Optimization of the box section of the main girder of the bridge crane with the rail placed above the web plate

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INDUSTRIAL APPLICATION

Optimization of the box section of the main girder of the bridge crane with the rail placed above the web plate

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Abstract The paper considers the problem of optimization of the box section of the main girder of the bridge crane. Reduction of the girder mass is set as the objective function. The method of Lagrange multipliers was used as the methodology for approximate determination of optimum dependences of geometrical parameters of the box section. The criteria of permissible stresses and strains, lateral stability and dynamic stiffness were applied as the constraint functions. The obtained results of optimization of geometrical parameters were verified on numerical examples and the comparison with some solutions of cranes was made. The comparative analysis of the optimization results and the solutions was the basis for recommendations which are significant for designers during construction of cranes.

Keywords Box section · Bridge crane · Optimization · Stress · Lateral stability

1 Introduction

The main task in the process of designing the carrying structure of the bridge crane is determination of optimum dimensions of the main girder box section. The main girder is the most responsible part of the bridge crane and therefore it is necessary, during optimization, to affect the increase in its carrying capacity with simultaneous reduction of its mass. The mass of the main girder has the largest share in the total mass of the bridge crane, so it is very important to perform its optimization in order to reduce the total costs of manufacturing the whole carrying structure. The analysis of cost structure for manufacturing metal structures made by Farkas 1984 showed that the participation of material costs in the total costs is the largest (30–73)%, and that the other costs are lower.

That is the reason why the selection of the optimum shape and geometrical parameters which influence the reduction of mass and costs of manufacturing is the subject of research of a lot of authors regardless of whether they deal specifically with cranes or carrying structures in general (Farkas 1984; Farkas and Jármai 1997; Farkas et al. 2005; Jarmai and Farkas 2001; Jarmai et al. 2003; Kaufmann et al. 2010; Farkas et al. 2010; Mijailović 2010; Mijailovic and Kastratovic 2009; Selmic et al. 2006a, b).

The optimization of the welded box girder performed by Jarmai and Farkas (2001) showed that regular placement of longitudinal stiffeners may result in savings up to the amount of (18–21)%. The inserted longitudinal stiffeners increase the stability of the box girder with the reduction of size of the cross section. Placement of additional longitudinal and transverse stiffeners also influences the improvement of the structure of the box girder in terms of savings in material up to 38.33%, which was confirmed by the finite element method (Qin and Zhu 2010).

The optimization of the box section of the main girder of the bridge crane was also carried out in the paper written by Pinca et al. (2009a, b), where during the optimization the constant height of the girder h was adopted, and the other geometrical parameters of the cross section were changed.

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In this way was shown that the mass can be reduced even up to 20.6, i.e. 8.46% without any risky exceeding of the permissible stress. It was also pointed out that selecting a finite element type influences the accuracy of calculation and reliability of results.

Zuberi et al. (2008) solved the problem of optimization of the longitudinal girder with box section and with the rail placed above the web using three constraint functions. The finite element method was used in that paper, 11 geometrical parameters were varied, and the importance of the possibility of using a large number of variables during optimization was emphasized. It can be noticed that optimization is performed either analytically or by means of the finite element method. The finite element method is suitable because a lot of variables are varied. The advantage of analytical methods is that they provide functional dependences of optimization results so that the analysis can define the influence of certain parameters on the reduction of mass. Most authors set permissible stress or two constraint functions: permissible stress and permissible deflection as the constraint function. The criterion of lateral stability has lately been increasingly applied as the constraint function (Zuberi et al. 2008; Farkas 2005; Jarmai and Farkas 2001; Jarmai et al. 2003; Mijailovic and Kastratovic 2009).

Niezgodziñski and Kubiak (2005) analyzed errors during manufacturing box girders of cranes due to welding and their later effects on the increase in stresses and strains. They proposed straightening and regeneration which refers to welding straps to the corresponding zones for overcoming the problem. A similar conclusion was reached by Blum and Haremski (2010), who also proposed welding of additional stiffeners, i.e. plates welded in the bottom flange as a solution of the problem, but they added a plate at midspan, halfway of the web height that should prevent its horizontal deformations. The paper also pointed to the occurrence of initial crack both in operation and due to the technology of manufacturing and its influence on later damages of the structure. Fatigue of the structure due to technological loads may be increased even up to 30%. Local buckling and the point of placing stiffeners was treated by Farkas (1986), too, but he emphasized the importance of changing the type of material. He showed that for the D class of cranes changing the type of material might result in savings of (16–18)%, i.e. (22–24)%. For the C class savings are considerably smaller. Jarmai (1990) also showed that replacement of the material S230 with steel S355 resulted in considerable savings in mass. Savings are also achieved by manufacturing an asymmetric girder where a thinner plate is inserted at the point which is not exposed to direct wheel-rail pressure. Savings are accomplished in the mass of girder material as well as in the energy for moving the equipment for its manufacturing and preparation of surfaces.

Taking into account all these restrictions and parameters analyzed in the papers, justification of creating a multicriteria optimization method and use of the FE method was proved (Kaufmann et al. 2010). Having in mind all the above mentioned results and conclusions, the aim of this paper is to define optimum values of geometrical parameters of the box girder cross-section that will lead to the reduction of its mass. It is also necessary to define more closely the relations between the main parameters of the cross section which make the starting point for those who design box girders.

2 Mathematical formulation of the optimization problem

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area. Minimization of the mass corresponds to minimization of the volume, i.e. the area of the cross section of the girder, where the given boundary conditions must be satisfied. The optimization problem defined in this way can be given the following general mathematical formulation:

$$\text{minimize } f(\boldsymbol{X}), \tag{1}$$

$$\text{subject to: } g_j(\boldsymbol{X}) \leq 0,\ j = 1, ..., m, \tag{2}$$

where: $f(\boldsymbol{X})$ the objective function, $g_j(\boldsymbol{X}) \leq 0$ the constraint function, m is the number of constraints.

Here $\boldsymbol{X} = \{x_1, \ldots, x_D\}^T$ **r** represents the design vector made of D design variables. Design variables are the values that should be defined during the optimization procedure.

In this paper optimization for the following cases of constraints was performed:

$$g_1 = \sigma_e - \sigma_o \leq 0 \text{ - the strength criterion,} \tag{3}$$

$$g_2 = T - T_d \leq 0; \text{ - the criterion of dynamic stiffness,} \tag{4}$$

$$g_3 = \sigma_r - \sigma_k \leq 0; \text{ - the criterion of lateral stability,} \tag{5}$$

$$g_{41} = f_v - f_{v,dop} \leq 0,$$
$$g_{42} = f_h - f_{h,dop} \leq 0; \text{ - the criterion of stiffness,} \tag{6}$$

where:

– σ_e, σ_0 - the maximum equivalent and permissible stress of the girder,

- T, T_d - the calculation and permissible times of damping of oscillations,
- σ_r, σ_k - the calculation and stresses in lateral buckling of the girder,
- $f_v, f_{v,0}, , f_h, , f_{h,0}$ - the calculation and permissible deflections of the girder in the vertical and horizontal planes.

The Lagrange function is defined in the following way:

$$\Phi(\boldsymbol{X}) = f(\boldsymbol{X}) + \sum_{j=1}^{m} \lambda_j \cdot g_j(\boldsymbol{X}) \tag{7}$$

where λ_j is the known Lagrange multiplier. Optimal values of parameters are determined:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_1}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_1}{\partial b} \wedge g_1 = 0 \text{ - the criterion of permissible stress,} \tag{8}$$

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_2}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_2}{\partial b} \wedge g_2 = 0 \text{ - the criterion of dynamic stiffness,} \tag{9}$$

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_3}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_3}{\partial b} \wedge g_3 = 0 \text{ - the criterion of lateral stability,} \tag{10}$$

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_4}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_4}{\partial b} \wedge g_4 = 0 \text{ - the criterion of stiffness of the girder.} \tag{11}$$

3 Objective and constraint functions

3.1 Objective function

The objective function is represented by the area of the cross section of the box girder. The paper treats two optimization parameters (h, b).

The vector of the given parameters is:

$$\vec{x} = (M_{cv}, M_{ch}, Q, L, \sigma_o, G_k, k_a, \ldots), \tag{12}$$

where:

- M_{cv} and M_{ch} - are the bending moments in the vertical and horizontal planes,
- Q - the carrying capacity of the crane,
- L - the span of the crane,
- G_k - the mass of the crane cab,
- k_a - the dynamic coefficient of crane load in the horizontal plane.

The dynamic coefficient of crane load in the horizontal plane (k_a) was adopted according (Ostrić and Tošić 2005), but the way of its selection does not reduce the generality of consideration.

The area of the cross section (Fig. 1), i.e. the objective function, is:

$$A(h, b) = f(h, b) = \frac{2}{s} \cdot \left(e \cdot b \cdot h + h^2\right), \tag{13}$$

where:

$e = t_1/t_2$ the ratio between thicknesses of plates at the flange and at the web,
$s = h/t_2$ the ratio between the height and thickness of the plate at the web,
$k = h/b$ the ratio between the height and width of the girder.

