

# ANALYSIS AND RESYNTHESIS OF THE HANDPAN SOUND

**Eyal Alon**

AudioLab, Department of Electronics,  
University of York, UK  
ea553@york.ac.uk

**Dr. Damian T. Murphy**

AudioLab, Department of Electronics,  
University of York, UK  
damian.murphy@york.ac.uk

## ABSTRACT

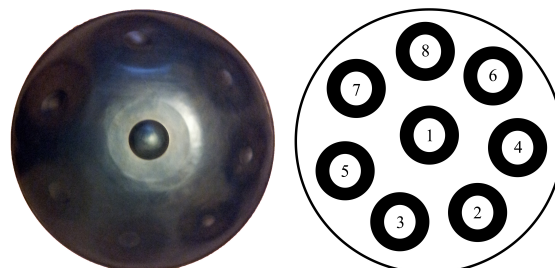
Handpan is a term used to describe a group of struck metallic musical instruments, which are similar in shape and sound to the Hang<sup>1</sup> (developed by PANArt in 2000). The handpan is a hand played instrument, which consists of two hemispherical steel shells that are fastened together along the circumference. The instrument usually contains a minimum of eight notes and is played by delivering rapid and gentle strikes to the note areas. An experimental procedure has been designed and implemented to record, analyse, and resynthesise the handpan sound. Four instruments from three different makers were used for the analysis, giving insight into common handpan sound features, and the origin of signature amplitude modulation characteristics of the handpan. Subjective listening tests were conducted aiming to estimate the minimum number of signature partials required to sufficiently resynthesise the handpan sound.

## 1. INTRODUCTION

Handpan is a term used to describe a group of struck metallic percussion instruments, which are similar in shape and sound to the Hang<sup>1</sup> (developed by PANArt Ltd. in January 2000 [3]). The handpan is a hand played instrument, which consists of two hemispherical steel shells that are fastened together along the circumference. The instrument is played by delivering rapid and gentle finger strikes to individual notes. Similar to steel pan notes, the frequencies produced from the Hang's principal modes of vibration in each note area have a 1:2:3 ratio [4]. An additional frequency component found in the spectrum of the Hang at approximately 85 Hz is associated with the cavity (Helmholtz) resonance frequency.

In October 2014, there were approximately 80 handpan makers worldwide [5]. Some notable makers are Pantheon Steel [6], Zen Handpans [7], CFoulke [8], and Saraz Handpans [9]. As seen in Figure 1, the handpan typically consists of eight or more notes. Amongst makers and players, the notes are commonly known as “note-fields” due to the fact that strikes delivered to different areas of the

<sup>1</sup> Hang® is a registered trademark and should not be used to describe other musical instruments such as handpans [1], nor should the term handpan be used to refer to the Hang® [2].



**Figure 1.** Top view of a handpan with eight note-fields.

note-field will emphasize specific harmonics, resulting in a different timbre. An objective standard for classification of handpan quality does not yet exist, however some discussion of this amongst makers and enthusiasts has occurred [10]. Furthermore, no standard exists for handpan making or tuning so each maker creates instruments with different materials, tools, dimensions, and shell and note-field architectures.

This paper presents the design and implementation of an experimental procedure to measure, analyse, and resynthesise the signature handpan sound. Results from a listening test conducted in order to assess the quality of the resynthesised signals go some way towards determining the minimum number of partials required to sufficiently resynthesise the handpan sound.

## 2. MEASUREMENT AND ANALYSIS

A handpan frame was constructed from extruded aluminium rods and was designed to support the handpan, excitation mechanism, and microphone securely within the anechoic chamber when making measurements. This required the frame to be strong enough to provide support whilst at the same time minimise the influence of the frame itself on the measurements. It was noted from previous research on the Hang that varying the spacing of the player's knees can influence the tuning of the Helmholtz resonance frequency by effectively changing the acoustical “length” of the neck [11]. Investigation of the effects of the handpan cavity on the overall sound is beyond the scope of this paper and should be considered for future work. The size of the frame was adjusted to provide support as close to the rim of the handpan as possible, as well as to ensure no obstructions between the microphone and each of the note-fields. The Note-Field Excitation Mechanism (NFEM) was formed of a torsional spring (2.7mm wire diameter, 30mm

body length) fixed at one end and attached to a rounded rubber tip at the other. The NFEM was used by pulling the rubber tipped end of the spring back to a fixed position and then releasing to generate a strike to an individual note-field. This method of excitation was preferred over sinusoidal excitation or finger force as it allowed excitation of various positions with repeatable strikes that are similar in nature to finger strikes.

