Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI: 10.21474/IJAR01/17102
DOI URL: http://dx.doi.org/10.21474/IJAR01/17102

RESEARCH ARTICLE

EXPONENTIAL–TIME DIFFERENCING METHOD FOR SOLVING REACTOR POINT KINETICS EQUATIONS WITH TEMPERATURE AND POWER FEEDBACK AND ITS VALIDATION BY ANALYSINGTHERMAL REACTOR BENCHMARKS

P. Ravindra Babu[1], Usha Pal[2], R. Karthikeyan[2], R.N. Sen[2] and M.M.A. Razak[3]

1. Bhabha Atomic Research Centre, Visakhapatnam, India.
2. Bhabha Atomic Research Centre, Mumbai, India.
3. Bhabha Atomic Research Centre, Kalpakkam, India.

Manuscript Info

Manuscript History
Received: 15 April 2023
Final Accepted: 19 May 2023
Published: June 2023

Abstract

Exponential Time Differencing method with Taylor Series Expansion (ETD-TS) is used for solving nonlinear systems of point kinetics equations (PKEs). Integral forms of PKE are expanded using Taylor Series Expansion method. Time step value and Taylor series expansion order (n) is chosen to minimize the truncation error.PKEs are highly stiff and nonlinear in nature in presence of thermal feedback. So, advanced method (ETD-TS) is used in the present work to estimate the power transients. Thermal Reactors benchmark with stepandcompensated ramp reactivity insertions are considered for estimation of power transients with feedback. Comparison of results with other accurate numerical methods likeCATS (Convergence Accelerated Taylor Series), SCS (Shifted Chebyshev Series), ITS2 (Integral Taylor Series)etc confirms the validity and accuracy of this method.Time step of 10^{-2} – 10^{-4}sec is considered for estimationof power and reactivity transients.

Introduction:-

The solution of linear Boltzmann transport equation is coupled with precursor concentration equationsthat are in different time scales. To estimate the accurate value of neutron densities and precursor concentrations, temperature and power feedbacks needs to be applied. Due to the presence of feedback, nonlinearity is introduced in PKEs.PKEs are derived from the time dependent neutron transport equations. After, considering weight based approximations, these integro-differential equations are converted to ODEs. For reactor safety, it is important to estimate the power values at different time intervals by solving point kinetics equations. These equations provide useful information on the dynamics of the reactor core during transients that involve considerable power change in the nuclear power plant. Temperature feedback effect is considered in PKEs for improving the accuracy of results. Different numerical schemes are available in literature to solve PKEs with temperature and power feedback. Recent examples are, methods based on Reactivity Piecewise Constant Approximations(Kinardand Allen,2004), ET technique, based on Backward Euler (ET-BE) and Crank Nicholson (ET-CN) approximations (Nahla,2011), The Converged Accelerated Taylor Series Method (CATS) (Ganapol and Picca et al, 2012), The Enhanced Piecewise Constant Algorithm, EPCA (Picca etal,2013), Integral Taylor Series, ITS-2 (Serigo.Q et al, 2016) and Shifted Chebyshev Series,SCS (Hamda,2018.Each of these methods are having different levels of accuracy, computational time and complexity in implementing the algorithm, The numerical scheme used in this paper is based on Exponential Time Differencing

Corresponding Author:- P. Ravindra Babu
Address:- Bhabha Atomic Research Centre, Visakhapatnam, India.

with Taylor series expansion (ETD-TS), which involves the exact integration of PKEs followed by multiplying by integrating factor. The integrand is expanded withTaylor series expansion method. Recently, (M.M.A.Razak etal., 2015) used ETD method with Taylor series for solving the PKEs with step and ramp reactivity insertions in the absence of feedback, however in this paper we had used ETD-TS method for estimation of power transients by solving PKEs in the presence of temperature and power feedback. The coefficient matrix 'A' used here is 8×8 matrix, here the last column of the coefficient matrix is corresponds to the function U(t) which is dependent on temperature of the reactor core, which is different from (M.M.A.Razak etal., 2015) ETD-TS algorithm.

