**International Journal of Mathematics and Computer Research**

**ISSN: 2320-7167**

**Volume 11 Issue 09 September 2023, Page no. 3721-3728**

**Index Copernicus ICV: 57.55, Impact Factor: 7.362**

**DOI: 10.47191/ijmcr/v11i9.04**


# Some Contribution to Product of Strong Nuetrosophic Graphs

**Balasubramanian K R[1], Rajeswari K[2]**

[1]PG and Research Department of Mathematics, H. H. The Rajah's College (Autonomous), Pudukkottai, Tamilnadu, India.

[2]Department of Mathematics, Kalaignar Karunanidhi Government Arts College for Women (Autonomous), Pudukkottai, Tamilnadu, India.

| ARTICLE INFO | ABSTRACT |
|---|---|
| Published Online: 05 September 2023 Corresponding Author: Rajeswari K | In this work we developed the α- cut worthy(Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and Examples. |

**KEYWORDS:** Strong Neutrosophic Graphs, α- cut worthy(Level) Graphs, Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs.

Mathematics Subject Classification 2020: 05C07, 05C72, 05C76.

## 1. INTRODUCTION

Graph Theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects, a graph in this context is made up of vertices which are connected by edges. Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields, including operations research, computer science, and social network analysis. Rosenfeld introduced fuzzy graph in 1975. The operations of Cartesian product, compositions of fuzzy graphs were defined by J.N. Mordeson and C.S. Peng [1]. Developed the degree of a node in some fuzzy graphs, A. Nagoorgani and K. Radha [3]. The degree of a Node in fuzzy graphs using these operations was discussed by A. Nagoorgani and K. Radha. F.smarandache Single Valued Neotrosophic Graphs and three regions: Truth (or) acceptance (T), rejection (F), and (neutrality) indeterminacy (I) degrees both to Nodes and Lines. In this chapter we discuss the basic concepts of α- cut worthy (Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and examples of Neutrosophic Graphs by level graphs.

## 2. PRELIMINARIES

**Definition 2.1.**

A Neutrosophic Graph is of the form $G = \langle N, L \rangle$ Where,

(i) $N = \{a_i, a_j, a_k, \ldots a_z\}$ such that $\lambda_T : N \to [a,b]$, $\lambda_I : N \to [a,b]$ and $\lambda_F : N \to [a,b]$ denote the degree of membership, degree of membership and non- membership of the element $a_i \in N$, respectively, with $a = 0$ and $b = 1$.

$0 \le \lambda_T(a_i) + \lambda_I(a_i) + \lambda_F(a_i) \le 3$ for every $a_i \in N$, $(i = 1, 2, \ldots n)$

(ii) $L \subseteq NxN$ Where $\eta_T : NxN \to [a,b]$, $\eta_I : NxN \to [a,b]$, and $\eta_F : NxN \to [a,b]$, are such that

$\eta_T(a_ia_j) \le \min[\lambda_T(a_i), \lambda_T(a_j),]$

$\eta_I(a_ia_j) \le \min[\lambda_I(a_i), \lambda_I(a_j)]$,

$\eta_F(a_ia_j) \le \max[\lambda_F(a_i), \lambda_F(a_j)]$, and

$0 \le \eta_T(a_ia_j) + \eta_I(a_ia_j) + \eta_F(a_ia_j) \le 3$, for every $(a_ia_j) \in L$ $(i,j = 1,2,\ldots,n)$

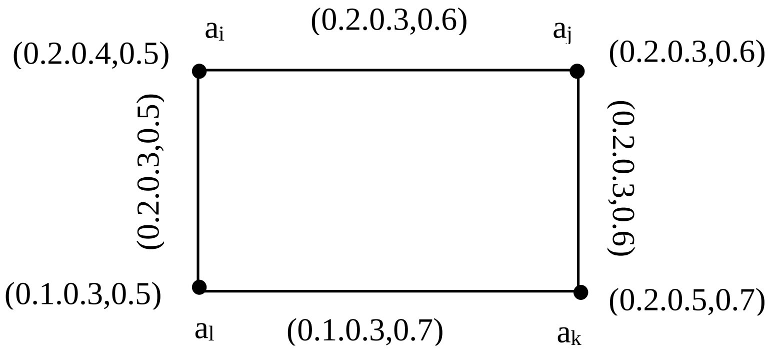


**Figure 1: Neutrosophic Graph**

**Definition 2.2.**

A Neutrosophic Graph NG = < N, L > with the triplet ( $\lambda_T, \lambda_I, \lambda_F$ ) and ( $\eta_T, \eta_I, \eta_F$ ) is called Strong Neutrosophic Graph if

$$\eta_T(a_ia_j) = \min[\lambda_T(a_i), \lambda_T(a_j)]$$
$$\eta_I(a_ia_j) = \min[\lambda_I(a_i), \lambda_I(a_j)]$$
$$\eta_F(a_ia_j) = \max[\lambda_F(a_i), \lambda_F(a_j)], \text{ for all } (a_ia_j) \in L.$$

