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One-step deterministic multipartite entanglement purification with linear optics

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ABSTRACT

We present a one-step deterministic multipartite entanglement purification scheme for an N-photon system in a Greenberger–Horne–Zeilinger state with linear optical elements. The parties in quantum communication can in principle obtain a maximally entangled state from each N-photon system with a success probability of 100%. That is, it does not consume the less-entangled photon systems largely, which is far different from other multipartite entanglement purification schemes. This feature maybe make this scheme more feasible in practical applications.

1. Introduction

The distribution of entanglement between distant locations plays an important role in quantum information processing [1], such as quantum teleportation [2], quantum dense coding [3–5] and quantum cryptograph [6–11]. Without entanglement, any quantum computation and long-distance quantum communication would become no more efficient than the classical ones. Bipartite maximally entangled states, such as the Bell diagonal states $|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, are the basic entanglement form. Here $|0\rangle \equiv |H\rangle$ and $|1\rangle \equiv |V\rangle$ represent the horizontal and the vertical polarizations of photons, respectively. They are the two eigenvectors of the basis σ_z. However, a multipartite entangled state exhibits various characters more than a bipartite entangled one. For instance, the tripartite system can be entangled into two kinds of tripartite systems, i.e., a Greenberger–Horne–Zeilinger (GHZ) state or a W state. By local operations and classical communications (LOCC), these two types of entangled states cannot be converted to each other [12]. Now multipartite entangled states also provide the superpower in quantum computation and quantum communication. For instance, the controlled teleportation [13,14], quantum secret sharing [15–17], and quantum state sharing [18–22] all resort to multipartite entanglement.

In order to share a maximally entangled state among some distant locations, the parties in quantum communication should transmit the entangled state over a practical quantum channel such as an optical fiber. Unfortunately, the entangled state will degrade and become a mixed state due to the dissipative effects of the noisy quantum channel. If the fidelity of the entangled state decreases, some quantum communication processes will become insecure. Entanglement purification [23–40] provides us a powerful tool to recover a subset of maximally entangled states from a set of mixed entangled states. By far, most of entanglement purification schemes are focused on bipartite entangled quantum systems, only a little have been studied for multipartite quantum entangled systems because of their complicated structures.

The first multipartite entanglement purification protocol (MEPP) was presented by Murao et al. [36] in 1998. Their protocol is used to purify GHZ states with some controlled-NOT (CNOT) gates. In their protocol, the whole purification protocol is divided into two steps. One is used to purify the bit-flip error and the other is used to purify the phase-flip error. After performing the CNOT operations and measurements, by communicating the results and

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selecting the subensemble of the initial ensemble of pairs, the parties can finally improve the fidelity of the remained mixed entangled systems. This protocol works in a probabilistic way as the parties in quantum communication can only obtain a subset of high-fidelity multipartite entangled states from a large set of less-entangled states probabilistically, as the same as almost all the entanglement purification protocols for two-photon systems [23–30]. This protocol has been extended to high-dimensional multipartite quantum systems by Cheong et al. [40] in 2007. In their protocol, the two-dimensional Hadamard operation is extended to quantum Fourier transformation and the CNOT gates are substituted by the generalized XOR gates in high-dimensional quantum systems. However, the CNOT gate is much difficult in current experiment, and the maximal success probability for achieving a CNOT gate is only 1/4 with only linear elements and single photon sources [41]. Recently, we also proposed a MEPP [37] with cross-Kerr nonlinearities. In this protocol, we use cross-Kerr nonlinearities to construct a quantum nondemolition detector (QND) [42] which has the functions of both parity-check measurements and single-photon detections. With QNDs, we can perform our MEPP repeatedly and get some high-fidelity GHZ states. However, the cross-Kerr nonlinearity is too small in nature, most of works based on cross-Kerr nonlinearity focus on the study in theory [43]. This protocol cannot be realized easily at present. Moreover, the previous works on multipartite entanglement purification [36–38] are essentially used to get a high-fidelity entangled state and none can get a maximally pure entangled state. In 2002, Simon and Pan [27] showed that entanglement purification can be performed between different degrees of freedom of photons. They have used the spatial entanglement to purify the polarization entanglement to correct the bit-flip errors in entangled photon pairs. In 2010, the concept of deterministic entanglement purification was proposed for entangled photon pairs with hyperentanglement [31]. Subsequently, the schemes for deterministic bipartite polarization entanglement purification by using spatial entanglement were proposed [32,33]. In deterministic entanglement purification, the parties in quantum communication can in principle obtain a maximally entangled state from each photon pair transmitted and do not consume the entangled photon systems after they have been transmitted over a noisy channel in theory.