Point kinetics equations and ETD-TS method

Point kinetics equations are coupled equations and has to be solved simultaneously. ETD – TS method has four different steps:

Multiplying the PKE's with exponential Integrating factor

1. Integrating the PKEs within the time limits 't_n' to 't_{n+1}'
2. Expanding the neutron density and precursor concentrations using the Taylor series expansion
3. Estimation of neutron density and precursor concentrations at $n+1^{th}$ time step.

$$\frac{dP(t)}{dt} = \frac{(\rho(t)-\beta)}{\Lambda}P(t) + \sum_{i=1}^{6}\lambda_i * C_i(t) \ldots\ldots(1) \qquad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda}P(t) - \lambda_i * C_i(t) \qquad (2)$$

P(t) = power in watts; $C_i(t)$ = i^{th} group precursors concentration (atoms/cm^3) ;
β= effective value of beta;Λ = mean neutron generation time;
λi = decay constant of i^{th} group precursor;βi = i^{th} group precursor beta value;

For simplicity P(t) which is the neutron number density can be assumed as power of the reactor by assuming praportinal

From Eq.(1) and Eq.(2) we can write PKE with feedback in the matrix form as shown below:

$$\begin{pmatrix} dP(t)/dt \\ dC1(t)/dt \\ dC2(t)/dt \\ dC3(t)/dt \\ dC4(t)/dt \\ dC5(t)/dt \\ dC6(t)/dt \\ dTemp(t)/dt \end{pmatrix} = \begin{pmatrix} \frac{(\rho(t)-\beta)}{\Lambda} & \lambda1 & \lambda2 & \lambda3 & \lambda4 & \lambda5 & \lambda6 & 0.0 \\ \beta_1/\lambda1 & -\lambda1 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_2/\lambda2 & 0.0 & -\lambda2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_3/\lambda3 & 0.0 & 0.0 & -\lambda3 & 0.0 & 0.0 & 0.0 & 0.0 \\ \beta_4/\lambda4 & 0.0 & 0.0 & 0.0 & -\lambda4 & 0.0 & 0.0 & 0.0 \\ \beta_5/\lambda5 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda5 & 0.0 & 0.0 \\ \beta_6/\lambda6 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -\lambda6 & 0.0 \\ U(t) & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} \begin{pmatrix} P(t) \\ C1(t) \\ C2(t) \\ C3(t) \\ C4(t) \\ C5(t) \\ C6(t) \\ Temp(t) \end{pmatrix}$$

Right hand side of above 8×8 square matrix is the coefficient matrix 'A(t)'.

The general form of first order stiff ODE (Cox,2002)is written as:

$$\frac{du}{dt} = cu + F(u,t) \qquad (3)$$

Here 'c' is stiffness coefficient.The value of $u(t_{n+1})$ can be written as follows after applying the procedure of exponential differencing method(Cox,2002):

$$u(t_{n+1}) = u(t_n)e^{ch} + e^{ch}\int_0^h e^{-c\tau}F(u(t_n+\tau),t_n+\tau)d\tau \quad (4)$$

$$P(t_n+1) = P(tn)e^{\frac{-h}{\Lambda}} + \frac{1.0}{\Lambda}\int_0^h P(tn+\tau)e^{\frac{\tau-h}{\Lambda}}\left(1.0-\beta+\rho(t_n+\tau)\right)d\tau$$

$$+\int_0^h e^{\frac{\tau-h}{\Lambda}}\sum_{i=1}^{6}\lambda_i C_i(t_n+\tau)\,d\tau \quad (5)$$

$$C_i(t_n+1) = C_i(t_n)e^{\frac{-h}{\Lambda}} + \left(\frac{\beta_i}{\Lambda}\right)\int_0^h e^{\frac{\tau-h}{\Lambda}}P(t_n+\tau)d\tau + \left(\frac{1.0}{\Lambda}-\lambda_i\right)\int e^{\frac{\tau-h}{\Lambda}}C_i(t_n+\tau)d\tau \qquad (6)$$

