Predicting Lotto Numbers

A natural experiment on the gambler's fallacy and the hot hand fallacy

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1. Introduction

Drawing the right inference from observing noisy data is difficult. People tend to see patterns in data even when, in fact, there are none and the data come from independent and identically distributed (i.i.d.) draws. This paper focuses on two — apparently contradictory — fallacies in inference from noisy data that have been found to be pervasive in the literature [Tversky and Kahneman, 1971]. One is the 'gambler's fallacy' (GF) according to which people tend to believe in frequent reversals, i.e. that a particular event is less likely to occur because it occurred recently. In state lotteries, for example, after a number has been drawn, the amount bet on that number falls sharply [Clotfelter and Cook, 1993, Terrell, 1994], and roulette players expect that a black number is 'due' after observing a sequence of red numbers [Croson and Sundali, 2005].[1] The other is the 'hot-hand fallacy' (HHF) according to which people believe in the continuation of a trend, i.e. that a particular event is more likely to occur because a recent streak of such events occurred. A much-cited case in point is that bets on basketball players who scored unusually well in the recent past tend to be too high [Gilovich et al., 1985, Camerer, 1989]. And Guryan and Kearney [2008] provide evidence for a 'lucky store effect', the observation that lotto vendors sell many more tickets if they have sold a ticket that won a large prize a week earlier.[2]

Theoretical models by Rabin [2002] and Rabin and Vayanos [2010] (henceforth RRV) show that the two apparently contradictory fallacies can be reconciled with reference to the 'law of small numbers' [Tversky and Kahneman, 1971].[3] According to

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1. Frequent reversals have also been observed in various laboratory experiments where participants are asked to generate a random sequence as in a coin toss [e.g., Bar-Hillel and Wagenaar, 1991, Rapoport and Budescu, 1997].

2. False hot-hand beliefs also arise in laboratory experiments with an ambiguous data-generating process [Offerman and Sonnemans, 2004] or among participants who let 'experts' bet for them on outcomes of random draws [Huber et al., 2010].

3. In fact, the Marquis de Laplace [1812] had already suggested that both fallacies (he calls them illusions) are likely to occur in lotto gambling: "When a number in the lottery of France has not been drawn for a long time the crowd is eager to cover it with stakes. They judge since the number has not been drawn for a long time that it ought at the next drawing to be drawn in preference to others. So common an error appears to me to rest upon an illusion (...). By an illusion contrary to the preceding ones one seeks in the past drawings of the lottery of France the numbers most often

this 'law', people hold the fallacious belief that small samples should be representative of the population and should therefore 'look like' large samples. RRV show that a person who falls prey to the GF in the short run is also prone to develop the HHF in the long run. We provide supportive evidence for such 'fallacy reversal' using data from a state lottery.

The intuition behind fallacy reversal is as follows. A person who believes that small samples should 'look like' large samples expects frequent reversals in short random sequences, and thus falls prey to the GF in the short run. If such a person is uncertain about the true probability underlying a sequence of events, she starts to doubt about the true probability when observing a long streak because this observation does not correspond to what she believes a random sequence should look like. As a consequence, this person revises her estimate of the true probability, starts to believe in the continuation of the streak, and thus develops the HHF. Uncertainty about the true probability is key for a fallacy reversal to occur according to this theory, and it also seems to be key in practice. Asparouhova et al. [2009] ask participants in a lab experiment to predict the next observation in a random-walk process. The authors show that the HHF becomes more prevalent (compared to the GF) as subjects perceive the data-generating process to be less random.[4]

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drawn, in order to form combinations upon which one thinks to place the stake to advantage. But (...) the past ought to have no influence upon the future." Cited after Truscott and Emory, 1902: 161f.

4. Lab experiments in psychology show that the GF is mostly observed when events are believed to be totally random while the HHF arises when events are perceived to be at least partly driven by a systematic factor, involving, for example, human skill [see Oskarsson et al., 2009, for a review]. Guryan and Kearney [2008] provide an explanation along these lines for their 'lucky store effect'.

The law of small numbers as modeled by RRV not only reconciles gambling behavior and betting in sports markets, but may also provide a unifying framework to account for several anomalies observed in financial markets. One of these anomalies is that asset prices underreact to news in the short run and overreact over long time horizons.[5] In a nutshell, the reasoning is as follows. If many investors are prone to the GF, and believe that short sequences of unexpectedly high earnings will quickly reverse in the future, stock prices will underreact to news about earnings. The same investors, if uncertain about the process behind earnings sequences, attribute long streaks of unexpectedly high earnings to an underlying fundamental and expect such streaks to continue, leading to overreaction of stock prices in the long run (see Rabin and Vayanos, 2010, page 28).[6] Among investors who are confident that earnings are i.i.d., underreaction persists and may even become stronger, the longer the streak.[7]

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De Bondt and Thaler [1985] and Jegadeesh and Titman [1993] are seminal papers documenting long-run overreaction and short-run momentum, respectively. More recent evidence for underreaction to earnings announcements in the short run comes from Cohen and Frazzini [2008], DellaVigna and Pollet [2009], and Hirshleifer et al. [2009]. See Barberis et al. [1998] for an alternative model that reconciles short-run underreaction and long-run overreaction.

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These aggregate patterns may additionally be driven by biases in the processing of private information among investors [e.g., Daniel et al., 1998]. Such biases are unlikely in lotto gambling given that all information about outcomes of lotto draws is public.

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For example, Loh and Warachka [2012] show that stock returns underreact to 'news' (earnings surprises) in the short and in the long run. They suggest that overreaction to streaks of high earnings is absent because investors in their sample are confident that the long-term distribution of earnings surprises is i.i.d..

We study the GF and the HHF among lotto players by inferring players' beliefs about winning numbers from the numbers they choose on their lotto tickets, and by relating these choices to recent histories of lotto draws. Using this lotto data set has a number of advantages over other data sources as is explained next.

Identifying biases as the GF and the HHF and how they may relate using financial or sports market data is fraught with difficulties. A particularly important problem is that the data-generating process in these markets is not known to the researcher but has to be somehow estimated. They do not provide a clean test environment because these data are not only driven by randomness but also by ability, skill, or other systematic factors that are often difficult to measure. For example, long streaks of unexpectedly high earnings in a firm can be due to chance but they can also be driven by a more fundamental cause like superior management, so it is not clear whether investing in stocks with a long history of high earnings should be seen as a hot-hand *fallacy* or as a hot-hand *reality*.[8] In contrast, lotto gambling is an ideal test environment to study these fallacies and how they relate. It provides a natural experiment because lotto draws follow a known, truly random process (with a fixed probability for each number), and draws are thus independent and truly exogenous. In fact, the true randomness of the game is tightly controlled (often by government regulation) and made transparent to players (e.g., by drawing balls from an urn and by airing the draws on TV).

[8] For example, persistence has been observed in the performance of mutual funds [Hendricks et al., 1993, Carhart, 1997] and superior hedge funds [Jagannathan et al., 2010]. And in sports, a belief in a hot hand is not necessarily fallacious but may be 'real'. See Bar-Eli et al. [2006] for an overview and Yaari and David [2012] and Miller and Sanjurjo [2014] for recent studies.

A key novelty of our data is that we can track players' choices (the numbers they bet on) over time. The data come from lotto played over the Internet in Denmark where the law requires gamblers to be uniquely identified. This identification allows us to measure how individual players react with their number choices to recent draws and provides three advantages. First, in contrast to aggregate lottery data [see Clotfelter and Cook, 1993, Terrell, 1994], our data allows us to identify whether changes in the aggregate amount bet on recently drawn numbers are caused by the GF, or by players who stop playing after they have won. Second, our data allows us to test whether players prone to the GF are also prone to the HHF as numbers get 'hotter'. Such a test was not possible with data used in the previous lottery studies because no streaks of 'hot' numbers were observed.[9] Third, the main advantage as compared to Sundali and Croson [2006], who use data from videotaped roulette players, is that players in our data set have a unique ID and can thus be clearly identified. Our dataset contains many more observations, and due to the vast popularity of lotto play, players are arguably less selected than gamblers in casinos.