In this Letter, we present a one-step deterministic multipartite entanglement purification protocol (DMEPP) for an N-photon system in a GHZ state with linear optical elements. The parties in quantum communication can obtain the maximally entangled N-photon systems without largely consuming the less-entangled systems but rather only the spatial entanglements by postselection. Compared with the previous MEPPs [36–38], the present DMEPP has some advantages. First, it works in a deterministic way. That is, the parties in quantum communication can obtain a maximally entangled state from each entangled photon system after it has been transmitted over a noisy channel, by performing this protocol only one time with the success probability of 100% in principle, which is far different from the previous MEPPs [36–38] as they can only get some mixed states with a higher fidelity, by consuming a large number of less-entanglement photon systems. Second, it does not require the photon systems to be entangled in the polarization degree of freedom before they are transmitted over noisy channels but spatial entanglements. Moreover, the present DMEPP may be feasible at present as a good mode overlap on the PBSs and the phase stability have been achieved in the previous works [26,27] and the spatial entanglement for an N-photon system can be obtained by converting the polarization entanglement into the spatial degree of freedom with only linear optical elements. This DMEPP may be very useful in long-distance multi-party quantum communication in future.

2. One-step deterministic three-photon entanglement purification

To show how this DMEPP works for quantum systems in a GHZ state explicitly, we first take three-photon GHZ-state systems as an example for describing its principle and then we discuss the case for N-photon GHZ-state systems.

Generally speaking, the three-photon GHZ states for a spin 1/2 system can be described as

$$|\Phi^{\pm}\rangle_{ABC} = \frac{1}{\sqrt{2}}\big(|000\rangle \pm |111\rangle\big)_{ABC}, \tag{1}$$

$$|\Phi_1^{\pm}\rangle_{ABC} = \frac{1}{\sqrt{2}}\big(|100\rangle \pm |011\rangle\big)_{ABC}, \tag{2}$$

$$|\Phi_2^{\pm}\rangle_{ABC} = \frac{1}{\sqrt{2}}\big(|010\rangle \pm |101\rangle\big)_{ABC}, \tag{3}$$

$$|\Phi_3^{\pm}\rangle_{ABC} = \frac{1}{\sqrt{2}}\big(|001\rangle \pm |110\rangle\big)_{ABC}. \tag{4}$$

Here the subscripts A, B, and C denote the three photons transmitted to the three remote parties, say Alice, Bob, and Charlie, respectively.

Let us assume that the initial three-photon GHZ state which will be transmitted to the three parities is $|\Phi^+\rangle_{ABC}$. In order to share the polarization entangled state $|\Phi^+\rangle_{ABC}$, the three parties add the entanglement in another degree of freedom in their transmission of photons. That is, the spatial entanglement of an entangled photon system is added for its transmission, similar to the two-photon polarization entanglement purification using spatial entanglement by Simon and Pan in Ref. [27]. So the initial state of each three-photon system which will be transmitted over a noisy channel can be written as

$$|\phi_{ABC}^{+}\rangle = \frac{1}{2}\big(|000\rangle + |111\rangle\big)_{ABC} \otimes \big(|a_1a_2a_3\rangle + |b_1b_2b_3\rangle\big)_{ABC}. \tag{5}$$

We denote the spatial entangled state (or called it the path entangled state) of a three-photon system as $|\Psi\rangle_s = \frac{1}{\sqrt{2}}(|a_1a_2a_3\rangle + |b_1b_2b_3\rangle)_{ABC}$, similar to the two-photon spatial state in Ref. [27]. Here a_i and b_i ($i = 1, 2, 3$) are the two spatial modes (i.e., the upper path a and the lower path b) for the i-th photon (i.e., A, B, or C), shown in Fig. 1. The state shown in Eq. (5) is given a term as hyperentanglement, which has been used to complete the Bell-state analysis [44–48].

