Mathematical Description of the Regensburg Model Scenario Types RM 1 – 6

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Content

Indexes Tables and Figures

1 Introduction

The Regensburg Model Scenario Types RM $1 - 6$ are used to derive plausible emission paths that meet a certain budget. The emission paths are essentially determined indirectly by an assumption about the property of the annual changes. This is the innovative core of the RM Scenario Types.

We pursue two approaches:

1. Determination of the course of the annual reduction rates (RM 1 - 5).

2. Determination of a constant annual reduction amount (RM-6).

In the indirect determination of emission paths using annual reduction rates with a monotonic trajectory, the following four **basic types** can be distinguished:

(1) Initial less than proportional increase[1] in annual reduction rates (RM-2, RM-4) ► concave

(2) Initial over-proportional increase in annual reduction rates (RM-5) ► convex

(3) Linear increase in annual reduction rates (RM-3) ► linear

(4) Constant annual reduction rate (RM-1) ► constant

The RM Scenario Types are used in our **tools** to derive plausible global or national emission paths.

The **Excel tools** (see Chapter 10 References) can be downloaded from our website: http://save-the-climate.info.

Here is an overview of our **web applications**: https://www.climate-calculator.info

2 Constraints to be specified

B budget for a certain period (budget period); here: 2020 - 2100

E_{BY} emissions in the base year (BY); here: $BY = 2019$

E_{min} minimum of emissions in the budget period;
       a negative value represents the potential for net negative emissions

RR_{BY+1} rate of change in the first year of the budget period in RM $2 - 5$ (start change rate);
       first year here: 2020; in scenario type RM-2, only a negative value is possible

TV threshold from which the method is changed in order to map net negative emissions
       in a pragmatic way (from this value a constant annual reduction amount is used)

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[1] "Increase" refers to the absolute amount of the reduction rates.

3 Formulae Regensburg Model Scenario Types

3.1 Determination of paths via annual rates of change (scenario types RM 1 – 5)

$$E_t = \begin{cases} \max\left(E_{min};\ E_{t-1} * (1 + \mathbf{RR_t})\right) & for\ E_{t-1} > TV\ ^2 \\ \max\left(E_{min};\ E_{t-1} + (E_{t-1} - E_{t-2})\right) & for\ E_{t-1} \leq TV\ ^3 \end{cases}$$

where:

E_t emissions in the year t; here: 2020 – 2100

The **reduction rates** (annual rates of change) in the individual scenario types are based on the following formulae:

name scenario type	formula	basic function type	constraint	course of the reduction rates
RM-2-exp[4]	$\mathbf{RR_t} = RR_{t-1} * (1 + \mathbf{a})$	e^x	$\mathbf{a} \geq 0$	► concave
RM-4-quadr[5]	$\mathbf{RR_t} = \mathbf{a} * (t - (BY + 1))^2 + RR_{BY+1}$	$y = ax^2 + b$	$\mathbf{a} \leq 0$	
RM-5-rad[6]	$\mathbf{RR_t} = \mathbf{a} * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$	$y = a\sqrt{x} + b$	$\mathbf{a} \leq 0$	► convex
RM-3-lin	$\mathbf{RR_t} = \mathbf{a} * \left(t - (BY + 1)\right) + RR_{BY+1} = RR_{t-1} + \mathbf{a}$	$y = ax + b$	$\mathbf{a} \leq 0$	► linear
RM-1-const	$\mathbf{RR_t} = \mathbf{a}$	$y = a$	$\mathbf{a} \leq 0$	► constant

Table 1: Formulae RM Scenario Types 1 - 5[7]

The free parameter $\mathbf{a}$ is determined for each scenario type using an iterative solution method so that the budget (B) is adhered to. In the Excel tools, the integrated target value search ("goal seek") is used for this purpose, which is embedded in a macro[8] that ensures that the constraint for $\mathbf{a}$ is also met.[9]

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[2] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application of $\mathbf{RR_t}$).

[3] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application from the last absolute reduction amount; the emission path is then a straight line).

[4] In this scenario type, the free parameter $\mathbf{a}$ can be called the escalation rate applied to the reduction rate of the previous year. This scenario type can also be represented using the following formula: $RR_{BY+1} * e^{(t-(BY+1))*\ln(1+a)}$.

[5] Basic function type: $y = ax^2 + b$. The term $[t - (BY +1)]$ is set for x in a variable transformation in order to be able to calculate with years. For $t = 2020$ the value of the term is 0. The term thus takes the values 0, 1, …, 80 for the period 2020 - 2100 considered here.