To know the optimal value of the ratio between the height and width of the girder k is of particular significance for the designer, especially in the initial design phase so that its determination is the subject of research in a large number of papers (Farkas and Jármai 1997; Gašić et al. 2011a, b; Savković 2005; Selmic et al. 2006b).

The expressions for the moments of inertia around the x and y axes are:

$$I_x = \frac{1}{6} \cdot \frac{h^4}{s} + \frac{1}{2} \cdot e \cdot b \cdot \frac{(s+e)^2}{s^3} \cdot h^3, \tag{14}$$

$$I_y = \frac{1}{6} \cdot e \cdot \frac{h}{s} \cdot b^3 + \frac{1}{2} \cdot \frac{h^2}{s} \cdot \frac{(f \cdot b \cdot s + h)^2}{s^2}, \tag{15}$$

where: $f = \frac{b_1}{b} < 1$ - the ratio between the distance of web plates and the width of flange plates of the box girder.

Since the expressions for the moments of inertia (I_x, I_y) and the section moduli (W_x, W_y) are complex, it is common to take approximate values of expressions by neglecting the members of the lower order (Gašić et al. 2011a, b; Savković

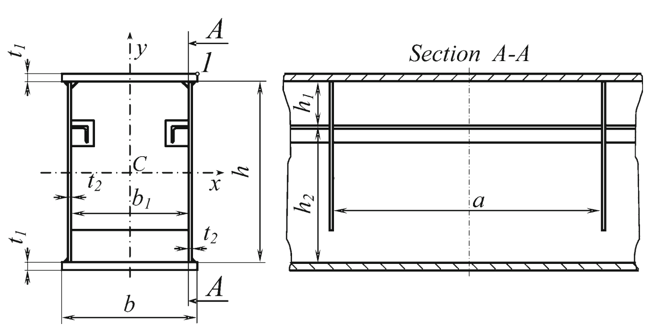

Fig. 1 The box section of the main girder of the bridge crane and elements of the box profile relevant for testing of the local stability

2005; Selmic et al. 2006b; Mijailovic and Kastratovic 2009; Mijailović 2010):

$$I_x = \beta_x^2 \cdot h^2 \cdot A,\ W_x = \alpha_x \cdot h \cdot A, \tag{16a}$$

$$I_y = \beta_y^2 \cdot b^2 \cdot A;\ W_y = \alpha_y \cdot b \cdot A, \tag{16b}$$

where:

β_x, β_y the dimensionless coefficient of the moment of inertia for the x and y-axes,
α_x, α_y the dimensionless coefficient of the resistance moment of inertia for the x and y-axes.

The coefficients β_x and α_x are obtained from the conditions of equality of the (14) and the expression (16a):

$$\beta_x = \frac{1}{2 \cdot s} \cdot \sqrt{\frac{k \cdot s^2 + 3 \cdot e \cdot (s+e)^2}{3 \cdot (e+k)}},\ \alpha_x = \frac{2 \cdot s}{s + 2 \cdot e} \cdot \beta_x^2. \tag{17}$$

Using the fact that e much smaller than s and k much smaller than s the coefficients with the form β_x and α_x can be simplified:

$$\beta`_x \cong \frac{1}{2} \cdot \sqrt{\frac{k + 3 \cdot e}{3 \cdot (e+k)}}, \alpha`_x \cong \frac{k + 3 \cdot e}{6 \cdot (e+k)}. \tag{18}$$

This approximation can be graphically represented (Fig. 2), where it is seen that deviations are negligible in the considered range of parameters k.

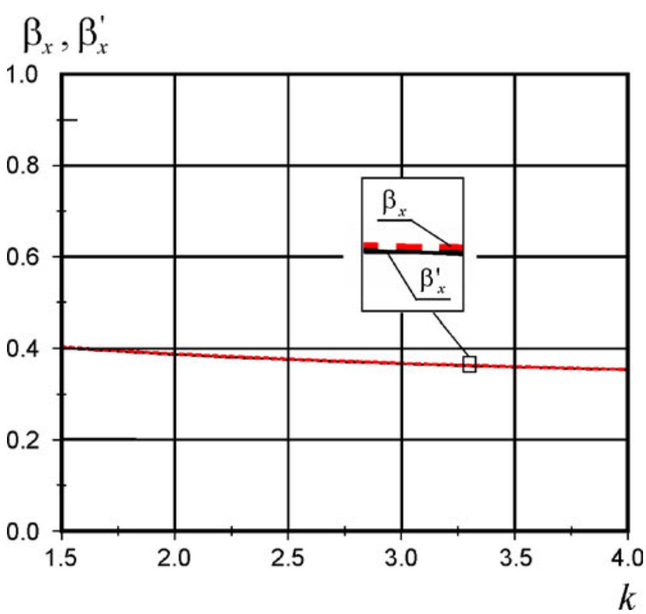

Fig. 2 Approximation of the coefficient of the moment of inertia around the x-axis

By repeating the procedure for the moment of inertia and the section moduli for the y – axis, the following values of coefficients are obtained in a simpler form:

$$\beta`_y \approx \frac{1}{2} \cdot \sqrt{\frac{3 \cdot k \cdot f^2 + e}{3 \cdot (e+k)}}, \alpha`_y = \frac{3 \cdot k \cdot f^2 + e}{6 \cdot (e+k)}. \tag{19}$$

3.2 Constraint functions

3.2.1 Criterion of permissible stress

The maximum equivalent stress which occurs in the main girder of the bridge crane is at point 1 (Fig. 1). The constraint function according to this criterion is:

$$g_1 = g_1(h, b) = \sigma_e - \sigma_0 = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \sigma_0 \le 0, \tag{20}$$

where: c - the coefficient of influence of the dead weight of the girder on the bending moment.

In order to apply the method of Lagrange multipliers, for the criterion of permissible stress, it is necessary to find the corresponding partial derivatives, in accordance with the expression (8):

$$\frac{\partial g_1}{\partial b} = -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{M_{ch}}{\alpha_y} \cdot \frac{1}{A} \cdot \frac{1}{b^2} + \frac{k_a \cdot c}{\alpha_y} \cdot \frac{1}{b^2}\right];$$

$$\frac{\partial g_1}{\partial h} = -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h} + \frac{M_{cv}}{\alpha_x} \cdot \frac{1}{A} \cdot \frac{1}{h^2} + \frac{c}{\alpha_x} \cdot \frac{1}{h^2} + \frac{M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h}\right]; \tag{21}$$

$$\frac{\partial A}{\partial b} = \frac{2}{s} \cdot e \cdot h; \quad \frac{\partial A}{\partial h} = \frac{2}{s} \cdot (e \cdot b + 2 \cdot h);$$

By replacing the expression (21) in (8), after rearrangement, the following relation is obtained:

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2 \cdot A} \cdot \frac{\partial A}{\partial b} = \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b^2 \cdot A} \cdot \frac{\partial A}{\partial h}. \tag{22}$$

From (20) and (22) it is possible to find the optimum values of dimensions h and b, i.e. their ratio k according to this criterion. The bending moment due to dead weight is considerably smaller than the moment caused by an external load, so that the following approximation can be performed:

$$\frac{M_{cv} + c \cdot A}{M_{ch} + k_a \cdot c \cdot A} \approx \frac{M'_{cv}}{M'_{ch}}, \tag{23}$$

On the basis of Fig. 3, the ratio $\left(M'_{cv}/M'_{ch}\right)$ is obtained:

$$\frac{M'_{cv}}{M'_{ch}} = \frac{1}{k_a} \cdot \frac{R \cdot (L - e_1)^2 + 2 \cdot G_k \cdot e_k \cdot (L + e_1)}{R_h \cdot (L - e_1)^2 + 2 \cdot G_k \cdot e_k \cdot (L + e_1)}. \quad (24)$$

It is common that the distance between the cab and the crane runway is (Ostrić and Tošić 2005) $e_k \leq 2\,m$ and it is considerably smaller than the span of the bridge crane L, so it is shown that the influence of the cab is not important for this analysis. Using the conclusions from previous analysis, as well as the recommendations (Ostrić and Tošić 2005), it can be written that:

$$\frac{M'_{cv}}{M'_{ch}} = \frac{1}{k_a} \cdot \frac{R}{R_h} = \frac{1}{k_a} \cdot c_1. \quad (25)$$

The member c_1 (Ostrić and Tošić 2005) depends on the carrying capacity and the classification class and it reads:

$$c_1 = \frac{R}{R_h} = \frac{f_1(Q)}{f_2(Q)} = \frac{\psi \cdot Q + m_o + K \cdot Q^\alpha}{Q + m_o + K \cdot Q^\alpha}. \quad (26)$$

where: K - the coefficient of influence of the classification class on the mass of the trolley, ψ - the dynamic coefficient of the influence of load oscillation in the vertical plane, α - the coefficient of influence of the load mass on the mass of the trolley, m_o - the assumed mass of the trolley in the first approximation.

