THE PROOF: GOLDBACH'S CONJECTURE

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Abstract. In this paper, I want to present the proof to 'The Goldbach's Conjecture'. I have used fundamental concepts of number theory and fundamental methods of proof like contradiction to prove the conjecture.

1. Introduction

Conjecture:- *Every even natural number greater than 2 is the sum of two prime numbers.*

Proof. Assuming the above mentioned conjecture to be false, there must be any even integer greater than 2 which cannot be expressed as the sum of 2 prime numbers.

Let the even integer be of the form 2m ($m \in N$, $m > 1$). Therefore, as per our asumption, we need to show that the sum of 2 primes is not equal to atleast one value of $2m$ to claim that our assumption was correct:-

$$p + p' \neq 2m(p, p' \in prime) \tag{1}$$

- Prime numbers are both odd as well as even(i.e. 2) but as we know that sum of an odd and even number yields to odd number, we shall consider 2 cases.

Case I:- (Even Prime + Even Prime)

- Let $p, p' = 2$. (as 2 is the only even prime.)

$$p + p' = 2 + 2 = 4 = 2m \tag{2}$$

- As 4 is an even number, it is of the form $2m$.

Case II:- (Odd Prime + Odd Prime)

- **Any odd prime is of the form $4k \pm 1$. ($k \in N$)**

Proof. Let n be any odd prime.If we divide any n by 4, we get, $n = 4q + r$ ($q, r \in Z$) where $0 \leq r < 4$ i.e., $r = 0, 1, 2, 3$. Clearly, 4n is never prime and $4n+2 = 2(2n+1)$ cannot be prime unless n=0 (since, 4 and 2 cannot be factors

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of an odd prime). Thus, An odd prime n is either of the form $4q+1$ or $4q+3$ (or $4q'-1$ where $q'=q+1$) i.e. it is of the form $4q \pm 1$.

- Let p = $4k \pm 1$ and p' = $4k' \pm 1 (k, k' \in N)$.

  Therefore, when two odd primes are added, 4 cases can arise.

$$p + p' = 4k \pm 1 + 4k' \pm 1 \tag{3}$$

Case A:-

$$p + p' = 4k + 1 + 4k' + 1 = 2(2k + 2k' + 1) = 2m \tag{4}$$

Case B:-

$$p + p' = 4k - 1 + 4k' - 1 = 2(2k + 2k' - 1) = 2m \tag{5}$$

Case C:-

$$p + p' = 4k + 1 + 4k' - 1 = 2(2k + 2k') = 2m \tag{6}$$

Case D:-

$$p + p' = 4k - 1 + 4k' + 1 = 2(2k + 2k') = 2m \tag{7}$$

- In all the cases, the outcome is always of the form 2m i.e. it is even and thus no values of 2m exists which follows our assumption. Therefore, every even natural number greater than 2 can be represented as the sum of two prime numbers. This yields to a contradiction. Therefore, our assumption was wrong.
- Thus, the above mentioned conjecture (Goldbach's conjecture) is proved.

□

References

[1] M. Burton, *Elementary Number Theory*, McGraw-Hill, 2010.