[6] Basic function type: $y = a\sqrt{x} + b$. The term $[t - (BY +1) - 0.5]$ is set for x. 0.5 serves to smooth the course at the beginning (see Chapter 9.1). The term $[t - (BY +1)]$ represents a variable transformation in order to be able to calculate with years. x thus takes the values 0.5, 1.5, …, 79.5 in the period 2021 - 2100 considered here.

[7] In the scenario types RM 2, 5 and 3 for $t = BY+1$ the predefined RR_{BY+1} (see Chapter 2) must be used. Thus, the equations above hold for $t > BY+1$ (here: $t > 2020$).

[8] We have published the macro separately (Wolfsteiner, 2023).

[9] If no solution can be found with the given framework data, RR_{BY+1} is varied in the Excel tools and B in the corresponding web apps.

3.2 Determination of paths via annual reduction amount (scenario type RM-6)

RM-6-abs: $E_t = \max\left(E_{min};\; E_{t-1} + RA\right)$ [10]

The free parameter RA (constant annual reduction amount) is determined using an iterative solution method so that the budget (B) is adhered to.[11]

3.3 Phases for determining the paths

This usually leads to the following **three phases** for determining the paths (see Figure 3 for illustration):[12]

1. Application of the annual **reduction rates** (RM 1 - 5) or **reduction amount** (RM-6).

2. RM $1 - 5$ if $E_{t-1} \leq TV$: The last reduction amount from phase 1 is used as the constant reduction amount until E_min is reached. In this phase, the emission path is a straight line.

3. Minimum for the annual emissions (E_min) is used until 2100.

4 Overview of the RM Scenario Types

basic type (see Chapter 1)	Scenario Type	course of the annual reduction rates		basic function type	course of the annual reduction amounts	course of the emission paths
(4)	RM-1-const	constant	─	$y = constant$	concave	convex
(3)	RM-3-lin	linear	╲	$y = ax + b$	u-shaped	s-shaped (first concave then convex)
(1)	RM-2-exp	concave	╮	$y = e^x$	u-shaped	
	RM-4-quadr			$y = ax^2 + b$	u-shaped	
(2)	RM-5-rad	convex	╲	$y = a\sqrt{x} + b$	u-shaped	
-	RM-6-abs	concave	╮	-	constant	linear

Table 2: RM Scenario Types overview[13]

In principle, there are several options for mapping the basic types (1) and (2) using a specific function (see Chapter 9.2 for another examples). However, the results of the functions used in basic types (1) and (2) differ only to a limited extent when the budget is tight. This is also shown by comparing the results of RM-2 and RM-4 (cf. Figure 1). Therefore, the scenario types RM 1 and RM 3 - 6 represent the range of plausible possibilities well.

For RM-1 with a constant annual reduction rate and for RM-6 with a constant annual reduction amount, the starting rate of change is endogenous. In the other scenario types, RR_{BY+1} can be specified freely[14] resp. at a realistic level.

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[10] "Max" means here, take the larger value. Either E_{min} or E_t (which results from the application of the constant annual reduction amount: RA).

[11] We also offer a formula-based solution for the scenario type RM-6 (cf. Wittmann & Wolfsteiner, 2023).

[12] When actual emissions are available after the base year, there is another phase with actual values (cf. Chapter 8).

[13] See also Chapter 7 for a graphic illustration of the differences.

[14] However, only a negative value is possible for the scenario type RM-2.

5 Choice of a RM Scenario Type

The following questions can play a role in the selection of a scenario type:[15]

1. Which change rates are realistic and when?

2. Do initially slowly increasing reduction rates (RM-2/4 and RM-6) imply an unjustifiable duty for the future, as these later require very high reduction rates?

3. Could high later reduction rates even make sense because they provide a greater lead time for the necessary investments? The investments could then rather be made within the framework of normal investment cycles. However, this requires a very credible climate policy backed by effective instruments.

4. Do initially rapidly increasing reduction rates (RM-3 and RM-5) convey a more credible climate protection policy that creates planning security for public and private investments in a fossil-free future?

Overall, in many cases, scenario types RM-2 and RM-4 are likely to be the most realistic, with RM-4 having the advantage that a positive starting change rate can also be specified. However, these scenario types require a high level of credibility of climate policy or climate policy instruments so that investments in a fossil-free future are made in the right time.

6 Overshoot

If net negative emissions are allowed ($E_{min} < 0$), the budget may be temporarily exceeded. This overshoot will then be offset by net negative emissions by 2100.

See this paper on the limitations of an overshoot (Wolfsteiner & Wittmann, 2023e).

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[15] Cf. corresponding excursus in (Sargl, et al., 2023b) and corresponding chapter in (Sargl, et al., 2023a).