Eqns.(5) and (6) can be obtained after integration of (1) and (2) and multiplying both sides of equation (1) and (2) with integrating factor et/Λ(M.M.A.Razak et al,2015,Cox et al,2002):

With Taylor series expansion power at time $t_n+\tau$ can be written as shown below:

$P(t_n + \tau) = \sum_{i=0}^{n} \frac{\partial^i P(t_n)}{\partial \tau^i} \frac{\tau^i}{i!}$(7)

DifferentialCoefficients of power procurer concentrations, Reactivity and Temperature functions are obtained using the above matrix equations as follows:

$$\frac{dF(t_n)}{dt} = A(t_n)F(t_n)$$

While estimating the higher order derivatives as we are choosing the smaller time steps, derivativtives of $A(t_n)$ can be neglected. matrix equation can be written as follows:

$\frac{d^2F(t_n)}{dt^2} = A^2(t_n)F(t_n)$ ..... $\frac{d^nF(t_n)}{dt^2} = A^n(t_n)F(t_n)$

These Higher order derivatives are used in the taylor series expansion of $P(t_n + \tau)$, $C_i(tn + \tau)$,$\rho(tn + \tau)$ functions estimations in Eq(5) and Eq(6).

TemperatureFeedback and it'sNumerical Solution

Step change in Reactivity with thermal feedback (adiabatic model), which is mentioned in (Hetrick, 1971; Stacey, 2001) is shown below:

$\rho(tn) = \rho_0 - \alpha[Temp(t) - Temp_0]$------- (8)

$$\frac{dTemp(t)}{dt} = Hn(t) \text{ ..... } (9)$$

Here, Temp(t) is the reactor temperature at time 't' and $Temp_0$ is the initial temperature of the reactor 'α' is the Temperature coefficient of reactivity, H=AKc, whose units are K/MW.s is the reciprocal of the thermal capacity of the reactor and units of A' are MW.cm3. Differtinating Eq. (8) and substituting in Eq. (9), we obtain:

$$\frac{d\rho(t)}{dt} = -\alpha Hn(t) \text{ ..... } (10)$$

After applying the procedure of (COX et al,2002) Eq. (9) & (10) can be written in the following integral form:

$\rho(t_{n+1}) = \rho(t_n)e^{-h/\Lambda} + e^{-\frac{h}{\Lambda}}(-\alpha H \int_0^h n(t_n + \tau)e^{\frac{\tau}{\Lambda}}d\tau + \frac{1.0}{\Lambda}\int_0^h \rho(t_n + \tau)e^{\frac{\tau}{\Lambda}}d\tau)$ ... (11)

$$Temp(t_{n+1}) = Temp(t_n)e^{-h/\Lambda} + e^{-\frac{h}{\Lambda}}(H\int_0^h n(t_n + \tau)e^{\frac{\tau}{\Lambda}}d\tau + \frac{1.0}{\Lambda}\int_0^h Temp(t_n + \tau)e^{\frac{\tau}{\Lambda}}d\tau)). (12)$$

Equations (11)&(12) are solved using Taylor series expansion method, These equations can be used for estimation of temperature and reactivity values at 'tn+1' sec.

Temperature Feedback and it's Numerical Solution Power Transients of thermal reactor caused by step reactivity with Temperature Feedback

ETD-TS method has been applied to solve the point kinetics equations for thermal reactors, in presence of Newtonian temperature feedback. Six groups of delayed neutrons are consideredin this problem. The decay constants of neutron precursors, delayed neutron fractions and feedback parameters for thermal reactor are taken as $\lambda1 = 0.0124s-1$, $\lambda2 = 0.0305s-1$, $\lambda3 = 0.111$ sec-1, $\lambda4 = 0.301$ sec-1, $\lambda5 = 1.13$ sec-1,. $\lambda6 = 3.00$ sec-1, $\beta1 = 0.00021$, $\beta2 = 0.00141$, $\beta3 = 0.00127$, $\beta4 = 0.00255$, $\beta5 = 0.00074$, $\beta6 = 0.00027$, $\beta eff = 0.00645$, $\Lambda = 0.00005$ (sec), $\alpha = 0.00005$ (K-1) and Kc = 0.05 (K/MWs).