Using the relations (22), (23) and (25), it is obtained that:

$$k = \sqrt{\frac{e \cdot \alpha_y}{\alpha_x} \cdot \frac{M'_{cv}}{M'_{ch}}}. \quad (27)$$

The expression (27) represents the optimum value of the ratio between the parameters h and b obtained according to the criterion of permissible stress. Using the obtained dependences from the constraint function according to the criterion of permissible stress, the objective function can be written in the following form:

$$A_\sigma = A(h) \geq \frac{\frac{M_{cv}}{\alpha_x} + \frac{M_{ch}}{\alpha_y} \cdot k}{\sigma_0 \cdot h - \frac{c}{\alpha_x} - \frac{k_a \cdot c}{\alpha_y} \cdot k}. \quad (28)$$

3.2.2 The criterion of dynamic stiffness

In order to determine the optimum ratio of optimization parameters according to the criterion of dynamic stiffness, it is necessary to analyze oscillation of the main girder in the vertical plane. The analysis procedure was performed in compliance with the Ostrić and Tošić (2005). A simple girder with its mass concentrated at midspan is taken to be the model of oscillation (Fig. 4).

The mass m_1 is determined according to the expression (Ostrić and Tošić 2005):

$$m_1 = 0{,}5 \cdot (Q + m_0) + 0{,}486 \cdot m_m, \quad (29)$$

where: $0{,}486 \cdot m_m$ - the reduced continual mass of the girder at midspan for the assumed function of displacement of the elastic line of the adopted discrete dynamic model for the simple girder.

The time of damping of oscillation is determined from the expression (Ostrić and Tošić 2005):

$$T = 3 \cdot \frac{\tau}{\gamma_d} \leq T_d, \quad (30)$$

where:

$\tau = 2 \cdot \pi \cdot \sqrt{\delta_{11} \cdot m_1}$ - the period of oscillation (s), (31)

$\delta_{11} = \frac{1,0 \cdot L^3}{48 \cdot E \cdot I_x}$ - the deflection of the girder caused by the action of the unit force, (32)

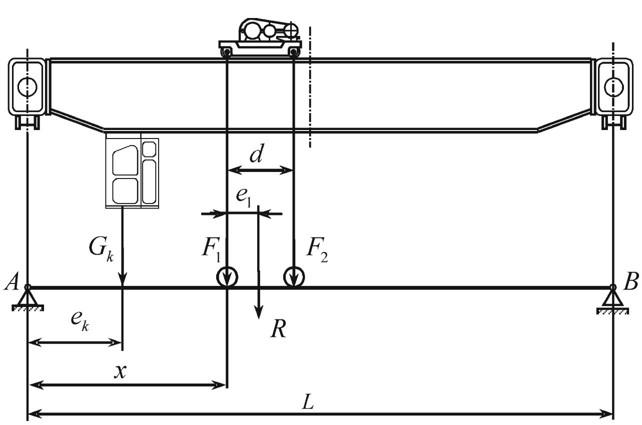

Fig. 3 Loads of the main girder of the bridge crane

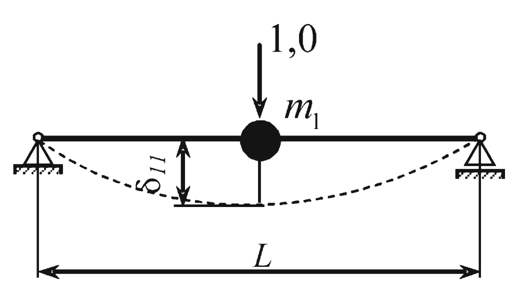

Fig. 4 The model of oscillation of the main girder with concentrated mass of the load and the trolley

T_d the permissible time of damping of oscillation (s), which depends on the purpose of the crane. For general purpose cranes it ranges within: 12–15 (s), for cranes transporting molten metal: 8–10 (s).

γ_d the logarithmic decrement which shows the rate of damping of oscillation, which depends on the ratio between the height of the girder $h$and the span L.

The relation (29) can be written as:

$$m_1 = 0{,}5 \cdot (Q + m_0) + 0{,}486 \cdot m_m$$
$$= M + r \cdot L \cdot A, \tag{33}$$

where: $M = 0{,}5 \cdot (Q + m_0)$ - the constant part of the expression, $0,486 \cdot m_m = r \cdot L \cdot A$ - the variable part of the expression.

If the denotation:

$$C_d = \frac{1{,}0}{48 \cdot E \cdot \beta_x^2}, \tag{34}$$

is introduced and if the relations (31), (32) and (33) are replaced in (30), the constraint function for the criterion of dynamic stiffness is:

$$g_2(h, b) = \frac{6 \cdot \pi}{\gamma_d} \cdot \sqrt{\frac{C_d \cdot M \cdot L^3}{h^2 \cdot A} + \frac{C_d \cdot r \cdot L^4}{h^2}} - T_d \le 0. \tag{35}$$

In order to apply the method of Lagrange multipliers for the criterion of dynamic stiffness, it is necessary to find the corresponding partial derivatives, in accordance with the expression (9):

$$\frac{\partial g_2}{\partial b} = \frac{6 \cdot \pi}{\gamma_d} \cdot \frac{1}{\sqrt{\delta_{11} \cdot m_1}} \cdot \frac{C_d \cdot M \cdot L^3}{h^2} \cdot \left(-\frac{1}{A^2}\right) \cdot \frac{\partial A}{\partial b};$$
$$\frac{\partial g_2}{\partial h} = \frac{6 \cdot \pi}{\gamma_d} \cdot \frac{1}{\sqrt{\delta_{11} \cdot m_1}} \cdot \left[\frac{C_d \cdot M \cdot L^3}{h^2} \cdot \left(-\frac{1}{A^2}\right) \cdot \frac{\partial A}{\partial h} - \frac{2 \cdot C_d \cdot M \cdot L^3}{h^3 \cdot A} - \frac{2 \cdot C_d \cdot r \cdot L^4}{h^3}\right]; \tag{36}$$

By replacing the expression (36) in (9), after rearrangement, it is obtained that:

$$\frac{e + k}{k} - \frac{(1 - 2 \cdot t_2 \cdot e)}{2 \cdot [G(m) - 1]} = 0, \tag{37}$$

where: $G(m) = \frac{0{,}486 \cdot m_m}{0{,}5 \cdot (m_Q + m_k)}$.

By using the obtained dependences from the constraint function according to the criterion of dynamic stiffness, the objective function can be written in the following form:

$$A_d = A(h) \ge \frac{C_d \cdot M \cdot L^3}{\left(\frac{T_d \cdot \gamma_d}{6 \cdot \pi}\right)^2 \cdot h^2 - C_d \cdot r \cdot L^4}. \tag{38}$$

3.2.3 The criterion of local buckling of plate fields subjected to compressive stresses

Testing of the box girder stability was carried out in accordance with the European standard (prEN 13001–3–1:2010). According to this standard, it is necessary to check the stability of the flange plate with the width b_1 and the thickness t_1 (Fig. 1), the stability of the web plate above the longitudinal stiffener (length a, height h_1 and thickness t_2 – Fig. 1) as well as the stability of the web plate under the longitudinal stiffener (length a, height h_2 and thickness t_2 – Fig. 1).

The criterion of local buckling of top flange plate of the box girder Testing of the stability of the flange plate segment (Fig. 1) subjected to the action of normal compressive stress in the x direction was carried out in compliance with the standard prEN 13001–3–1:2010.

This criterion is fulfilled if the following condition is satisfied:

$$\left|\sigma_{Sd,x}\right| \le f_{b,Rd,x} = \frac{\kappa_x \cdot f_{yk}}{\gamma_m}, \tag{39}$$

where: $\left|\sigma_{Sd,x}\right| = |-\nu_1 \cdot (\sigma_{zV1} + f \cdot \sigma_{zH1})|$ - the design value of the compressive stress in the x direction, $f_{yk} = f_y$ - the minimum yield stress of the plate material, γ_m - the general resistance factor, κ_x - a reduction factor according to (40):

$$\kappa_x = c_e \cdot \left(\frac{1}{\lambda_x} - \frac{0,22}{\lambda_x^2}\right) \le 1 \text{ for } \lambda_x > 0,673;$$
$$\kappa_x = 1 \text{ for } \lambda_x \le 0,673, \tag{40}$$

where: λ_x - the non-dimensional plate slenderness according to (41):

$$\lambda_x = \sqrt{\frac{f_{yk}}{K\sigma \cdot \sigma_e}} \tag{41}$$

with:

$$c_e = 1,25 - 0,12 \cdot \psi_e, \tag{42}$$

ψ_e - the edge stress ratio of the plate, relative to the maximum compressive stress, $K\sigma$ - a buckling factor given in Table 15 (prEN 13001–3–1:2010), σ_e - a reference stress according to (43):

$$\sigma_e = \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{t_1}{b_1}\right)^2. \tag{43}$$

The coefficient ψ_e is defined by the ratio of factored stresses:

$$\psi_e = \frac{\sigma_2}{\sigma_1} = \frac{\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} - f \cdot k}{\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + f \cdot k} \tag{44}$$

where: σ_1, σ_2 - the stresses due to the factored load.