During a practical transmission, the channel noise leads the initial state in the polarization degree of freedom to become a mixed state. For example, the photons traveling from the source to the three parities will suffer from the depolarization of noisy channels, consisting of both bit-flip errors and phase-flip errors. The spatial entanglement can also be affected. Fortunately, the bit-flip error of spatial part does not exist and the phase-flip error can be eliminated by controlling the lengths of the channels exactly. From the view of the outcome of the measurement on the photon systems in the polarization degree of freedom with a product basis, say $\sigma_z^A \otimes \sigma_z^B \otimes \sigma_z^C$, the state of the polarization part after the transmission over noisy channels can be described as

$$\begin{aligned}\rho_p = {} & F_0|000\rangle\langle 000| + F_1|001\rangle\langle 001| + F_2|010\rangle\langle 010| \\ & + F_3|011\rangle\langle 011| + F_4|100\rangle\langle 100| + F_5|101\rangle\langle 101| \\ & + F_6|110\rangle\langle 110| + F_7|111\rangle\langle 111|,\end{aligned} \tag{6}$$

where F_i ($i = 0, 1, 2, \ldots$) are the probabilities of the eight three-photon product states $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ when Alice, Bob, and Charlie measure their photons with the basis σ_z, and $F_0 + F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 = 1$. The

