A network approach to harmonic evolution and complexity in western classical music

Marco Buongiorno Nardelli[1,2]

[1] Department of Physics and Division of Composition Studies,
University of North Texas, Denton, TX 76203, USA
[2] Santa Fe Institute, Santa Fe, NM 87501, USA
mbn@unt.edu

Abstract. I recently introduced the concept of dynamical score network to represent the harmonic progressions in any composition. Through a process of chord slicing, I obtain a representation of the score as a complex network, where every chord is a node and each progression (voice leading) links successive chords. In this paper, I use this representation to extract quantitative information about harmonic complexity from the analysis of the topology of these networks using state of the art statistical mechanics techniques. Since complex networks support the communication of information by encoding the structure of allowed messages, we can quantify the information associated with locating specific addresses through the measure of the entropy of such network. In doing so I can then introduce a measure of complexity that can be used to quantify harmonic evolution when applied to an extensive corpus of scores spanning 500 years of western classical music.

Keywords: music complexity; computational music theory; music analysis; music composition; music information retrieval; music evolution; music innovation

1 Introduction

In the article *Topology of Networks in Generalized Musical Spaces*, published on the Leonardo Music Journal, [1] I have introduced the concept of harmony as a network representation of the musical structures built out of all possible combinatorial pitch class sets in any arbitrary temperament. Inspired by a long tradition of network representations of musical structures such as the circle of fifths [2], the Tonnentz [3], and recent works on the spiral array model of pitch space, [4] the geometry of musical chords [5] and generalized voice-leading spaces [6] [7], I interpret the harmonic structure of a composition as a large-scale complex network whose topological properties uncover its underlying organizational principles and demonstrates how classifications or rule-based frameworks (such as common-practice harmony, for instance) can be interpreted as emerging phenomena in a complex network system. Since the conclusions of that study serves as foundations for the present paper, let me review some of its principal results.

Network analysis methods exploit the use of graphs or networks as convenient tools for modelling relations in large data sets. If the elements of a data set are thought of as "nodes", then the emergence of pairwise relations between them, "edges", yields a network representation of the underlying set. Similarly to social networks, biological networks and other well-known real-world complex networks, entire dataset of musical structures can be treated as networks, where each individual musical entity (pitch class set (pcs), chord, rhythmic progression, etc.) is represented by a node, and a pair of nodes is connected by a link if the respective two objects exhibit a certain level of similarity according to a specified quantitative metric. Pairwise similarity relations between nodes are thus defined through the introduction of a measure of "distance" in the network: a "metric" [8]. As in more well-known social or biological networks, individual nodes are connected if they share a certain property or characteristic (i.e., in a social network people are connected according to their acquaintances, collaborations, common interests, etc.) Clearly, different properties of interest can determine whether a pair of nodes is connected; therefore, different networks connecting the same set of nodes can be generated.

In this paper I construct networks where nodes are the individual chords that can be extracted from a score, and edges are built between successive chords in the progression: nodes are connected if they appear as neighbours in the sequence. Naturally, nodes are visited numerous times, and the score evolution implies a directionality of the links. The networks are thus "directed", and each edge will have a weight (strength) proportional to the times the link is visited.

Given a network, we can perform many statistical operations that shed light on the internal structure of the data. In this work I will consider only two of such measures, degree centrality and modularity class. [9] The degree of a node is measured by the number of edges that depart from it. It is a local measure of the relative "importance" of a node in the network. Modularity is a measure of the strength of division of a network into communities: high modularity (above 0.6 in a scale from 0 to 1) corresponds to networks that have a clearly visible community structure. [10]. Isolating communities through modularity measures provides a way to operate within regions of higher similarity.

In a more recent work [11] I have proposed that this score network can be viewed both as a as a static graph that represents all the existing chord changes in a composition, or as a dynamical system, a time series of a non-stationary signal, and as such, it can be partitioned, as for community structures, using time series analysis and change point detection. This dual representation (static and dynamical) offers novel ways to quantify the harmonic complexity of a single score or a full corpus without relying on comparisons with pre-determined reference sets.

