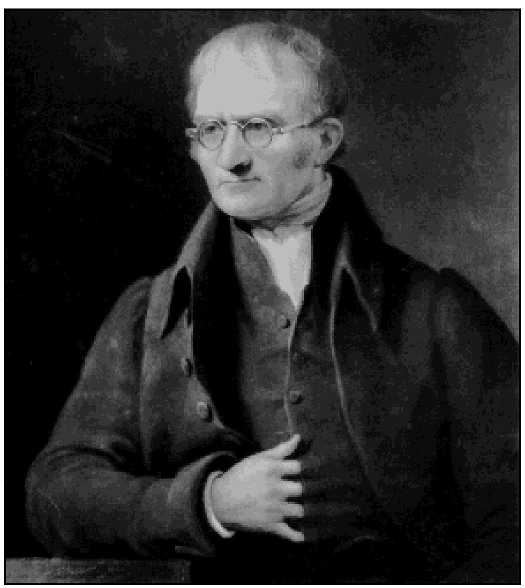


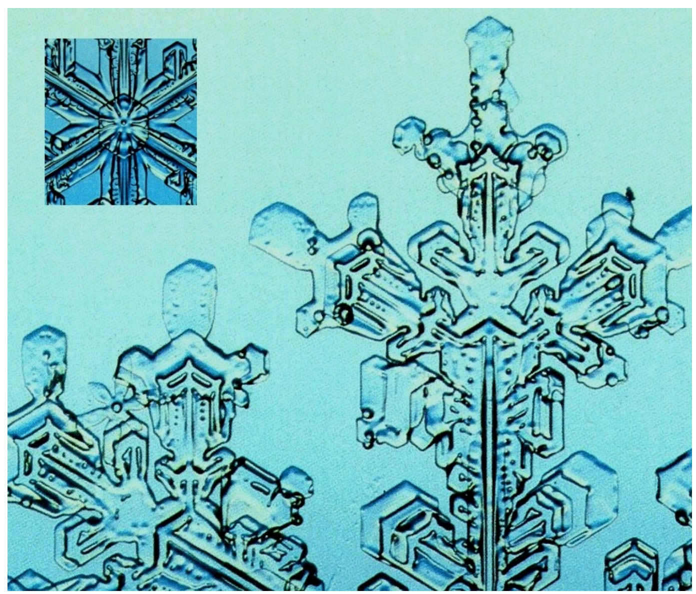


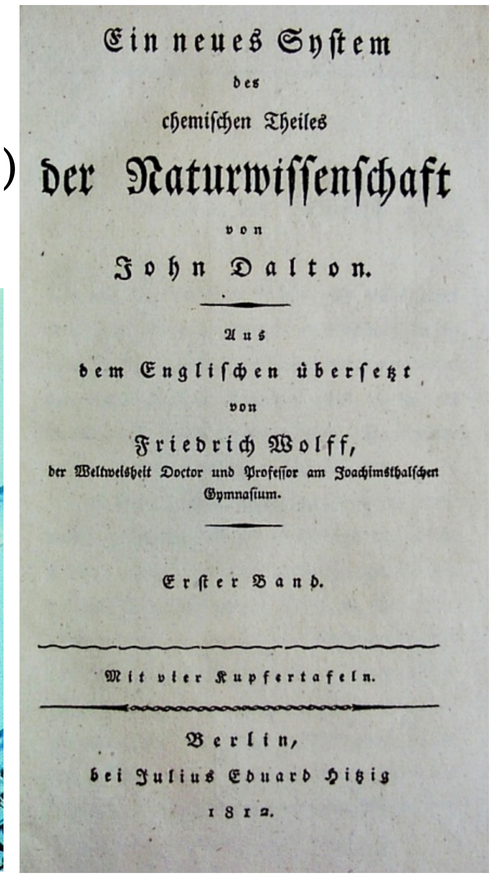
1. A short history of ice: a mechanicists point of view

the resolution of the hexagonal structure of ice involved ...

Dalton (1808), expressing the symmetry of ice crystals as a consequence of the atomistic structure of matter (supporters: Boyle, Newton, Lomonosov,...)







Ein neues System
des
chemischen Theiles
der Naturwissenschaft
von
John Dalton.

Aus
dem Englischen übersetzt
von
Friedrich Wolff,
der Weltweisheit Doctor und Professor am Joachimsthalschen Gymnasium.

Erster Band.

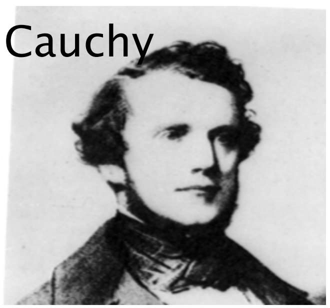
Mit vier Kupfertafeln.

Berlin,
bei Julius Eduard Hitzig
1812.

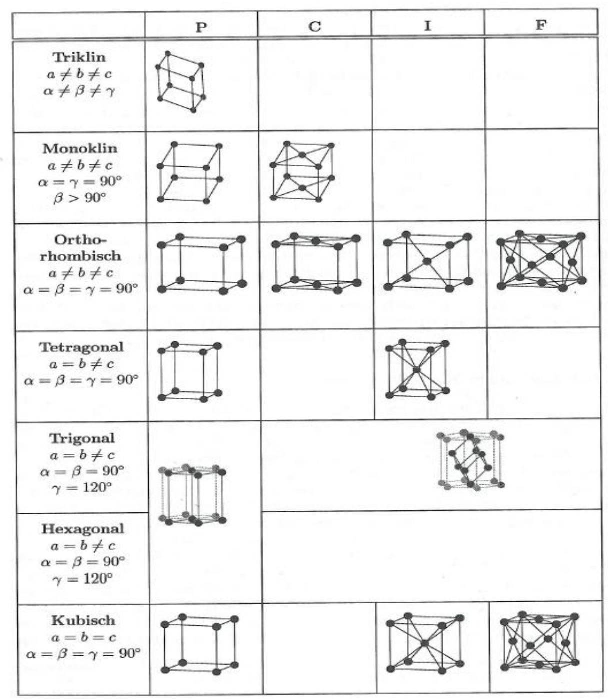
1. A short history of ice: a mechanicists point of view

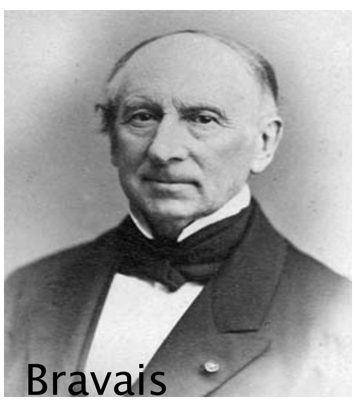
the resolution of the hexagonal structure of ice involved ...