A previous study of the steel pan used sandbags to minimize radiation from surrounding notes [12]. Some steel pan makers use magnets to achieve a similar effect as the cross-talk between notes can interfere with the tuning process [13]. In order to determine the signature sound of an isolated handpan note-field, and to estimate the contribution of surrounding notes to the overall handpan sound, each note-field sound was measured in two configurations:

- Damped: Magnetic absorbing pads were placed to cover all note-fields other than the one currently being recorded, in order to suppress their vibration and contribution to the recorded signal.
- Undamped: No magnetic absorbing pads were used to dampen surrounding note-fields.

The measurement procedure for an individual handpan note-field was implemented in the following sequence:

1. Securely place the handpan inside the frame.
2. Position the microphone and NFEM appropriately.
3. Adjust recording levels to avoid clipping.
4. Deliver a strike to the note-field allowing the sound to decay to an inaudible level.
5. Place magnetic absorbing pads on all surrounding note-fields prior to delivering an additional strike.

Eight strikes were delivered to each note-field in each configuration and were allowed to decay for ten seconds prior to the following strike. Once the handpan and microphone were positioned securely, they were not moved until all notes of the handpan were recorded. Table 1 provides a key of measurements taken for all four investigated instruments. All note-fields were measured in both undamped and damped configurations.

## 2.1 Identification of Signature Partial

In the context of this paper, a signature partial is defined as one of a number of highest magnitude detected peaks in the spectrum of the handpan sound. The Energy Decay Relief (EDR) method is useful for smoothing transient features and amplitude modulations present in a signal [14,15], thus allowing an easier identification of the handpan’s signature partial frequencies and corresponding decay rates. For each recorded handpan note, a single EDR analysis frame was used to extract the frequency values of signature partials. This frame was chosen as the first to follow the transient onset of the recorded note (approximately 4-10 ms), in order to avoid erroneous frequency selection due to the broadband nature of the note onset.

Inst. 1	Inst. 2	Inst. 3	Inst. 4
A <sub>3</sub>	Ab <sub>3</sub>	A# <sub>3</sub>	A <sub>4</sub>
B <sub>2</sub>	B <sub>3</sub>	A <sub>3</sub>	B <sub>4</sub>
B <sub>3</sub>	C <sub>3</sub>	A <sub>4</sub>	C <sub>4</sub>
C# <sub>4</sub>	C <sub>4</sub>	D <sub>3</sub>	C <sub>5</sub>
D <sub>4</sub>	D <sub>4</sub>	D <sub>4</sub>	D <sub>4</sub>
E <sub>4</sub>	Eb <sub>4</sub>	E <sub>4</sub>	E <sub>4</sub>
F# <sub>3</sub>	G <sub>3</sub>	F <sub>4</sub>	F <sub>4</sub>
F# <sub>4</sub>	G <sub>4</sub>	G <sub>4</sub>	G <sub>3</sub>
-	-	-	G <sub>4</sub>

**Table 1.** A key of note measurements taken for all four instruments investigated in this project.

To improve the accuracy of the identified frequency values associated with each peak, a parabolic interpolation method was used [16]. Table 2 shows three detected partials (in order of descending magnitude from left to right) for all eight note-fields of Instrument 3, and their corresponding frequency value ratios. For seven out of eight note-fields, the three highest magnitude partials detected have an approximate 1:2:3 frequency value ratio. The third highest magnitude partial of the undamped D<sub>3</sub> note-field is approximately 4 times the value (i.e. the double octave) of the fundamental frequency, which produces an approximate 1:2:4 frequency ratio. The presence of this partial seems to suggest a strong coupling between the D<sub>3</sub> and D<sub>4</sub> note-fields. This emphasises the D<sub>5</sub> frequency (c. 592 Hz), which is the octave partial of the D<sub>4</sub> note-field. This suggestion is strengthened by examining the three highest magnitude partials detected from the damped D<sub>3</sub> signal: 152.2 Hz, 298.1 Hz, and 449.2 Hz which have a frequency ratio of approximately 1:2:3.