7 Exemplary annual change rates and paths RM Scenario Types[16]

In the following figures, the RM scenario types have been applied to global paths. A global CO2 budget of 550 Gt 2020 - 2100 was assumed in Figure 1 and in Figure 2. In Figure 1, the starting rate of change (RR_{2020}) was set at -0.5% and E_min at -0.8 Gt. In Figure 2, a starting rate of change of +1.5% (actual value 2019) was chosen. In Figure 3, a global CO2 budget of 400 Gt was assumed and E_min was set at -3.8 Gt ($RR_{2020} = +0.5\%$).
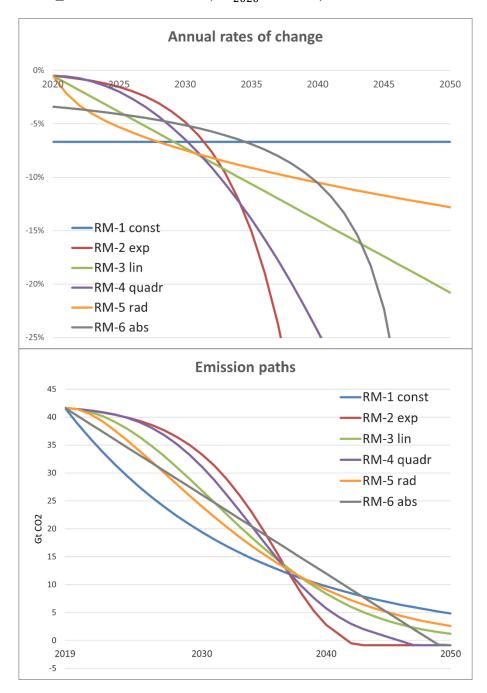

Figure 1: RM Scenario Types 1 - 6

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[16] The RM Scenario Types can also be well graphically understood with our web app: http://paths.climate-calculator.info.
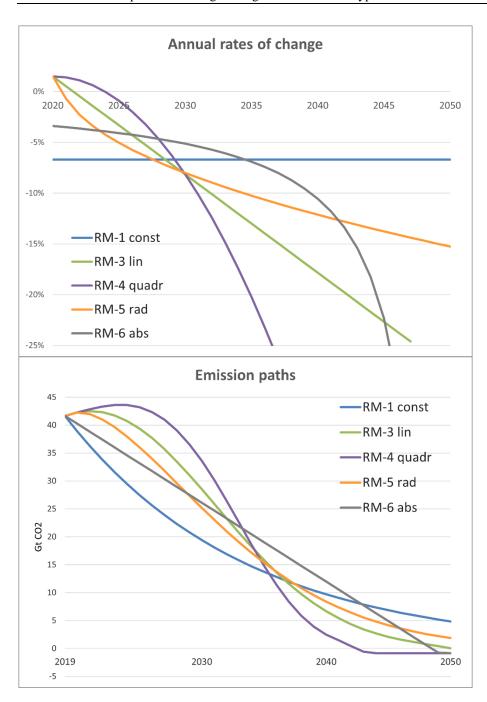
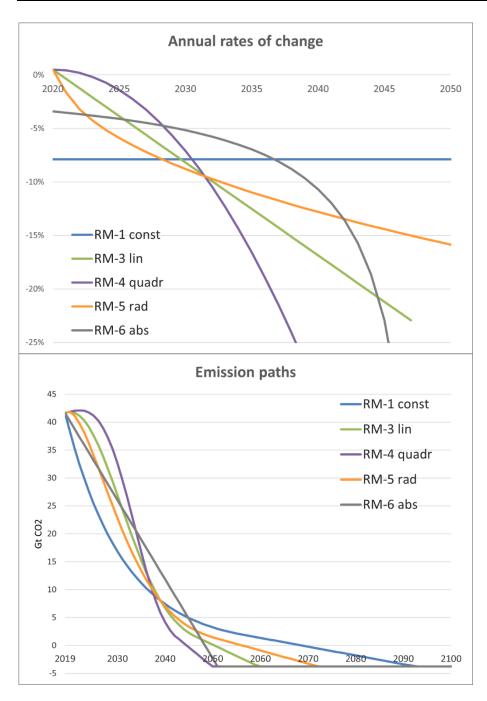*Figure 2: RM Scenario Types with positive starting change rate RM 3 – 5*

Figure 3: RM Scenario Types with high net negative emissions

8 Actual emissions after the base year 2019

It has long been discussed that global emissions should fall from 2020 on at the latest in order to meet climate targets. For this reason, we have chosen **2019** as the **base year**. This also argues in favour of retaining the 2019 base year, even if actual emissions data are available after 2019. Furthermore, the years after 2019 are less suitable as base years due to exceptional circumstances (e.g. pandemic, Ukraine war).