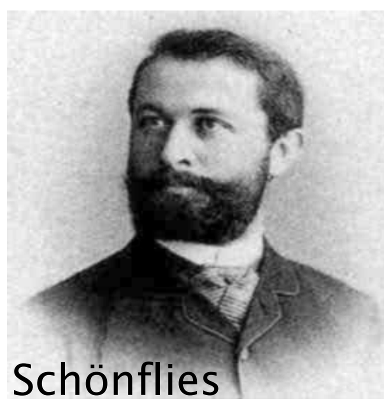
19th century mathematicians Bravais, Cauchy & Schönflies,...
developing the group theoretical foundations of crystallography



Bravais' 14 lattices





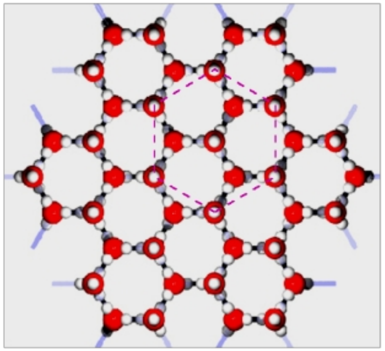


1. A short history of ice: a mechanicists point of view

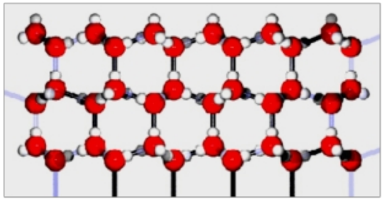
the resolution of the hexagonal structure of ice involved ...

Bragg & Bragg (1922), Pauling (1935), presenting the atomistic structure of hexagonal ice

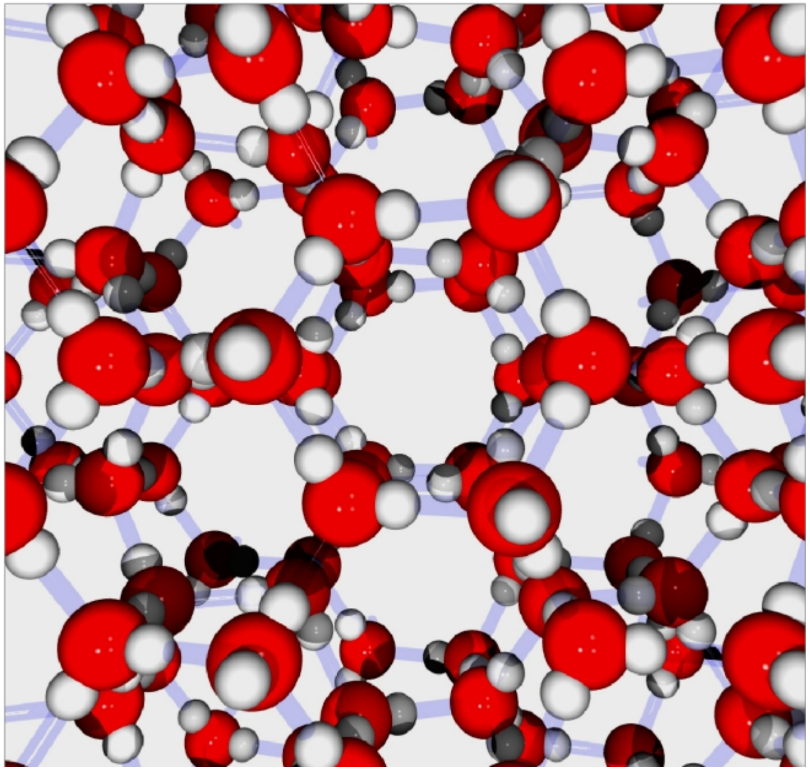
(hypothetical, perfect crystal)



