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ON LINEAR DIOPHANTINE EQUATION

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Abstract:

In this paper, the concept of Diophantine equation are discussed and the theorem relating to the linear Diophantine equations of two variables *x* and *y* are presented.

Introduction:

A Scientific approach to the study of numbers can be traced back to pythagores (6^{th} century B.C) and his disciples. Further development of number theory was due to the great mathematician fermat in 17^{th} century. Now, theory of number has developed into a vast and beautiful branch of mathematics, having link with almost every branch of this science. An equation a x + by = c the first degree in two unknowns X and Y with the condition that X and Y should be integers is called "Diophantine Equations" in honour of Diophantur, a Greek mathematician.

Linear Diophantine Equations:

Diophantine Equations:

The famous mathematician Diophantus of third century A.D. worked on Diophantine equation of second and higher orders. A number of riddlers and puzzles came into existence in finding out the solution of Diophantine equation of the first and second degree our problem is to find all the positive fractions that behave in this way. That is, I have to determine *x*, y, z in such a way that $(10x\ y)/(10y + z) = x/z$.

This reduces to $(10x + y)\, z = x\,(10y + z)$

$$\Rightarrow (y - x)z = 10\,(y - z)\,x.$$

But we are only interested in solutions such that *x*, y & z are positive integers less than 10. The equation $(y-x)\, z = 10\,(y - z)\, x$ is an indeterminate equation. It has many algebraic solutions, such a problem is called Diophantine problem and we have to solve a Diophantine equation. This particular problem is merely a curiosity, but there are many important Diophantine equations. In general, the added restrictions are that the solutions are to be integers or sometimes, rational. Frequently the solutions are also to be positive.

Linear Diophantine Equations of Two Variables:

The Equations $ax + by = C$

Diophantine equation influenced the development of number theory. Any linear equation in two variables having integral co-efficients can be put in the form $ax + by = C$. The problem is trivial unless neither a nor b is zero. So we can suppose $a \neq 0$, $b \neq 0$, we let d denote (a, b).

Definition:

The general form of a linear Diophantine equation in two unknowns *X* and Y is

$$ax + by = C \qquad (1)$$

With the restriction that the solutions be only in integers, If $x = x_0$, $y = y_0$ satisfy (1) we write the solution as (x_0, y_0)

We now present some simple but interesting examples.

Example:

$6x + 5y = 22$ is a linear Diophantine equation one of it solution is $x = 2$, $y = 2$ since, $(6 x 2) + (5 x 2) = 22$. The following theorem gives a standard criterion for the solvability of the linear Diophantine equation.

Theorem:

The equation $ax + by = c$ is solvable in integers if and only if 'd' divides 'c' where d is the greatest common divisor of a and b.

Proof:

Let d/c. Since (a, b) = d, there exist a integers x_1 and y_1 such that

$$ax_1 + by_1 = d \qquad (2)$$

Multiplying both sides by c/d in (2) we get.

$$a\,(c/d)\, x_1 + b\,(c/d)\, y_1 = c$$

$$\Rightarrow x = (c/d)\, x_1,$$

$$y = (c/d)\, y_1 \text{ is a solution of (2)}$$

Conversely, let (x_0, y_0) be a solution of (2). Then $ax_0 + by_0 = C$. Now d divides a and b $\Rightarrow$ d/c.

Remark: The equation $ax+by= c$ is solvable if and only if $(a, b) =1$.

Remark: If *x* and y are solutions of $ax+ by = c$, then $x = X + Bt$, $y= Y-At$ is also a solution, for every value of the integers t and A and B are defined by $a = Ad$ and $b = Bd$. Where d is the greatest common divisor of a and b.

We present below a direct proof to the above statement for,

$$
\begin{aligned}
ax + by &= a(X + Bt) + b(Y - At) \\
&= aX + aBt + by - Abt \\
&= ax + by + aBt - bAt \\
&= C + t(aB - bA) \\
&= C + 0 \text{ (Since d = Ad and b = Bd)} \\
&= C
\end{aligned}
$$

Examples: Consider the following equation

$$112x + 70y = 168 \qquad (3)$$

We know that (112, 70) = 14.

$$(3) \div 14 \Rightarrow 8x + 5y = 12 \qquad (4)$$

$$\Rightarrow \quad (8, 5) = 1.$$

Therefore (4) has a solution. The solution of (4) is x =4 and y = - 4.
Therefore x = 4 and y = - 4 is a solution of (3).

References:

1. Dickson. L. E, History of the Theory of Numbers, Vol.2, Chelsea, New York, 1952.
2. Kirch. A. M, Elementary Number Theory, In tent Educational publisher, New York, 1974.
3. Nagell. T, Introduction to Number Theory, Stockholm and New York, 1951.
4. Ore. O, Number Theory and its History, McGraw – Hill. New York. 1948.
5. Uspensky. J. V, and M.A Hear let, Elementary Number Theory, McGraw-Hill Bool company, INC. New York and London. 1939.