Transient power, reactivity and temperature values initiated by step reactivity addition of 1.0 $, 1.5$ and 2.0 $ with temperature feedback was studied using ETD-TS method. The results are summarized in Table 1, 2 and 3 respectively for 1.0 $, 1.5$ and 2.0 $. Results are compared with other numerical methods ET-BE, ET-CN, ITS2 and CATS method values. It is observed that even with higher step reactivity addition of 2 $, ETD-TS method is more accurate than ET-BE and ET-CN methods while estimating the transient power and reactivity values. Estimated values using ETD-TS method are giving identical results when compared with highly accurate i.e, ITS2 method values. Integrands in the Equations (4)-(6) & (8) expanded up to five Taylor series terms for better accuracy of ETD-TS method.

In the Table.1 ET-BE, ET-CN and NAM methods are using fixed time step of h=0.001 sec, which is much larger than the time step sizes h=0.01 sec used in the ETD-TS method. computational time for ETD-TS method is 26 sec

for 1.0, 1.5 and 2.0$ step reactivity insertion case, with Intel ® Core ™ i9 computer and processer speed 2.8 GHz, which is less when compared with ITS2 method i.e,28.0 45.0, 52 sec.ITS2 method takes more computational time when compared with ETD-TS method. But, whereas ETD-TS method uses constant computational time for all the three cases.

Table 1:- Comparison of transient values of power and reactivity values initiated due to step reactivity 1.0 $.
* Numbers in the parenthesis denotes powers of 10.

Time (sec)	**Power (Watt)**				**ρ($)**				**Temp ETD-TS Method (^{0}K)**
	ET-BE Method Ref.	**ET-CN Method Ref.**	**ITS-2 Method Ref.**	**ETD-TS Method(h=0.01 sec)n=5**	**ET-BE Method Ref.**	**ET-CN Method Ref.**	**ITS2 Method (Ref.)**	**ETD-TS Method**	
0.0	1.00000	1.0000	1.0000	1.0000	1.00000	1.00000	1.00000	1.00000	293.0000
10.0	1.3210E+02	1.3206E+02	1.3203E+02	1.3203E+02	-0.18579	-0.18604	-0.18629	-0.18629	446.0327
20.0	5.1727E+01	5.1716E+01	5.1699E+01	5.1699E+01	-0.50585	-0.50595	-0.50622	-0.50623	487.3038
30.0	2.8189E+01	2.8187E+01	2.8174E+01	2.8174E+01	-0.65332	-0.65321	-0.65361	-0.65361	506.3163
40.0	1.8154E+01	1.8155E+01	1.8146E+01	1.8146E+01	-0.74087	-0.74070	-0.74110	-0.74110	517.6022
50.0	1.2788E+01	1.2786E+01	1.2779E+01	1.2779E+01	-0.79982	-0.80018	-0.80010	-0.80010	525.2134
60.0	9.4828E+00	9.4898E+00	9.4749E+00	9.4749E+00	-0.84241	-0.84122	-0.84277	-0.84277	530.7180
70.0	7.2485E+00	7.2324E+00	7.2444E+00	7.2444E+00	-0.87463	-0.87818	-0.87491	-0.87491	534.8647
80.0	5.6467E+00	5.6534E+00	5.6462E+00	5.6462E+00	-0.89997	-0.89771	-0.89974	-0.89974	538.0667
90.0	4.4588E+00	4.4684E+00	4.4568E+00	4.4568E+00	-0.91892	-0.91619	-0.91921	-0.91921	540.5786
100.0	3.5507E+00	3.5531E+00	3.5501E+00	3.5501E+00	-0.93474	-0.93468	-0.93465	-0.93465	542.5704

Table 2:- Comparison of transient values of power and reactivity values initiated due to step reactivity (1.5 $).