view along *c*-axis

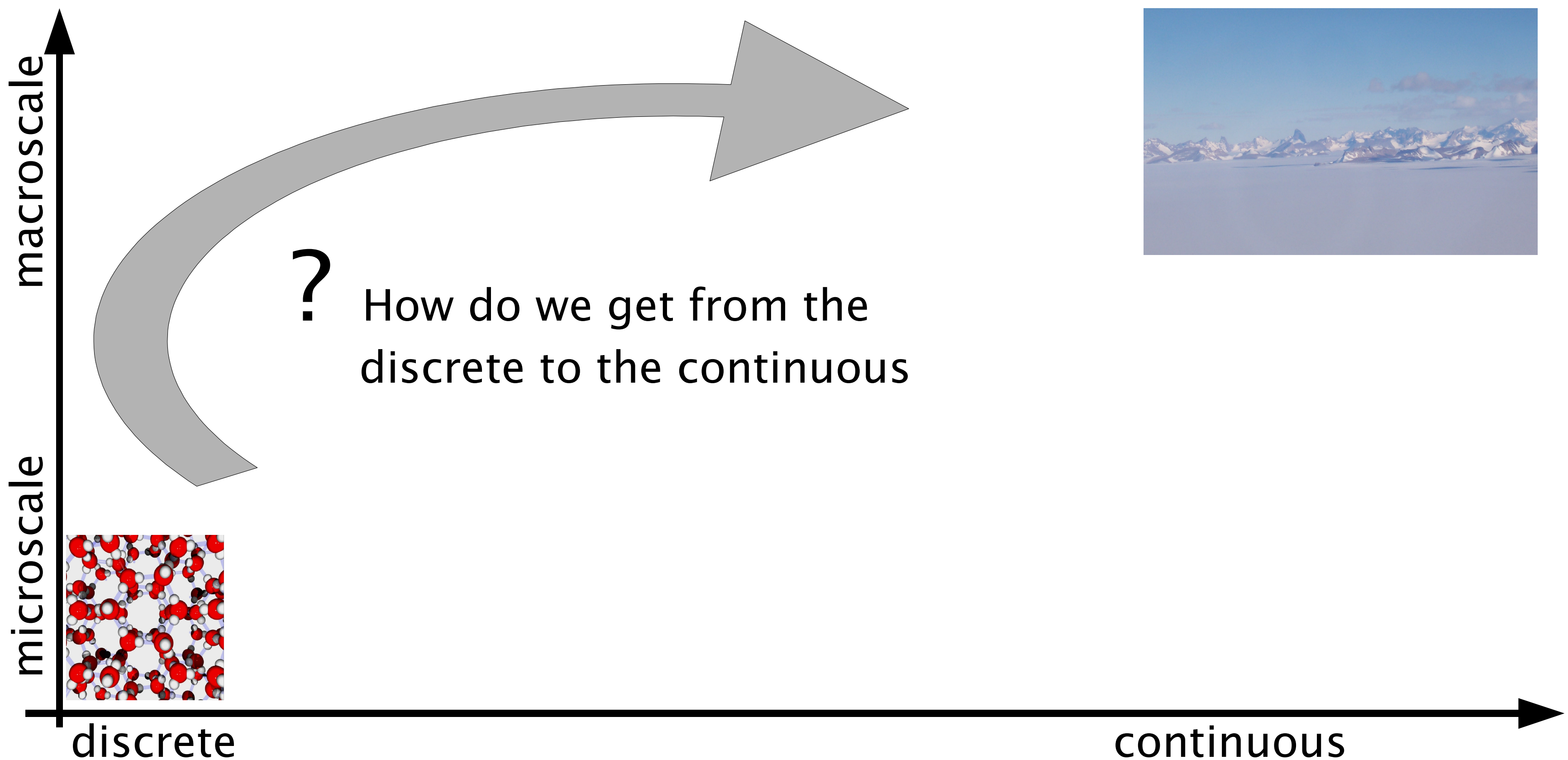


transversal view

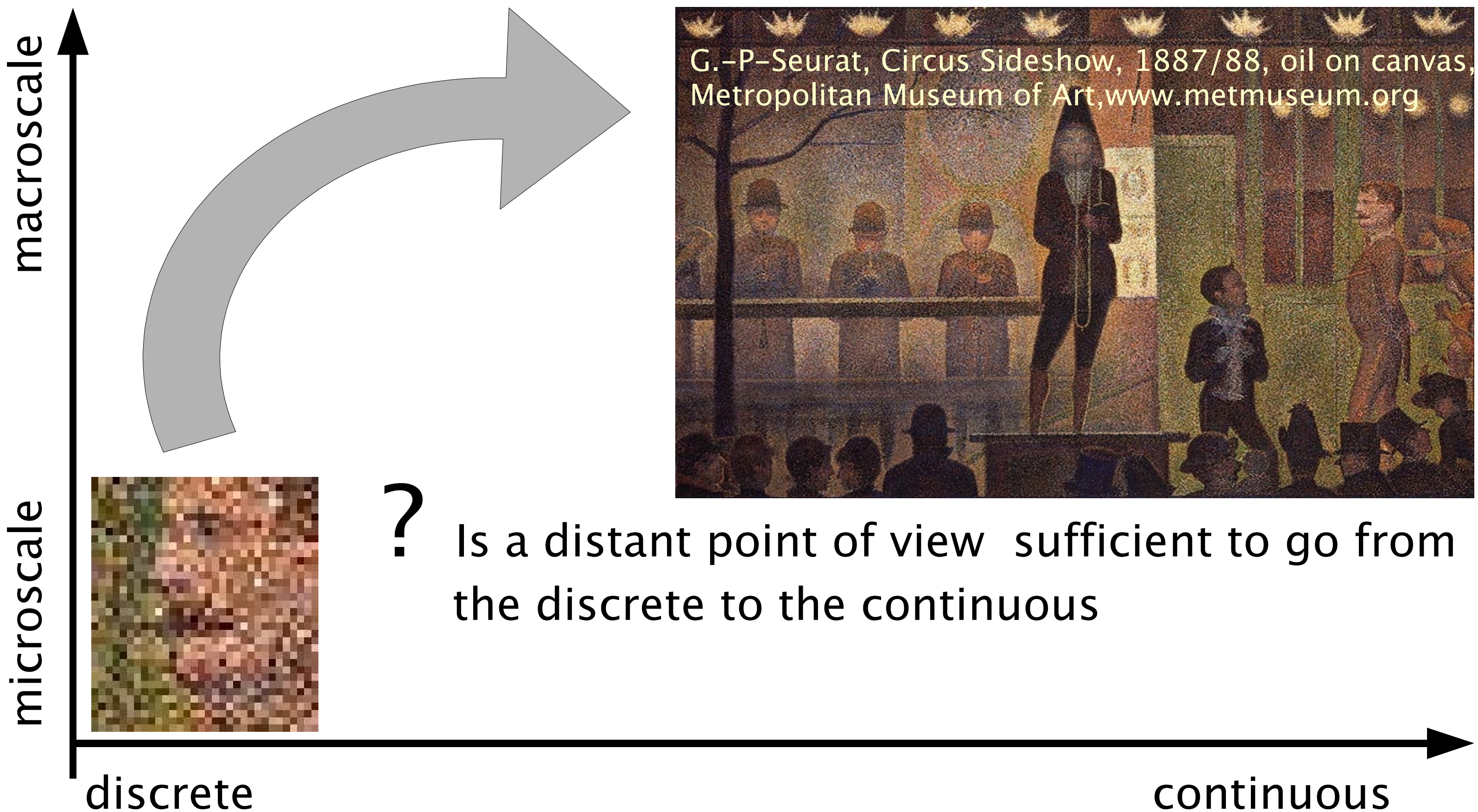


3D view along *c* axis

2. From the discrete to the continuous: homogenization and the continuum hypothesis



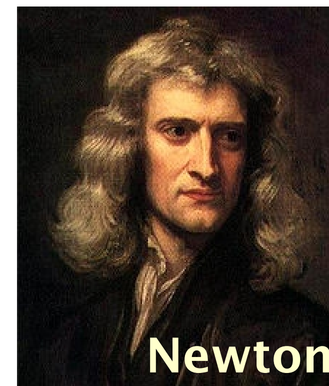
2. From the discrete to the continuous: homogenization and the continuum hypothesis



Is a distant point of view sufficient to go from the discrete to the continuous

2. From the discrete to the continuous: homogenization and the continuum hypothesis

Mathematics plays a prominent role in the evolution of continuum mechanics:



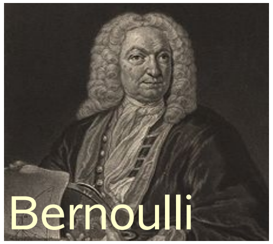
Mathematics in the 17th century:

- Leibniz (1660) & Newton (1666/1693): "*Calculus*"
- rapidly developing disciplines: theories of infinite series, ordinary and partial differential equations, calculus of variations, differential geometry
- applications: cartography, navigation, ballistics, marine & mechanical engineering, mechanics, astronomy