For average values, this ratio can also be approximately written by the expression:

$$\psi_p \cong 0,83 - 0,06 \cdot k \tag{45}$$

Based on the given ratios, and with the corresponding transformations, the following relations are obtained:

$$c_p \cong 1,15 + 0,0072 \cdot k \tag{46}$$

$$K\sigma_p = \frac{8,2}{1,88 - 0,06 \cdot k} \tag{47}$$

$$\sigma_e = \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{e \cdot k}{s \cdot f}\right)^2 \tag{48}$$

$$Ko = \frac{1}{\pi} \cdot \sqrt{\frac{12 \cdot (1 - \nu^2) \cdot f_y}{E}}, \tag{49}$$

$$\lambda_{xp} \cong \frac{Ko}{\sqrt{K\sigma_p}} \cdot \frac{s \cdot f}{e \cdot k}. \tag{50}$$

For the defined range of the ratio k, the change of relations (40) and (50) can be seen for the adopted values of classification class 2m/M5 (FEM 9.511/ISO 4301–1), girder material S235JRG2, $s = 210$, $f = 0,74$ and $e = 1,2 \div 2,0$ (Fig. 5), i.e. classification class 2m/M5 (FEM 9.511/ISO 4301–1), girder material S235JRG2, $e = 1,2$, $f = 0,74$ and $s = 160 \div 210$ (Fig. 5). It is seen that the factor κ_x takes the value 1 for the defined range of the ratio k.

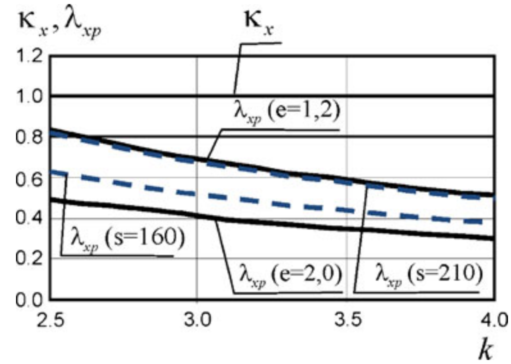

Fig. 5 Change of the coefficients λ_{xp} and κ_x as the function of the parameter k

The local stability of the flange plate (39) is satisfied if the following condition is fulfilled:

$$\sigma_{zV1} + \sigma_{zH1} \geq \frac{1}{\kappa_x} \cdot (\sigma_{zV1} + f \cdot \sigma_{zH1})$$
$$\Leftrightarrow \frac{M_{VI}}{W_{x1}} + \frac{M_{HI}}{W_{y1}} \geq \frac{1}{\kappa_x} \cdot \left(\frac{M_{VI}}{W_{x1}} + f \cdot \frac{M_{HI}}{W_{y1}}\right) \tag{51}$$

Rearranging the expression (51) with the corresponding transformations, it is obtained that:

$$f_l(L) \geq f_d(L),$$

$$f_l = 2{,}17 \cdot \kappa_x \cdot (Q + m_k) \cdot \frac{(L - e_1)^2}{L} \cdot \left(\alpha_y \cdot c_1 + \alpha_x \cdot k_a \cdot k\right) + 40{,}82 \cdot \kappa_x \cdot L^2 \cdot \left(\alpha_y + \alpha_x \cdot k_a \cdot k\right) \cdot \frac{(e + k)}{s \cdot k} \cdot \left(\frac{x}{L}\right)^2$$

$$f_d = 2{,}17 \cdot (Q + m_k) \cdot \frac{(L - e_1)^2}{L} \cdot \left(\alpha_y \cdot c_1 + f \cdot \alpha_x \cdot k_a \cdot k\right) + 40{,}82 \cdot L^2 \cdot \left(\alpha_y + f \cdot \alpha_x \cdot k_a \cdot k\right) \cdot \frac{(e + k)}{s \cdot k} \cdot \left(\frac{x}{L}\right)^2 \tag{52}$$

where: $\frac{x}{L} = \frac{L}{h} = 14 \div 20$- the recommended value of the ratio between the span and the height of the main girder (Ostrić and Tošić 2005). Testing of the fulfillment of conditions of stability for the flange plate can be established for the adopted values of the classification class 2m/M5 (FEM 9.511/ISO 4301–1), $s = 210$, $e = 1{,}33$ and $f = 0{,}74$ the variable parameters: carrying capacity (Fig. 6), ratio between the width and height of the girder k (Fig. 6) and the value X (Fig. 6).

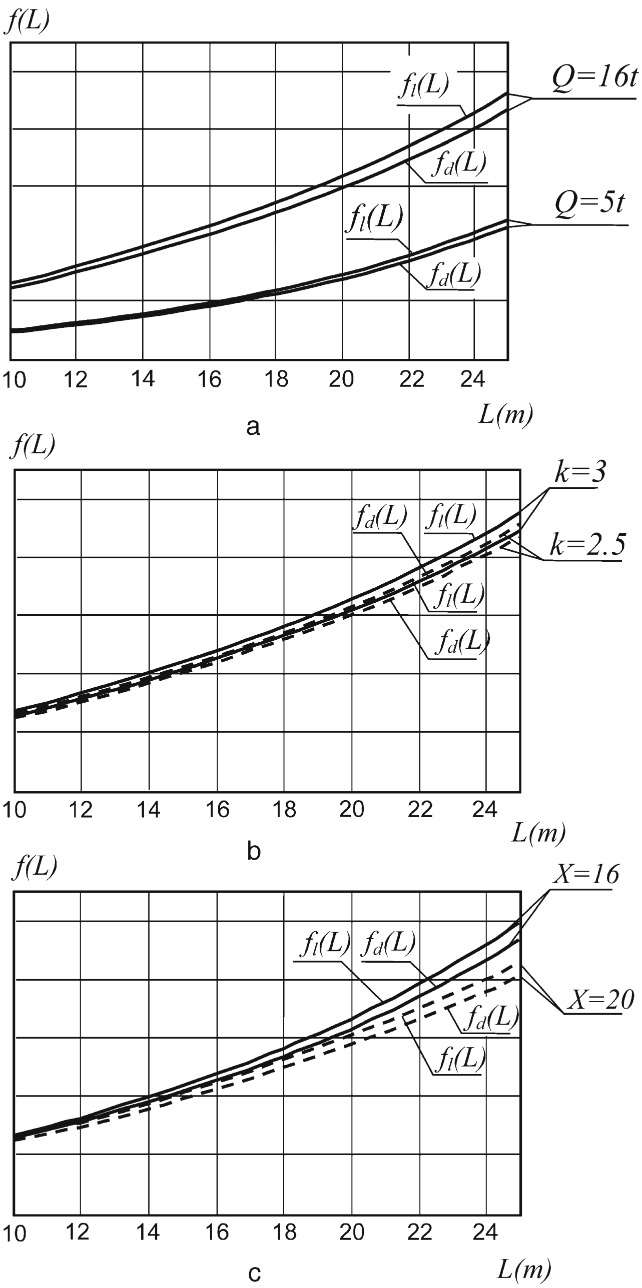

Fig. 6 Testing of the stability of the flange plate for variable parameters: **a**) carrying capacity **b**) ratio between the width and height of the girder k and **c**) the value of the coefficient X

It is shown that the condition of stability of the flange plate for the mentioned parameters is fulfilled.

The constraint function (5) for stability of the flange plate reads:

$$g_{31} = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + f \cdot \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \kappa_x \cdot \sigma_0 \leq 0. \tag{53}$$

In order to apply the method of Lagrange multipliers, it is necessary to find the corresponding partial derivatives, in accordance with the expression (10):

$$\frac{\partial g_{31}}{\partial b} = -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{f \cdot M_{ch}}{\alpha_y} \cdot \frac{1}{A} \cdot \frac{1}{b^2} + \frac{f \cdot k_a \cdot c}{\alpha_y} \cdot \frac{1}{b^2}\right];$$

$$\frac{\partial g_{31}}{\partial h} = -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h} + \frac{M_{cv}}{\alpha_x} \cdot \frac{1}{A} \cdot \frac{1}{h^2} + \frac{c}{\alpha_x} \cdot \frac{1}{h^2} + \frac{f \cdot M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h}\right]; \tag{54}$$

Using the relation (23) as well as the derivatives $\partial A/\partial b$ and $\partial A/\partial h$, it is obtained that:

$$k = \sqrt{\frac{e \cdot \alpha_y}{f \cdot \alpha_x} \cdot \frac{c}{k_a}}. \tag{55}$$

The expression (55) represents the optimum value of the ratio between the parameters h and b according to the criterion of lateral stability of the flange plate. It is seen that, according to this criterion, the optimum value k takes higher values in relation to the strength criterion because $f < 1$. Using the obtained dependencies from the constraint function according to the criterion of stability of the flange plate, the constraint function can be written in the following form:

$$A = A(h) \geq \frac{\frac{M_{cv}}{\alpha_x} + \frac{f \cdot M_{ch}}{\alpha_y} \cdot k}{\kappa_x \cdot \sigma_0 \cdot h - \frac{c}{\alpha_x} - \frac{f \cdot k_a \cdot c}{\alpha_y} \cdot k} \tag{56}$$

The criterion of local buckling of web plate of the box girder Testing of the stability of the web plate segment (Fig. 1) subjected to the action of normal stresses in the x and y directions was carried out in compliance with the standard prEN 13001–3–1:2010.