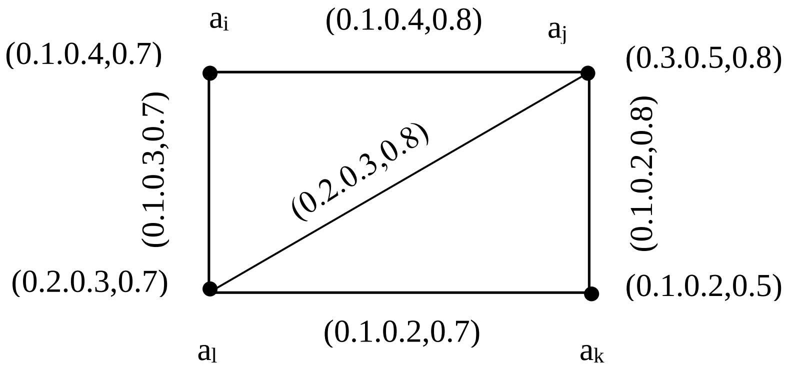


**Figure 2: StrongNeutrosophic Graph**

## 3. MAIN RESULT

**Homomorphic product of Strong Neutrosophic Graphs**

**Definition 1.** Let $(SNG)_{1_H} = (\lambda_1, \eta_I)$ and $(SNG)_{2_H} = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1_H})^* = (N_1, L_1)$ and $((SNG)_{2_H})^* = (N_2, L_2)$ . Then the Homomorphic Product of Strong Neutrosophic Graphs is defined $(SNG)_{1_H}$ *and* $(SNG)_{2_H}$ is a pair of functions $(\lambda_1 \Diamond \lambda_2, \eta_1 \Diamond \eta_2,)$ with underlying node set

$\lambda_1 \Diamond \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set

$\eta_1 \Diamond \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \in L_2 \text{ or } a_ia_j \in L_1, b_ib_j \notin L_2\}$ with

$\lambda_1 \Diamond \lambda_2)(a_i, b_i) = \min((\lambda_1)_T(a_i), (\lambda_2)_T(b_i))$, where $a_i \in N_1$ and $b_i \in N_2$ .

$(\eta_1 \Diamond \eta_2)((a_i, b_i)(a_j, b_j)) = \min((\lambda_1)_T(a_i), (\eta_2)_T(b_ib_j))$, if $a_i = a_j$ and $b_ib_j \in L_2$ .

$(\eta_1 \Diamond \eta_2)((a_i, b_i)(a_j, b_j)) = \min((\eta_1)_T(a_i, a_j), (\lambda_2)_T(b_i), (\lambda_2)_T(b_j))$, if $a_ia_j \in L_1$ and $b_ib_j \notin L_2$ .

**Boxdot Product of Strong Neutrosophic Graph:**

**Definition 2.** Let $(SNG)_{1_B} = (\lambda_1, \eta_I)$ and $(SNG)_{2_B} = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1_B})^* = (N_1, L_1)$ and $((SNG)_{2_B})^* = (N_2, L_2)$ . Then the Box dot Product of Strong Neutrosophic Graphs is defined $(SNG)_{1_B}$ *and* $(SNG)_{2_B}$ is a pair of functions

$(\lambda_1 \boxdot \lambda_2, \eta_1 \boxdot \eta_2,)$ with underlying node set

$\lambda_1 \boxdot \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set
$\eta_1 \boxdot \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \notin L_2 \text{ or } a_ia_j \in L_1, b_ib_j \notin L_2\}$ with
$(\lambda_1 \boxdot \lambda_2)_T(a_i, b_i) = \min(\ (\lambda_1)_T(a_i), (\lambda_2)_T(b_i)$, where $a_i \in N_1$ and $b_i \in N_2$.
$(\eta_1 \boxdot \eta_2)_T((a_i, b_i)(a_j, b_j)) = \min(\ (\lambda_1)_T(a_i), (\lambda_2)_T(b_i), (\lambda_2)_T(b_j)$, if $a_i = a_j$ and $b_ib_j \notin L_2$.
$(\eta_1 \boxdot \eta_2)_T((a_i, b_i)(a_j, b_j)) = \min(\ \eta_1(a_i, a_j)_T, \lambda_2(b_i), \lambda_2(b_j)$ if $a_ia_j \in L_1$ and $b_ib_j \notin L_2$.