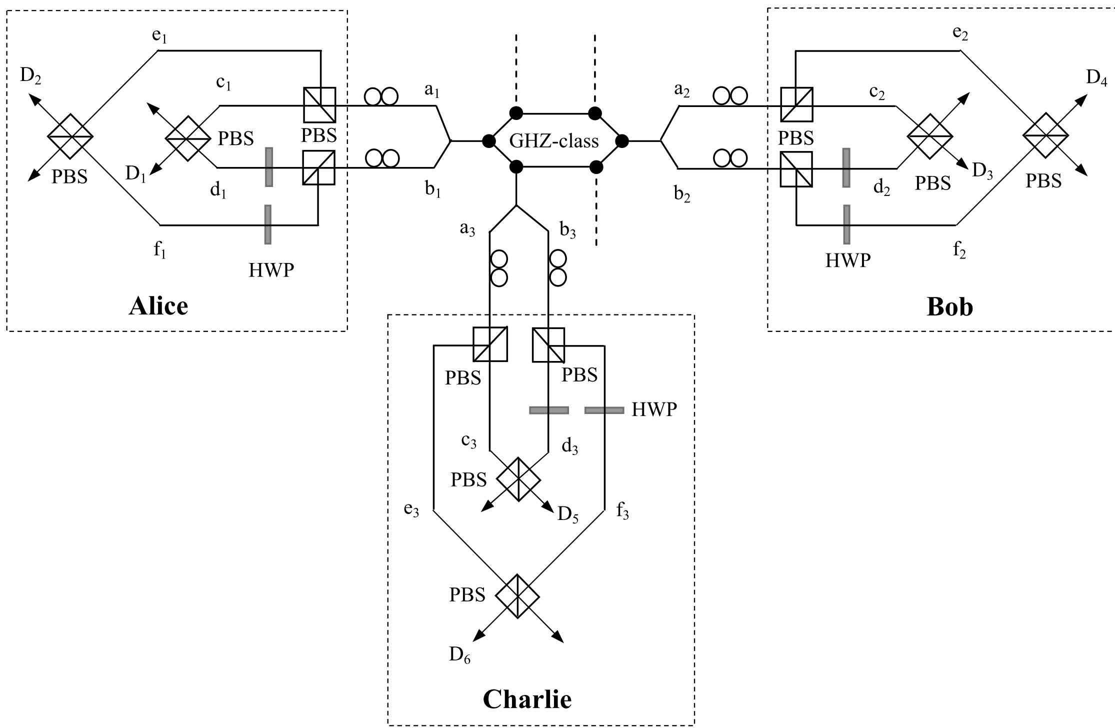

Fig. 1. Schematic illustration for the principle of the present one-step deterministic multipartite entanglement purification. PBS represents a polarizing beam splitter and it is used to transfer $|H\rangle$ polarization photon and reflet $|V\rangle$ polarization photon. HWP represents a half wave plate and it can convert $|H\rangle$ into $|V\rangle$, and $|V\rangle$ into $|H\rangle$. For three-photon entanglement purification, only six detectors are required here. After the detection for each spatial mode, the parties in quantum communication can obtain a maximally entangled polarization state from each photon system in a deterministic way in principle. The present protocol can be extended to a more generalized case for N-photon systems ($N > 3$). Here the dashed lines with black dots represent the similar devices for other parties and the rectangles with dashed lines represent the laboratories controlled by the parties in quantum communication. The circles with real lines represent the optical fibers. D_i ($i = 1, 2, 3, \dots, 2N$) represent the output modes for photons and the parties can obtain the standard N-photon GHZ state $|\Phi^+\rangle_{A\cdots YZ}$ if they add a bit-flip operation σ_x on each of the output modes D_{2k} ($k = 1, 2, \dots, N$).

whole mixed state of each three-photon system after the transmission over noisy channels can be described as

$$\rho = \rho_p \otimes \rho_s, \tag{7}$$

where $\rho_s = |\Psi_s\rangle\langle\Psi_s|$.

Similar to entanglement purification protocols [23–33] for two-photon systems, the mixed state shown in Eq. (7) can be viewed as a probabilistic mixture of eight pure states: the three-photon system is in the state $|000\rangle \otimes |\Psi\rangle_s$ with a probability of F_0, in the state $|001\rangle \otimes |\Psi\rangle_s$ with a probability of F_1, and so on.

The principle of our one-step DMEPP is shown in Fig. 1. We first consider the item $|000\rangle \otimes |\Psi\rangle_s$. After the PBSs and HWPs shown in Fig. 1, it evolves as

$$\begin{aligned}
|000\rangle \otimes |\Psi\rangle_s &= |000\rangle \otimes \frac{1}{\sqrt{2}}\big(|a_1a_2a_3\rangle + |b_1b_2b_3\rangle\big) \\
&= \frac{1}{\sqrt{2}}\big(|0_{a_1}0_{a_2}0_{a_3}\rangle + |0_{b_1}0_{b_2}0_{b_3}\rangle\big) \\
&\xrightarrow{PBSs+HWPs} \frac{1}{\sqrt{2}}\big(|0_{c_1}0_{c_2}0_{c_3}\rangle + |1_{d_1}1_{d_2}1_{d_3}\rangle\big)_{ABC} \\
&= \frac{1}{\sqrt{2}}\big(|000\rangle + |111\rangle\big)_{D_1,D_3,D_5}.
\end{aligned} \tag{8}$$

From Eq. (8), $|000\rangle \otimes |\Psi_s\rangle$ will become the three-photon maximally entangled state $|\Phi^+\rangle_{ABC}$ and the three photon will emit from the output modes D_1, D_3 and D_5, respectively.