We find evidence of statistically significant fallacy reversal. Consistent with the GF, players place on average about 2% fewer bets on numbers drawn in the previous week compared to numbers not drawn, as long as those numbers are not 'hot'. Consistent with the HHF, players bet on average more money on numbers, as they get 'hotter', i.e. as they have won more often in the recent past. In particular, players bet about 1% more on numbers drawn in the previous week for each additional week

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Both Clotfelter and Cook (1993), and Terrell (1994) study three-digit numbers (ranging from 000 to 999) which means that the occurrence of (even short) streaks is highly unlikely. For example, the chance of observing a streak of length two is 1/1'000'000.

they have been drawn before. While many players choose the same numbers week after week, we find that these results are driven by frequent players who change their number choices, rather than by players who stop betting after their numbers have won. Our findings, in combination with those of Asparouhova et al. [2009], suggest that the hot-hand logic may also work in contexts that have previously not been associated with it (e.g., coin flips, roulette, or lotto play).

In an individual-level analysis we show that the two fallacies are systematically related as predicted by RRV. We find that the players who exhibit the GF also tend to be prone to the HHF. To illustrate, out of those players who react significantly to the recent drawing history, 55% exhibit both fallacies. This is more than twice the percentage one would expect if players choose numbers randomly (25%).

The data analyzed here has been used in Suetens and Tyran [2012] with a particular focus on the GF and on gender issues. That study shows that male players are prone to the GF while female players are not. The current paper provides a much broader analysis and studies various issues that were not touched upon previously. In particular, the current paper studies both fallacies and how they relate. To the best of our knowledge, this paper is the first to study fallacy reversal in the aggregate and at the individual level using lotto data.

The paper is organized as follows. Section 2 describes the lotto data and defines the main variables of interest. Section 3 analyzes the aggregate reaction of lotto players to the recent drawing history of lotto numbers. Section 4 uses individual-level data to investigate the relation between the two biases. Section 5 concludes.

2. The data

We analyze data from lotto played on Saturdays in Denmark over the Internet (henceforth lotto for short) covering 28 weeks in 2005. Lotto is organized by a state monopoly (Danske Spil). Every Saturday, 7 balls are drawn from an urn containing 36 balls numbered from 1 to 36, which is aired on state TV. Information about the winning numbers is also posted on the website of Danske Spil, that is, the same website where players buy tickets. However, we do not know whether players pay attention to this information.[10] The price of a lotto ticket is about 0.4 Euro (DKK 3).[11]

The payout rate is set to 45% by law and the remainder of the revenues is earmarked for 'good causes' or goes to the general government budget. Lotto has a parimutuel structure as the payout rates are fixed per prize category and the prize money per category is shared among the winners in that category. One quarter of all payouts are reserved for the jackpot (7 correct numbers), and there are four graded prizes for having selected fewer correct numbers. If no-one wins the jackpot, it is rolled over to the next week. In our data set, the average jackpot was about 534'000 Euro (4 million DKK), and the highest jackpot, net of taxes, was 1.4 million Euro (10.2 million DKK).[12] Given that a lotto ticket costs DKK 3, and the chances to win

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There are also a number of other websites, unrelated to Danske Spil, that give information about the history of the number drawings (for example, which numbers are 'due', or which numbers are 'hot'). However, we do not know whether players consult these websites.

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The following paragraph describes the rules of lotto at the time the data was collected. The prize structure has been modified since to yield higher jackpots.

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are about 1:8 million (see footnote 15), there was thus no week where the jackpot was even close to being large enough to make gambling profitable in expectation.

Lotto is normally played in Denmark by purchasing hard-copy lotto tickets in vending booths like drugstores and supermarkets. Since 2002, lotto can also be played over the Internet. Lotto numbers can be picked in various ways in Denmark. Traditionally, players manually select 7 out of 36 numbers on each ticket they buy. However, we analyze numbers picked in 'Systemlotto'. Here, players select between 8 and 31 numbers manually and let the lotto agency choose combinations of 7 out of these numbers.[13] Our data has been provided directly by the state lottery agency and is unlikely to contain any error. All players in our dataset are identified by a unique ID-number which allows the lottery agency to track the choices of players over time.[14] We also received information about the gender and age of (most) players. In total, 189'531 persons have played lotto over the Internet at least once in the second half of 2005. More than half of these (100'386) manually select their numbers in the traditional way, and 25'807 select numbers using Systemlotto.

Prizes above DKK 200 are subject to a special tax of 15% but are otherwise exempt from income tax.

[13] Other ways to play are 'Quicklotto' where all numbers are selected randomly by the lotto agency and 'Lucky-lotto' where players select up to 6 numbers manually and let the lotto agency choose the remaining numbers.

[14] We do not have information about the exact birth dates of the players so that, unfortunately, in our analyses we are not able to test whether players based their number selection on their birth date.

We focus our analysis on Systemlotto rather than the traditional manual selection. The main reason is that in Systemlotto players choose *numbers*, rather than *combinations* of numbers, as in the traditional manual selection. They choose fewer unique numbers than players who select in the traditional way, which suggests that they are more likely to believe that a particular number is going to win. To illustrate, Systemlotto players pick less than half among the 36 available numbers (14 numbers in an average week, 8 in a modal week). In manual selection, players pick most available numbers (29 in an average week, 32 in a modal week). Also, the selection screens look different for traditional manual selection and Systemlotto. In Systemlotto the choice of particular numbers is emphasized because players simply enter between 8 and 31 numbers. With traditional manual selection they enter their choice of combinations of numbers on a grid which may give rise to particular visual patterns like crosses, see [Simon, 1999]. Using Systemlotto thus avoids confound between choosing numbers randomly and non-randomly according to visual patterns versus the GF/HHF. Moreover, in our sample we observe particular *numbers* winning repeatedly but not particular *combinations* of numbers winning repeatedly.[15] Given that a key focus of our study is on how players react to a particular drawing outcome being more or less frequent in the recent past, it is natural to focus on data where we can believe that players choose numbers rather than combinations of numbers.

An interesting question is whether players who think they can predict lotto numbers would want to play Systemlotto at all, or prefer to play the traditional lotto where 7 numbers are picked manually. Under the assumption that players rank the

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While there are only 36 numbers, there are about 8 million ways to combine 7 out of 36 numbers $(36!/(7!(36-7)!) = 8'347'680)$.

numbers in expected likelihood, and prefer that the 7 most likely numbers appear with certainty on their ticket, they would prefer to pick numbers randomly. However, RRV do not make this assumption. They do not specify how players' number expectations translate into betting behavior. For example, a player may believe that $x > 7$ numbers are 'sufficiently' likely as compared to the $36 - x$ other numbers, and therefore bet on combinations of these x numbers using Systemlotto. If instead the player thinks only $x \leq 7$ are sufficiently likely as compared to the $36 - x$ other numbers, he would prefer to pick the numbers by hand and add $7 - x$ other (random) numbers.

An advantage of our data set compared to laboratory data is that it reflects behavior of a heterogeneous pool of people and behavior is observed in a 'natural' situation. In fact, lotto is quite popular in Denmark. For example, according to the lotto agency, about 75% of the adult Danish population have played lotto at least once. Yet, Systemlotto players are clearly not representative for the Danish population or even for the pool of internet lotto players. People playing Systemlotto buy on average about twice as many tickets as other internet players (29 vs. 14 tickets per week; the medians are 19 and 10, respectively). Systemlotto is also especially popular with male players: 82% of the players are male compared to 73% for other selection devices.

2.1. Dependent variables

Our empirical strategy is to make inferences about the (unobservable) belief in one's ability to predict winning lotto numbers more accurately than pure chance from observable reactions to previous drawings. That is, we infer that players believe recently drawn numbers are more likely to win if they systematically prefer them and *vice versa* if they avoid them. A player is said to be more confident that a particular number is going to win if the player is more likely to pick the number, and/or if he or she places more bets on it (i.e. buys more tickets including this number).