The case when, in addition to vertical stiffeners at midspan, a row of horizontal stiffeners is also placed, the horizontal stiffeners being placed at the distance of $(0,25 \div 0,33) \cdot h$, was considered.

The criterion of stability of the web plate in area 1 is fulfilled if the following condition is satisfied:

$$\left(\frac{\left|\sigma_{Sd1,x}\right|}{f_{b,Rd1,x}}\right)^{e_{1x}} + \left(\frac{\left|\sigma_{Sd1,y}\right|}{f_{b,Rd1,y}}\right)^{e_{1x}} - \left(\kappa_{1x} \cdot \kappa_{1y}\right)^6 \cdot \left(\frac{\left|\sigma_{Sd1,x} \cdot \sigma_{Sd1,y}\right|}{f_{b,Rd1,x} \cdot f_{b,Rd1,y}}\right) \leq 1 \tag{57}$$

where: $\left|\sigma_{Sd1,x}\right|, \left|\sigma_{Sd1,y}\right|$ - the design value of the compressive stress in the x and y directions at point 1, $f_{b,Rd1,x}$, $f_{b,Rd1,y}$ - the limit design compressive stresses,

$e_{1x} = 1 + \kappa_{1x}^4$, $e_{1y} = 1 + \kappa_{1y}^4$.

κ_{1x} - a reduction factor for area 1 according to (58):

$$\kappa_{1x} = c_{1e} \cdot \left(\frac{1}{\lambda_{1x}} - \frac{0,22}{\lambda_{1x}^2} \right) \leq 1 \text{ for } \lambda_{1x} > 0,63;$$

$$\kappa_{1x} = 1 \text{ for } \lambda_{1x} \leq 0,63, \tag{58}$$

where: λ_{1x} - the non-dimensional plate slenderness for area 1 according to the previous equations.

The coefficient ψ_{1e} is defined by the ratio of the factored stresses:

$$\psi_{1e} = \frac{\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + 2 \cdot f \cdot k}{2 \cdot \left(\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + f \cdot k \right)} \tag{59}$$

For average values, this ratio can approximately be written as:

$$\psi_{1p} \cong 0,54 + 0,015 \cdot k \tag{60}$$

Based on the given ratios, and with the corresponding transformations, the following relations are obtained:

$$c_{1p} \cong 1,185 - 0,0018 \cdot k \tag{61}$$

$$K\sigma_{1p} = \frac{8,2}{1,59 + 0,015 \cdot k} \tag{62}$$

$$\lambda_{1xp} \cong \frac{Ko}{\sqrt{K\sigma_{1p}}} \cdot \frac{s}{4} \tag{63}$$

For the defined range of the ratio k, the change of relations (58) and (63) can be seen for the adopted values of classification class 2m/M5 (FEM 9.511/ISO 4301–1), girder material S235JRG2, $s = 160 \div 210$, and $f = 0,74$, analogously to procedure done for relations (40) and (50) and Fig. 5.

The criterion of stability of the web plate in area 2 is fulfilled if the following condition is satisfied:

$$\left(\frac{\left|\sigma_{Sd2,x}\right|}{f_{b,Rd2,x}} \right)^{e_{2x}} + \left(\frac{\left|\sigma_{Sd2,y}\right|}{f_{b,Rd2,y}} \right)^{e_{2y}} - \left(\kappa_{2x} \cdot \kappa_{2y}\right)^6 \cdot \left(\frac{\left|\sigma_{Sd2,x} \cdot \sigma_{Sd2,y}\right|}{f_{b,Rd2,x} \cdot f_{b,Rd2,y}} \right) \leq 1 \tag{64}$$

All expressions in (64) correspond to the expressions in (57), and the index “2” refers to area 2 (Fig. 1).

The coefficient ψ_{2e} is defined by the ratio of the factored stresses:

$$\psi_{2e} = \frac{\sigma_3}{\sigma_4} \approx \frac{\sigma_2}{\sigma_4} = -\frac{\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + 2 \cdot f \cdot k}{2 \cdot \left(\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} - f \cdot k \right)} \tag{65}$$

$$\sigma_3 = \sigma_{Sd2,x} = -\nu_1 \cdot \left(\frac{\sigma_{zV1}}{2} + f \cdot \sigma_{zH1} \right)$$

For average values, this ratio can approximately be written by the expression:

$$\psi_{2p} \approx -(0,6 + 0,01 \cdot k) \tag{66}$$

Based on the given ratios, and with the corresponding transformations, the following relations are obtained:

$$c_{2p} = 1,25 \tag{67}$$

$$K\sigma_{2p} = 15,1 + 1,8 \cdot k + 0,0978 \cdot k^2 \tag{68}$$

$$\lambda_{2xp} \cong \frac{Ko}{\sqrt{K\sigma_{2p}}} \cdot \frac{3 \cdot s}{4} \tag{69}$$

For the defined range of the ratio k, the change of relations (58) and (69) can be seen for the adopted values of classification class 2m/M5 (FEM 9.511/ISO 4301–1), girder material S235JRG2, $s = 160 \div 210$ and $f = 0{,}74$, analogously to procedure done for relations (40) and (50) and Fig. 5.

The factors for the y axis will be analyzed when the load in the y direction is observed.

The normal stress in the $y$direction due to the action of wheel pressure on the web plate is determined according to the following expression (prEN 13001–3–1:2010):

$$\left|\sigma_{Sd,y}\right| = \sigma_y = -\nu_1 \cdot \frac{\gamma \cdot F_1}{t_2 \cdot l_r}, \tag{70}$$

where: l_r - the effective distribution length given in Annex C.4, prEN 13001–3–1:2010 according to (71):

$$l_r = 2 \cdot h_e \cdot tg\kappa + \lambda, \tag{71}$$

where: h_e - the distance between the section under consideration and the contact level of the acting load, κ - is the dispersion angle; κ shall be set to $\kappa \leq 45°$; for further work it is adopted that $\kappa = 45°$,

$$\lambda = 0,2 \cdot r_t, \text{ - the length of the contact area,} \tag{72}$$

where: r_t - the radius of the wheel.

The dominant action of wheel pressure on the web plate is in area 1.

The expression for the boundary compressive stress due to pressure (57) in the direction of the y-axis reads:

$$f_{b,Rd1,y} = \frac{\kappa_{1y} \cdot f_{yk}}{\gamma_m}, \tag{73}$$

where: κ_{1y} – a reduction factor for area 1 according to (74)

$$\kappa_{1y} = 1,13 \cdot \left(\frac{1}{\lambda_{1y}} - \frac{0,22}{\lambda_{1y}^2} \right) \leq 1 \;\text{ for }\; \lambda_{1y} > 0,831;$$
$$\kappa_{1y} = 1 \;\text{ for }\; \lambda_{1y} \leq 0,831, \tag{74}$$

where: λ_{1y} - the non-dimensional plate slenderness for area 1, $K\sigma_{1y}$ - a buckling factor for area 1 given in Table 15 (prEN 13001–3–1:2010), c_{1r} - the width over which the transverse load is distributed (corresponds to c_{1r}).

$$\frac{c_{1r}}{a} = \frac{12,15 \cdot s \cdot f + 2 \cdot b_1 \cdot e \cdot k}{2 \cdot b_1 \cdot k \cdot s} \tag{75}$$

It is shown that this ratio is smaller than 0.1 and knowing that $\alpha_{1e} > 4$, it is obtained that $K\sigma_{1y} \cong 0,5$, so it is obtained that:

$$\lambda_{1y} = \frac{Ko}{\sqrt{K\sigma_{1y}/\frac{c_{1r}}{a}}} \cdot \frac{s}{4} \tag{76}$$

Because of the small value of the ratio (76), the factor κ_{1y} takes the value $\kappa_{1y} = 1$ even for rather high values of s.