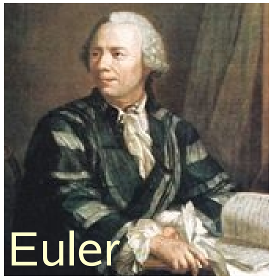
2. From the discrete to the continuous: homogenization and the continuum hypothesis

Mathematics plays a prominent role in the evolution of continuum mechanics:

Mathematics in the 18th century:

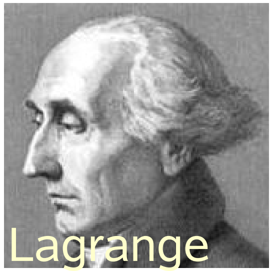


- Johan Bernoulli, *Hydraulica*, (dated) 1732



- Euler, *Introductio in analysin infinitorum* 1748,

  Institutiones calculi differentialis, 1755

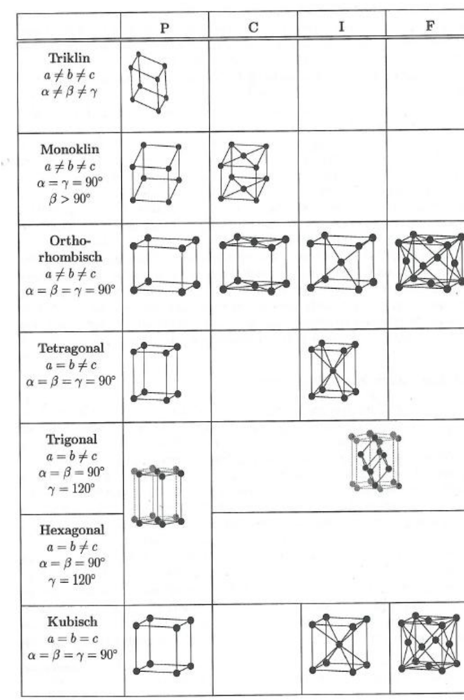


- Lagrange, *Mecanique analytique*, 1788

2. From the discrete to the continuous: homogenization and the continuum hypothesis

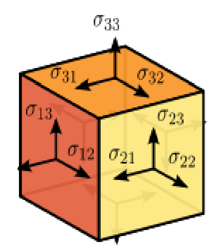
Mathematics plays a prominent role in the evolution of continuum mechanics:

Mathematics in the 19th century: Cauchy



- *Cours d'analyse*, 1821
- *Le calcul infinitesimal*, 1823

- Invention of the stress tensor



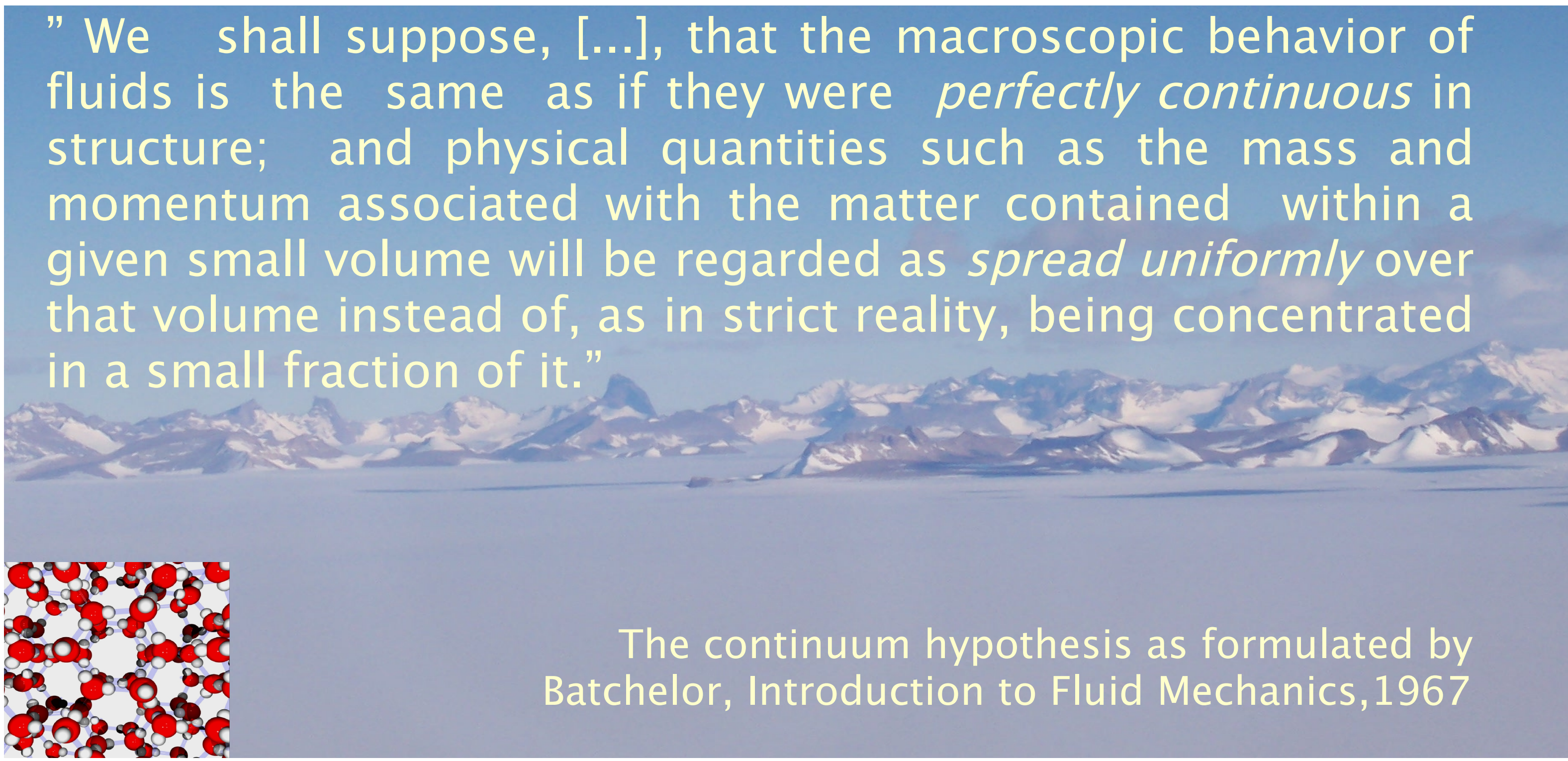
- Combination of the stress tensor with Eulers laws of mechanics

→ general framework for the description of the motion of any continuous medium

2. From the discrete to the continuous: homogenization and the continuum hypothesis

macroscale

”We shall suppose, [...], that the macroscopic behavior of fluids is the same as if they were *perfectly continuous* in structure; and physical quantities such as the mass and momentum associated with the matter contained within a given small volume will be regarded as *spread uniformly* over that volume instead of, as in strict reality, being concentrated in a small fraction of it.”