**Star Product of Strong Neutrosophic Graph:**

**Definition 3.** Let $(SNG)_{1_S} = (\lambda_1, \eta_I)$ and $(SNG)_{2_S} = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1_S})^* = (N_1, L_1)$ and $((SNG)_{2_S})^* = (N_2, L_2)$. Then the Star Product of Strong Neutrosophic Graphs is define$(SNG)_{1_S}$ *and* $(SNG)_{2_S}$ is a pair of functions
$(\lambda_1 * \lambda_2, \eta_1 * \eta_2,)$ with underlying node set
$\lambda_1 * \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set
$\eta_1 * \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \notin L_2 \text{ or } a_ia_j \in L_1, b_ib_j \in L_2\}$ with
$(\lambda_1 * \lambda_2)_T(a_i, b_i) = \min(\ (\lambda_1)_T(a_i), (\lambda_2)_T(b_i))$, where $a_i \in N_1$ and $b_i \in N_2$.
$(\eta_1 * \eta_2)_T((a_i, b_i)(a_j, b_j)) = \min(\ (\lambda_1)_T(a_i), (\lambda_2)_T(b_i), (\lambda_2)_T(b_j)$, if $a_i = a_j$ and $b_ib_j \notin L_2$.
$(\eta_1 * \eta_2)_T((a_i, b_i)(a_j, b_j)) = \min(\ (\eta_1)_T(a_i, a_j), (\eta_2)_T(b_ib_j))$ if $a_ia_j \in L_1$ and $b_ib_j \notin L_2$.
and also for the indeterminancy, falsity.

**Definition 4.**
Let $\lambda = \{< c, \lambda_T(c), \lambda_I(c), \lambda_F(c)>, c \in N\}$, The α- cut worthy set of a Neutrosophic set λ of the set N is the crisp set $\lambda_\alpha$ is given by
$\lambda_\alpha = \{c \in N$:either $(\lambda_T(c) \geq \alpha, \lambda_I(c) \geq \alpha$ and $\lambda_F(c) \leq 1 - \alpha\}$. where $\alpha \in [0, 1]$.
Let $\eta = \{<ab, \eta_T(cc'), \eta_I(cc'), \eta_F(cc')>\}$, The α- cut worthy set of a Neutrosophic set η of the set $L \subseteq N \times N$ is the crisp set $\eta_\alpha$ is given by
$\eta_\alpha = \{ cc' \in L$: either $(\eta_T(cc') \geq \alpha, \eta_I(cc') \geq \alpha$ and $\eta_F(cc') \leq 1 - \alpha\}$.
where $\alpha \in [0, 1]$.

**Example: 1.**
In the Neutrosophic Graph NG = (λ,η) on non-empty set
$N = \{a_i, a_j, a_k\}$ as shown in Figure 4.2.

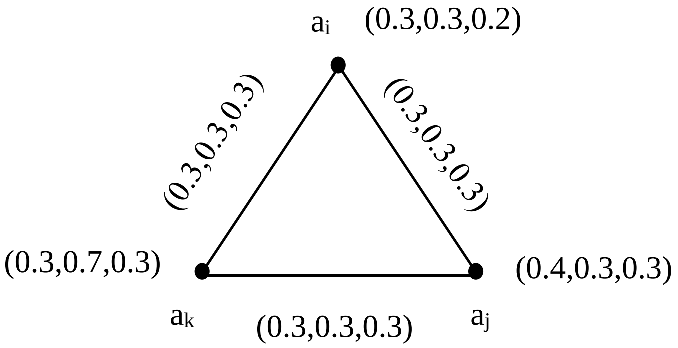


**Figure 3: Neutrosophic Graph NG = (λ, η)**

Let α = 0.3. We have $\lambda_{0.3} = \{a_i, a_j, a_k\}$, $\eta_{0.3} = \{a_ia_j, a_ja_k, a_ka_i\}$. Clearly, the 0.3 - cut worthy graph $G_{0.3} = (\lambda_{0.3}, \eta_{0.3})$is a crisp graph $G^* = (N, L)$.

**Proposition 1.** The cut worthy graph $NG_\alpha = (\lambda_\alpha, \eta_\alpha)$ is a crisp graph.

**Theorem: 1.**
**If any two Homomorphic Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs**

Let $SNG_1: (\lambda_1, \eta_1)$ and $SNG_2: (\lambda_2, \eta_2)$ be two Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*: (N_1, L_1)$ and $(SNG_2)^*: (N_2, L_2)$ respectively. Then SNG = (λ,η) is the Homomorphic Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each $\alpha \in [0, 1]$. The α- cut worthy graph $SNG_\alpha$ is the Homomorphic Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and

$(SNG_2)_\alpha$.

**Proof:**

Let NG = $(\lambda,\eta)$ be the Homomorphic Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each $\alpha \in [0, 1]$, if $(a_i,b_i) \in \lambda_\alpha$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} = (\lambda)_T(a_i, b_i) \geq \alpha$$
$$\min\{(\lambda_1)_I(a_i), (\lambda_2)_I(b_i)\} = (\lambda)_I(a_i, b) \geq \alpha$$
$$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} = (\lambda)_F(a_i, b_i) \leq 1 - \alpha$$
$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$

so, $(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. i.e,$(a_i,b_i) \in (\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha$. Therefore $\lambda_\alpha \subseteq (\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha$.