	ET-BE Method Ref.	**ET-CN Method Ref.**	**ITS2 Method Ref.**	**ETD-TS Method(h=0.01 sec)n=5**	**ET-BE Method Ref.**	**ET-CN Method Ref.**	**ITS2 Method (Ref.)**	**ETD-TS Method**	
0.0	1.00000	1.0000	1.0000	1.0000	1.50000	1.50000	1.50000	1.50000	293.0000
10.0	1.08546E+02	1.08231E+02	1.07911E+02	1.07911E+02	-0.82008	-0.82562	-0.83150	-0.83152	593.7647
20.0	4.18325E+01	4.17205E+01	4.16042E+01	4.16042E+01	-1.07899	-1.08380	-1.08895	-1.08897	626.9756
30.0	2.34123E+01	2.33593E+01	2.32989E+01	2.32989E+01	-1.19961	-1.20381	-1.20902	-1.20903	642.4644
40.0	1.53686E+01	1.53360E+01	1.53034E+01	1.53034E+01	-1.27331	-1.27732	-1.28210	-1.28211	651.8907
50.0	1.09320E+01	1.09096E+01	1.08901E+01	1.08901E+01	-1.32384	-1.32773	-1.33213	-1.33214	658.3454
60.0	8.13017E+00	8.11509E+00	8.10103E+00	8.10104E+00	-1.36068	-1.36469	-1.36857	-1.36858	663.0460
70.0	6.20461E+00	6.19337E+00	6.18269E+00	6.18269E+00	-1.38849	-1.39177	-1.39604	-1.39605	666.5895
80.0	4.81225E+00	4.81004E+00	4.79331E+00	4.79331E+00	-1.40904	-1.41025	-1.41717	-1.41718	669.3160
90.0	3.76957E+00	3.76577E+00	3.75561E+00	3.75561E+00	-1.42606	-1.42874	-1.43365	-1.43366	671.4411
100.0	2.97798E+00	2.96556E+00	2.96607E+00	2.96607E+00	-1.43869	-1.44722	-1.44661	-1.44662	673.1127

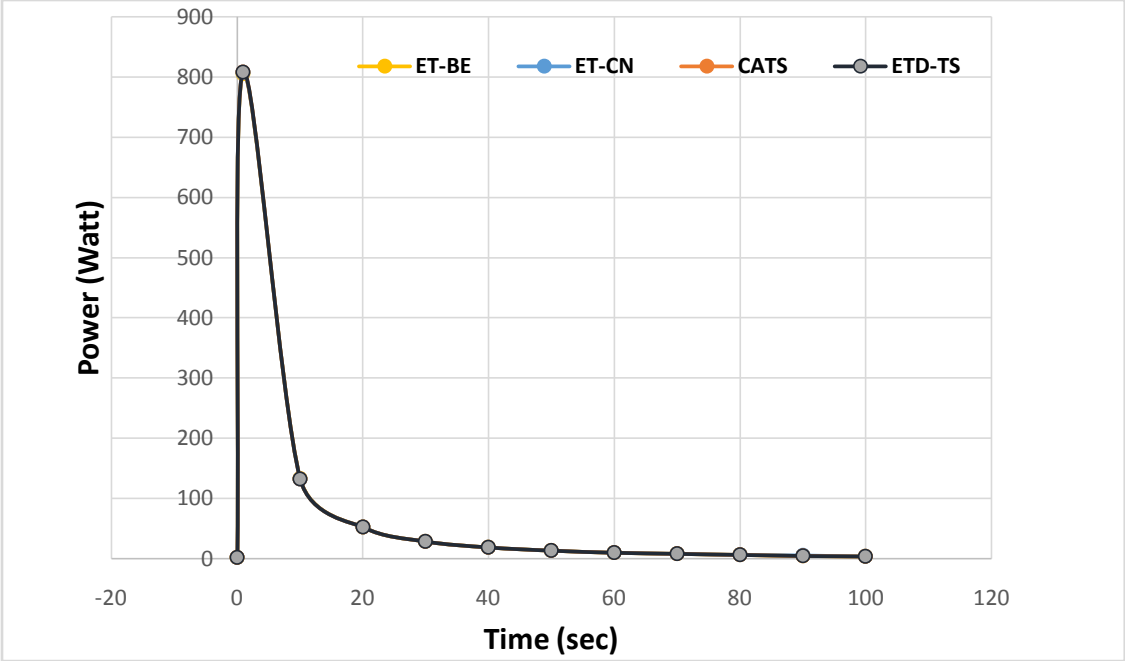

Fig.1:- Comparison of Transient Power (Watt) with initial stepreactivity (1.0$).