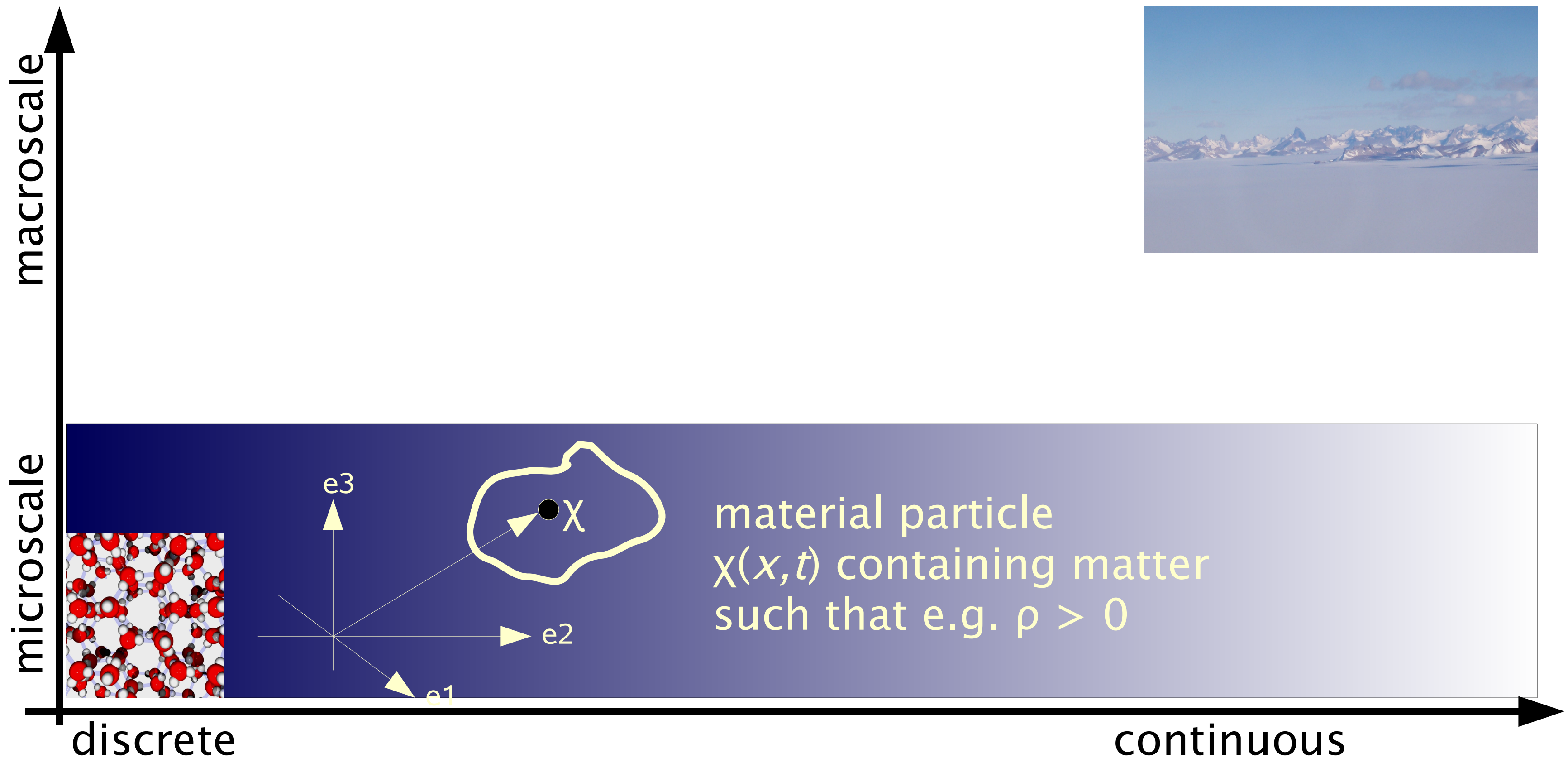
The continuum hypothesis as formulated by Batchelor, Introduction to Fluid Mechanics,1967

microscale

discrete

continuous

2. From the discrete to the continuous: homogenization and the continuum hypothesis



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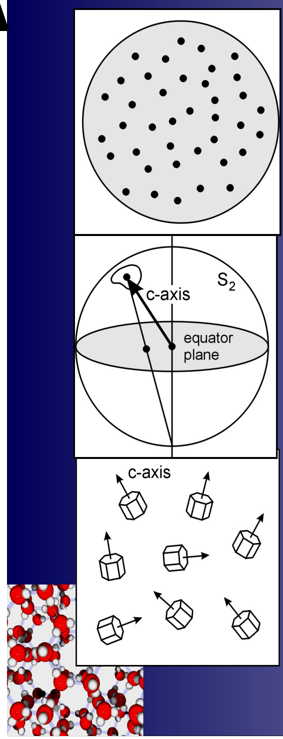
2. From the discrete to the continuous: homogenization and the continuum hypothesis

macroscale

microscale



”Smallest given volume” in large natural ice masses: polycrystals

(unresolved) randomly oriented hexagonal ice crystals implying mechanically isotropic material behavior

discrete

continuous

Stockholms universitet

2. From the discrete to the continuous: homogenization and the continuum hypothesis

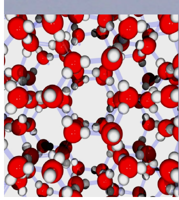
macroscale

Postulate: Large natural ice masses are treated as a

- single-constitutent
 (no impurities [dust, minerals], no water...),
- homogeneous (no bubbles, ...),
- polycrystalline,
- isotropic

continuous material.

microscale



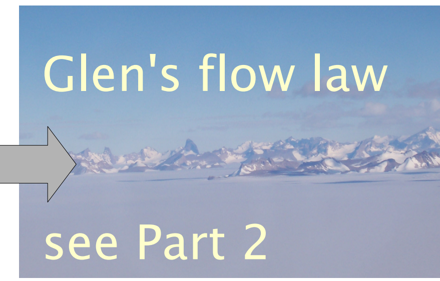
discrete

continuous

3. παντα ρει – everything flows: Rheology 1

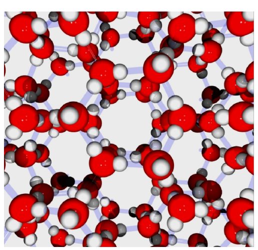
macroscale

Postulate:
the flow behavior of a large natural ice mass is modeled by regarding it as an incompressible, viscous, heat-conducting non-Newtonian fluid