Let $(a_i,b_i) \in (\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha$. then $a_i \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T(a_i), (\lambda_2)_T(b_i) \geq \alpha, \min(\lambda_1)_I(a_i), (\lambda_2)_I(b_i) \geq \alpha,$

$\max(\lambda_1)_F(a_i), (\lambda_2)_F(b_i) \leq 1 - \alpha.$

Since $(\lambda,\eta)$ is the Homomorphic Product of $SNG_1$ and $SNG_2$.

$(\lambda)_T(a_i, b_i) \geq \alpha$, $(\lambda)_I(a_i, b_i) \geq \alpha$, $(\lambda)_F(a_i, b_i) \leq 1 - \alpha$. i.e., $(a_i) \in (\lambda_1)_\alpha$.

Therefore $(\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha \subseteq \lambda_\alpha$ and so $(\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha = \lambda_\alpha$

To prove $\eta_\alpha$= L, L is a Line set of the Homomorphic Strong Neutrosophic graph $(SNG_1)_\alpha \lozenge (SNG_2)_\alpha$ $\forall\ \alpha \in [0, 1]$. Then $(a_i, b_i)(a_j, b_j)) \in \eta_\alpha$.

Then$(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \leq 1 - \alpha$.

Since $(\lambda,\eta)$ is the Homomorphic Product of $NG_1$ and $NG_2$.

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$
$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$
$$\eta_F(a_i, b_i)(a_j, b)) = \min\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} \leq 1 - \alpha$$
$$\text{if } a_i a_j \in L_1 \text{ and } b_i b_j \in L.$$

Similarly, to the node set

$$(\eta_1 \lozenge \eta_2))_T((a_i, b_i)(a_j, b_j)) = \min\{((\lambda_1)_T(a_i),(\lambda_1)_T(a_j),(\lambda_2)_T(a_i)),(\lambda_2)_T(a_j)\},$$
$$\text{if } a_1a_2 \notin L_1 \text{and} b_1b_2 \notin L_2.$$

Similarly, the results also apply for the intermediate and falsity values.

Conversely,

Suppose that $NG_\alpha: (\lambda_\alpha, \eta_\alpha)$is the Homomorphic Product of Neutrosophic Graphs $((SNG_1)_\alpha = ((\lambda_1)_\alpha, (\eta_1)_\alpha)$ and $(SNG_2)_\alpha = ((\lambda_2)_\alpha, (\eta_2)_\alpha)$ for each $\alpha \in [0, 1]$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$$

$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} \leq 1 - \alpha.$ if $a_i \in N_1$ and $b_i \in N_2$ .

$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$., by hypothesis $(a_i, b_i) \in (\lambda)_\alpha$

$$(\lambda)_T(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_I(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_F(a_i, b_i) \leq 1 - \alpha = \max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\}$$

Take $(\lambda)_T(a_i, b_i) = \beta, (\lambda)_I(a_i, b_i) = \beta, (\lambda)_F(a_i, b_i) = 1 - \beta$, then $(a_i,b_i) \in (\lambda)_\beta$

Since $(\lambda_\beta, \eta_\beta)$ is the Homomorphic Product of Neutrosophic Graphs

$(SNG_1)_\beta = ((\lambda_1)_\beta, (\eta_1)_\beta)$ and $(SNG_2)_\beta = ((\lambda_2)_\beta, (\eta_2)_\beta)$

Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.

Hence , $(\lambda_1)_T(a_i) \geq \beta, , (\lambda_1)_I(a_i) \geq \beta, , (\lambda_1)_F(a_i) \leq 1 - \beta$, and

$(\lambda_2)_T(b_i) \geq \beta, (\lambda_2)_I(b_i) \geq \beta, , (\lambda_2)_F(b_i) \leq 1 - \beta,$

It follows that

$(\lambda)_T(a_i, b_i) = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i, b_i) \in N_1 \lozenge N_2$

$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$

Hence , $(\lambda_1)_T(a_i, a_j) \geq \alpha, (\lambda_1)_I(a_i, a_j) \geq \alpha, (\lambda_1)_F(a_i, a_j) \leq 1 - \alpha$ and

$(\lambda_2)_T(b_i, b_j) \geq \alpha, (\lambda_2)_I(b_i, b_j) \geq \alpha, (\lambda_2)_F(b_i, b_j) \leq 1 - \alpha,$

$(\eta_1)_T(a_i, a_j) \geq \beta, (\eta_1)_I(a_i, a_j) \geq \beta, (\eta_1)_F(a_i, a_j) \leq 1 - \beta$, and

$(\eta_2)_T(b_i, b_j) \geq \beta, (\eta_2)_I(b_i, b_j) \geq \beta, (\eta_2)_F(b_i, b_j) \leq 1 - \beta,$