If a bit-flip error occurs on the first qubit after the transmission over a noisy channel, the item $|000\rangle$ becomes $|100\rangle$ with the probability of F_4. With the setup shown in Fig. 1, the whole state of the three-photon system evolves as

$$\begin{aligned}
|100\rangle \otimes |\Psi_s\rangle &= |100\rangle \otimes \frac{1}{\sqrt{2}}\big(|a_1a_2a_3\rangle + |b_1b_2b_3\rangle\big) \\
&= \frac{1}{\sqrt{2}}\big(|1_{a_1}0_{a_2}0_{a_3}\rangle + |1_{b_1}0_{b_2}0_{b_3}\rangle\big) \\
&\xrightarrow{PBSs+HWPs} \frac{1}{\sqrt{2}}\big(|1_{e_1}0_{c_2}0_{c_3}\rangle + |0_{f_1}1_{d_2}1_{d_3}\rangle\big)_{ABC} \\
&= \frac{1}{\sqrt{2}}\big(|100\rangle + |011\rangle\big)_{D_2,D_3,D_5}.
\end{aligned} \tag{9}$$

Alice, Bob, and Charlie will obtain the state $|\Phi_1^+\rangle_{ABC}$ and the three photons will emit from the output modes D_2, D_3 and D_5, respectively. Alice needs only perform a bit-flip operation on her qubit to transform $|\Phi_1^+\rangle_{ABC}$ to $|\Phi^+\rangle_{ABC}$. Followed by the same principle, the item $|001\rangle \otimes |\Psi\rangle_s$ will become $\frac{1}{\sqrt{2}}(|001\rangle + |110\rangle)_{D_1,D_3,D_6}$ and the three photons will emit from the output modes D_1, D_3 and D_6, respectively. The other items can also be used to get three-photon maximally entangled states following the same principle. The relation between output modes and the maximally entangled states emitted from PBSs are shown in Table 1. With different output modes, the three parties can get different maximally entangled states. With some suitable unitary operations, the three parties can in principle obtain the standard GHZ state $|\Phi^+\rangle_{ABC}$, shown in Table 1. Here $I^A = |0\rangle\langle 0| + |1\rangle\langle 1|$ and $\sigma_x^A = |1\rangle\langle 0| + |0\rangle\langle 1|$ represent the unit operation and the bit-flip operation on the photon A, respectively. From Table 1, one can see that Alice, Bob and Charlie can obtain the standard GHZ state $|\Phi^+\rangle_{ABC}$ if they perform their bit-flip operations σ_x with HWPs on the output modes D_2, D_4 and D_6, respectively.

Table 1
The relation between the output modes and the purified entangled states.

Initial item	$\|000\rangle$	$\|001\rangle$	$\|010\rangle$	$\|100\rangle$	$\|011\rangle$	$\|101\rangle$	$\|110\rangle$	$\|111\rangle$
Output mode	$D_1D_3D_5$	$D_1D_3D_6$	$D_1D_4D_5$	$D_2D_3D_5$	$D_1D_4D_6$	$D_2D_3D_6$	$D_2D_4D_5$	$D_2D_4D_6$
Final state	$\|\Phi^+\rangle$	$\|\Phi_3^+\rangle$	$\|\Phi_2^+\rangle$	$\|\Phi_1^+\rangle$	$\|\Phi_1^+\rangle$	$\|\Phi_2^+\rangle$	$\|\Phi_3^+\rangle$	$\|\Phi^+\rangle$
Operations	$I^AI^BI^C$	$I^AI^B\sigma_x^C$	$I^A\sigma_x^BI^C$	$\sigma_x^AI^BI^C$	$I^A\sigma_x^B\sigma_x^C$	$\sigma_x^AI^B\sigma_x^C$	$\sigma_x^A\sigma_x^BI^C$	$\sigma_x^A\sigma_x^B\sigma_x^C$

3. One-step deterministic N-photon entanglement purification

It is straightforward to extend the present DMEPP to the case with an N-photon GHZ state. An N-photon hyperentangled GHZ state in both the polarization and the spatial-mode degrees of freedom of photons can be described as

$$|\phi_N^+\rangle = \frac{1}{2}\big(|0\cdots00\rangle + |1\cdots11\rangle\big)_{A\cdots YZ} \otimes \big(|a_1\cdots a_{N-1}a_N\rangle + |b_1\cdots b_{N-1}b_N\rangle\big)_{A\cdots YZ}. \tag{10}$$