## 2.2 Decay Rate Estimation

Upon detection of the signature partials it is desirable to estimate their corresponding decay rates. Musical instrument decay times have previously been estimated in dB/sec [17] or by calculating quality factors [18, 19]. Estimation of modal decay rates can also be achieved by calculating T60 values [20] using EDR plots [15]. In the context of this paper, the PD<sub>60</sub> is defined as the amount of time it takes for a partial to decay by 60 dB from its initial magnitude value. The highest magnitude partial for an individual note-field measurement was used to determine the -60 dB threshold. To implement this, MATLAB’s `polyfit` function was used to calculate the coefficients of a 2<sup>nd</sup> degree polynomial that best fits a section of the corresponding decay curve (using a least-squares method [21]). In order to select the appropriate section for calculation of the polynomial, the gradient of the selected frequency bin over time was calculated. Where this gradient approaches zero represents the point where the decay curve reaches the noise floor, and this can be seen as a suitable end point for best-fit calculation.

Table 3 displays the mean PD<sub>60</sub> decay times, standard deviations, and minimum and maximum values of the three highest magnitude partials for individual instruments, note

Note-field	Partial 1, freq. ratio	Partial 2, freq. ratio	Partial 3, freq. ratio
A# <sub>3</sub>	234.8 Hz 1	697.7 Hz 2.97	463.3 Hz 1.97
A <sub>3</sub>	226.3 Hz 1	444.6 Hz 1.96	664.3 Hz 2.94
A <sub>4</sub>	444.1 Hz 1	884.3 Hz 1.99	1325 Hz 2.98
D <sub>3</sub>	152.1 Hz 1	297.7 Hz 1.96	592.3 Hz 3.89
D <sub>4</sub>	591.8 Hz 1.99	297.1 Hz 1	884.1 Hz 2.98
E <sub>4</sub>	334.6 Hz 1	664.1 Hz 1.98	993.4 Hz 2.97
F <sub>4</sub>	700.6 Hz 1.99	352 Hz 1	1053 Hz 2.99
G <sub>4</sub>	395.5 Hz 1	788.3 Hz 1.99	1175 Hz 2.97

**Table 2.** Three signature partials and corresponding frequency ratios (relative to the fundamental frequency), of all eight undamped note-fields of Instrument 3. Partial are sorted in order of descending magnitude from left to right.

groups, and all instruments. The three note groups are: low (B<sub>2</sub>-B<sub>3</sub>), mid (C<sub>4</sub>-E<sub>4</sub>), and high (F<sub>4</sub>-C<sub>5</sub>). Generally, the mean PD<sub>60</sub> values decrease for higher register note groups. Despite this, the longest measured PD<sub>60</sub> value (5.9 s) is from the mid note group. Instrument 1 and Instrument 2 have very similar results for all parameters, possibly due to the fact that they are both from the same handpan maker. Instrument 3 and Instrument 4 have relatively short average PD<sub>60</sub> values, which could be due to the fact that they contain more higher register notes compared to Instrument 1 and Instrument 2.

Instrument/ note group	Mean PD <sub>60</sub> (s)	Standard deviation	Min (s)	Max (s)
Instrument 1	3.3	1.1	1.7	5.9
Instrument 2	3.3	1.2	1.6	5.9
Instrument 3	2.8	0.7	1.4	4.0
Instrument 4	2.1	0.5	0.9	3.4
Low (B <sub>2</sub> -B <sub>3</sub> )	3.2	0.9	1.6	5.1
Mid (C <sub>4</sub> -E <sub>4</sub> )	3.0	1.2	1.2	5.9
High (F <sub>4</sub> -C <sub>5</sub> )	2.5	0.9	0.9	4.2
All instruments	2.9	1.0	0.9	5.9

**Table 3.** Mean PD<sub>60</sub> decay times, standard deviations, and minimum and maximum values of the three highest magnitude partials for individual instruments, note groups, and all instruments.

### 2.3 Amplitude Modulations

Several partials in many of the measured handpan signals exhibit amplitude modulations. In order to calculate the

rate of modulation, an algorithm was developed that finds the local minima in the spectrogram of a given partial and calculates the mean number of samples between the minima. The rate of modulation is then estimated by calculating the inverse of the mean number of samples. Table 4 displays the estimated amplitude modulation rates for several partials from Instrument 1.