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T\} = \alpha, \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \alpha$

$\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} = 1 - \alpha$

$\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}=\beta, \min\{(\eta_1)_I\,(a_i,a_j),(\eta_2)_I(b_i,b_j)\}=\beta \quad \max\{(\eta_1)_F\,(a_i,a_j),(\eta_2)_F\}=1-\beta$

$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \alpha \;=\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) \geq \alpha \;=\min\{(\eta_1)_I\,(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,n_i)(a_j,b_j)) \leq 1-\alpha \;=\max\{(\eta_1)_F\,(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L_2$$
$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \beta \;=\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)a_j,b_j)) \geq \beta \;=\min\{(\eta_1)_I\,(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) \leq 1-\beta \;=\max\{(\eta_1)_F\,(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L_2$$

$$\eta_T(a_i,b_i)(a_j,b_j)) \;=\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) \;=\min\{(\eta_1)_I\,(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) \;=\max\{(\eta_1)_F\,(a_i,a_j),(\eta_2)_F(b_i,b_j)\}$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L_2.$$

**Theorem: 2.**

**If any two Bot dot Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs**

Let $SNG_1:(\lambda_1,\eta_1)$ and $SNG_2:(\lambda_2,\eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*:(N_1,L_1)$ and $(SNG_2)^*:(N_2,L_2)$ respectively. Then SNG = (λ,η) is the bot dot Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each α ∈ [0, 1]. The α- cut worthy graph $SNG_\alpha$ is the box dot Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and $(SNG_2)_\alpha$.

**Proof:**

Let SNG = (λ,η) be the Box dot Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each α ∈ [0, 1], if $(a_i,b_i) \in \lambda_\alpha$.

$$\min\{(\lambda_1)_T\,(a_i),(\lambda_2)_T(b_i)\}=(\lambda)_T(a_i,b_i)\geq\alpha$$
$$\min\{(\lambda_1)_I\,(a_i),(\lambda_2)_I(b_i)\}=(\lambda)_I(a_i,b)\geq\alpha$$
$$\max\{(\lambda_1)_F\,(a_i),(\lambda_2)_F(b_i)\}=(\lambda)_F(a_i,b_i)\leq 1-\alpha$$
$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$

so, $(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. i.e, $(a_i,b_i) \in (\lambda_1)_\alpha \boxdot (\lambda_2)_\alpha$. Therefore $\lambda_\alpha \subseteq (\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha$.

Let $(a_i,b_i) \in (\lambda_1)_\alpha \boxdot (\lambda_2)_\alpha$. then $a_i \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T\,(a_i),(\lambda_2)_T(b_i)\geq\alpha, \min(\lambda_1)_I\,(a_i),(\lambda_2)_I(b_i)\geq\alpha,$

$\max(\lambda_1)_F\,(a_i),(\lambda_2)_F(b_i)\leq 1-\alpha.$

Since (λ,η) is the Box dot Product of $SNG_1$ and $SNG_2$. $(\lambda)_T(a_i,b_i)\geq\alpha$, $(\lambda)_I(a_i,b_i)\geq\alpha$, $(\lambda)_F(a_i,b_i)\leq 1-\alpha$. i.e., $(a_i) \in (\lambda_1)_\alpha$.

Therefore $(\lambda_1)_\alpha \boxdot (\lambda_2)_\alpha \subseteq \lambda_\alpha$ and so $(\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha = \lambda_\alpha$

To prove $\eta_\alpha$= L, L is a Line set of the Box dot Strong Neutrosophic Graph $(NG_1)_\alpha \boxdot (NG_2)_\alpha$ ∀ α ∈ [0, 1]. Then $(a_i,b_i)(a_j,b_j)) \in \eta_\alpha$.

Then$(a_i,b_i)(a_j,b_j))\geq\alpha$, $(a_i,b_i)(a_j,b_j))\geq\alpha, (a_i,b_i)(a_j,b_j))\leq 1-\alpha$.

Since (λ,η) is the Box dot Product of $SNG_1$ and $SNG_2$.

$$\eta_T(a_i,b_i)(a_j,b_j))=\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}\geq\alpha$$
$$\eta_T(a_i,b_i)(a_j,b_j))=\min\{(\eta_1)_T\,(a_i,a_j),(\eta_2)_T(b_i,b_j)\}\geq\alpha$$
$$\eta_F(a_i,b_i)(a_j,b))=\min\{(\eta_1)_F\,(a_i,a_j),(\eta_2)_F(b_i,b_j)\}\leq 1-\alpha$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L.$$

Similarly, to the node set

$$(\eta_1 \boxdot \eta_2))_T((a_i,b_i)(a_j,b_j)) = \min\{((\lambda_1)_T(a_i),(\lambda_1)_T(a_j),(\lambda_2)_T(a_i)),(\lambda_2)_T(a_j)\},$$
$$\text{if } a_1a_2 \notin L_1 \text{and} b_1b_2 \notin L_2.$$

Similarly, the results also apply for the intermediate and falsity values.