As all the values for the expression (57) are now known, the following relation can be written:

$$\sqrt{\left|\sigma_{Sd1,x}\right|^2 + \left|\sigma_{Sd1,y}\right|^2 - \left|\sigma_{Sd1,x} \cdot \sigma_{Sd1,y}\right|} \leq f_{b,Rd1} \tag{77}$$

Also, the following conditions must be fulfilled:

$$\left|\sigma_{Sd1,x}\right| \leq f_{b,Rd1}, \quad \left|\sigma_{Sd1,y}\right| \leq f_{b,Rd1} \tag{78}$$

If it is assumed that $\left|\sigma_{Sd1,x}\right|$ is the critical stress in relation to the others and if the expression (77) is considered, the following relation is obtained:

$$\left|\sigma_{Sd1,x} \cdot \sigma_{Sd1,y}\right| \geq \left|\sigma_{Sd1,y}\right|^2 \Leftrightarrow \left|\sigma_{Sd1,x}\right| \geq \left|\sigma_{Sd1,y}\right| \tag{79}$$

and it should be proved.

By applying the previous procedure as with the relation (51), the following expression is obtained:

$$f_{1l}(L) \geq f_{1d}(L),$$
$$f_{1l}(L) = 2,17 \cdot (Q + m_k) \cdot \frac{(L - e_1)^2}{L} \cdot \left(\alpha_y \cdot c_1 + f \cdot \alpha_x \cdot k_a \cdot k\right) + 40,82 \cdot L^2 \cdot \left(\alpha_y + f \cdot \alpha_x \cdot k \cdot k_a\right) \cdot \frac{(e + k)}{s \cdot k} \cdot \left(\frac{L}{X}\right)^2$$
$$f_{1d}(L) = 1120 \cdot \alpha_x \cdot \alpha_y \cdot (\psi \cdot Q + m_k) \cdot \frac{e + k}{(12,15 + 1,4 \cdot e) \cdot k} \cdot \left(\frac{L}{X}\right)^2$$

Testing of the fulfillment of the conditions of stability of the web plate can be established for the adopted values of the classification class 2m/M5 (FEM 9.511/ISO 4301–1), $s = 180$, $e = 1{,}33$ and $f = 0{,}74$ and the variable parameters: carrying capacity, ratio between the width and height of the girder k and the value X, in accordance with previous procedure ((52) – Fig. 6).

It is shown that the condition of stability of the web plate is fulfilled in area 1 for the mentioned parameters. Also, it is shown that the stress that occurs is not higher that the allowed one according to the strength criterion, and the stability itself is satisfied.

The expression for l_{2r}, the action of wheel pressure on the web plate in area 2, reads:

$$l_{2r} = 12,15 + 2 \cdot e \cdot \frac{h}{s} + \frac{h}{2} \tag{80}$$

The members in the expression for the equivalent stress in relation to the y axis for area 2 are:

$$\sigma_{Sd2,y} = -\nu_1 \cdot \frac{\gamma \cdot F_1}{t_2 \cdot l_{2r}}, \tag{81}$$

$$f_{b,Rd2,y} = \frac{\kappa_{2y} \cdot f_{yk}}{\gamma_m}, \tag{82}$$

where: κ_{2y} - a reduction factor for area 2 according to (83):

$$\kappa_{2y} = 1{,}13 \cdot \left(\frac{1}{\lambda_{2y}} - \frac{0{,}22}{\lambda_{2y}^2} \right) \leq 1 \;\text{ for }\; \lambda_{2y} > 0{,}831;$$
$$\kappa_{2y} = 1 \;\text{ for }\; \lambda_{2y} \leq 0{,}831, \tag{83}$$

where: λ_{2x} - the non-dimensional plate slenderness for area 2. $K\sigma_{2y}$ - a buckling factor for area 2 given in Table 15 (prEN 13001–3–1:2010), c_{2r} - the width over which the transverse load is distributed (corresponds to l_{2r}):

$$\frac{c_{2r}}{a} = \frac{24{,}3 \cdot s \cdot f + 4 \cdot b_1 \cdot e \cdot k + b_1 \cdot k \cdot s}{4 \cdot b_1 \cdot k \cdot s} \tag{84}$$

It is shown that this ratio is close to the value of 0.3 and knowing that $\alpha_{2e} = 8/3$, it is obtained that $K\sigma_{2y} \approx 1,2$, so it is obtained that:

$$\lambda_{2y} = \frac{Ko}{\sqrt{K\sigma_{2y}/\frac{c_{2r}}{a}}} \cdot \frac{3 \cdot s}{4} \tag{85}$$

Based on the expressions (80) and (81), the change of stress due to the trolley wheel pressure in area 2 is obtained:

$$\sigma_{Sd2,y} = -\nu_1 \cdot \frac{2 \cdot \gamma \cdot F_1 \cdot s^2}{24{,}3 \cdot s \cdot h + 4 \cdot e \cdot h^2 + s \cdot h^2} \tag{86}$$

As $(\kappa_{2x} \cdot \kappa_{2y})^6$ much smaller than 1 and as all the values from the expression (64) are known, it obtains the form:

$$\left(\frac{|\sigma_{Sd2,x}|}{f_{b,Rd2,x}}\right)^{e_{2x}} + \left(\frac{|\sigma_{Sd2,y}|}{f_{b,Rd2,y}}\right)^{e_{2y}} \leq 1, \tag{87}$$

The ratio between the maximum compressive stress of the girder and the compression in this area is:

$$\psi_2 = \frac{\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + 2 \cdot f \cdot k}{2 \cdot \left(\frac{c_1}{k_a} \cdot \frac{\alpha_y}{\alpha_x} + k\right)} \tag{88}$$

The expression (87) now becomes:

$$\left(\frac{\psi_2}{\kappa_{2x}}\right)^{1+\kappa_{2x}^4} + \left(\frac{|\sigma_{Sd2,y}| \cdot \gamma_m}{\kappa_{2y} \cdot f_y}\right)^{1+\kappa_{2y}^4} \leq 1 \tag{89}$$

The change of this dependence in the function can be established for the adopted values of the classification class 2m/M5 (FEM 9.511/ISO 4301–1), girder material S235JRG2, $e = 1{,}33$, $f = 0{,}74$, and $s = 210$, i.e. $s = 160$ (Fig. 7), and the variable parameters: the carrying capacity Q, the width b_1 and the ratio between the width and the height of the girder k. For higher carrying capacities it is better to adopt a bigger width b_1 (Fig. 7) if lower values for k are expected. For lower carrying capacities ($Q \leq 8t$) it is more rational to use smaller widths for b_1. Where higher values for k are expected it is possible to use higher values for s. For lower values for s this condition is satisfied for any k (Fig. 7).

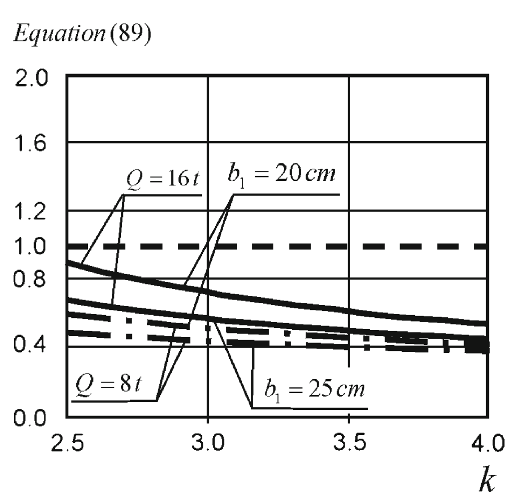

Fig. 7 Change of the value of the function (89) depending on the carrying capacity Q and the width b_1

It is shown that area 1 is critical in testing of the local stability of web plates. By applying the method of Lagrange multipliers, for the constraint function (5) reads:

$$g_{32} = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + f \cdot \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \kappa_{1x} \cdot \sigma_0 \leq 0. \tag{90}$$

Using the relation (23) as well as the derivatives $\partial A/\partial b$ and $\partial A/\partial h$, it is obtained that:

$$k = \sqrt{\frac{e \cdot \alpha_y}{f \cdot \alpha_x} \cdot \frac{c_1}{k_a}}. \tag{91}$$

Using the obtained dependencies from the constraint function according to the criterion of stability of the web plate, the objective function can be written in the following form:

$$A_{vl} = A(h) \geq \frac{\frac{M_{cv}}{\alpha_x} + \frac{f \cdot M_{ch}}{\alpha_y} \cdot k}{\kappa_{1x} \cdot \sigma_0 \cdot h - \frac{c}{\alpha_x} - \frac{f \cdot k_a \cdot c}{\alpha_y} \cdot k} \tag{92}$$

3.2.4 The criterion of girder stiffness

In order to satisfy this criterion, it is necessary that the deflections in the corresponding plane have the values smaller than the permissible ones. The maximum values of deflection must be within the following limits:

$$f_v = \frac{F_1 \cdot L^3}{48 \cdot E \cdot I_x} \cdot \left[1 + w \cdot \left(1 - 6 \cdot p^2\right)\right] \leq f_{v,dop} = K_v \cdot L \text{ - the deflection in the vertical plane,} \tag{93}$$

$$f_h = \frac{k_a \cdot F_{1h} \cdot L^3}{48 \cdot E \cdot I_y} \cdot \left[1 + w \cdot \left(1 - 6 \cdot p^2\right)\right] \leq f_{h,dop} = K_h \cdot L \text{ - the deflection in the horizontal plane,} \tag{94}$$

where (Fig. 3): $w = \frac{F_2}{F_1} \leq 1$, $p = \frac{d}{L}$, d – the distance between the wheels of the trolley.