Note-field	Frequency bin (Hz)	AM rate (Hz)
A <sub>3</sub>	662	3.3
A <sub>3</sub>	438	8.6
B <sub>3</sub>	248	3.9
F# <sub>3</sub>	373	3.1
F# <sub>4</sub>	374	3.3

**Table 4.** Estimated amplitude modulation rates for several partials from Instrument 1.

### 2.4 Undamped and Damped Measurements

Upon excitation of the handpan’s note-field, surrounding note-fields are also excited. This phenomenon is known as “sympathetic vibration” and has been previously investigated in other musical instruments such as the harp [22]. Comparing spectrograms of signals produced in the undamped and damped configurations provided insight regarding the origin of the handpan’s signature amplitude modulation characteristics, and the contribution of surrounding note-fields to the overall handpan sound.

The significant reduction in amplitude modulation depth on partials in the damped signals, when compared to their corresponding undamped signals strengthens the following hypothesis: The signature amplitude modulation characteristics in the handpan sound are due to a slight mismatch in tuning of signature partials on separate note-fields. High-resolution methods such as ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), can be used to identify the frequency values of these closely spaced partials [22] and this will be a subject of future work.

## 3. RESYNTHESIS

The time domain waveform of the handpan sound can be thought of as having two stages: attack and release. The attack is associated with a transient, broadband onset whilst the release is associated with a mostly sinusoidal steady state decay.

### 3.1 Steady State

The signature partials and corresponding decay rates identified in Section 2 were used to resynthesise the steady state stage of the handpan sound. The frequencies of the partials detected were used to set the oscillators for resynthesis, whilst the PD<sub>60</sub> decay times were used to calculate the required exponential decay time constant. The phase,  $\phi(n)$ , is given by:

$$\phi(n) = 2\pi \cdot f_{sin} \cdot t(n) \quad (1)$$

where  $f_{sin}$  is the frequency of the signature partial, and  $t(n)$  is the time value at sample number  $n$  (sample rate = 44.1 kHz). The initial peak magnitude,  $A$ , used for resynthesis of an individual partial is given by:

$$A = EDR_{max} \cdot 10^{\frac{A_{dB}}{20}} \quad (2)$$

where  $EDR_{max}$  is the maximum value of the EDR, and  $A_{dB}$  is the initial magnitude (in decibels) of the 2<sup>nd</sup> degree polynomial described in Section 2.2. The initial sinusoidal vector,  $y_{sin}(n)$ , is given by:

$$y_{sin}(n) = A \cdot \sin(\phi(n)) \quad (3)$$

The exponential decay time constant,  $\tau$ , for the highest magnitude partial is given by:

$$\tau = \frac{PD_{60}}{-3} \quad (4)$$

where  $PD_{60}$  is the estimated  $PD_{60}$  decay time for the signature partial. The exponentially decaying sinusoidal vector,  $y_r(n)$ , is given by:

$$y_r(n) = y_{sin}(n) \cdot e^{\frac{t(n)}{\tau}} \quad (5)$$

The summed resynthesised signal, containing  $k$  desired partials,  $y_{allr}(n)$ , is given by:

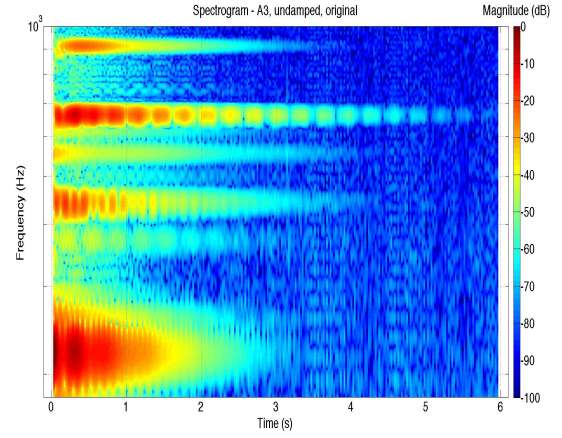
$$y_{allr}(n) = y_{r1}(n) + y_{r2}(n) + \dots + y_{rk}(n) \quad (6)$$