Conversely,

Suppose that $SNG_\alpha:(\lambda_\alpha,\eta_\alpha)$ is the Box dot Product of Strong Neutrosophic Graphs $((NG_1)_\alpha = ((\lambda_1)_\alpha,(\eta_1)_\alpha)$ and $(NG_2)_\alpha = ((\lambda_2)_\alpha,(\eta_2)_\alpha)$ for each α ∈ [0, 1].

$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$

$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} \leq 1-\alpha.$ if $a_i \in N_1$ and $b_i \in N_2$ .

$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$., by hypothesis $(a_i, b_i) \in (\lambda)_\alpha$

$$(\lambda)_T(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_I(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_F(a_i, b_i) \leq 1-\alpha = \max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\}$$

Take $(\lambda)_T(a_i, b_i) = \beta, (\lambda)_I(a_i, b_i) = \beta, (\lambda)_F(a_i, b_i) = 1-\beta$, then $(a_i,b_i) \in (\lambda)_\beta$

Since $(\lambda_\beta, \eta_\beta)$ is the box dot Product of Strong Neutrosophic Graphs

$(SNG_1)_\beta = ((\lambda_1)_\beta, (\eta_1)_\beta)$ and $(SNG_2)_\beta = ((\lambda_2)_\beta, (\eta_2)_\beta)$

Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.

Hence , $(\lambda_1)_T(a_i) \geq \beta, , (\lambda_1)_I(a_i) \geq \beta, , (\lambda_1)_F(a_i) \leq 1-\beta$, and

$(\lambda_2)_T(b_i) \geq \beta, (\lambda_2)_I(b_i) \geq \beta, , (\lambda_2)_F(b_i) \leq 1-\beta.$

It follows that

$(\lambda)_T(a_i, b_i) = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i, b_i) \in N_1 \boxdot N_2$

if $a_i \in N_1$ and $b_i \in N_2$

Hence , $(\lambda_1)_T(a_i, a_j) \geq \alpha, (\lambda_1)_I(a_i, a_j) \geq \alpha, (\lambda_1)_F(a_i, a_j) \leq 1-\alpha$ and

$(\lambda_2)_T(b_i, b_j) \geq \alpha, (\lambda_2)_I(b_i, b_j) \geq \alpha, (\lambda_2)_F(b_i, b_j) \leq 1-\alpha,$

$(\eta_1)_T(a_i, a_j) \geq \beta, (\eta_1)_I(a_i, a_j) \geq \beta, (\eta_1)_F(a_i, a_j) \leq 1-\beta$, and

$(\eta_2)_T(b_i, b_j) \geq \beta, (\eta_2)_I(b_i, b_j) \geq \beta, (\eta_2)_F(b_i, b_j) \leq 1-\beta,$

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T\} = \alpha, \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \alpha$

$\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} = 1-\alpha$

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} = \beta, \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \beta$ $\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F\} = 1-\beta$

$$\eta_T(a_i, b_i)(a_j, b_j)) \geq \alpha = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$
$$\eta_I(a_i, b_i)(a_j, b_j)) \geq \alpha = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$
$$\eta_F(a_i, n_i)(a_j, b_j)) \leq 1-\alpha = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}.$$

if $a_i a_j \in L_1$ and $b_i b_j \in L_2$

$$\eta_T(a_i, b_i)(a_j, b_j)) \geq \beta = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$
$$\eta_I(a_i, b_i)a_j, b_j)) \geq \beta = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$
$$\eta_F(a_i, b_i)(a_j, b_j)) \leq 1-\beta = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}.$$

if $a_i a_j \in L_1$ and $b_i b_j \in L_2$

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$
$$\eta_I(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$
$$\eta_F(a_i, b_i)(a_j, b_j)) = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}$$

if $a_i a_j \in L_1$ and $b_i b_j \in L_2$.

**Theorem: 3.**

**If any two Star Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs**

Let $SNG_1: (\lambda_1, \eta_1)$ and $SNG_2: (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*: (N_1, L_1)$ and $(SNG_2)^*: (N_2, L_2)$ respectively. Then SNG = $(\lambda,\eta)$ is the Star Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each $\alpha \in [0, 1]$. The $\alpha$- cut worthy graph $SNG_\alpha$ is the Star Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and $(SNG_2)_\alpha$.