? Is rheology a matter of scale

microscale



discrete

continuous

3. παντα ρει – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum

”single-constituent, homogeneous”

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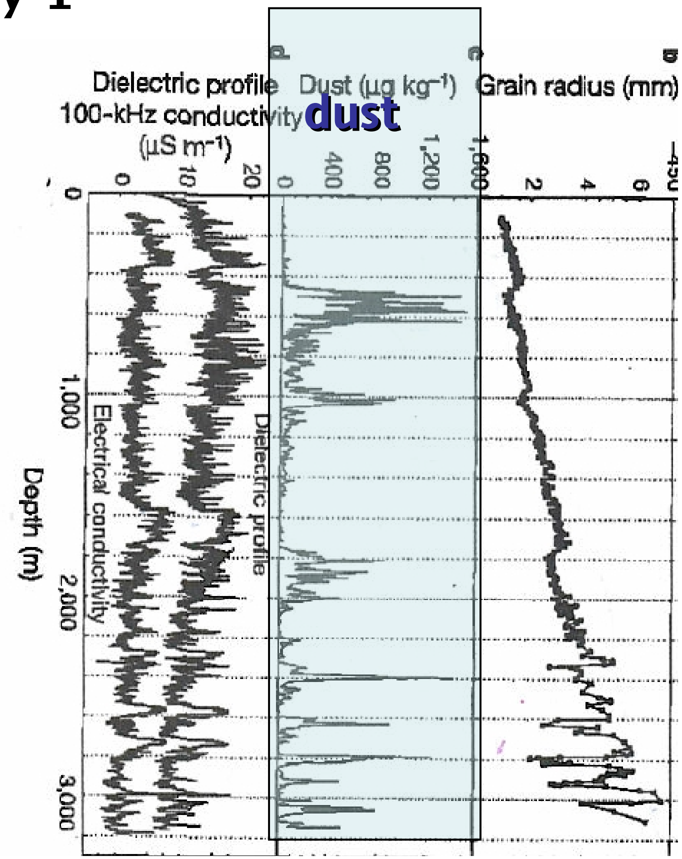
The rheology (the flow law) has been formulated for the idealized continuum

”single-constituent, homogeneous”



From: „8 glacial cycles from an antarctic ice core“, EPICAcommunity members, Nature (2004)

3. παντα ρει – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum

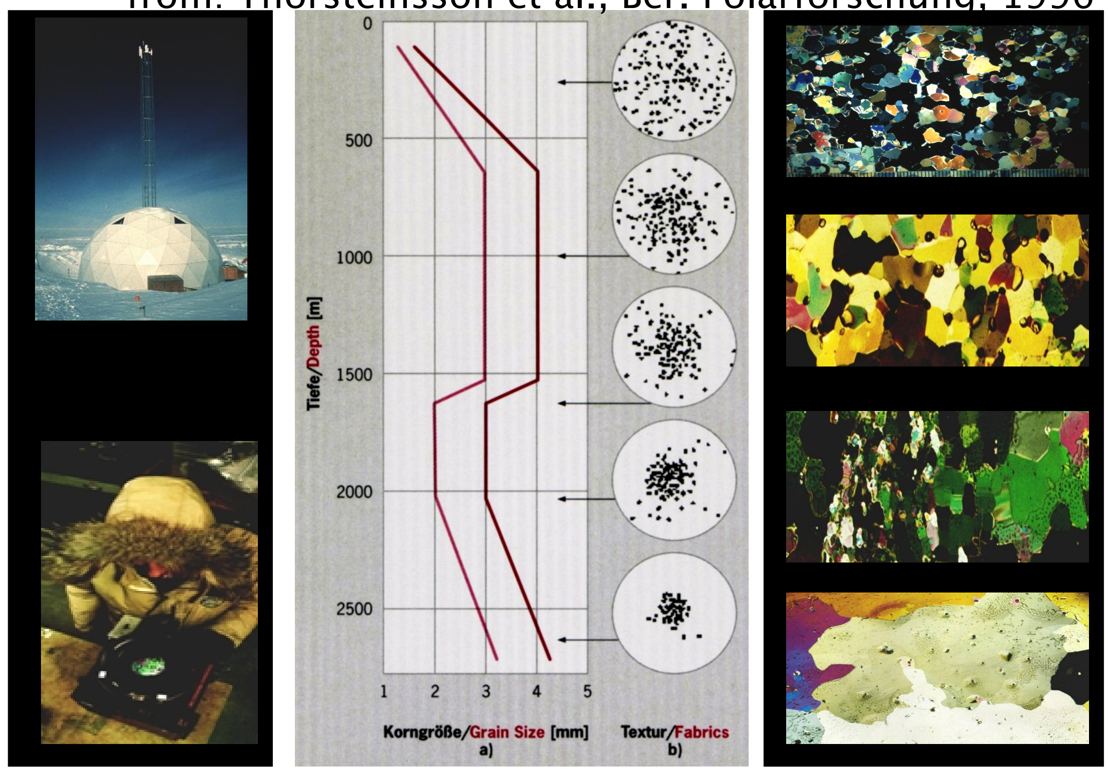
”isotropic”

3. παντα ρει – everything flows: Rheology 1

The rheology (the flow law) has been formulated for the idealized continuum

from: Thorsteinsson et al., Ber. Polarforschung, 1996

”isotropic”



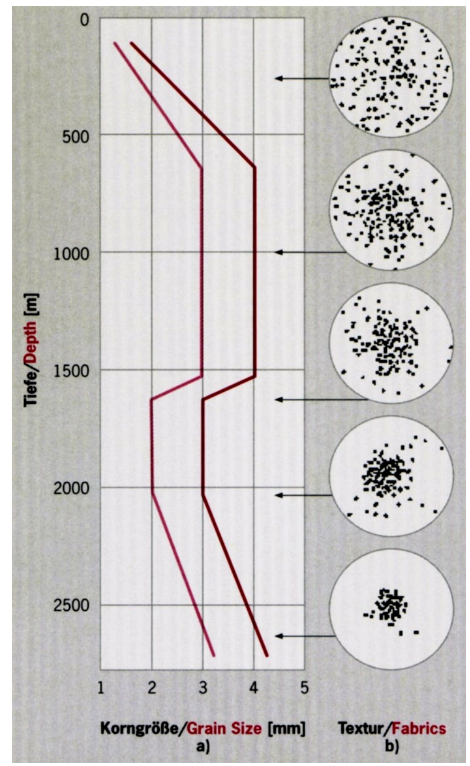
3. παντα ρει – everything flows: Rheology 1