If it is adopted that the coefficients K_v and K_h are equal, the constraint function has the form:

$$g_4(h, b) = \frac{g_{42}}{g_{41}} = \frac{k_a \cdot F_{1h} \cdot \beta_x^2 \cdot h^2}{F_1 \cdot \beta_y^2 \cdot b^2} - 1 \leq 0. \tag{95}$$

In order to apply the method of Lagrange multipliers, for the criterion of girder stiffness, it is necessary to find the corresponding partial derivatives, in accordance with the expression (11):

$$\frac{\partial g_4}{\partial b} = -2 \cdot \frac{k_a \cdot F_{1h} \cdot \beta_x^2 \cdot h^2}{F_1 \cdot \beta_y^2 \cdot b^3}, \quad \frac{\partial g_4}{\partial h} = \frac{2 \cdot k_a \cdot F_{1h} \cdot \beta_x^2 \cdot h}{F_1 \cdot \beta_y^2 \cdot b^2}. \tag{96}$$

By replacing the expression (96) in (11), it is obtained that:

$$\frac{\partial A}{\partial b} / \frac{\partial A}{\partial h} = -\frac{h}{b} \Leftrightarrow e \cdot b = -h. \quad (97)$$

In the expression (97) a negative solution which is not within the set of real solutions ($b, h > 0$) is obtained.

By using the obtained dependences from the constraint function according to the criterion of stiffness, the objective function can be written in the following form:

$$A_f = A(h) \geq \frac{F_{1h} \cdot L^2 \cdot \left[1 + w \cdot \left(1 - 6 \cdot p^2\right)\right]}{48 \cdot E \cdot \beta_x^2 \cdot K_f \cdot h^2}. \quad (98)$$

4 Numerical representation of the results obtained

4.1 Analysis of the optimization results

Using the expression (27) the optimum value of the parameter k according to the criterion of permissible stress is obtained as a function of the values e and k_a. The coefficient f does not considerably influence the parameter k and does not have to be treated in optimization because it depends on the adopted concept of the box girder, and in practice it is within the range: $f \in (0.85 \div 0.90)$. The coefficient e is adopted and depending on the crane manufacturer it is most frequently within the range $e \in (1.2 \div 2.0)$. The dynamic coefficient k_a depends on the selection of crane trolley. In the initial phase of design, without precise knowing of acceleration values in load lifting, it may be adopted that its value is within the range (Ostrić and Tošić 2005): $k_a \in (0.10 \div 0.15)$. Taking into account the mentioned recommendations, the value of the coefficient k is presented in Table 1.

The expression (28) represents the objective function obtained from the constraint function according to the criterion of permissible stress and together with the objective function (13) it can be graphically represented. The expression (28) is represented by a dashed line, and the expression (13) is represented by a continuous line (Fig. 8), for the corresponding types of material (S235), classification class 2m/M5 (FEM 9.511/ISO 4301–1), where the values of the parameters L = 20 m and Q = 12.5 t are adopted. At the intersection of these curves, on the abscissa, there is an optimum height $h$for the constraint function according to

Table 1 Optimum values of the parameter k as a function of the coefficients e and k_a

	$k_a = 0.15$					$k_a = 0.125$					$k_a = 0.10$				
e	1.2	1.4	1.6	1.8	2.0	1.2	1.4	1.6	1.8	2.0	1.2	1.4	1.6	1.8	2.0
k	3.1	3.3	3.5	3.6	3.8	3.4	3.6	3.9	4.1	4.2	3.9	4.1	4.4	4.6	4.8

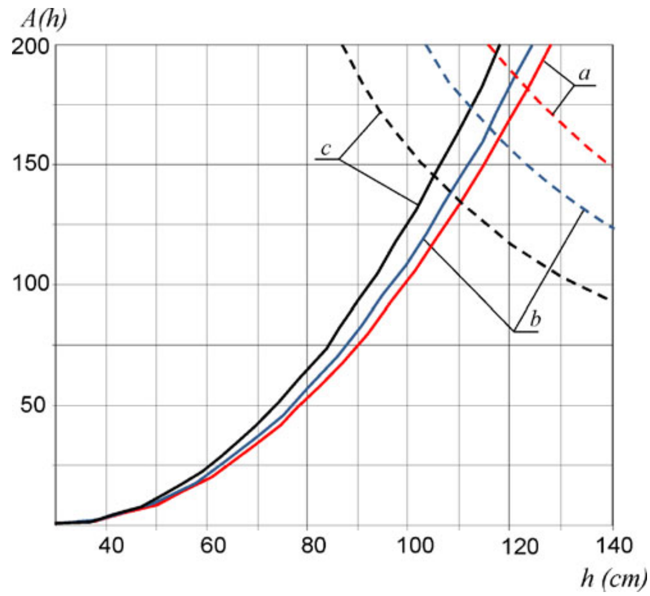

Fig. 8 Optimum values of the girder height and the objective function according to the strength criterion **a** S235JRG2, **b** S275JR, **c** S355JR

the criterion of permissible stress. Figure 8 shows how the position of the intersection point changes depending on the selection of material. By changing the parameters L, Q and k, as well as the type of material, the optimum value of the objective function is easily obtained (13).

Using the expression (37) the optimum value of the parameter k according to the criterion of dynamic stiffness is obtained. The value of this parameter depends, to the greatest extent, on the member $G(m)$ and it is difficult to be determined in advance. Its value is defined by the span of the bridge crane and the mass of the trolley, and these parameters are determined by the investor and cannot be the subject of optimization. Taking into account that the optimum value of the parameter k, to the criterion of permissible stress, is within the range (2.5÷4.5) and using (37), the value of the member $G(m)$ in the interval −0.04÷0.53 is obtained (Table 2).

Table 2 The values of the member $G(m)$ as a function of the parameter k and the coefficient e

	$k = 2.5$					$k = 3$				
e	1.20	1.40	1.60	1.80	2.00	1.20	1.40	1.60	1.80	2.00
$G(m)$	0.53	0.43	0.33	0.24	0.17	0.50	0.38	0.28	0.19	0.11
	$k = 3.5$					$k = 4$				
e	1.20	1.40	1.60	1.80	2.00	1.20	1.40	1.60	1.80	2.00
$G(m)$	0.48	0.35	0.25	0.14	0.05	0.46	0.33	0.21	0.10	0.0
	$k = 4.5$									
e	1.20	1.40	1.60	1.80	2.00					
$G(m)$	0.44	0.31	0.19	0.07	−0.04					

Table 3 Optimum values of the parameter $k$as a function of the coefficient e

	$k_a = 0.15$					$k_a = 0.125$					$k_a = 0.10$				
e	1.2	1.4	1.6	1.8	2.0	1.2	1.4	1.6	1.8	2.0	1.2	1.4	1.6	1.8	2.0
k	3.3	3.5	3.7	3.9	4.1	3.6	3.9	4.1	4.4	4.5	4.2	4.4	4.7	4.9	5.1

The expression (38) represents the objective function obtained from the constraint function according to the criterion of dynamic stiffness and together with the objective function (13) it can be graphically represented (analogously to previous procedure and Fig. 8). Using the expression (55) the optimum value of the parameter $k$according to the criterion of lateral stability is obtained. The optimum values of the parameter k as a function of the member e are presented in Table 3.

The expression (56) represents the objective function obtained from the constraint function according to the criterion of lateral stability and together with the objective function (13) it can also be graphically represented (analogously to previous procedure and Fig. 8).

The expression (98) represents the objective function obtained from the constraint function according to the criterion of stiffness and together with the objective function (13) it can be graphically represented.

4.2 Comparative presentation of the obtained results for the corresponding spans and carrying capacities of cranes

In order to perform a comparative analysis of optimization results, it is necessary to define the initial parameters of cranes which refer to their geometrical characteristics, classification class and carrying capacity. These are the data which the designer receives from the investor as the project task.