**Proof:**

Let SNG = $(\lambda,\eta)$ be the Star Product of Strong Neutrosophic Graphs $SNG_1$ and $SNG_2$ for each $\alpha \in [0, 1]$, if $(a_i,b_i) \in \lambda_\alpha$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} = (\lambda)_T(a_i, b_i) \geq \alpha$$
$$\min\{(\lambda_1)_I(a_i), (\lambda_2)_I(b_i)\} = (\lambda)_I(a_i, b) \geq \alpha$$
$$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} = (\lambda)_F(a_i, b_i) \leq 1-\alpha$$

if $a_i \in N_1$ and $b_i \in N_2$

so, $(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. i.e, $(a_i,b_i) \in (\lambda_1)_\alpha * (\lambda_2)_\alpha$. Therefore $\lambda_\alpha \subseteq (\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha$.

Let $(a_i,b_i) \in (\lambda_1)_\alpha * (\lambda_2)_\alpha$.. then $a_i \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T(a_i), (\lambda_2)_T(b_i) \geq \alpha, \min(\lambda_1)_I(a_i), (\lambda_2)_I(b_i) \geq \alpha,$

$\max(\lambda_1)_F(a_i), (\lambda_2)_F(b_i) \leq 1-\alpha.$

Since (λ,η) is the Star Product of $SNG_1$ and $SNG_2$. $(\lambda)_T(a_i, b_i) \geq \alpha$, $(\lambda)_I(a_i, b_i) \geq \alpha$, $(\lambda)_F(a_i, b_i) \leq 1-\alpha$. i.e., $(a_i) \in (\lambda_1)_\alpha$.

Therefore $(\lambda_1)_\alpha * (\lambda_2)_\alpha \subseteq \lambda_\alpha$ and so $(\lambda_1)_\alpha \lozenge (\lambda_2)_\alpha = \lambda_\alpha$

To prove $\eta_\alpha = L$, L is a Line set of the Box dot Strong Neutrosophic Graph

$(SNG_1)_\alpha * (SNG_2)_\alpha\ \forall\ \alpha \in [0, 1]$. Then $(a_i, b_i)(a_j, b_j)) \in \eta_\alpha$.

Then $(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \leq 1-\alpha$.

Since (λ,η) is the Box dot Product of $SNG_1$ and $SNG_2$.

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$

$$\eta_F(a_i, b_i)(a_j, b)) = \min\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} \leq 1-\alpha$$

if $a_i a_j \in L_1$ and $b_i b_j \in L$.

Similarly, to the node set

$(\eta_1 * \eta_2))_T((a_i, b_i)(a_j, b_j)) = \min\{((\lambda_1)_T(a_i), (\lambda_1)_T(a_j), (\lambda_2)_T(a_i)), (\lambda_2)_T(a_j)\}$,

if $a_1a_2 \notin L_1$ and $b_1b_2 \notin L_2$.

Similarly, the results also apply for the intermediate and falsity values.

Conversely,

Suppose that $SNG_\alpha: (\lambda_\alpha, \eta_\alpha)$ is the Box dot Product of Strong Neutrosophic Graphs

$((SNG_1)_\alpha = ((\lambda_1)_\alpha, (\eta_1)_\alpha)$ and $(SNG_2)_\alpha = ((\lambda_2)_\alpha, (\eta_2)_\alpha)$ for each $\alpha \in [0, 1]$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$$

$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} \leq 1-\alpha$. if $a_i \in N_1$ and $b_i \in N_2$.

$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$, by hypothesis $(a_i, b_i) \in (\lambda)_\alpha$

$$(\lambda)_T(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$

$$(\lambda)_I(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$

$$(\lambda)_F(a_i, b_i) \leq 1-\alpha = \max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\}$$

Take $(\lambda)_T(a_i, b_i) = \beta$, $(\lambda)_I(a_i, b_i) = \beta$, $(\lambda)_F(a_i, b_i) = 1-\beta$, then $(a_i, b_i) \in (\lambda)_\beta$

Since $(\lambda_\beta, \eta_\beta)$ is the box dot Product of Strong Neutrosophic Graphs

$(NG_1)_\beta = ((\lambda_1)_\beta, (\eta_1)_\beta)$ and $(NG_2)_\beta = ((\lambda_2)_\beta, (\eta_2)_\beta)$

Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.

Hence, $(\lambda_1)_T(a_i) \geq \beta$, $(\lambda_1)_I(a_i) \geq \beta$, $(\lambda_1)_F(a_i) \leq 1-\beta$, and

$(\lambda_2)_T(b_i) \geq \beta$, $(\lambda_2)_I(b_i) \geq \beta$, $(\lambda_2)_F(b_i) \leq 1-\beta$,

It follows that

$(\lambda)_T(a_i, b_i) = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i, b_i) \in N_1 * N_2$

if $a_i \in N_1$ and $b_i \in N_2$

Hence, $(\lambda_1)_T(a_i, a_j) \geq \alpha$, $(\lambda_1)_I(a_i, a_j) \geq \alpha$, $(\lambda_1)_F(a_i, a_j) \leq 1-\alpha$ and