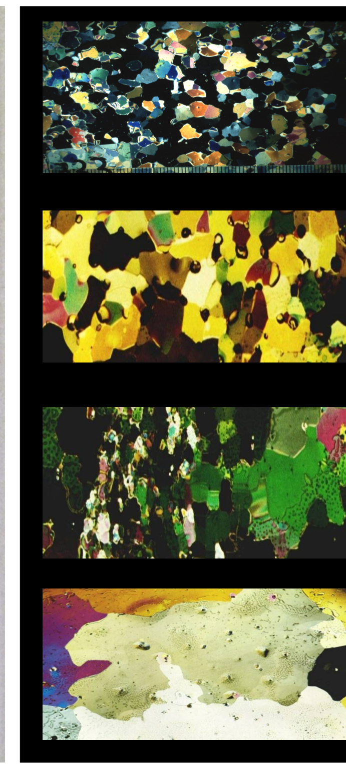
The rheology (the flow law) has been formulated for the idealized continuum

”isotropic”

we observe *anisotropy*:

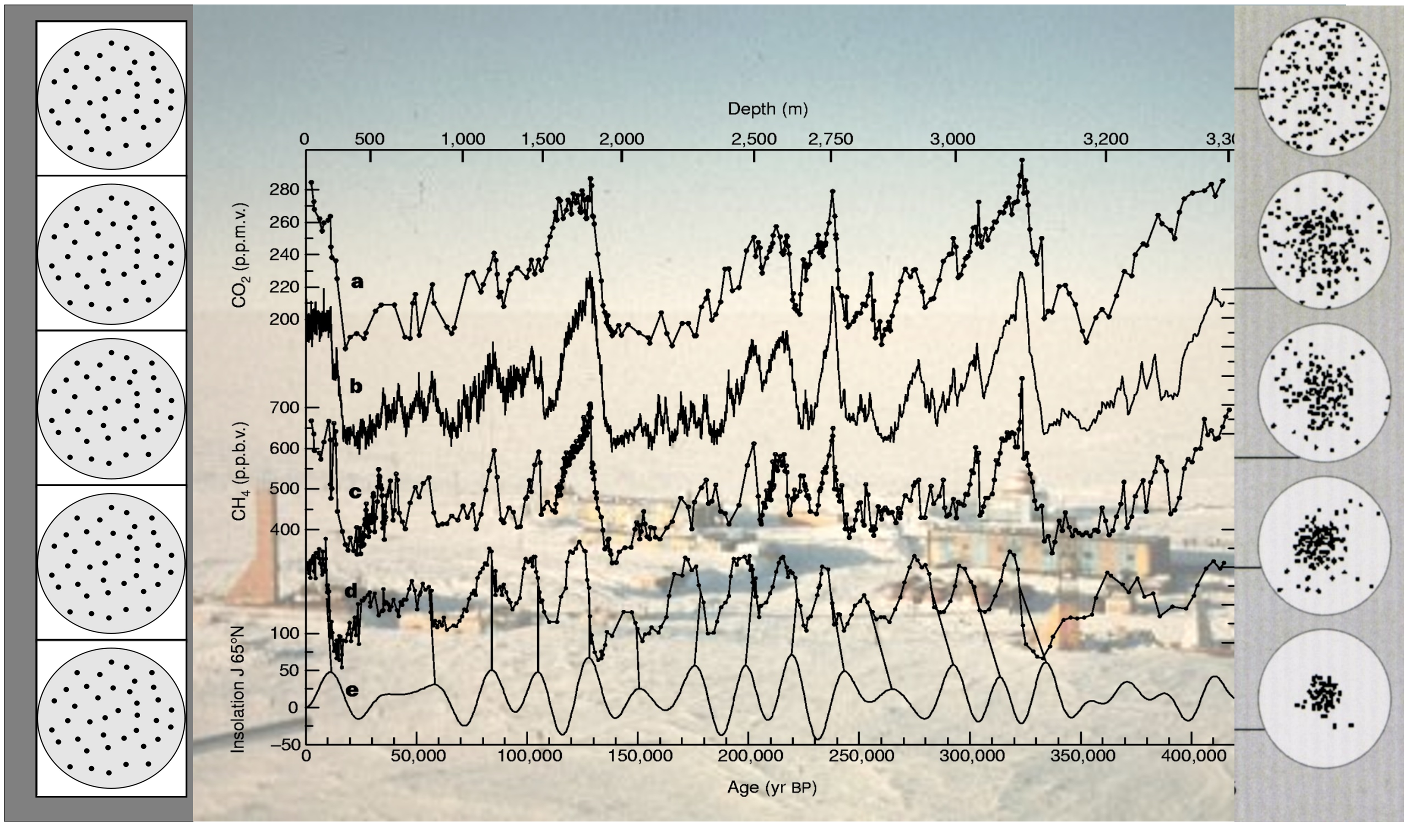
αν = not
ισοσ = identical
τρεπειν = turn around





3. παντα ρει – everything flows: Rheology 1

Isotropy vs. Anisotropy: Why should we bother ?