2 Methods

2.1 Network models

I will make use of two principal software libraries for computational music analysis, both written in the Python language: MUSICNTWRK (at www.musicntwrk.com) and music21 (at https://web.mit.edu/music21). MUSICNTWRK is an open source python library for pitch class set and rhythmic sequences classification and manipulation, the generation of networks in generalized music and sound spaces, deep learning algorithms for timbre recognition, and the sonification of arbitrary data [11]. music21, developed at MIT [12], is an object-oriented toolkit for analysing, searching, and transforming music in symbolic (score-based) forms of great versatility, whose modularity allows a seamless integration with MUSICNTWRK and other applications.

Scores are read in musicxml format by the readSCORE function of MUSCNTWRK, where their harmonic content (and other relevant information, like in which bar the chord is found) is extracted using the music21 parser and converter (using the "chordify" method). With this we obtain easily the full sequence of pcs, chord by chord, where each change to a new pitch results in a new chord. Upon "chordification", each pcs is reduced to its normal form. While such "quantization" of pcs is quite adequate for the analysis of pieces with a harmonic movement where each vertical pcs plays some functional values (for instance in the corpus of J.S. Bach's chorales), for compositions where there is more contrapuntal development, the number of individual pcs in the sequence can become very large, without providing necessarily more detailed information, since many such chords are only modifications via passing notes or vagrant harmonies. To circumvent this problem and make the analysis more manageable without losing any functional value, we have devised a "filtering" algorithm based on the cumulative measure of how many times an individual pcs appears in the sequence. All pcs with a frequency lower than a threshold are eliminated.

Starting from digitalized scores (in musicxml or MIDI format) I then construct networks where nodes are the individual chords, and edges are built between successive chords in the progression: nodes are connected if they appear as neighbors in the sequence. Naturally, nodes are visited numerous times, and the score evolution implies a directionality of the links. The networks are thus directed, and each edge will have a weight (strength) proportional to the times the link is visited. In Figure 1 I show the network of one score from the L. van Beethoven's corpus.

I have analyzed an extensive corpus of scores by composers spanning five centuries of western classical music: Josquin des Prez (1450-1521), G.P. da Palestrina (1525-1594), Claudio Monteverdi (1567-1643), J.S. Bach (1685-1750), J. Haydn (1732-1809), W.A. Mozart (1756-1791), L. van Beethoven (1770-1827), J. Brahms (1833-1897) and G. Mahler (1860-1911).

2.2 Conditional Degree Matrix

The local metrics usually used in networks theory fall short of capturing the richness of the vast majority of natural network topologies. At the same time, one of the most commonly used (local) characteristics is a node's degree. Based on this attribute, I propose to use what has been named ``conditional degree matrix" D [13] to characterize the topology of the harmonic networks.

This matrix captures the classical node distribution and shows the existing architecture between the network nodes taking into account their different degrees. Each element of the matrix, $D_{i,j}$ is defined as the number of nodes of degree i connected to nodes of degree j, $N_{i,j}$, divided (normalized) by the number of total nodes, N_t, that is:

$$D_{i,j} = \frac{N_{i,j}}{N_t}.$$

This definition produces a symmetric matrix and ensures that D is properly normalized. More generally, directed and weighted networks would result in non-symmetric matrices.

The structure of the D matrix allows to estimate the complexity of a given network and provides more information than the classical degree distribution:
D effectively acts as a probability matrix and can be the input for the evaluation of other metrics such as entropy, divergence, and complexity among others.

One of the essential properties of this matrix is that it allows to explore the characteristics of the degree of connections of each node with its environment (its close neighborhood) in a direct way. Its importance can be understood in terms of information diffusion: the rows i of this matrix show the probability that nodes of degree i are connected with nodes of another degree j. Their frequency will finally be reflected in each of its elements $D_{i,j}$.

2.3 Kullback-Leibler divergence

To extract quantitative information from the network topology I use the the Kullback-Leibler (KL) divergence as metric. In both information theory and probability theory, the Kullback-Leibler divergence is used as a measure indicating the difference between two probability functions. In general terms, KL measures the expected number of extra bits or excess surprise from using Q as a model when the data distribution is P.