### 3.2 Transient Stage

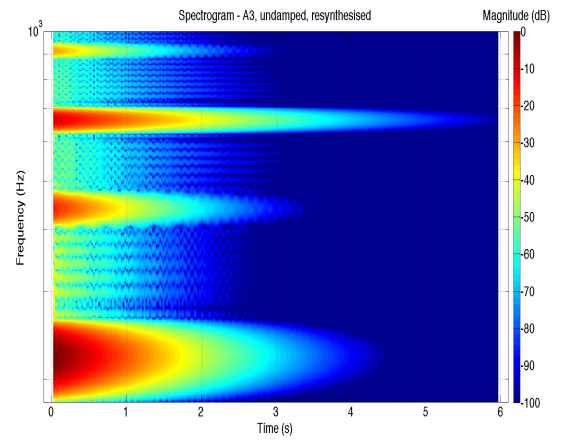
Attack transients are essential for the discrimination and identification of various musical instruments [23]. Whilst transient analysis is beyond the scope of this paper, in order to increase the level of similarity between the resynthesised and original handpan signals, some method of transient modelling is required. In an attempt to isolate the transient portion of a struck note, all note-fields and the port hole of a handpan were covered with magnetic absorbing pads. The port hole was covered in order to reduce the presence of the Helmholtz resonance frequency in the measured signal. Then, the NFEM was used to strike the interstitial area of the handpan in between two note-fields. The signal was cropped at 10 ms, tapered and zero padded to match the length of the resynthesised steady state handpan signal. The transient and steady state signal were then convolved to produce an attack with a higher degree of similarity to the original handpan sound. The convolution of two signals can be interpreted as the multiplication of their spectrum [24], so any spectral component that is not present in both input signals, will not be present in the output signal. Convolution was preferred over simple addition of the transient and steady state signals following informal listening tests, the results of which showed that the convolved signals sounded better than their corresponding summed signals.

### 3.3 Amplitude Modulations

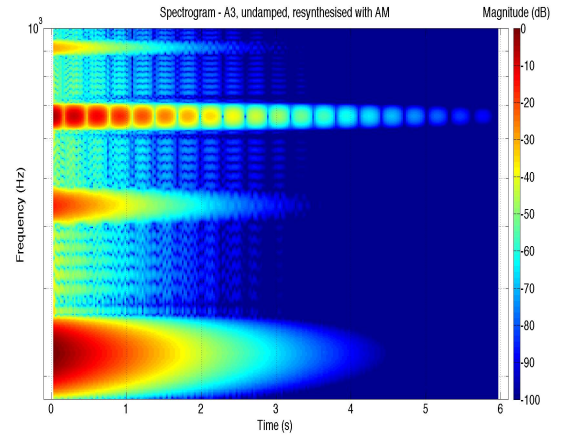
As detailed in Section 2.3, the signature handpan sound can exhibit amplitude modulations on individual or multiple partials at different modulation depths and rates. To



(a)

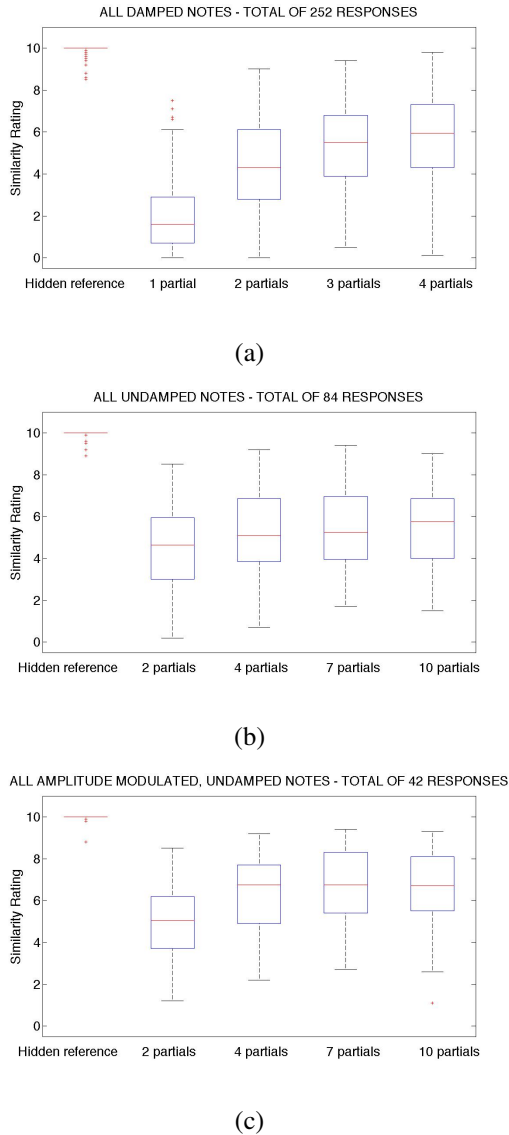


(b)



(c)

**Figure 2.** Spectrograms of the: (a) original; (b) resynthesised; and (c) resynthesised with AM signals. The amplitude modulated partial's frequency value shown in (c) is 662 Hz, with a modulation rate of approximately 3.3 Hz. The amplitude modulating oscillator's frequency value was set to 665.3 Hz.