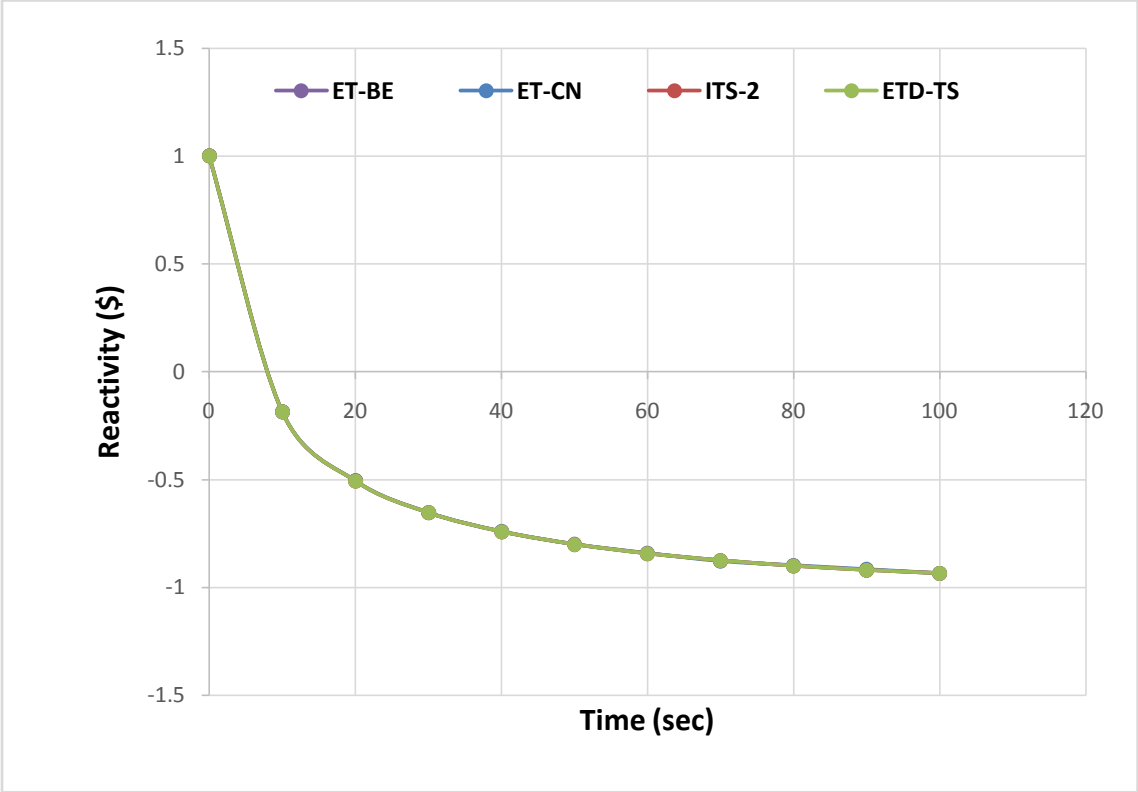

Fig.2:- Comparison of Transient Reactivity values with initial step reactivity (1.0$).