We use two proxies for the confidence of a player that a particular number is going to win. The first is called NumberBet and simply indicates whether or not a player has bet in a particular week on a particular number. While NumberBet does not take into account how much money a player bets in total, the second measure for confidence, called MoneyBet does. This measure reflects how the total amount of money a player bets in a particular week is distributed over the lotto numbers. More specifically, the two measures (which serve as our independent variables below) are defined as follows.

$$\text{NumberBet}_{ijt} = \begin{cases} 1 & \text{if} \quad \text{if player } i \text{ picks number } j \text{ in week } t, \\ 0 & \text{if} \quad \text{otherwise} . \end{cases} \tag{1}$$

Our second dependent variable, MoneyBet, measures how much money a player bets on a particular number relative to other numbers. Recall that in Systemlotto, players pick one or more sets containing 8 to 31 numbers. For each chosen set, the lotto agency generates at least 8 tickets with different combinations of subsets of 7 out of the chosen numbers.[16] The relative weight put on a number thus depends on

[16] The exact relation between the total number of tickets/combinations generated by Systemlotto out of a set of chosen numbers and the total number of lotto numbers a player chooses in the set depends on which of three 'systems' players use to generate tickets/combinations. See section A.1 in the Online Appendix for details. We do not have information about the subsets of 7 numbers that eventually end up on the tickets.

how many numbers are chosen in total. Therefore, we define our second dependent variable as the total amount of money bet in t by player i multiplied by the relative weight put on the number by the player in t:

$$\text{MoneyBet}_{ijt} = \text{Total amount bet in DKK}_{it} \times \text{Weight}_{ijt}, \tag{2}$$

$$\text{with Weight}_{ijt} = \frac{\#\text{ of times number } j \text{ is picked in week } t \text{ by player } i}{\sum_j \#\text{ of times number } j \text{ is picked in week } t \text{ by player } i}.$$

The following three examples illustrate the interpretation of the variable MoneyBet.

EXAMPLE 1. *Player A chooses a set of 10 numbers and Player B chooses a set of 24 numbers. Both A and B buy 120 tickets (so each spends DKK 360) generated out of their chosen sets.*

EXAMPLE 2. *Player C chooses a set of 10 numbers from 1 to 10 and a set of 10 numbers from 5 to 14. For each set, 8 tickets are generated. Player C thus buys 16 tickets in total (spends DKK 48).*

EXAMPLE 3. *Player D chooses a set of 12 numbers from 1 to 12 in week 1, from which 12 tickets are generated (spends DKK 36). In week 2 player D chooses the same set of numbers, but now opts for 48 tickets (DKK 144).*

Table 1. **Summary statistics of dependent variables**

	Full sample		**Active players**		**Sample of changers**	
Dep. var.:	NBet	MBet	NBet	MBet	NBet	MBet
Minimum	0	0	0	0	0	0
Maximum	1	1665.21	1	1665.21	1	1665.21
Mean	.15	1.10	.37	2.76	.42	3.54
St. Dev.	.36	4.29	.48	6.45	.49	8.06
Median	0	0	0	0	0	0
75% Percentile	0	0	1	3.00	1	4.80
90% Percentile	1	3.00	1	9.00	1	11.70
95% Percentile	1	6.00	1	12.00	1	13.80
99% Percentile	1	15.23	1	27.60	1	36.00
N	26'013'456		9'148'464		4'419'612	
# Players	25'807		17'318		8'224	

Notes: The table reports summary statistics of the dependent variables as defined in eqs. 1 and 2 for the three samples we use in our analysis. NBet refers to NumberBet and MBet to MoneyBet. In the full sample, data points from players who do not participate in the lottery game in a particular week are coded as zero (for all numbers in that week). In the sample of active players, which is a subset of the full sample, these data points are coded as missing so are not part of the sample. The changers are a subset of the sample of active players. This sample is obtained by excluding players who always bet the same amount on the same numbers across the 28 weeks of study. The unit of observation is a player (i) who bets in a given week (t) on a given lotto number (j).

In example 1, both players buy the same number (120) of tickets. Yet, it is plausible to assume that player A is more confident that (some of) his 10 numbers are going to win than player B who picks 24 numbers. The variable MoneyBet gives each of the 10 numbers chosen by player A a larger weight (of 1/10) than each of the 24 numbers chosen by player B (1/24). In example 2, player C chooses two sets which partly overlap since the numbers 5 to 10 are elements of both sets. It seems plausible to assume that player C is more confident that one of the numbers contained in both sets (5 to 10) is going to win than one of the numbers contained in only one of the sets (1 to 4 and 11 to 14). The variable MoneyBet gives more weight to numbers occurring in overlapping sets than to numbers occurring in only one set. Finally, in example 3, where the amount bet by player D is multiplied by four, the variable MoneyBet is multiplied by four as well.

Table 1 provides summary statistics of the dependent variables for the three samples we use in our analysis (NBet refers to NumberBet and MBet to MoneyBet). The first is what we call the 'full sample' which includes at any time t all (N = 25'807) subjects who play at least once over the entire observation period (28 weeks). The second is what we call the sample of 'active players'. This sample contains at any point in time only the players who are active in the sense that they participate in lotto gambling in this particular period. Technically speaking, the difference between the two samples is that data points from players who do not participate in the lottery in a given period are coded as zeroes in the full sample while they are coded as missing (i.e. are not part of the sample) in the active sample. Thus, the active sample is a subsample of the full sample, and varies in size from period to period. One way to think about the difference between the two samples is that inclusion in the 'full sample' of player i in period t is unconditional on betting in t, while inclusion in the 'active' sample is conditional on betting in this period.

The third sample is a subsample of the sample of active players. It contains only players who change their bets (as defined in eq. 2) at least once over the entire observation period. Technically speaking, this 'sample of changers' is obtained from the sample of active players by excluding players who pick the same numbers week after week. For the samples of active players and of the changers, inclusion of a lagged dependent variable in the regressions (see below) implies that only data points are taken into account where players play in at least two consecutive weeks. This has the additional advantage that these data come from players for whom we can be sure that they visit the Danske Spil website two weeks in a row, and hence presumably know whether one of their chosen numbers has won in the previous week.

To illustrate, in the full sample the mean amount bet on a number by a player per week is DKK 1.10. Since there are 36 lotto numbers in total, this corresponds to a total amount bet by player per week of DKK 39.68 (about 5.30 Euro). The mean amount bet in the active sample is more than twice as large, namely DKK 2.76, corresponding to a total of DKK 99.44 (about 13.25 Euro). Finally, 'changers' bet even more with an average of DKK 3.54 per week per number, so DKK 127 (about 17.00 Euro) in total per week. The table also provides information about the distribution of bets. In all samples, the median NumberBet is 0, meaning that fewer than half of the numbers are selected for each player and week. In about 5% of the cases, a player bets almost one Euro (DKK 6.00) on a given number in a given week for the full sample, and the corresponding numbers are about double that amount in the sample of active players (DKK 12.00) and the sample of changers (DKK 13.80), respectively.

2.2. Main independent variables

Our aim is to study whether players systematically react to drawings in previous weeks and, if so, whether players who fall prey to the GF also develop the HHF. To test for the presence of GF, i.e. whether players bet less on numbers drawn in the previous week, we define the variable Drawn_{jt-1} as follows:

$$\text{Drawn}_{jt-1} = \begin{cases} 0 & \text{if number } j \text{ has not been drawn in week } t-1, \\ 1 & \text{if number } j \text{ has been drawn in week } t-1. \end{cases} \tag{3}$$

To study whether the HHF is present, we test whether players bet more on 'hot' numbers. More specifically, we test whether players bet more on numbers that have been drawn frequently in x weeks preceding week $t-1$. We use this measure rather than a literal 'streak', i.e., the number of consecutive weeks a number has been drawn, because long streaks are rare in lotto by the nature of randomness. In our sample, the maximum length of weeks with consecutive draws of a particular number is 4, and such a streak occurs only once. Clearly, a trade-off is involved in the choice of x: in order to obtain sufficient power, x must not be too small. But to capture *recent* drawing history, x must not be too large. Therefore, we chose $x = 5$.[17] Thus, our second independent variable Hotness_{jt-1} measures how often number j has been drawn in weeks $t-2$ to $t-6$:

$$\text{Hotness}_{jt-1} = k \text{ if number } j \text{ has been drawn } k \text{ times in weeks } t-2 \text{ to } t-6. \quad (4)$$

Figure A2 in the Online Appendix illustrates the properties of the variable Hotness_{jt-1}. The probability that a number is drawn k times in 5 weeks quickly decays as k increases from $k = 1$ onwards. More crucially, given that it is much less likely that a number is drawn in week $t-1$ than that it is not drawn (7/36 vs. 29/36), the hotness of numbers drawn in week $t-1$ has a different interpretation than the hotness of numbers not drawn. For example, the probability that a number is drawn

17. Our results do not hinge on this choice. We get very similar results with $x = 4$ or 6, see Online Appendix tables A1 and A2.

twice in weeks $t-2$ to $t-6$ and drawn in $t-1$ is just 3.8% compared to 15.9% if not drawn in $t-1$. So the 'hotness perception' of a number with hotness equal to 2 will be much stronger if it is drawn in week $t-1$ than if not drawn.