The Kullback-Leibler divergence for the conditional degree matrix is defined as:

$$KL = \sum_{i,j} D_{i,j} log\left(D_{i,j}/Q_{i,j}\right)$$

where D is our CDM, and the reference matrix Q is defined as the mean of all the D for the whole corpus:

$$Qi,j = \frac{1}{N} \sum_{n} D_{i,j}^{n},$$

and N is the total number of score networks.

Since the KL divergence quantifies how much the topology of any individual network "diverges" from the average of the reference corpus, it provides a way to quantify the difference in the distribution of observed degrees and in particular, the way in which the occurrence and distribution of hubs (as chords that are more important in the harmonic progression of a piece) characterizes the harmonic structure of the composition.

2.4 Diffusion Entropy Analysis

Diffusion Entropy Analysis (DEA) is a time-series analysis method for detecting temporal complexity in a dataset; such as heartbeat rhythm [14] [15] [16] a seismograph [17], or financial markets [18]. DEA uses a moving window method to convert the time-series into a diffusion trajectory, then uses the deviation of this diffusion from that of ordinary brownian motion as a measure of the temporal complexity in the data. It is thus appropriate to analyze the score time series and derive quantitative estimates of complexity.

Diffusion Entropy Analysis was first introduced by Scafetta and Grigolini [19] as a method of statistical analysis of time-series based on the Shannon entropy of the diffusion process to determine the scaling exponent of a complex dynamic system. It was later refined with the introduction of ``stripes'' (MDEA) by Culbreth $et\ al.$ [20] in the context of detecting crucial events. While the reader should refer to the publications above for a full treatment of DEA, here I use the realization that the scaling of the diffusion coefficient δ obtained in DEA provides a measure of complexity of the time-series, measured through the statistics of occurrences of crucial events. Here, δ ranges between 0.5 and 1.0: for a completely non-complex process, such as a random walk, MDEA yields $\delta = 0.5$. For a process at criticality, MDEA yields $\delta = 1$. Therefore, δ represents a measure of the ``strength'' of the complexity present in the process: the closer δ is to 1 the closer the process is to criticality.

In Fig. 2, we show a MDEA analysis of the first movement of Beethoven's string quartet Op. 127 n. 12, that was extensively discussed in [21]: $\delta \approx 0.7$ indicates a "medium" level of complexity as observed in other composition of the same period as it will be discussed extensively below.

MDEA analysis has been carried out using the module DEA implemented in the MUSICNTWRK library [11].

3 Results and discussion

I have applied the above metrics to our selected corpus of composition and the results are summarized in Fig. 3 and 4.

In Fig. 3 I show the KL divergence calculated for the full corpus of compositions. Here the values are referenced to the average of the corpus, that is, I am capturing how much the topology of a given piece deviates from the cumulative average. The results point to a clear distinction between earlier polyphony (des Prez, Bach) and later chromaticism (Mahler). There is a marked transition starting in the XIX century and culminating with the works of Brahms and Mahler. It is important to note that this metric provides a somewhat indirect measure of complexity as a relative difference between compositions. Of course, more work is needed to understand this metric further: although large, the corpora I have analyzed are still small in statistical terms, and more analysis should be done by extending the corpora to a larger repertoire and/or using simulated data as toy models of the musical practice of selected composers.

For a more direct evaluation of complexity, we turn now to the MDEA results. By applying the procedure outlined in Fig. 2 to the full corpus, I have extracted the values of δ for each piece and collected the results in Fig. 4.