The other values of parameters in this phase are:

$$e = 1.33, \quad f = 0.85, \quad \psi = 1.15, \quad k_a = 0.1,$$

$$e_k = 2.3\,m, \quad G_k = 15\,kN, \quad T_d = 15\,s.$$

The analysis was performed for the classification class 2m/M5 (FEM 9.511/ISO 4301–1), which is, according to the (Ostrić and Tošić 2005), most frequently used in practice. The following values hold for it:

$$\gamma = 1.05, \quad \alpha = 1.20, \quad K = 0.08,$$

$$m_o = 1.20, \quad K_f = \frac{1}{600}.$$

The analysis was performed for steel S235JRG2. In order to perform a comparative analysis, it is necessary to take into consideration the recommendations specified in the standard as well as those given by crane manufacturers (Catalogues 1996). Serbian crane manufacturers recommend that the minimum value of the width b_1 should be $b_1 > 20$ cm, wherefrom it is obtained that:

$$k \leq \frac{f \cdot h}{20}. \tag{99}$$

This recommendation does not hold with other world manufacturers but it does not reduce the generality of application of the procedure. If the expressions (13) and (28) are made equal, the dependence of the parameter k according to the criterion of permissible stress is obtained:

$$k = F\left(s, e, h, M_{cv}, M_{ch}, \alpha_x, \alpha_y\right), \tag{100}$$

which can be graphically represented (Fig. 9). If the expressions (13) and (38) are made equal, the dependence of the parameter k according to the criterion of dynamic stiffness is obtained (Fig. 9):

$$k = F\left(s, e, h, T_d, C_d, r, L, \gamma_d, M\right). \tag{101}$$

If the expressions (13) and (56) are made equal, the dependence of the parameter k according to the criterion of lateral stability is obtained (Fig. 9):

$$k = F\left(s,\ e,\ f,\ h,\ Q,\ M_{cv},\ M_{ch},\ \alpha_x,\ \alpha_y, k_a,\ f_y,\ \nu,\ E\right). \tag{102}$$

If the expressions (13) and (98) are made equal, the dependence of the parameter k according to the criterion of girder stiffness (Fig. 9):

$$k = F\left(E, e, \beta_x, K_f, h, F_{1h}, w, p, L\right). \tag{103}$$

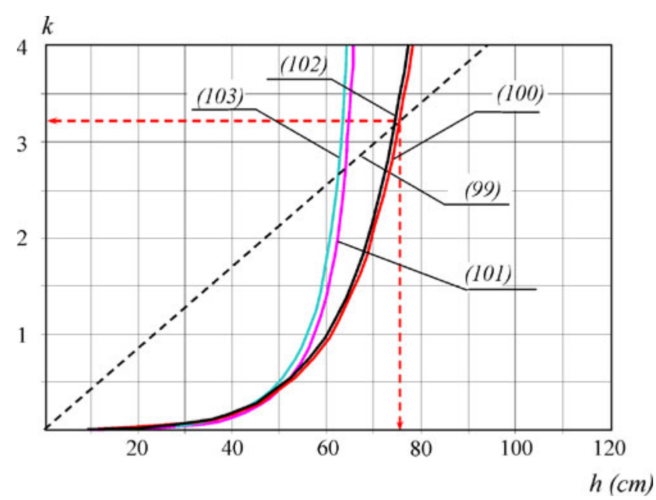

Fig. 9 Multicriteria determination of the optimum value of the parameter k for the crane span $L = 10$ m and the carrying capacity $Q = 8t$

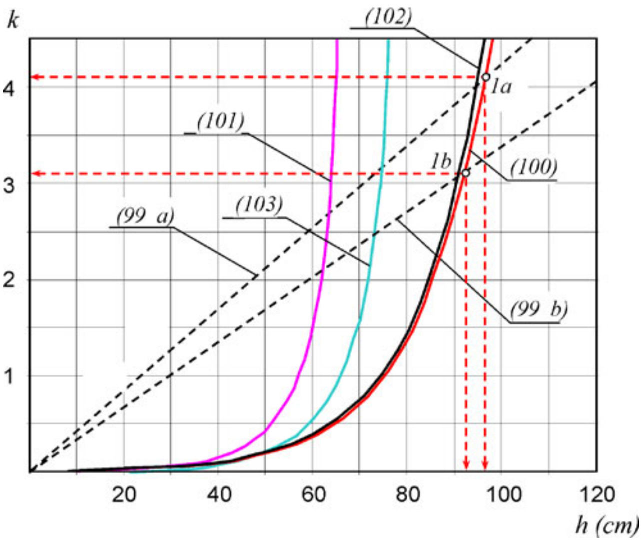

Fig. 10 Multicriteria determination of the optimum value of the parameter k for the crane span $L = 10$ m and the carrying capacity $Q = 16t$: **a** $b_1 = 20$ cm, **b** $b_1 = 25$ cm

For the crane span $L = 10\ m$ and the carrying capacity $Q = 8t$, using the above mentioned relations, the strength criterion is relevant and the optimum value of the parameter k is at the point of intersection between the relations (99) and (100) (Fig. 9).

The procedure thus performed enables fast and efficient determination of the optimum value of the parameter k according to the critical function. The value of the width b_1 does not influence the optimization procedure but it influences the obtained values of the optimization parameter k (Fig. 10).

Table 5 Basic characteristics of geometrical parameters after the optimization carried out with a certain percent of deviation from the objective function

No.	Q (t)	L (m)	h (cm)	b (cm)	t_1 (mm)	t_2 (mm)	A (cm^2)	Saving %
1.	16	22	130	30	8	6	204	26.3
2.	10	16	95	25	7	5	130	19.8
3.	10	20	105	25	7	5	140	22.2
4.	20	11,6	86	26	6	5	117.2	4.8
5.	10	17,3	105	26	6	5	136.2	29.4
6.	10	20	110	29	8	6	178.4	14.2
7.	16	13,7	95	25	8	6	154	8.9
8.	10	16	95	25	7	5	130	20.3

4.3 Comparative presentation of the obtained results for the corresponding spans and carrying capacities of cranes

Verification of the performed method of optimization of geometrical parameters was carried out by its comparison with several solutions of bridge cranes from different manufacturers. The vector of the given parameters consists of: $Q(t)$, $L(m)$, type of material of the main girder and the classification class. Table 4 shows the basic geometrical

Table 4 Basic geometrical characteristics of some solutions of bridge cranes

R.b.	Q (t)	L (m)	Material	Classification class (FEM/ISO)	h (cm)	b (cm)	t_1 (mm)	t_2 (mm)	A (cm^2)	Location	Manufacturer
1.	16	22	S235JRG2	2m/M5	98	50	12	8	276,8	Holcim, Paraćin	Tecon Engineering, Beograd
2.	10	16	S235JRG2	2m/M5	75	45	8	6	162	IMK-14, Kruševac	MIN, Niš
3.	10	20	S235JRG2	2m/M5	90	45	8	6	180	IMK-14, Kruševac	MIN, Niš
4.	20	11,6	S275JR	2m/M5	80	35	6	5	123,2	PPT, Trstenik	Atmos, Hoèe
5.	10	17,3	S235JRG2	2m/M5	84	46	10	6	192,9	Wagon factory Kraljevo	ILR Železnik
6.	10	20	S235JRG2	2m/M5	90	50	10	6	208	Wagon factory Kraljevo	ILR Železnik
7.	16	13,7	S235JRG2	2m/M5	78	30	10	8/6	169,2	Granit ad, Skoplje	Tecon Engineering, Beograd
8.	10	16	S235JRG2	2m/M5	80	42	8	6	163.2	IMK-14, Kruševac	IMK-14, Kruševac

characteristics taken from existing solutions. The manufacturer and location of the installed crane are also mentioned. Table 5 presents the values of geometrical characteristics after the optimization method performed for the same given parameters.

By the analysis of obtained results and their mutual comparison it is concluded that significant savings in the girder mass can be made after the optimization is carried out. The size of savings differs and depends on the manufacturer and vectors of the given parameters. For all solutions presented in Tables 4 and 5 the objective function and all constraints are satisfied.

5 Conclusion

The paper defined optimum dimensions of the box section of the main girder of the bridge crane in an analytical form, by using the method of Lagrange multipliers. The objective function is the minimum mass, i.e. the minimum area of the cross section, where the given constraints are satisfied: permissible stress, lateral stability, dynamic stiffness and permissible deflection. The results obtained may be of great use to the engineer-designer, particularly in the first phase of the design procedure when the basic dimensions of the main girder of the bridge crane, as its most responsible part, are defined. Using the obtained optimum values of geometrical parameters of the main girder, considerable savings in the material consumed is made thus reducing its price, which is shown by comparison with certain solutions of cranes under the same explotiation conditions. For the examples mentioned, savings in the material range between 4.8% and 29.4%.

Justification of applying the method of Lagrange multipliers was also shown because the optimization results were obtained in an analytical form, which allows drawing conclusions on the influence of certain parameters and directions of further research concerning the reduction of mass.

The conclusion is that further research should be directed toward a multicriteria analysis where it is necessary to include additional constraint functions, such as: material fatigue, influence of manufacturing technology, optimization of the ratio of plate thicknesses, types of material, conditions of crane operation.

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