Here a_i and b_i ($i = 1, 2, \ldots, N-1, N$) are the two spatial modes for the i-th photon (i.e., $A, \ldots, Y$, or Z), shown in Fig. 1. If the polarization part of each N-photon system suffers from the channel noise after the system is transmitted over a long-distance noisy channel, with the set of normal orthogonal basis $\{|0\cdots00\rangle, |0\cdots01\rangle, |0\cdots10\rangle, \ldots, |1\cdots11\rangle\}$, its state in the polarization degree of freedom can be written as

$$\rho_P^N = f_1|0\cdots00\rangle\langle0\cdots00| + f_2|0\cdots01\rangle\langle0\cdots01| + \cdots + f_{2^N}|1\cdots11\rangle\langle1\cdots11|, \tag{11}$$

where $f_1 + f_2 + \cdots + f_{2^N} = 1$. Similar to the case with three-photon systems, let us assume that the bit-flip error of spatial part in a multipartite entangled photon system does not exist and its phase-flip error can be eliminated by controlling the lengths of the channels exactly. Therefor, after the transmission over optical-fiber channels, the whole state of an N-photon system can be described as

$$\rho^N = \rho_P^N \otimes \rho_S^N. \tag{12}$$

Here we denote the spatial entangled state of an N-photon system as $|\Psi\rangle_s^N = \frac{1}{\sqrt{2}}(|a_1\cdots a_{N-1}a_N\rangle + |b_1\cdots b_{N-1}b_N\rangle)_{A\cdots YZ}$ and $\rho_S^N = |\Psi\rangle_s^N\langle\Psi|$.

The mixed state shown in Eq. (12) can be viewed as a probabilistic mixture of 2^N pure states: the N-photon system is in the state $|0\cdots00\rangle \otimes |\Psi\rangle_s^N$ with a probability of f_1, in the state $|0\cdots01\rangle \otimes |\Psi\rangle_s^N$ with a probability of f_2, and so on.

We first consider the item $|0\cdots00\rangle \otimes |\Psi\rangle_s^N$. After the PBSs and HWPs shown in Fig. 1, the initial item evolves as

$$\begin{aligned}
&|0\cdots00\rangle \otimes |\Psi\rangle_s^N \\
&\quad= |0\cdots00\rangle \otimes \frac{1}{\sqrt{2}}\big(|a_1\cdots a_{N-1}a_N\rangle + |b_1\cdots b_{N-1}b_N\rangle\big) \\
&\quad= \frac{1}{\sqrt{2}}\big(|0_{a_1}\cdots 0_{a_{N-1}}0_{a_N}\rangle + |0_{b_1}\cdots 0_{b_{N-1}}0_{b_N}\rangle\big) \\
&\quad\xrightarrow{\text{PBSs+HWPs}} \frac{1}{\sqrt{2}}\big(|0_{c_1}\cdots 0_{c_{N-1}}0_{c_N}\rangle + |1_{d_1}\cdots 1_{d_{N-1}}1_{d_N}\rangle\big) \\
&\quad= \frac{1}{\sqrt{2}}\big(|0\cdots00\rangle + |1\cdots11\rangle\big)_{D_1,\ldots,D_{2N-3},D_{2N-1}}.
\end{aligned} \tag{13}$$

That is, $|0\cdots00\rangle \otimes |\Psi_s^N\rangle$ will become the N-photon maximally entangled state $|\Phi^+\rangle_{A\cdots YZ} = \frac{1}{\sqrt{2}}(|0\cdots00\rangle + |1\cdots11\rangle)_{A\cdots YZ}$ and the N-photons will emit from the output modes $D_1, \ldots, D_{2N-3}$, and D_{2N-1}, respectively. For other items, we can get the similar outcome. In detail, for the item $|i\cdots jk\rangle \otimes |\Psi\rangle_s^N$ ($i, j, k \in \{0, 1\}$), after the PBSs and HWPs shown in Fig. 1, it evolves as