To test whether the lotto drawings are truly random, we ran a number of Chi-square tests that compare the observed and expected distributions of variables. These tests provide reassuring results. For example, we cannot reject the null that the observed distribution of drawings over all 28 weeks is different from the uniform distribution ($\chi^2 = 22.57$; critical value of 53.20 with 35 degrees of freedom and $\alpha = .05$). We also test whether the hotness variable is distributed as expected. Figure A2 in the Online Appendix shows that observed hotness in our sample matches expected hotness closely. Chi-square tests confirm this impression ($\chi^2 = 2.50$ and $\chi^2 = 2.37$ for Hotness given Drawn = 1 and Drawn = 0, respectively; the critical value is 11.14 with 4 degrees of freedom and $\alpha = .05$).

2.3. Control variables

Our regressions control for various factors that plausibly affect betting behavior. First, betting is known to go up in weeks when the jackpot is 'rolled over' from the previous week [e.g., Perez, 2013]. To illustrate, in an average rollover week, the total amount bet in our sample is about 23% higher than in a non-rollover week (DKK 1.16 mio. vs. DKK 0.95 mio., or about 155'100 vs. 126'200 Euro). We control for a rollover effect in our regressions by including a binary variable indicating whether week t is a rollover week. Notice that rollovers can occur in several consecutive weeks, when nobody wins the jackpot. Second, it is well-known that low numbers are generally more popular than high numbers, and this is also the case in our sample. For example, the lowest 5 numbers (1 to 5) are picked more than 30% more often than the highest 5 numbers (32 to 36).[18] We control for such effects by including fixed number effects.

Finally, to control for the fact that some (perhaps idiosyncratically 'lucky') numbers are more popular with particular players than others, we include a lagged dependent variable in the regressions. For example, consider a player who always chooses lotto number 22, say. Suppose number 22 happens to be one of the hot numbers, i.e. has been drawn frequently in the recent past. If we do not correct for the player's idiosyncratic preference, we would wrongly conclude that the player exhibits the HHF. Such cases are qualitatively important given that out of the 17'318 players who have at least two consecutive observations, 9'094 players do not change how they bet on numbers at all over the period of study, and given that those who change, typically keep the majority of numbers the same. In the regressions with NumberBet as the dependent variable, we additionally control for the total number of numbers chosen by a player in week t, and for the total number of tickets bought by a player in week t. These two variables are expected to be positively associated with NumberBet because it is more probable that a player picks a particular number in week t, the more numbers she picks and the larger the number of tickets she buys in t.

3. Aggregate data analysis

This section analyzes how players bet as a function of recent drawings by reporting the results from pooled regressions. We estimate the following regression models

[18] The popularity of the lotto numbers is documented in Figure A1 of the Online Appendix.

separately for the three samples explained in Table 1:

$$\mathrm{DV}_{ijt} = \beta_0 + \beta_1 \mathrm{Drawn}_{jt-1} + \beta_2 \mathrm{Hotness}_{jt-1} + \Gamma \mathrm{Controls}_{ijt-1} + \varepsilon_{ijt}, \text{ and} \tag{5}$$

$$\begin{aligned}\mathrm{DV}_{ijt} = \beta_0 + \beta_1 \mathrm{Drawn}_{jt-1} + \beta_2 \mathrm{Hotness}_{jt-1} + \beta_3 \mathrm{Drawn}_{jt-1} \mathrm{Hotness}_{jt-1} + \\ \Gamma \mathrm{Controls}_{ijt-1} + \varepsilon_{ijt}, \text{ with } i = 1, ..., N; j = 1, ..., 36; t = 1, ..., T_i\end{aligned} \tag{6}$$

The dependent variable (DV) is either $\mathrm{NumberBet}_{ijt}$, as defined in (1) or $\mathrm{MoneyBet}_{ijt}$, as defined in (2). Drawn_{jt-1} is defined as in eq. 3 and $\mathrm{Hotness}_{jt-1}$ as in eq. 4. Note that the only difference between the two specifications is the inclusion of an interaction effect between Drawn and Hotness. This interaction effect is supposed to measure the differential effects of Hotness given that a number was drawn in the previous week or not (cf. the discussion of figure 1).

Figure 1 illustrates the estimated effects of the recent drawing history on our two indicators of betting. The panels in the left column (a) show the effect on NumberBet, i.e. the probability to pick a particular number, and the panels in the right column (b) show the effect on MoneyBet, i.e. how much money a typical player bets on a particular number, relative to other numbers. The figure is based on specification (6) of which results are reported in Table 2.

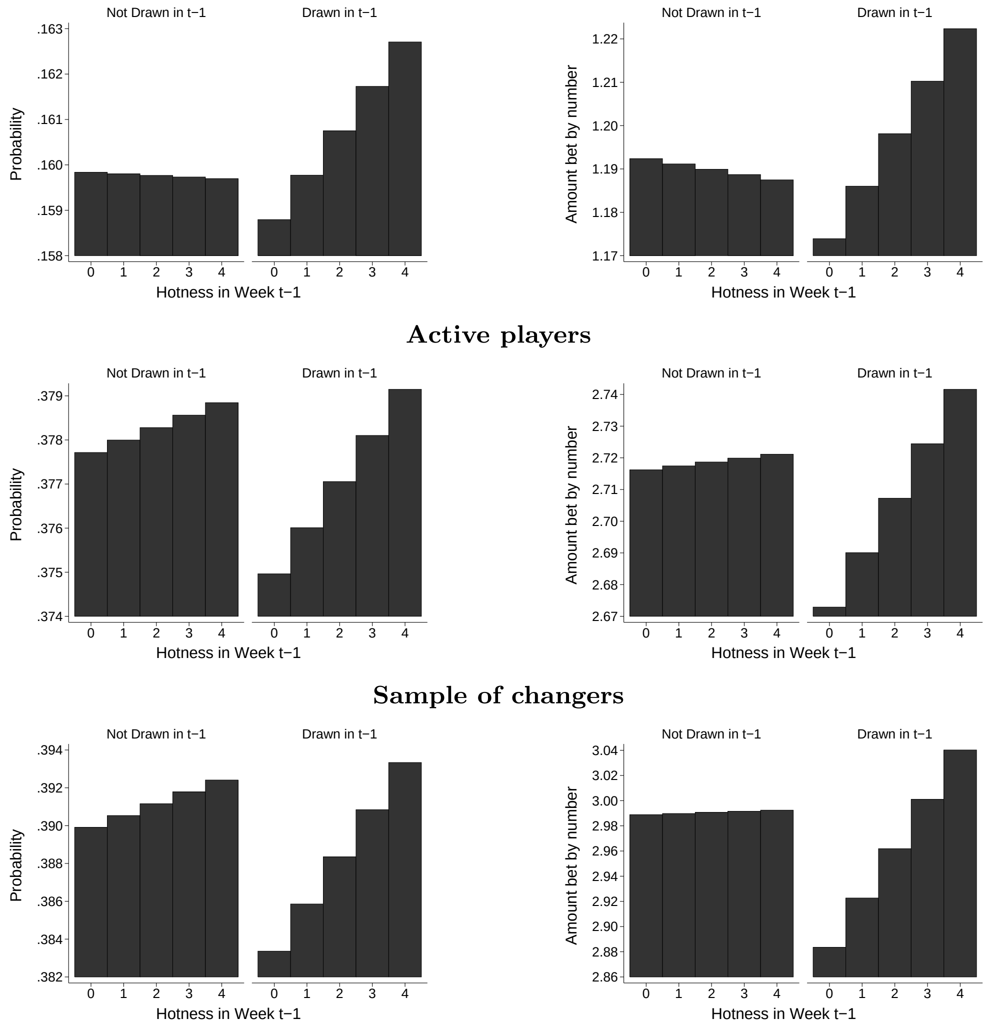
The panels illustrate the GF and the HHF differentiated by whether a number has been drawn in the previous week (right part in each panel) or has not been drawn (left part) as follows. The GF-effect is illustrated by comparing two bars at a given level of Hotness within a panel. For example, comparing the two bars at Hotness = 0 in the

Figure 1. **Estimated betting behavior as a function of the recent drawing history**
(a) NumberBet **(b) MoneyBet**