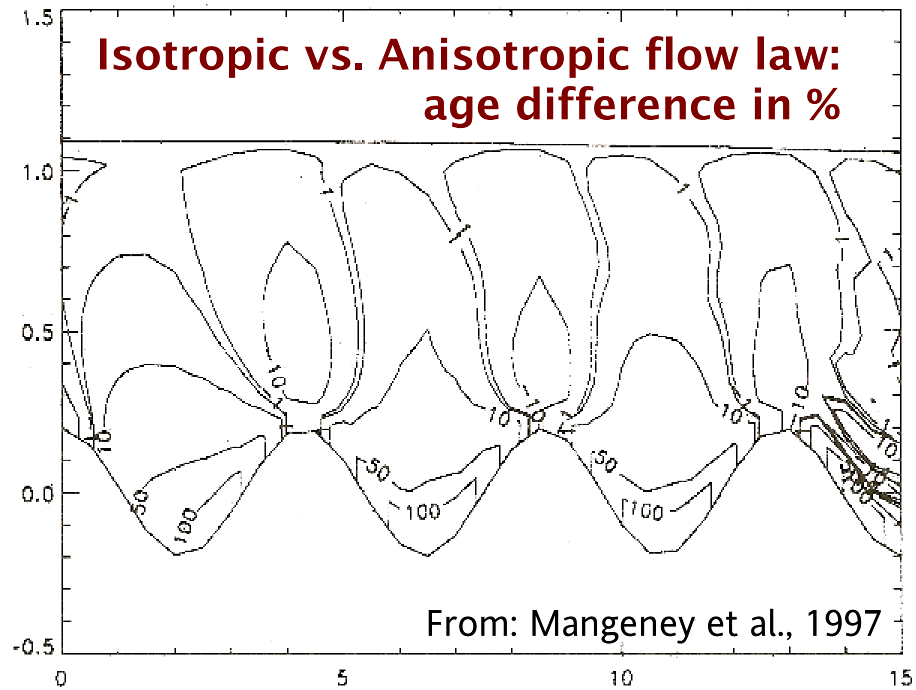


from: Petit et. al, Nature (399), 1999

3. παντα ρει – everything flows: Rheology 1

Evolving anisotropy (distribution of c-axis orientation) changes the mechanical response of ice and results in *altered*

- flow velocities
- particle positions
- depth-age relation for ice cores
- reconstructions of past climates



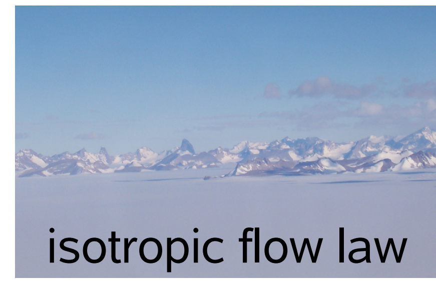
numerical determination of ice age in cores based on models with/without induced anisotropy may differ by 40–100 kyr

3. παντα ρει – everything flows: Rheology 1

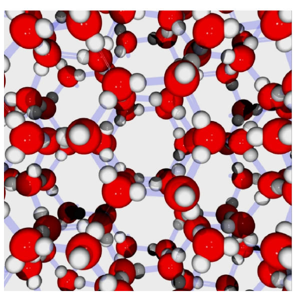
macroscale

deformation of ice sheets takes place

• under the action of gravity
• in the presence of high temperatures
• in response to processes at their boundaries
• ...



microscale

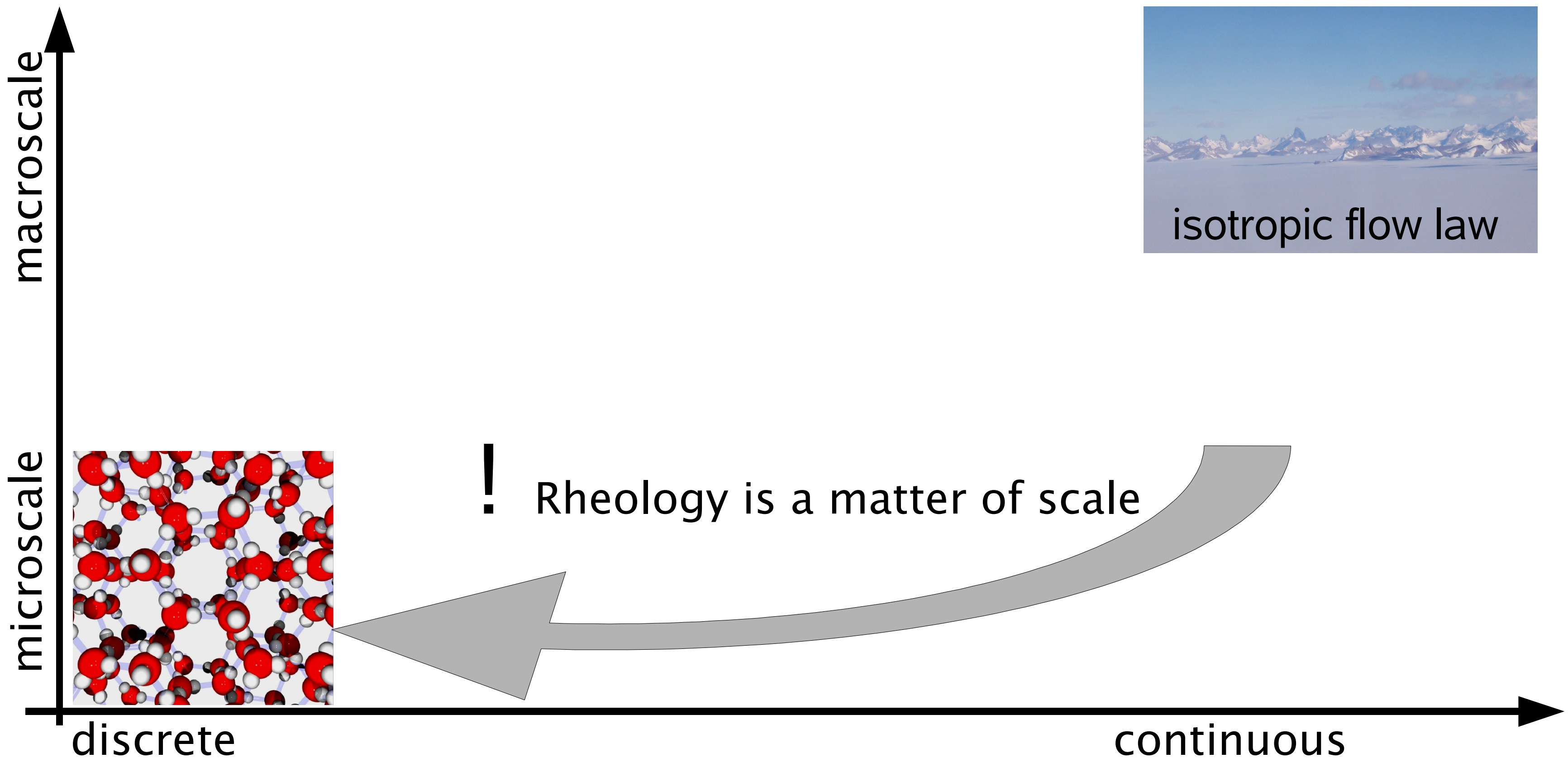


! Rheology ...

• depends on Flow–Structure–Environment–Interplay (FSEI)
  [the entire is more than the sum of its parts]
• requires consideration of *induced* anisotropy

discrete → continuous

3. παντα ρει – everything flows: Rheology 1



3. παντα ρει – everything flows: Rheology 1

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Warning: a true anisotropic flow law is not obtained by introducing constant scalar enhancement factors to an isotropic flow law

Exercise: calculate the stresses !