Although the data points show a wide distribution around the averages for each composer, the results point to an increase of harmonic complexity over time, a result that agrees broadly with other analysis based on different metrics. These results allow us to discriminate further among composers and different time periods. We can, in principle divide this graph in three regions. The first region corresponds to the Renaissance and Early Baroque composers, where δ is consistently lower. Since this musical period is characterized by a modal approach to harmony, we can easily infer that modal harmony is characterized by a lower complexity, as observed in the scarcity of functional chords (functional chords are hubs in tonal harmony networks [21], a more homogeneous distribution of node degrees, and a lack of multiple tonal centers.[1]

The second section corresponds to the Common Practice period, that shows an average complexity measure of ≈ 0.7. This is when tonal harmony has matured into an established musical language. Once again in the third section, corresponding to XIX and early XX century, harmonic complexity increases to an average of 0.8. Once more, enhanced criticality and complexity correspond to a fragmentation of the tonal harmony language towards increase chromaticism, as it was observed in the CDM data above.

Finally, I have superimposed measures of complexity for pieces of the pop/rock repertoire as a suggestion for further analysis and discussion beyond the realm of classical music genres.

4 Conclusions

In conclusion, with this study I have built on the concept of network representation of musical spaces, introduced the idea of a composition as a dynamical score network, and discussed two complementary measures of music complexity. Although the results point unequivocally to an increase of harmonic complexity in western classical music, the present study is just an initial exploration of this fascinating topic. One of the challenges is the availability of large score corpora that would make the analysis of a single composer's production more coherent. I am currently working to expand the availability of corpora and I hope to extend this work in the future. Notwithstanding its limitations, this study demonstrates that combining the abstraction of a score as network with established mathematical and statistical techniques is a powerful tool for a quantitative analysis of music production that is independent of prior musicological or music

[1] It is important to note that here *complexity* must be interpreted as a statistical measure on the network topology associated to a dynamical system behavior, and not as a measure of how sophisticated a piece is or is perceived by a listener: a Palestina's Mass can be more challenging and sophisticated than a Bach's chorale, although their measured complexities might suggest otherwise.

theoretic information and opens the way to a novel interpretation of music as a dynamical process.

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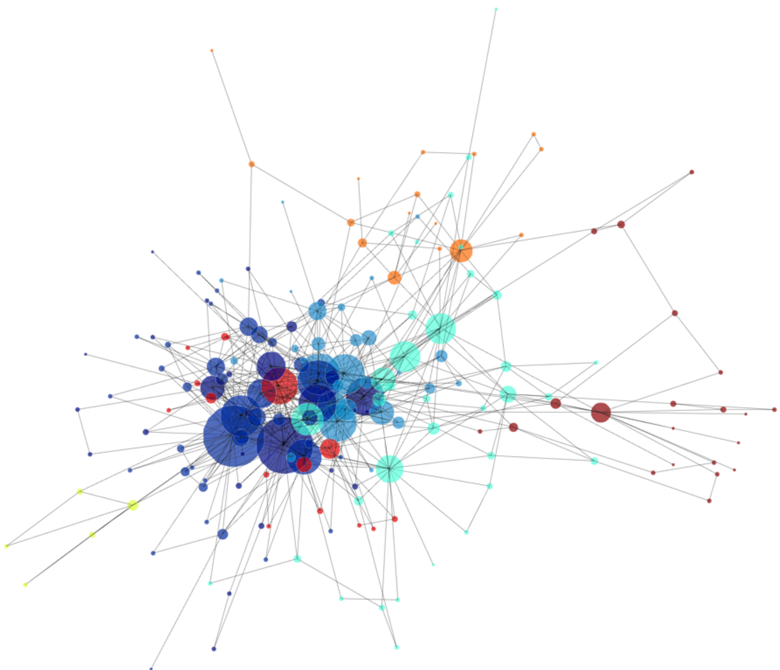

Fig. 1. Network structure of the third movement of Ludwig van Beethoven’s string quartet Op. 127 n. 12. Node size is proportional to degree, colors indicate the network community structure (the tonal region central to that particular section) and links correspond to voice leading (the transition from one chord to the next).