Full sample

Active players

Sample of changers



Notes: The panels on the left show the estimated probability to pick a particular number in week t depending on whether the number was drawn in week $t-1$ or not, as specified in eq. 6 with NumberBet as dependent variable. The probabilities are calculated at the mean of the independent variables (apart from the fixed number effects). The panels on the right show the estimated amount bet (in DKK) on a number in week t depending on whether the number was drawn in week $t-1$ or not, as specified in equation 6 with MoneyBet as dependent variable. The estimated amounts are calculated at the mean of the independent variables (apart from the fixed number effects). The first row refers to estimations for the full sample, the second and third row to the sample of active players, and the sample of changers, respectively.

upper left panel shows that the probability that a particular number is picked is lower when it has been drawn in the previous week than when it has not been drawn (just below .159 vs. just below .16, i.e. a drop of about 0.6%). The HHF-effect is illustrated by the sequence of bars within a part of a panel. For example, in the right part of the upper left panel, the probability to bet on a particular number increases from just below .159 when the number is not hot at all (Hotness = 0) to just below .163 (equivalent to an increase of 2.5%) when the number is very hot (Hotness = 4). These estimates are based on the full sample and reflect the case where a particular lotto number has been drawn in the previous week. The middle and bottom rows show the corresponding cases for the sample of active players and the sample of changers, respectively. The right column shows the corresponding cases for the MoneyBet.

Inspection of the figure reveals that the GF effects are weakest in the full sample (upper panels) — which includes many instances where players do not bet at all — but are much stronger for the sample of active players and changers. For example, the GF effect for the simple indicator of betting (variable NumberBet) increases from a meager 0.6% in the full sample to a difference of 2% for the sample of changers. The increases for MoneyBet are similar (from about 1.6% in the full sample to 3.3% in the sample of changers). We provide a possible explanation below, when discussing the regression results in detail.

Another result that can be gleaned from the figure is that the effect of Hotness is much more pronounced when a number has been drawn in the previous week vs. when it has not been drawn. This holds for both indicators of betting (compare the steepness of the sequence in the right vs. left half of each panel). This result is not surprising given that the interpretation of Hotness of a number depends on whether the number has been drawn in the previous week. That is, for the same value of our hotness variable, numbers *not* drawn in the previous week are, in fact, 'less hot' than numbers drawn in the previous week. The reason is that the probability that a number is drawn in week $t-1$ and x times in weeks $t-2$ to $t-6$ is much lower than the probability that the number is not drawn in week $t-1$ and drawn x times in weeks $t-2$ to $t-6$ (see Figure A2 in the Online Appendix).

Table 2 shows the estimation results for the three samples for specifications (5) and (6). We find robust evidence for the GF. With one exception to be discussed below, the effect of Drawn is negative across all specifications and samples. This finding shows that the players' tendency to avoid numbers drawn in the previous week is rather robust. We also find robust evidence for the HHF at the aggregate level across all samples. The effect of Hotness in specification (5) is positive, which suggests that players tend to bet more on numbers, the hotter they are. Also, the effect of Drawn x Hotness in specification (6) is always positive, which suggests that players bet more on numbers drawn in the previous week, the hotter they are.

The control variables all have the expected sign: players tend to pick the same numbers in subsequent weeks (see the coefficient on the lagged dep. var.), bets are higher in rollover weeks, and the probability of betting on a certain number is higher the more tickets one buys and the more numbers one chooses.

The evidence in support of the GF is clear and strong since players systematically avoid numbers and bet less money on numbers that have been drawn in the preceding week (i.e. the coefficients on Drawn in 5 are negative). This evidence is particularly strong when the drawn number is not 'hot' (i.e. the coefficients on Drawn in 6 are all negative). There is one exception to this pattern: the coefficient on Drawn is not (or weakly) significant for the full sample (see top panel of table 2, specification 5). Most

Table 2. **Pooled regression results**

Dep. var.:	NumberBet		MoneyBet	
	Eq. 5	Eq. 6	Eq. 5	Eq. 6
	Est. (s.e.)	Est. (s.e.)	Est. (s.e.)	Est. (s.e.)
		(a) Full sample		
Constant	.0103 (.0046)***	.0104 (.0046)***	.4355 (.0348)***	.4367 (.0348)***
Drawn	-.0001 (.0001)	-.0010 (.0002)***	-.0055 (.0029)*	-.0185 (.0039)***
Hotness	.0001 (.0000)***	-.0000 (.0000)	.0011 (.0006)**	-.0012 (.0007)*
Drawn x Hotness		.0010 (.0001)***		.0133 (.0020)***
Rollover	.0117 (.0010)***	.0117 (.0010)***	.2455 (.0077)***	.2458 (.0077)***
# Tickets	.0059 (.0002)***	.0059 (.0002)***		
# Numbers	.0006 (.0002)***	.0006 (.0002)***		
Lagged dep. var.	.6838 (.0148)***	.6838 (.0148)***	.6090 (.0266)***	.6090 (.0266)***
R^2	.639	.639	.368	.368
N		25'084'404		
# Players		25'807		
		(b) Active players		
Constant	.0177 (.0040)***	.0614 (.0084)***	.4326 (.0362)***	.4341 (.0362)***
Drawn	-.0020 (.0003)***	-.0028 (.0004)***	-.0279 (.0071)***	-.0433 (.0092)***
Hotness	.0004 (.0001)***	.0003 (.0001)***	.0040 (.0013)***	.0012 (.0014)
Drawn x Hotness		.0008 (.0002)***		.0159 (.0038)***
Rollover	.0048 (.0003)***	.0048 (.0003)***	.1350 (.0111)***	.1353 (.0111)***
# Tickets	.0001 (.0000)***	.0001 (.0000)***		
# Numbers	.0011 (.0004)***	.0011 (.0004)***		
Lagged dep. var.	.9141 (.0026)***	.9141 (.0026)***	.8497 (.0120)***	.8497 (.0120)***
R^2	.855	.855	.668	.668
N		8'525'016		
# Players		17'318		
		(c) Sample of changers		
Constant	.0612 (.0084)***	.0776 (.0098)***	.8477 (.0706)***	.8510 (.0706)***
Drawn	-.0047 (.0007)***	-.0065 (.0009)***	-.0684 (.0178)***	-.1054 (.0231)***
Hotness	.0010 (.0002)***	.0006 (.0002)***	.0076 (.0032)***	.0009 (.0036)
Drawn x Hotness		.0019 (.0005)***		.0383 (.0093)***
Rollover	.0115 (.0009)***	.0115 (.0009)***	.3499 (.0278)***	.3507 (.0278)***
# Tickets	.0002 (.0000)***	0002 (.0000)**		
# Numbers	.0015 (.0007)***	.0015 (.0007)***		
Lagged dep. var.	.7974 (.0061)***	.7974 (.0061)***	.7636 (.0184)***	.7636 (.0184)***
R^2	.672	.672	.518	.518
N		3'380'508		
# Players		8'224		

Notes: The table reports estimations of the regressions specified in eqs. 5 and 6 based on the full sample (panel (a)), the sample of active players (panel (b)), and the sample of changers (panel (c)). Estimated coefficients for the fixed number effects (jointly statistically significant) are not reported. Standard errors are robust to within-player dependency. The stars ***, ** or * indicate that the effect of the variable is statistically significant at the 1%, 5% or 10% level, respectively.

likely, the reason is that in the full sample data points are included that indicate non-participation rather than not betting on a particular number *given participation*. Indeed, a value of zero for the dependent variable in the full sample may come from either variation at the intensive margin (how players place their bets) or from variation at the extensive margin (whether players bet at all). Thus, there is some scope for confound in the full sample — the variation at the extensive margin may have canceled out the variation at the intensive margin — which is absent in the other samples. Overall, we conclude that the evidence for the GF on the basis of behavioral responses in betting (variation at the intensive margin) is clear and strong.

We now turn to the results on the HHF. Table 2 shows that the positive effect of Hotness in specification (5) is significant in all cases. Results based on specification (6) show that the positive effect of Hotness is almost entirely driven by Drawn x Hotness, meaning that players primarily tend to move to hot numbers drawn in the previous week. This tendency is much less pronounced (or even absent) if a previously hot number has not been drawn in the previous week.

