THE PROOF: GOLDBACH'S CONJECTURE

ANIKET BHATTACHARJEE

ABSTRACT. In this paper, I want to present the proof to one of the most famous conjecture - The Goldbach's Conjecture.

1. INTRODUCTION

Conjecture:- *Every even natural number greater than 2 is the sum of two prime numbers.*

Proof.

- Let C be the set of even natural numbers which follow the conjecture such that it is the sum of 2 prime numbers. Thus, $C \in \{4, 6, 8, 10, ...\}$.
- Prime numbers are both odd as well as even(i.e. 2). Therefore, 3 cases arise to prove the conjecture.

Case I:- (Even Prime)

- Considering the even prime number i.e. 2, the sum of two even primes is always 4.
- *The first element of set C is obtained in this case.*

Case II:- (Odd Prime)

- **Any odd prime is of the form $4k \pm 1$. ($k \in N$)**

Proof. Let n be any odd prime.If we divide any n by 4, we get, $n = 4q + r$ $(q, r \in Z)$ where $0 \leq r < 4$ i.e., $r = 0, 1, 2, 3$. Clearly, 4n is never prime and $4n+2 = 2(2n+1)$ cannot be prime unless n=0 (since, 4 and 2 cannot be factors of an odd prime). Thus, An odd prime n is either of the form $4q + 1$ or $4q + 3$ (or $4q' - 1$ where $q' = q + 1$) i.e. it is of the form $4q \pm 1$.

- Therefore, when two odd primes are added, 4 cases can arise $(m \in N)$

$$4k \pm 1 + 4k' \pm 1 (k, k' \in N) \tag{1}$$

Case A:-

$$= 4k + 1 + 4k' + 1 = 2(2k + 2k' + 1) = 2m \tag{2}$$

Date: July 11, 2023.

Case B:-

$$= 4k - 1 + 4k' - 1 = 2(2k + 2k' - 1) = 2m \tag{3}$$

Case C:-

$$= 4k + 1 + 4k' - 1 = 2(2k + 2k') = 2m \tag{4}$$

Case D:-

$$= 4k - 1 + 4k' + 1 = 2(2k + 2k') = 2m \tag{5}$$

- In all the 4 cases, the outcome is of the form 2m i.e. it is even and it is always true as addition of 2 odd (prime) numbers is always even.
- *On substituting different values to k and k', we obtain the rest of the values of set C.*

Case III: (Odd Prime + Even Prime)

- The sum of odd prime and even prime number is not possible as it gives an odd number which does not follow the conjecture which is based on even natural numbers.

Thus, the above mentioned conjecture (Goldbach's conjecture) is proved. □

References

[1] M. Burton, *Elementary Number Theory*, McGraw-Hill, 2010.