$$\begin{aligned}
&|i\cdots jk\rangle \otimes |\Psi\rangle_s^N \\
&\quad= |i\cdots jk\rangle \otimes \frac{1}{\sqrt{2}}\big(|a_1\cdots a_{N-1}a_N\rangle + |b_1\cdots b_{N-1}b_N\rangle\big) \\
&\quad= \frac{1}{\sqrt{2}}\big(|i_{a_1}\cdots j_{a_{N-1}}k_{a_N}\rangle + |i_{b_1}\cdots j_{b_{N-1}}k_{b_N}\rangle\big) \\
&\quad\xrightarrow{\text{PBSs+HWPs}} \frac{1}{\sqrt{2}}\big(|i_{c_1}\cdots j_{c_{N-1}}k_{c_N}\rangle \\
&\qquad+ |\bar{i}_{d_1}\cdots \bar{j}_{d_{N-1}}\bar{k}_{d_N}\rangle\big)_{D_{1+i},\ldots D_{2N-3+j},D_{2N-1+k}}.
\end{aligned} \tag{14}$$

Here $\bar{i} = 1 - i$, $\bar{j} = 1 - j$, and $\bar{k} = 1 - k$. That is, the parties can determinate the state of their N-photon system by detecting the outputs of their photons, as the same as the case with a three-photon system. With some unitary operations on the multipartite entangled system, the parties in quantum communication can obtain the standard N-photon GHZ state $|\Phi^+\rangle_{A\cdots YZ}$.

4. Discussion and summary

By far, we have briefly been talking about our DMEPP. It is interesting to compare this protocol with the conventional multipartite entanglement purification protocols (CMEPP), such as the ones in Refs. [36,37]. In Ref. [36], Murao et al. used the setups P_1 and P_2 to purify the phase-flip and bit-flip errors, respectively. As a phase-flip error cannot be purified directly, it should be transformed into a bit-flip error with a Hadamard operation on each qubit in Ref. [36]. In the present one-step DMEPP, there are no phase-flip errors in the mixed state in the polarization degree of freedom and only bit-flip errors exist, as shown in Eq. (11). Moreover, this protocol does not require the N-photon system to be entangled in the polarization degree of freedom initially. That is, the initial input state in Eq. (10) can be $\rho'^N = \rho_P'^N \otimes \rho_S^N$. Here $\rho_P'^N = f_1'|0\cdots00\rangle\langle0\cdots00| + f_2'|0\cdots01\rangle\langle0\cdots01| + \cdots + f_{2^N}'|1\cdots11\rangle\langle1\cdots11|$ means that the initial polarization state of each N-photon system can be an arbitrary mixed state and $f_1' + f_2' + \cdots + f_{2^N}' = 1$. This protocol requires only that the spatial part is the maximally entangled state. After performing the present DMEPP, a maximally entangled state can be obtained from each N-photon system, but the spatial entanglement is consumed. This DMEPP is essentially some a kind of entanglement transfer. That is, the spatial entanglement has been transferred into the polarization entanglement of an N-photon system. This transfer is completed between two different degrees of freedom in the N-photon system itself. In fact, the CMEPPs [36–38] can also be regarded as entanglement transfer. It is the transfer between the same degree of freedom, i.e., polarization entanglement, but different N-photon systems. However, the present DMEPP works in a deterministic way as the parties can obtain a maximally entangled state from each N-photon system in principle and the CMEPPs [36–38] work in a probabilistic way as the parties can only obtain a subset of high-fidelity entangled states from a large set of less-entangled states probabilistically.

In Eq. (5), we initially explain our DMEPP with a hyperentangled state for clarity. In order to share the state $|\Phi^+\rangle_{ABC}$, in a common way, parties should prepare it locally and then send the three photons to different locations. The spatial entanglement added here is used to purify the state $|\Phi^+\rangle_{ABC}$ if it suffers from the noise. In this way, it is similar to that in Ref. [27]. From the above discussion, one can see that this DMEPP is essentially a kind of entanglement transfer between two different degrees of freedom of a photon system, with the success probability of 100%. Once the spatial entanglement is transferred into the polarization entanglement completely, the initial polarization entanglement becomes redundant and can be removed. This is the reason that we do not require the initial state to be the hyperentangled one shown in Eq. (5), and we only need to prepare the spatial entanglement and depress the effect of noise on it when the photon system is transmitted over an optical-fiber channel.