If **actual emissions after 2019** are used in the respective tool, then these replace the values that would result from the formulae (see Figure 4).

The **starting rate of change** in scenario types RM 2 - 5 continues to be specified for 2020 in the tools (RR_{2020}), even if actual change rates are available for the years after 2020.

Since RR_{2020} provides the starting value for determining the change rates, the actual value from 2020, which is significantly determined by the temporary corona effect, would not lead to meaningful results. Therefore, a normalised value must be used. The 2019 actual rate of change can provide guidance in many cases.

The consequence of this procedure is that when the rates of change according to the formulae are applied for the first time, they are not equally high in the scenario types RM 2 - 5, as shown in Figure 4 with actual emissions up to and including 2021 (see in contrast corresponding Figure 2).
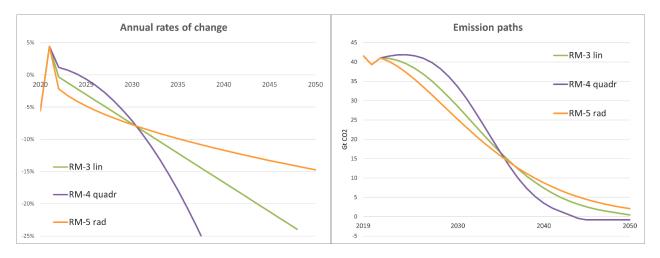

Figure 4: Consideration of actual emissions after 2019[17]

It would also be possible to specify the starting rate of change for the first year in which actual emissions are not yet known. Due to the pandemic and the Ukraine war, among other things, it would be difficult to find a good basis for this starting rate of change. Therefore, the year 2020 currently makes the most sense for the starting change rate.

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[17] RM-2 is not included because only a negative start change rate is possible with this scenario type. Here, as in Figure 2, a starting rate of change 2020 of +1.5% was assumed.

9 Attachment

9.1 Correction term RM-5

Why does the formula for the rates of change contain the term **0.5**?

$$RR_t = a * \sqrt{t - (BY + 1) - 0.5} + RR_{BY+1}$$

As shown in Figure 5 the interaction of the weighting factor a and the root without the correction term **0.5** in RM-5-rad would result in a relatively large step in the reduction rates from the first year in the budget period to the second year. With the correction term of **0.5**, this curve is "smoothed".
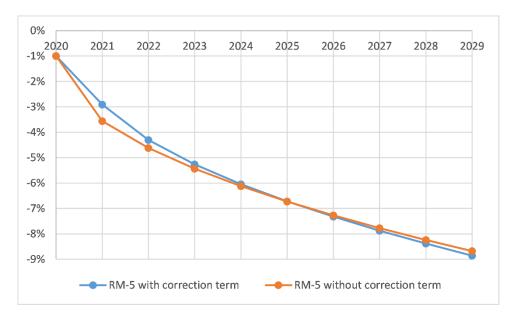

Figure 5: RM-5 correction term

9.2 Further possible scenario types

- Concave: $RR_t = RR_{BY+1} * e^{a*(t-(BY+1))}$
  This variant is almost congruent with RM-2-exp in the area used here.

- Convex: $RR_t = a * \ln(t - BY) + RR_{BY+1}$
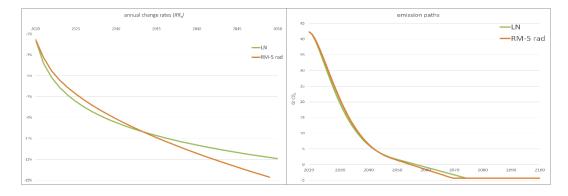 The Figure 6 show the difference to RM-5-rad:

Figure 6: Further scenario type LN

- In addition to monotonous courses, a u-shaped course would also be plausible for the reduction rates. This could be based on the assessment that after "harvesting the low-hanging fruits", the reduction rates will have to fall again. However, the lead time to the reductions, e.g., in the 2040s, can be seen as an opposing effect. This enables - with a credible climate protection policy - corresponding early long-term investments, which should make a continuous increase in the reduction rates possible. Therefor a credible climate protection policy should encourage early long-term investments, which should enable a continuous increase in the reduction rates.

- A function for the emission path can also be specified directly. See, for example, the corresponding chapters in: Resource Sharing Models - A mathematical description; published on zenodo (Wittmann & Wolfsteiner, 2023). In the RM Scenario Types, however, the focus is on the property of the annual changes. The focus on the necessary annual reduction rates makes clearer the challenge and makes it easier to choose a meaningful emission path (cf. Chapter 5).

10 References

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