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A note on explicit approximations to Colebrook's friction factor in rough pipes under highly turbulent cases

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ABSTRACT

New explicit approximations to the implicitly given Colebrook equation for flow friction factor are given. They are with improved accuracy compared with the one recently published [Shaikh et al.: Int. J. Heat Mass Tran. 88 (2015) 538–543]. The new approximations are highly accurate only in rough pipes under fully developed turbulent cases of flow.

1. Introduction

The Colebrook equation [1] is implicit in flow friction factor, λ (1):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.71 \cdot D}\right) \quad (1)$$

where:

λ-Darcy, i.e. Moody friction factor (dimensionless),
Re-Reynolds number (dimensionless),
ε/D-relative roughness of inner pipe surface (dimensionless).

The reason why to use an approximate formula [2,3], instead of the original Colebrook equation, is to avoid iterative procedure. In the approximations, the parameter λ is expressed explicitly, i.e. it is only on the left side of the "=" sign in equation.

The similar but more accurate approximations will be shown in addition to the recently published approximation by Shaikh et al. [4].[1] The presented approximations in this paper are recommended to be used when highly turbulent flow occurs in rough pipes [6–10].

2. Proposed approximation by Shaikh et al. [4]

In their recent paper, Shaikh et al. [4] published a new approximation to the Colebrook equation. Shaikh et al. [4] claim that their approximation (2) is valid for highly turbulent cases of flow in rough pipes where it gives very good results.

$$\lambda = 0.25\left[\log_{10}\left(\frac{2.51}{Re \cdot \left(1.14 - 2 \cdot \log_{10}\left(\frac{\varepsilon}{D}\right)\right)^{\alpha}} + \frac{\varepsilon}{3.71 \cdot D}\right)\right]^{-2} \quad (2)$$

In their case [4] in (2), $\alpha = -2$. Following the same accuracy check from Brkić [2,11], it is found that the error of the approximation (2) where $\alpha = -2$ will be the lowest for the highly turbulent zone under very rough conditions as claim by Shaikh et al. [4]. The relative error δ in this zone, i.e. where $10^{-2} < \varepsilon/D < 0.05$ and $10^6 < Re < 10^8$ is about 0.66% (the same as in [4]) as it can be seen from Fig. 1.

On the other hand, this approximation (2) where $\alpha = -2$, will produce maximal relative error about $\delta = 335.64\%$ regarding the whole domain of applicability of the Colebrook equation, i.e. for $10^{-6} < \varepsilon/D < 0.05$ and for $10^4 < Re < 10^8$ (the domain as reported in Brkić [2]), and $\delta = 186.12\%$ for $10^{-4} < \varepsilon/D < 0.05$ and for $10^4 < Re < 10^8$ (the domain as reported in Shaikh et al. [4]); which is the same value of the error in mesh of 740 check points used in Brkić [2], as reported in Shaikh et al. [4]; $\delta = 186.1135\%$ with their mesh of 1000×1000 check points.

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[1] It should be noted that the source of the Brkić approximation is not [2] as reported in [4], but [5].

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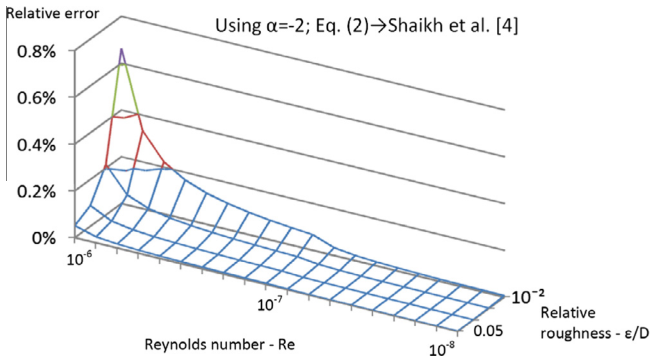

Fig. 1. Distribution of the relative error for the approximation by Shaikh et al. [4]; Eq. (2) when $\alpha = -2$.

3. New more accurate approximation

Eq. (2) can be transformed into (3).

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{2.51}{Re \cdot \beta} + \frac{\varepsilon}{3.71 \cdot D}\right) \tag{3}$$

Comparing the original Colebrook equation (1) [1] and the approximation (3) it is clear that $\beta = \sqrt{\lambda}$. Following algorithm from Brkić [3] and Shacham [12] for the iterative solution of the implicitly given Colebrook equation (1) [1], for the first iteration so called smooth term has to be set as $\frac{2.51}{Re \cdot \sqrt{\lambda}} = 0$; which means $\beta \to \infty$. This means that the Colebrook equation in the first iteration will contain only rough part $\frac{\varepsilon}{3.71 \cdot D}$, i.e. it will be as in (4) [13].

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{\varepsilon}{3.71 \cdot D}\right) = 1.14 - 2 \cdot \log_{10}\left(\frac{\varepsilon}{D}\right) \tag{4}$$

The new approximation produced in that way, using the second iteration, will be (3) where $\beta = \sqrt{\lambda}$ is set by (4); which means that $\alpha = -1$ in (2).