The finding that effects are much smaller in the full sample than in the other two samples shows that the GF and the HHF are mainly driven by variation at the intensive margin (how betting changes in response to drawings in previous weeks) rather than variation at the extensive margin (players entering or exiting the lottery, or changing their total bets).[19] In fact, regressions where the samples are split into frequent and infrequent players (playing in more than half vs. less than half of the weeks), or into relatively 'heavy' and 'light' players (betting more money vs. less money than the median player), show that the effects are strongest for frequent, heavy players (see Tables A3 to A6 in the Online Appendix).[20] Moreover, the GF

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This finding is confirmed in an estimation where the decision to play is regressed on Drawn and Hotness: these variables have no significant effect on the decision to play.

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Table 3. **The effect of arbitrage**

	(1)	(2)
Dep. var.: MoneyBet	Est. (s.e.)	Est. (s.e.)
Constant	.4352 (.0348)***	.4368 (.0348)***
Drawn	-.0049 (.0028)*	-.0209 (.0040)***
Hotness	.0012 (.0007)*	-.0018 (.0008)**
Rollover	.2460 (.0008)***	.2443 (.0078)***
Drawn x Rollover	-.0020 (.0032)	.0073 (.0052)
Hotness x Rollover	-.0001 (.0012)	.0017 (.0014)
Drawn x Hotness		.0160 (.0021)***
Drawn x Hotness x Rollover		-.0088 (.0044)**
Lagged dep. var.	.6090 (.0266)***	.6090 (.0266)***
R^2	.368	.368
N	25'084'404	
# Players	25'807	

Notes: The table reports estimations for the full sample of the regressions specified in eqs. 5 and 6, including interactions of Drawn and Hotness (and its interaction in specification (2)) with the rollover dummy. The dependent variable is MoneyBet. Estimated coefficients for the fixed number effects (jointly statistically significant) are not reported. Standard errors are robust to within-player dependency. The stars ***, ** or * indicate that the effect of the variable is statistically significant at the 1%, 5% or 10% level, respectively.

results are driven by male players [see Tables A7 and A8 in the Online Appendix, or Suetens and Tyran, 2012].

Our results are robust to alternative specifications of the Hotness variable (see Tables A1 and A2 in the Online Appendix), to the exclusion of 'lucky' numbers that are typically below 10 (see Table A9), and to switching to the sample of player who pick their number in the traditional way (see Table A10).[21]

Table 3 addresses the question of whether GF and HHF patterns are different in weeks where the main prize is rolled-over from the previous weeks. Because demand

As shown in Tables A4 and A6, infrequent or light players in the full sample tend to pick numbers drawn in the previous week rather than avoid them (positive effect of Drawn in specification (5)), in particular, numbers that are hot as well (positive effect of interaction term in specification (6)).

21. We also find that results are qualitatively similar when all potential birthdays are excluded (i.e. numbers for 1 to 31). It is not clear whether the same effects should be expected *a priori*, though, given that this exercise excludes 86% of the data points.

for lotto tickets does normally not increase in proportion to the increase in prize money, the expected payoff is higher in rollover weeks than in non-rollover weeks [see Perez, 2013]. It could be that this stimulates 'arbitrageurs' with a taste for gambling to enter, and that the behavior of these new entrants cancels out the patterns observed in their absence. For example, arbitrageurs may bet on unpopular numbers or choose numbers randomly, without taking into account the recent drawing history. We test for this possibility by running regressions for the full sample including interactions between the recent drawing history and the rollover variable, and using MoneyBet as a dependent variable (because NumberBet does not measure the amount bet).

Table 3 shows that while the interaction of Drawn and Hotness is positive in non-rollover weeks, it is significantly negative in rollover weeks (see coefficient on Drawn x Hotness x Rollover in specification (2) in the table). This may be due to arbitrageurs entering the lottery, and avoiding hot numbers that were drawn in the previous week. In specification (1) we do not find an interaction between the drawing history variables and rollover.[22]

To summarize, our results for the aggregate level indicate that there is a systematic relation between betting behavior and the recent drawing history. On average, players bet less on numbers drawn in the previous week than on numbers not drawn, as long as these numbers are not hot (between 1.6 and 3.8% less, depending on the sample under consideration). Players also bet more on drawn numbers, the hotter they are (marginal effect between 0.9 and 1.4% The effects are driven by male, frequent, heavy players, and the aggregate effects of biases tend to be mitigated in rollover weeks.

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As one would expect, such an effect is not observed for the sample of active players, where entry and exit of arbitrageurs cannot shape effects by definition.

4. Individual-level analyses

This section studies whether the two fallacies simply coexist in the aggregate because some players are prone to the GF and others to the HHF, or whether instead, the same players who are prone to the GF are also the ones who tend to be prone to the HHF if a number is sufficiently 'hot' (as predicted by RRV). Our empirical approach in this section involves estimating regressions for each individual. We focus on the variable MoneyBet as defined by eq. 2 since this is the richest proxy for a player's betting behavior. Given that we do not have enough data points at the player level to correct for fixed number effects, we focus on the change in MoneyBet. For each player, we thus estimate how much he changes his bet on a particular number as a function of the recent drawing history. The GF is operationalized as a decrease in bets on numbers drawn in the previous week compared to numbers not drawn. We define HHF-players as those who bet more on numbers drawn in the previous week, the hotter they are. Also, we focus on the active players, since the participation choice of players does not depend on the recent drawing history. In particular, we estimate the following regression for each player i who bets in week t:

$$\Delta\text{MoneyBet}_{ijt} = \beta_{0j} + \beta_{1i}\text{Drawn}_{jt-1} + \beta_{2i}\text{Hotness}_{jt-1} + \beta_{3i}\text{Rollover}_{t-1} + \varepsilon_{ijt}, \tag{7}$$

$$\text{with } i = 1, ..., N; j = 1, ..., 36; t = 1, ..., T_i.$$

According to the RRV-hypothesis, for players prone to the GF in the 'short' run and to the HHF in the 'long' run, the effect of Drawn is negative, and the effect of Hotness is positive in eq.(7). We control for rollover weeks by including a rollover dummy. We summarize the results from the regressions in two ways: (1) by plotting the estimated effects of Hotness as a function of those of Drawn, and (2) by classifying players according to the sign of the estimated effect of Drawn and Hotness.

Figure 2 plots the normalized individual-level Hotness estimates as a function of the normalized individual-level Drawn estimates.[23] The figure includes a regression line that allows for a structural break at the point where the Drawn estimate is zero. The structural break is included in order to allow for a different relation between the Hotness and Drawn estimates, depending on whether the Drawn estimate is negative or not.[24] The figure shows that, overall, there is a negative relation between the Hotness and Drawn estimates. This relation is consistent with the hypothesized behavioral pattern: players who decrease bets on numbers drawn in week $t-1$ also tend to increase bets on hot numbers. Importantly, the relation between the Hotness and Drawn estimates is significantly stronger — the regression line in Figure 2 is steeper — for the range where the Drawn estimates are negative. The tendency that players decrease bets on numbers drawn in week $t-1$ as well as increase bets on hot numbers is thus stronger than the tendency that players increase bets on numbers

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The high magnitudes of some of the estimates are mostly driven by players for whom the dependent variable has very little variation. For example, for a player who changes his number choices only once in the period of study, and for whom this change happens to coincide with a move away from a number drawn in the previous week, the estimated effects of Drawn will be absurdly large. We preferred to include all data points in the plot rather than using an arbitrary cut-off and omit data.

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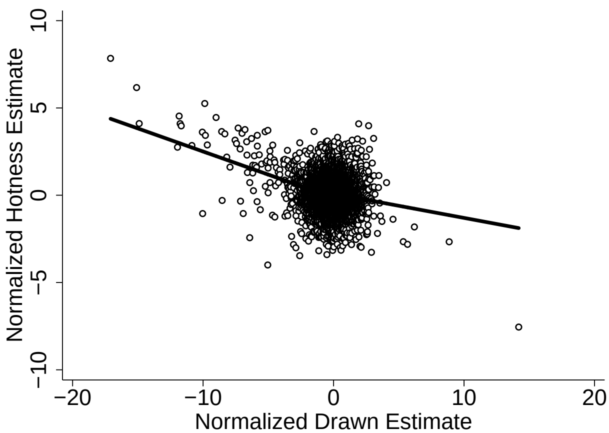
Because none of the estimates is exactly equal to 0, the structural break in the regression line does not appear at exactly 0.