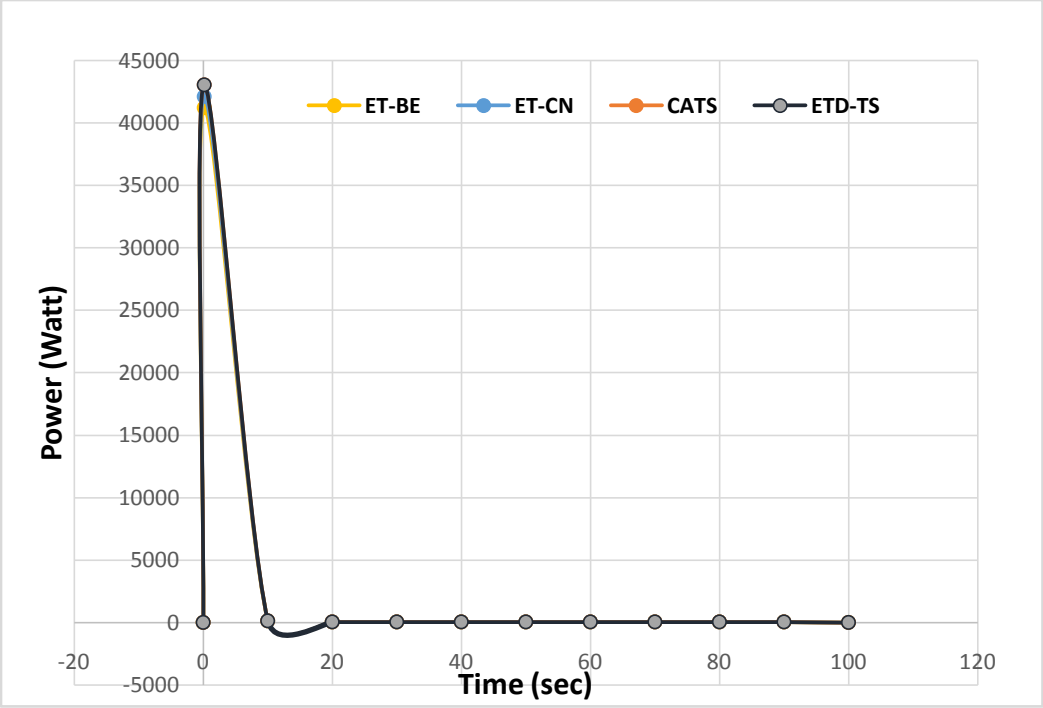

Fig.3:- Comparison of Transient Power (Watt) with initial step reactivity (1.5 $).