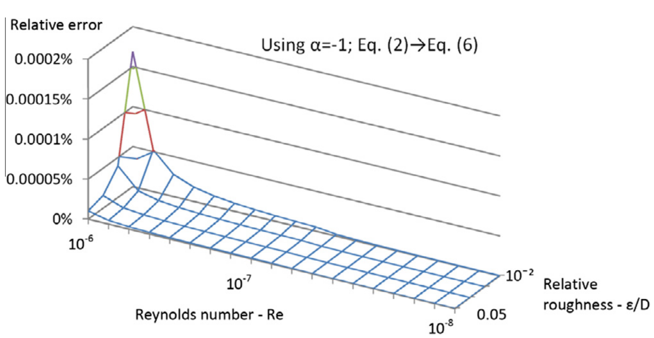

Fig. 2. Distribution of the relative error for the approximation (2) when $\alpha = -1$.

Using $\alpha = -1$ in (2), the relative error δ will be 0.0001692% (1.692×10^{-4}%) in the high turbulent zone of rough pipes, i.e. in the zone where $10^{-2} < \varepsilon/D < 0.05$ and $10^6 < Re < 10^8$, as it is shown in Fig. 2 (the error δ = 0.0001692% when $\alpha = -1$ will be lower about 3900 times compared with the one produced by the approximation by Shaikh et al. [1], δ = 0.66% when $\alpha = -2$). On the other hand, the maximal relative error δ will be; δ = 31.43% regarding the whole domain of applicability of the Colebrook equation, i.e. for $10^{-6} < \varepsilon/D < 0.05$ and for $10^4 < Re < 10^8$ (the domain as in Brkić [2]), and δ = 16.07% for $10^{-4} < \varepsilon/D < 0.05$ and for $10^4 < Re < 10^8$ (the domain as in Shaikh et al. [4]).

4. More accurate simple approximation

Having in mind simplicity [2,3,5,8,9,13–15], if $\beta \to \infty$ in (3), i.e. when $\alpha = \infty$ in (2), the relative error δ will be 0.16% in the high turbulent zone of rough pipes, i.e. in the zone where $10^{-2} < \varepsilon/D < 0.05$ and $10^6 < Re < 10^8$. This will produce (5):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{\varepsilon}{3.71 \cdot D}\right) \tag{5}$$

5. Conclusions

After conducting thorough analysis of the parameter α from (2), presented in Table 1, the following approximation can be recommended (6) for use in the high turbulent zone of rough pipes where it will not introduce the relative error more than 0.0001692%. It is equivalent to (2) when $\alpha = -1$.

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{2.51 \cdot \left(1.14 - 2 \cdot \log_{10}\left(\frac{\varepsilon}{D}\right)\right)}{Re} + \frac{\varepsilon}{3.71 \cdot D}\right) \tag{6}$$

Also having in mind simplicity, (5) can be used in the high turbulent zone of rough pipes where it will not introduce the relative error more than 0.16%. This simple approximation is equivalent to (2) when $\alpha = \infty$.

For $\alpha = -0.75$, the error of (7) in the whole domain of applicability of the Colebrook equation will be the lowest among all checked cases from Table 1, δ = 6.46%, while in the high turbulent zone of rough pipes will be also acceptable as it is around 0.054%.

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{2.51 \cdot \left(1.14 - 2 \cdot \log_{10}\left(\frac{\varepsilon}{D}\right)\right)^{0.75}}{Re} + \frac{\varepsilon}{3.71 \cdot D}\right) \tag{7}$$

6. Disclaimer

The views expressed are purely those of the writer and may not in any circumstance be regarded as stating an official position of the European Commission.

Table 1
Analysis of the influence of parameter α from (2) on accuracy of the approximations; the maximal relative error; δ.

Mesh	Re	ε/D	$\alpha = -1$ (%)	$\alpha = -2$ (%)	$\alpha = \infty$ (%)	$\alpha = 1$ (%)	$\alpha = 2$ (%)	$\alpha = 0.5$ (%)	$\alpha = -0.5$ (%)	$\alpha = -0.75$ (%)
I	10^4–10^8	10^{-6}–0.05	31.43	335.64	81.24	63.56	75.63	53.18	16.85	6.46
II	10^4–10^8	10^{-4}–0.05	16.08	186.12	61.42	55.70	60.63	48.60	16.43	2.27
III	10^5–10^8	10^{-3}–0.05	0.62	42.85	11.47	11.19	11.43	10.73	6.48	3.64
IV	10^6–10^8	10^{-2}–0.05	0.000169	0.664	0.161	0.155	0.160	0.147	0.09	0.054

I-the domain as reported in Brkić [2].
II and III-the domains as reported in Shaikh et al. [4].
IV-the domain from Shaikh et al. [4]; highly turbulent zone under very rough conditions.
$\alpha = -1 \to$ (6).
$\alpha = -2 \to$ Shaikh et al. [4].
$\alpha = \infty \to$ (5); simple form.
$\alpha = -0.75 \to$ (7); balanced.

Conflict of interest

None declared.

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