Figure 2. **Hotness estimates as a function of Drawn estimates**



Notes: The figure shows a scatter plot of the normalized estimates of Hotness in eq. 7 as a function of the normalized estimates of Drawn from the same regression equation. The plot is based on 7'323 observations (covering the same number of players). The black line is a regression line that allows for a structural break at the point where the estimate of Drawn is zero. It shows the predicted normalized Hotness estimates in the following regression: $\hat{\beta}_{2i} = \gamma_0 + \gamma_1 z\hat{\beta}_{1i} + \gamma_2 D_i + \gamma_3 D_i z\hat{\beta}_{1i}$ where $z\hat{\beta}_{1i}$ and $z\hat{\beta}_{2i}$ are the normalized individual-level Drawn and Hotness estimates from eq. 7, respectively, and $D_i = 1$ if $\hat{\beta}_{1i} > 0$. We used robust regression (command rreg in Stata). The regression results are $\hat{\gamma}_0 = -.135$, $\hat{\gamma}_1 = -.280$, $\hat{\gamma}_2 = .184$, and $\hat{\gamma}_3 = .161$ ($p < .001$ in all cases).

drawn in week $t-1$ as well as decrease bets on hot numbers. We take this as evidence supportive of the theoretical prediction that players who are prone to the GF also tend to be prone to the HHF.

Table 4 provides a classification of players based on the sign of the estimated effect of Drawn and Hotness. Panel (a) uses many but relatively noisy observations, while panel (b) uses few but highly informative ones. More specifically, panel (a) includes results using all players for whom eq. 7 can be estimated, and panel (b) uses only those players who are significantly biased at the 10% level.[25]

[25] While the absolute number of players for whom *both* effects (Drawn and Hotness in eq. 7) are significant at the 10% level is rather low (129), it is much higher than the number that would have been observed had all players chosen numbers randomly. Indeed, 129 is almost twice the number

TABLE 4. **Classification of players**

(A) **All players**

	Effect of Hotness			
Effect of Drawn	Negative	Positive	Total	Fisher p-value
Negative	1687 (23.04%)	2005 (27.38%)	3692 (50.42%)	$< .001$
Positive	1911 (26.10%)	1720 (23.49%)	3631 (49.58%)	.026
Total	3598 (49.13%)	3725 (50.87%)	7323	.294
Fisher p-value	.009	.001	.614	

(B) **Significantly biased players**

	Effect of Hotness			
Effect of Drawn	Negative	Positive	Total	Fisher p-value
Negative	17	71	88	$< .001$
Positive	21	20	41	.913
Total	38	91	129	.001
Fisher p-value	.646	$< .001$	.004	

Notes: The table reports numbers of players where $\hat{\beta}_{1i} < 0$ and $\hat{\beta}_{1i} > 0$ and $\hat{\beta}_{2i} < 0$ and $\hat{\beta}_{2i} > 0$ in eq. 7. The p-values come from Fisher exact tests that compare the observed distributions with the uniform distribution. Panel (a) includes results from all players and panel (b) from players for whom both coefficients are significantly different from zero at the 10% level.

The row variable in Table 4 refers to the effect of Drawn and the column variable to the effect of Hotness. The first row in panel (a) of Table 4 shows that 3'692 players out of 7'323 (50.4%) decrease their bets on numbers drawn in the previous week. Out of the 7'323 players, 3'725 players (50.9%) increase their bets on numbers drawn in the previous week, the more frequently they have been drawn in the previous six weeks. Overall, 2'005 players both decrease bets on numbers drawn in the previous week and increase bets on numbers the hotter they are (27.4% compared to 25% with random picking).

Panel (b) of the table shows how GF and HHF relate if we consider players whose estimated reactions are statistically significant at the 10% level.[26] The classification

that would have been observed had all players chosen numbers randomly (10% times 10% of 7'323 is about 73).

[26] The qualitative nature of this classification is the same if we use the 1% or 5% level instead.

follows the same logic as in panel (a), but yields a much sharper picture. We now find that 68.2% (88 out of 129) decrease their bets on last week's winners and that 70.5% (91 out of 129) of players increase their bets on numbers on 'hot' numbers. As a result, the 'overlap' between both fallacies is 55% (71 out of 129), which is more than twice the percentage when all players would randomize (25%). Thus, the majority of the significantly biased players are prone to both fallacies.

5. Discussion

Given that lotto drawings are truly random, it seems absurd to believe that anyone can predict next week's numbers. Yet, our data suggests that the lotto players studied here on average hold such beliefs and, curiously enough, the lotto agency itself describes the lotto as: 'a number game which is about predicting the correct numbers drawn' (translated from danskespil.dk, see 'rules of the game'). In particular, in line with recent economic theory, we find that players bet less on numbers drawn in the previous week (consistent with the gambler's fallacy), and bet more on these numbers, the 'hotter' they are (consistent with the hot-hand fallacy).

Below, we first discuss how our findings compare to results from related laboratory and lotto studies. We then briefly discuss to what extent our results from lotto may extrapolate to financial markets, and conclude with some caveats and suggestions for further research.

Our aggregate-level results on the gambler's fallacy are qualitatively in line with the findings in Clotfelter and Cook [1993] and Terrell [1994], but the quantitative reaction observed in our data set is about one order of magnitude smaller. These earlier studies find that bets drop sharply, by 36% and 18%, respectively, after a number was drawn. Of course, one needs to be careful with comparing quantitative reactions across studies because the nature of the lotteries is rather different. For example, these earlier studies concern three-digit number games with a high (1:1'000) chance of winning a low prize while we study the Danish state lotto with graded prizes and a low (about 1:8 mio.) chance to win a large jackpot. A difference between the two studies using three-digit games is that prizes are fixed in Clotfelter and Cook (1993) while they are shared among winners in Terell (1994). Since our data also comes from pari-mutuel betting (which implies that picking popular numbers is costly) the lower estimate by Terell, if any, is more pertinent. Another difference is that there is only an indirect relation between winning numbers and the amount of money won in our data: a player does not necessarily win a prize if he picks a number that happens to be drawn. He only does so if this number is part of a winning *combination*. In contrast, there is a one-to-one relation between winning numbers and winning prizes in three-digit lotteries. This direct relation may make the previous week's winning numbers more salient to the players. This salience is likely to have contributed to the strong reactions observed in the three-digit lotto studies which cannot discriminate between two possible responses to numbers drawn. The observed reaction in these studies is composed both of players who remain in the game and switch to a different number (consistent with the GF) and of players who quit the game if the number they picked wins. This participation effect may be particularly strong in the three-digit lotto but cannot be cleanly pinned down because these studies cannot, in contrast to our study, track individual players over time.

Our results with respect to fallacy reversal are both qualitatively and quantitatively in line with the laboratory evidence in Asparouhova et al. [2009]. Subjects try to predict the next observation in a random-walk process in this experiment. Again, results should be compared with care as the studies differ in many ways, including the key differences that our data come from a natural field experiment with many observations, and have a low probability of winning large prizes. Yet, the effects we observe are of the same magnitude and in the same range as theirs. For example, they find that players reduce their probability estimate of continuation by 1.8-2.6% after 'short' streaks, and increase it by 1.75% for each unit increase in streak length for 'long' streaks.[27] Our estimates are very similar to theirs. We find that, depending on the sample under consideration, players bet 1.6-3% less on numbers drawn in the previous week, and increase their bets by 0.9-1.4% for each unit increase in 'hotness'.[28]

The evidence we provide for the existence of the GF and the HHF is potentially relevant for a number of anomalies that seem to be common in financial decision-making and in financial markets. For example, it may explain why investors have a willingness to pay for useless 'expert' predictions of investment performance [Powdthavee and Riyanto, 2015], and if sufficiently prevalent, it may also explain

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Asparouhova et al. [2009] report that players reduce their probability estimate of continuation by 0.9% for each unit increase in streak length for short streaks. To make this number comparable, one needs to take the difference between the lotteries into account (in theirs, the probability of winning is 50%, meaning that streaks and the occurrence of hot numbers are very likely). We thus calculated the effect for streak lengths that would occur with a probability close to 7/36 (which is the probability a number is drawn in our lottery). In the 50% lottery, 7/36 lies between the probability of observing a streak of 2 (which occurs with probability .25) and a streak of 3 (which occurs with probability .125).