$(\lambda_2)_T(b_i, b_j) \geq \alpha$, $(\lambda_2)_I(b_i, b_j) \geq \alpha$, $(\lambda_2)_F(b_i, b_j) \leq 1-\alpha$,

$(\eta_1)_T(a_i, a_j) \geq \beta$, $(\eta_1)_I(a_i, a_j) \geq \beta$, $(\eta_1)_F(a_i, a_j) \leq 1-\beta$, and

$(\eta_2)_T(b_i, b_j) \geq \beta$, $(\eta_2)_I(b_i, b_j) \geq \beta$, $(\eta_2)_F(b_i, b_j) \leq 1-\beta$,

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T\} = \alpha$, $\min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \alpha$

$\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} = 1-\alpha$

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} = \beta$, $\min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \beta$ $\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F\} = 1-\beta$

$$\eta_T(a_i, b_i)(a_j, b_j)) \geq \alpha = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$

$$\eta_I(a_i, b_i)(a_j, b_j)) \geq \alpha = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$

$$\eta_F(a_i, n_i)(a_j, b_j)) \leq 1-\alpha = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}.$$

if $a_i a_j \in L_1$ and $b_i b_j \in L_2$

$$\eta_T(a_i, b_i)(a_j, b_j)) \geq \beta = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$

$$\eta_I(a_i, b_i)a_j, b_j)) \geq \beta = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$

$$\eta_F(a_i, b_i)(a_j, b_j)) \leq 1-\beta = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}.$$

if $a_i a_j \in L_1$ and $b_i b_j \in L_2$

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\}$$

$$\eta_I(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_I (a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$
$$\eta_F(a_i, b_i)(a_j, b_j)) = \max\{(\eta_1)_F (a_i, a_j), (\eta_2)_F(b_i, b_j)\}$$
if $a_i a_j \in L_1$ and $b_i b_j \in L_2$.

## CONCLUSION

In this paper, we have found the α- cut worthy (Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and examples of Strong Neutrosophic Graphs by α- cut worthy (Level) graphs.

## REFERENCES


1. Different Type of Product of Anti Fuzzy Graph. AIP – Conference Proceeding 2194,020128(2019) by Y. Trisant, Toto Nusantara, Desi Rahmadani, Abdullah Bin Gani and Heny Susanto. https://doi.org/10.1063/1.5139860

2. Mordeson, J.N., and P. Chang-Shyh, Operations on fuzzy graphs, *Information Sciences*, 79, pp. 159 −170, 1994.
3. M.Pal, S.Samanta and H.Rashmanlou, Some results on interval-valued fuzzy graphs, *International Journal of Computer Science and Electronics Engineering*, 3(3) (2015) 205-211.
4. Ozge Colakaglu, Hamza Menken, *On Corona Product of Two Fuzzy Graphs*, International Journal of Fuzzy Mathematical Archive, Vol. 10, No. 2, Pp. 95 – 100, 2016.
5. G.Nirmala and M.Vijaya, Fuzzy graphs on composition, tensor and normal products, *International Journal of Scientific and Research Publications*, 2(2012) 1-7.
6. Muhammad Akram and Gulfam Shahzadi, Operations on Single Valued Neutrosophic Graphs, Journal of Uncertain Systems, Vol.11, No.1, Pp. 1-26, 2017.
7. Gulistan M, Yaqoob N, Rashid Z, Smarandache F, Wahab H.A. A Study on Neutrosophic Cubic Graphs with Real Life Applications in Industries. *Symmetry* **2018**, *10*, 203. https://doi.org/10.3390/sym10060203
8. An approach to(μ, ν, ω) -single-valued neutrosophic submodules Muhammad Shazib Hameed , ZaheerAhmad , ShahbazAli , Muhammad Kamran & Alphonse-Roger Lula Babole. Scientifc Reports | (2023) 13:751 | https://doi.org/10.1038/s41598-022-18500-5
9. Şahin, m., & kargin, a. (2019). Single valued neutrosophic quadruple graphs. Asian Journal of Mathematics and Computer Research, 26(4), 243-250. Retrieved from https://www.ikppress.org/index.php/AJOMCOR/article/view/4817
10. Liu, R. (2020), Study on single-valued neutrosophic graph with application in shortest path problem. CAAI Trans. Intell. Technol., 5: 308-313. https://doi.org/10.1049/trit.2020.0111
11. Mohanta, K., Dey, A. & Pal, A. A note on different types of product of neutrosophic graphs. *Complex Intell. Syst.* **7**, 857–871 (2021). https://doi.org/10.1007/s40747-020-00238-0
12. Some operations on Domi Neutrosophic Graph Karthick Mohanta, Arindam Dey, Anitha Pal, Sankar Prasad Mondal, Tejinda singh Lakhwan, January 2022.Journal of Amlient Intelligence and Tumanized Computing 13(4): 1-19. https://doi.org/10.1007/s12652-021-02909-3