**Figure 3.** Boxplots produced for: (a) damped (252 responses); (b) undamped (84 responses); and (c) undamped with AM (42 responses) signals respectively.

include these characteristic amplitude modulations in the resynthesised signal, it is possible to exploit the fact that the linear superposition of two simple harmonic vibrations with similar frequencies leads to periodic amplitude vibrations [19]. Figure 2 displays spectrograms of the: (a) original; (b) resynthesised; and (c) resynthesised with AM signals. The amplitude modulated partial's frequency value shown in 2(c) is 662 Hz, with a modulation rate of approximately 3.3 Hz. The amplitude modulating oscillator's frequency value was set to 665.3 Hz. Comparing the 662 Hz partial in both 2(a) and 2(c) shows a high degree of amplitude modulation rate similarity.

#### 4. LISTENING TEST

In order to assess the quality of the resynthesised handpan sounds, a listening test was designed in order to judge the

degree of similarity between the handpan recordings and resynthesised versions created using different numbers of partials. As such the results should go some way toward indicating the number of partials required for the sufficient resynthesis of the handpan sound. Three groups of resynthesised handpan sounds were investigated: damped, undamped and undamped amplitude modulated. Three note registers (low, mid, high) were tested for the damped notes of two instruments.

Each question presented to participants contained five different stimuli. The stimuli for resynthesised damped handpan signals were: 1, 2, 3, and 4 partials, whereas the stimuli for resynthesised undamped handpan signals were: 2, 4, 7, and 10 partials. The fifth stimuli for both configurations was the hidden reference, which is an identical copy of the original audio signal. The difference in number of partials used for resynthesis of the undamped and damped signals is due to the observation that the damped signals contain less signature partials than their corresponding undamped signals.

Participants were asked to rate the similarity of each of the presented audio signals to the reference audio on a scale of 0-10 (with accuracy of a single decimal point). A score of 0 indicated that the corresponding audio sample was perfectly dissimilar to the reference audio, whilst a score of 10 indicated that the audio sample was perfectly similar to the reference audio.

The resynthesised audio samples required additional processing prior to implementation of the subjective listening tests. A section of background noise was cropped immediately before or after the original handpan audio signal and added to the resynthesised signal. Additionally, normalising was also required to bring the original and resynthesised audio signals to the same loudness level. This was achieved by calculating the RMS value for the original and resynthesised signals and scaling each signal appropriately to achieve the desired global RMS level.

MATLAB's `boxplot` function was used to analyse the results of the subjective listening test. Figure 3 shows boxplots of the listening test results for: (a) damped (252 responses); (b) undamped (84 responses); and (c) undamped with AM (42 responses) signals respectively.

Examining 3(a), which contains the boxplots produced for all damped note signals, shows a clear increase in the median similarity rating with increased amount of partials. Examining 3(b), which contains the boxplots produced for all undamped note signals, also shows a slight increase in median similarity rating with increased amount of partials, however this is not as significant for the results in 3(a). Examining 3(c), which contains the boxplots produced for all undamped, amplitude modulated signals shows an increase in median similarity rating for all stimuli, compared to 3(b). For instance, the median rating for the 4 partial stimulus in 3(b) is 5.1, whereas the median value is 6.75 for 3(c). This suggests that the amplitude modulations present in some of the handpan signals is a signature component and must be included in order to sufficiently resynthesise the handpan sound. Additionally, this suggests that addition of amplitude modulation in the resynthesised signal



reduces the number of partials required to achieve higher similarity ratings.

## 5. CONCLUSIONS

This paper presented the results of an experimental procedure to measure and analyse the handpan sound. Analysis and comparison of undamped and damped measurements strengthened the hypothesis that the signature amplitude modulation characteristics in the handpan sound are due to a slight mismatch in tuning of signature partials on separate note-fields. Based on the analysis results, resynthesised sounds were produced and compared to measured sounds in a listening test that aimed to determine the minimum number of partials required to sufficiently resynthesise the signature handpan sound. The results showed the highest median ratings given to resynthesised signals with 4-7 partials, and an additional oscillator used to model the handpan's signature amplitude modulations. Future work should focus on accurate modelling of the attack transient, investigation of the handpan cavity acoustics, and accurate identification of closely spaced partials using high-resolution methods.

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