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The results reported by Croson and Sundali [2005] and Sundali and Croson [2006] do not allow for a direct comparison of marginal effects.

why stock prices underreact to news, particularly if driven by small investors [e.g., Hvidkjaer, 2006].

We do find evidence of heterogeneous effects, suggesting that biases are more pronounced among frequent players, heavy players, and male players. While playing the lotto is common and widespread (about 75 % of the Danish adult population have played lotto at least once), it seems plausible that biased people are more likely to select into lotto gambling: people who believe that they can predict the lotto are more likely to play the lotto, and (all else equal) to bet more money. Analogously, it seems also plausible that people who think they can predict whether stocks go up or down are more likely to select into active trading in stock markets. From this perspective, it seems not entirely implausible that the biases observed in lotto may manifest themselves at least in some financial markets. Indeed, Kumar [2009] finds that people who play lotto also seem to be likely to invest in lotto-type stocks, such as low-priced stocks with high idiosyncratic volatility and skewness.

The extent to which biases observed in lotto gambling extrapolate to financial markets clearly also depends on possibilities to engage in arbitrage — rational investors may compensate the behavior of irrational ones such that biases have no effect on the aggregate [see, for example, Fehr and Tyran, 2005, for a discussion]. Although the scope for arbitrage seems rather limited in lotto gambling [Papachristou and Karamanis, 1998], we find evidence suggestive of 'arbitrage' in the sense that players entering the lotto in weeks with large jackpots tend to avoid hot numbers. As a result, some of the biases we observe in typical weeks are less pronounced in weeks with large jackpots.

Our lotto data has some advantages compared to a conventional laboratory study. For example, lotto gamblers come from all walks of life and their choices are not artificial in any way. However, because of its naturalness, our setting is also less controlled than a laboratory setting. This lack of control prevents us from addressing various interesting questions. For example, we do not know to which extent players have consulted information about past drawings. Also, the data at hand do not allow us to identify the relation between the gambler's and hot hand fallacies and socio-economic variables. We hope that future research will be able to address these issues.

Finally, we would like to notice that a fully-fleshed out model of the gambler's and hot hand fallacies would predict why people play the lottery, how much they bet, why they use a particular selection format, or why they switch between different selection formats. Developing such model is beyond the scope of this paper and is also left for future research.

References

Elena Asparouhova, Michael Hertzel, and Michael Lemmon. Inference from streaks in random outcomes: Experimental evidence on beliefs in regime shifting and the law of small numbers. *Management Science*, 55:1766–1782, 2009.

Micheal Bar-Eli, Simcha Avugos, and Markus Raab. Twenty years of "hot hand" research: Review and critique. *Psychology of Sport and Exercise*, 7:525–553, 2006.

Maya Bar-Hillel and Willem Wagenaar. The perception of randomness. *Advances in Applied Mathematics*, 12:428–454, 1991.

Nickolas Barberis, Andrei Shleifer, and Robert Vishny. A model of investor sentiment. *Journal of Financial Economics*, 49:307–343, 1998.

Colin Camerer. Does the basketball market believe in the hot hand? *American Economic Review*, 79:1257–1261, 1989.

Mark Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52: 57–82, 1997.

Charles Clotfelter and Philip Cook. The "gambler's fallacy" in lottery play. *Management Science*, 39:1521–1525, 1993.

Lauren Cohen and Andrea Frazzini. Economic links and predictable returns. *Journal of Finance*, 63:1977–2011, 2008.

Rachel Croson and James Sundali. The gambler's fallacy and the hot hand: Empirical data from casinos. *Journal of Risk and Uncertainty*, 30:195–209, 2005.

Kent Daniel, Daniel Hirshleifer, and Avanidhar Subrahmanyam. Investor psychology and security market under- and overreactions. *Journal of Finance*, 53:1839–1885, 1998.

Werner De Bondt and Richard Thaler. Does the stock market overreact? *Journal of Finance*, 40:793–805, 1985.

Stefano DellaVigna and Joshua Pollet. Investor inattention and Friday earnings announcements. *Journal of Finance*, 64:709–749, 2009.

Ernst Fehr and Jean-Robert Tyran. Individual irrationality and aggregate outcomes. *Journal of Economic Perspectives*, 19:43–66, 2005.

Thomas Gilovich, Robert Vallone, and Amos Tversky. The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology*, 17:295–314, 1985.

Johnathan Guryan and Mellisa Kearney. Gambling at lucky stores: Empirical evidence from state lottery sales. *American Economic Review*, 98:458–473, 2008.

Darryll Hendricks, Jayendu Patel, and Richard Zeckhauser. Hot hands in mutual funds: Short-run persistence of performance. *Journal of Finance*, 48:93–130, 1993.

David Hirshleifer, Sonya Lim, and Siew Teoh. Drive to distraction: Extraneous events and underreaction to earnings news. *Journal of Finance*, 64:2289–2326, 2009.

Jurgen Huber, Michael Kirchler, and Thomas Stöckl. The hot hand belief and the gambler's fallacy in investment decisions under risk. *Theory and Decision*, 68: 445–462, 2010.

Soeren Hvidkjaer. A trade-based analysis of momentum. *Review of Financial Studies*, 19:457–491, 2006.

Ravi Jagannathan, Alexey Malakhov, and Dimitry Novikov. Do hot hands exist among hedge fund managers? An empirical evaluation. *Journal of Finance*, 65: 215–255, 2010.

Narashimhan Jegadeesh and Sheridan Titman. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance*, 48:65–91, 1993.

Alok Kumar. Who gambles in the stock market? *Journal of Finance*, 64:1889–1933, 2009.

Pierre S. Marquis de Laplace. *A philosophical essay on probabilities.* Wiley and Sons, New York, Translated by FW Truscott and FL Emory from *Essai philosophique sur les probabilités, 1902*, 1812.

Roger Loh and Mitch Warachka. Streaks in earnings surprises and the cross-section of stock returns. *Management Science*, 58:1305–1321, 2012.

Joshua Miller and Adam Sanjurjo. A cold shower for the hot-hand fallacy. Technical report, IGIER Working Paper No 518, 2014.

Theo Offerman and Joep Sonnemans. What's causing overreaction? An experimental investigation of recency and the hot-hand effect. *Scandinavian Journal of Economics*, 106:533–553, 2004.

Aan Oskarsson, Leaf van Boven, GaryH McClelland, and Reid Hastie. What's next? Judging sequences of binary events. *Psychological Bulletin*, 135:262–285, 2009.

George Papachristou and Dimitri Karamanis. Investigating efficiency in betting markets: Evidence from the Greek 6/49 lotto. *Journal of Banking & Finance*, 22:1597–1615, 1998.

Levi Perez. The 'Who and Why' of lottery: Empirical highlights from the seminal economic literature. *Journal of Economic Surveys*, 27:915–940, 2013.

Nattavudh Powdthavee and Yohanes Riyanto. Would you pay for transparently useless advice? A test of boundary beliefs in the folly of predictions. *Review of Economics and Statistics*, forthcoming, 2015.

Matthew Rabin. Inference by believers in the law of small numbers. *Quarterly Journal of Economics*, 117:775–816, 2002.

Matthew Rabin and Dimitri Vayanos. The gambler's and hot-hand fallacies: Theory and applications. *Review of Economic Studies*, 77:730–778, 2010.

Amon Rapoport and DavidV Budescu. Randomization in individual choice behavior. *Psychological Review*, 104:603–617, 1997.

Johnathan Simon. An analysis of the distribution of combinations chosen by UK national lottery players. *Journal of Risk and Uncertainty*, 17:243–276, 1999.

Sigrid Suetens and Jean-Robert Tyran. The gambler's fallacy and gender. *Journal of Economic Behavior & Organization*, 83:118–124, 2012.

James Sundali and Rachel Croson. Biases in casino betting: The hot hand and the gambler's fallacy. *Judgment and Decision Making*, 1:1–12, 2006.

Dek Terrell. A test of the gambler's fallacy: Evidence from pari-mutuel games. *Journal of Risk and Uncertainty*, 8:309–317, 1994.

Amos Tversky and Daniel Kahneman. Belief in the law of small numbers. *Psychological Bulletin*, 76:105–110, 1971.

Gur Yaari and Gil David. "Hot hand" on strike: Bowling data indicates correlation to recent past results, not causality. *PLoS ONE*, 7:e30112, 2012. URL http://dxdoiorg/101371%2Fjournalpone0030112.