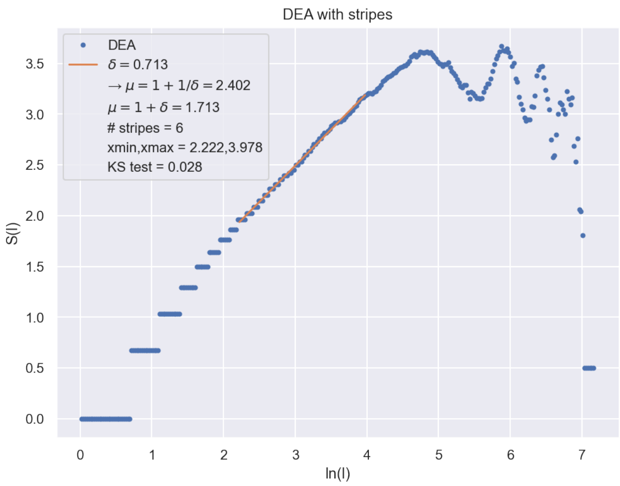

Fig. 2. Diffusion Entropy Analysis applied to the third movement of Beethoven's string quartet Op. 127 n. 12, whose network is shown in Fig. 1.

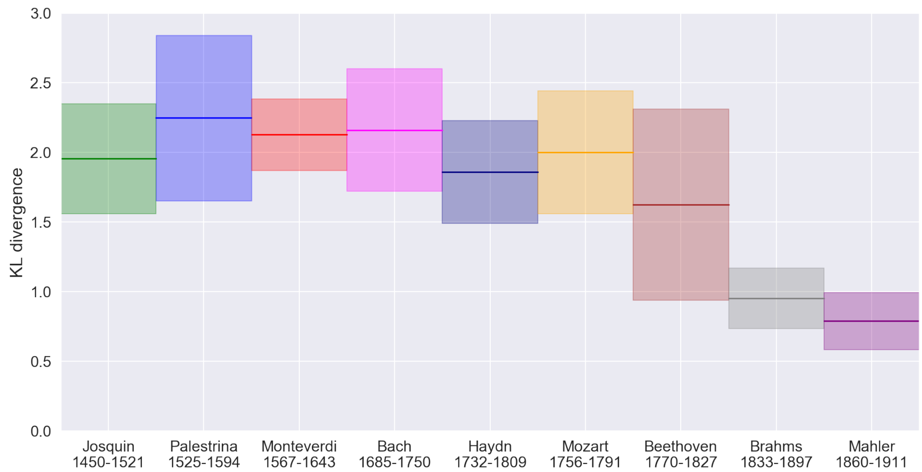

Fig. 3. Kullback-Leibler divergence. Horizontal lines are the average values of KL across the corpus of each composer; shaded areas indicate standard deviations.

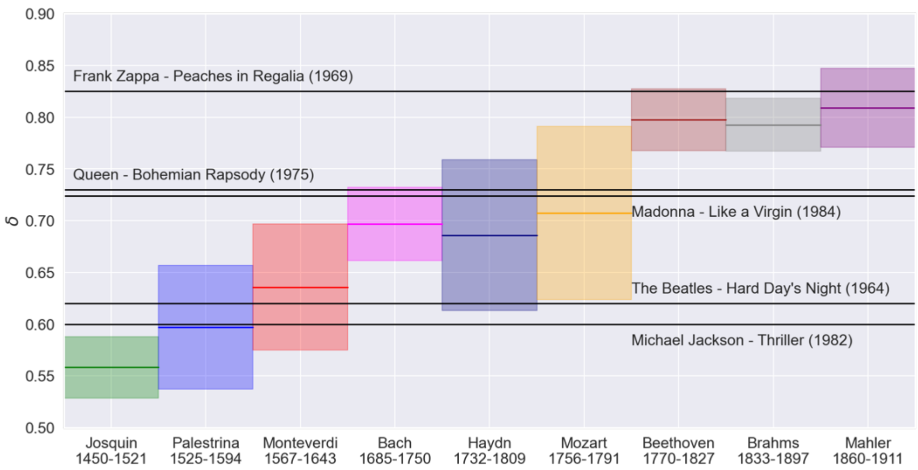

Fig. 4. Complexity for different composers as measured from the diffusion entropy analysis. Horizontal lines are the average values of δ across the corpus of each composer; shaded areas indicate standard deviations.

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