So far, we have fully described our DMEPP. Compared with the previous purification protocols, this present protocol has a success probability of 100% in theory. In Ref. [27], Simon and Pan have also exploited a spatial entanglement to purify a polarization entanglement of entangled photon pairs. However, they have used it to correct the bit-flip error only. The phase-flip error cannot be corrected directly and they have to use the conventional entanglement purification protocol to purify the phase-flip error by sacrificing a large number of less-entangled photon pairs. Meanwhile, we suppose the spatial entanglement do not suffer from the noise in the present DMEPP. In a practical transmission, the relative phase between different remote entangled pairs is sensitive to path length instabilities, which has to be kept constant [49].

In summary, we have presented a one-step deterministic multipartite entanglement purification scheme for an N-photon system in a Greenberger–Horne–Zeilinger state with linear optics. It has some advantages. First, it does not require CNOT gates and QNDs, which makes it more feasible than others. Second, it works in a deterministic way for an N-photon system and the parties in quantum communication can obtain a maximally entangled state from each photon system after it has been transmitted over a noisy channel, by performing this protocol only one time with the success probability of 100%, which is far different from the previous MEPPs [36–38] as they can only get a high-fidelity mixed state probabilistically by consuming a large number of less-entanglement photon systems. Third, it does not require the photon systems to be entangled in the polarization degree of freedom before they are transmitted over noisy channels. Moreover, this protocol may be feasible with current technology as only simple linear optical elements are required and a good mode overlap on the PBSs and the phase stability have been achieved in the previous works [26,27]. This DMEPP may be very useful in long-distance multi-party quantum communication in future.

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Appendix A

At present, it is not easy to prepare the spatial entanglement in Eq. (10) directly. Fortunately, this DMEPP dose not require the N-photon system be entangled in the polarization degree of freedom initially, which means that the initial state of each N-photon system in the polarization part can be a product one before it is transmitted over noisy channels. We can exploit the polarization entanglement to produce the spatial entanglement before the transmission over a noisy channel. We take a three-particle GHZ system as an example for describing the principle. Before transmission, we first prepare the three-photon entangled state $|\Phi^+\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{ABC}$. By using the setup shown in Fig. 2, the whole state evolves as

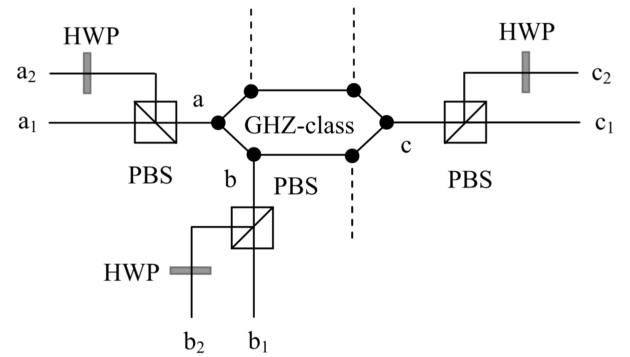

Fig. 2. The principle for the generation of the multipartite spatial entanglement from the polarization entanglement of an N-photon system. The dashed lines with black dots represent the similar devices for other parties in quantum communication.

$$|\Phi^+\rangle_{ABC} \xrightarrow{PBSs+HWPs} \frac{1}{\sqrt{2}}\big(|0_{a1}0_{b1}0_{c1}\rangle + |0_{a2}0_{b2}0_{c2}\rangle\big)$$
$$= \frac{1}{\sqrt{2}}|000\rangle \otimes \big(|a_1b_1c_1\rangle + |a_2b_2c_2\rangle\big). \tag{A.1}$$

Eq. (A.1) is a spatial entangled three-photon GHZ state.

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