Generalized Prime Number Sequence With Proof

Jihyeon Yoon*

July 28, 2023

Abstract

Prime numbers could be formalized with using notation of sequence.

1 Generalized Prime Number Sequence

Prime number[1] could be defined as following:

$$p \in \mathbb{N} \; is \; \textbf{prime} \iff \nexists n \in \mathbb{N} \; s.t. \; n \mid p \; and \; n \neq p, 1.$$

Generalized Prime Number Sequence(GPNS) is:

$$\begin{aligned}
&M_n = n(n+1)(n+2) \\
&\implies \forall p \geq 3 \implies p_n = k + \alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3 \cdots \\
&(\forall \alpha \in \mathbb{P} \cup \{0, 1\}, k \in \{0, 1, 3, 5\})
\end{aligned}$$

*Jihyeon Yoon(Independent researcher) is a korean medicine doctor. And he is a freelancer programmer also.
E-mail: flyingtext@nate.com
Yeongdeungpo-gu, Seoul, 07429,
Seoul, South Korea
ORCiD: https://orcid.org/0000-0001-9610-0994

2 Proof

Assuming that, prime number p, p', p'' always can be expressed into this cubic equation form:

$$p'' = (n^3 + an^2 + bn)p' + p \ (a, b, n \in \mathrm{N})$$

As primes are all odd number except 2, it would be for prime number larger than 2:

$$Odd = Even + Odd$$

And this leads to:

$$0 \equiv (n^3 + an^2 + bn) \mod 2$$

If so, with transformation:

$$Assuming\ that,\ (n^3 + an^2 + bn) = (n - c)(n - d)(n - e) \Rightarrow c \cdot d \cdot e = 0$$
$$\Rightarrow (n - c)(n - d)(n - e) = n(n - f)(n - g)$$

And to make this number even:

$$n\ is\ odd, \Rightarrow^{\forall} f,^{\forall} g,\ (0 \equiv (n - f) \mod 2)\ or\ (0 \equiv (n - g) \mod 2)$$
$$n\ is\ even \Rightarrow^{\forall} a,^{\forall} b,\ 0 \equiv (n^3 + an^2 + bn) \mod 2$$

Then again, assuming n is odd:

$$(n^3 - (f + g)n^2 + fgn)p' + p = p''$$

At least example with $p = 2$, $p' > 2$, this equation always gives a chance as p to be odd number:

$$2(n^3 - (f + g)n^2 + fgn) + p' = p''$$

And with $p > 2$, $p' > 2$:

$$(2k - 1)(n^3 - (f + g)n^2 + fgn) + (2k' - 1) = (2k'' - 1)$$
$$(p = 2k - 1, p' = 2k' - 1, p'' = 2k'' - 1)$$

And for this condition, Equation $(n^3 - (f+g)n^2 + fgn)$ always has to be even. And by given condition:

$$(Odd)^3 - (f+g)(Odd)^2 + fg(Odd) \Rightarrow \begin{cases} (f+g)\ is\ odd,\ fg\ is\ even \\ (f+g)\ is\ even,\ fg\ is\ odd \end{cases}$$
$$\Rightarrow (f+g)\ is\ odd,\ fg\ is\ even$$

And for least working case with $n = 1$, $p = 2$, $p' = 3$:

$$2(1 - (f+g) + fg) + 3 = p''$$

As $f, g \in \mathbf{N}$, p'' must be at least 3. And:

$$\min_{f,g \in \mathbf{N}} (-1 = fg - (f+g)) \Rightarrow f = 1, g = 2 \quad (without\ specific\ order)$$

Again, with foremost cubic equation:

$$p'' = (n^3 - 3n^2 + 2n)p' + p$$

Then, with making upper $n^3 - 3n^2 + 2n$ equation larger than 0 in any n in natural number, let's assume q-th and $(q+1)$-th prime number as:

$$p_{q+1} = (t^3 + 3t^2 + 2t)p_q + p_u\ (t, q, u \in \mathbf{N})$$

As for middle conclusion, if p_q should be expressed in $(t^3 + at^2 + bt)p' + p\ (a, b \in \mathrm{N})$ form, it should take upper form.

Next, let's assume that there is a prime number p_x that could not be expressed in upper form:

$$p_x = (t^3 + 3t^2 + 2t)p_q + p_u + r \neq (t^3 + 3t^2 + 2t)p_m + p_l \quad (r, m, l \in \mathbf{N}, r \neq 0, r \notin \mathbf{P})$$

In this case, r has to be an even number. As repeating assumption:

$$\begin{aligned} p_x &= (t^3 + 3t^2 + 2t)p_q + p_u + r = (t^3 + 3t^2 + 2t)p_q + (j^3 + 3j^2 + 2j)p_s + p_v + r \\ &= (t^3 + 3t^2 + 2t)p_q + (j^3 + 3j^2 + 2j)p_s + \cdots + p_y + r \end{aligned}$$

In this equation, $p_y + r$ has to be less than 6, and it contradicts the upper assumption in any case.

3 Conclusion

There is a rule in prime number sequence.

References

[1] Jean-Marie De Koninck and Nicolas Doyon. *The Life of Primes in 37 Episodes*. American Mathematical Soc., 5 2021.