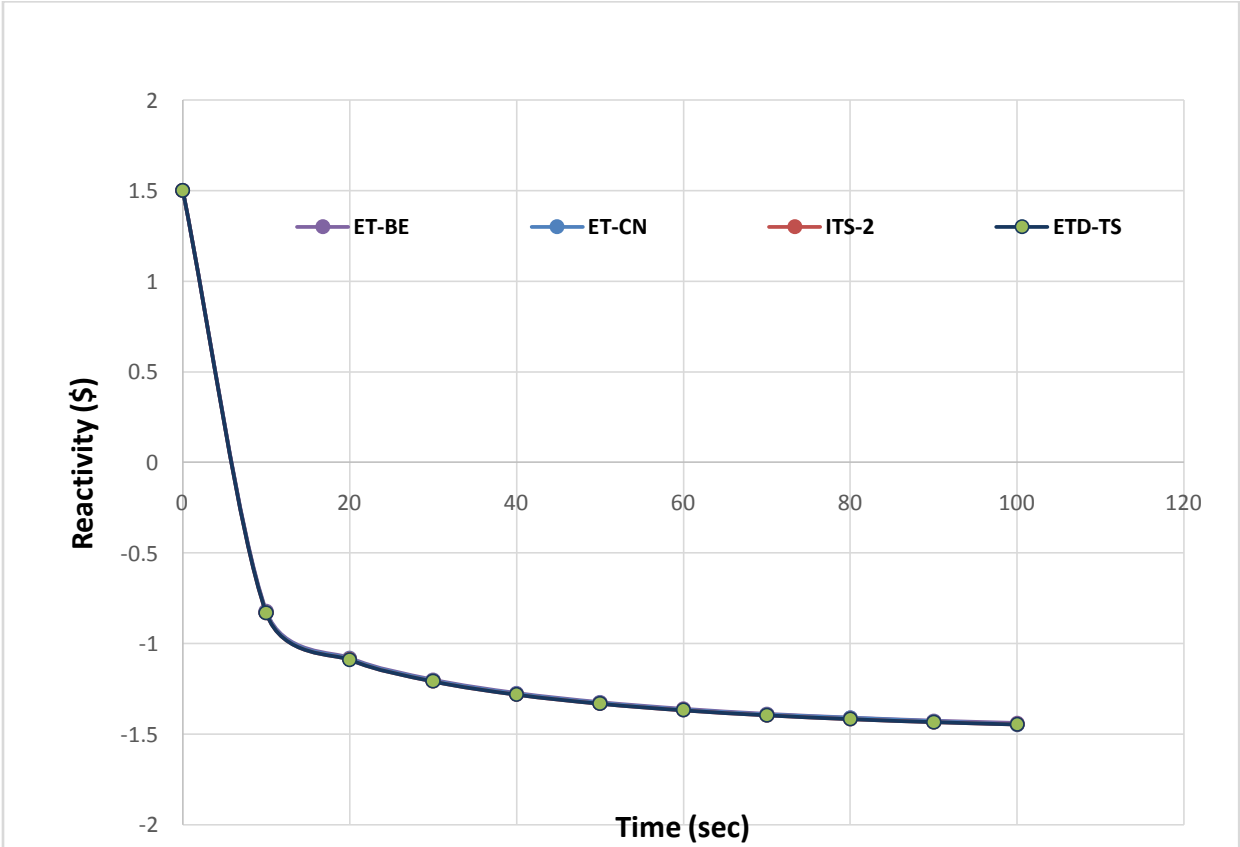

Fig.4:- Comparison of Transient Reactivity values with initial step reactivity (1.5$).

Table 3:- Comparison of transient values of power and reactivity values initiated due to step reactivity (2.0 $).

Time (sec)	Power (Watt)				ρ($)				Temp ETD-TS Method (^{0}K)
	ET-BE Method Ref.	ET-CN Method Ref.	ITS2 Method Ref.	ETD-TS Method(h=0.01 sec)n=5	ET-BE Method Ref.	ET-CN Method Ref.	ITS2 Method (Ref.)	ETD-TS Method	
0.0	1.00000	1.0000	1.0000	1.0000	2.00000	2.00000	2.00000	2.00000	293.0000
10.0	1.05111E+02	1.04263E+02	1.03381E+02	1.03381E+02	-1.31740	-1.34201	-1.36963	-1.36965	727.6849
20.0	3.97746E+01	3.94644E+01	3.91388E+01	3.91389E+01	-1.56502	-1.58776	-1.61333	-1.61335	759.1225
30.0	2.23353E+01	2.21750E+01	2.20038E+01	2.20038E+01	-1.68012	-1.70182	-1.72647	-1.72648	773.7162
40.0	1.46961E+01	1.45965E+01	1.44937E+01	1.44937E+01	-1.75046	-1.77101	-1.79558	-1.79559	782.6320
50.0	1.04532E+01	1.03871E+01	1.03186E+01	1.03186E+01	-1.79866	-1.81841	-1.84299	-1.84299	788.7476
60.0	7.76180E+00	7.70673E+00	7.66332E+00	7.66336E+00	-1.83328	-1.85537	-1.87749	-1.87750	793.1987
70.0	5.90482E+00	5.86898E+00	5.82940E+00	5.82943E+00	-1.85932	-1.87842	-1.90344	-1.90345	796.5459
80.0	4.55980E+00	4.53523E+00	4.49943E+00	4.49945E+00	-1.87896	-1.89690	-1.92333	-1.92334	799.1112
90.0	3.55555E+00	3.53251E+00	3.50742E+00	3.50744E+00	-1.89498	-1.91538	-1.93876	-1.93876	801.1013
100.0	2.79554E+00	2.76655E+00	2.75512E+00	2.75514E+00	-1.90635	-1.93386	-1.95083	-1.95084	802.6584

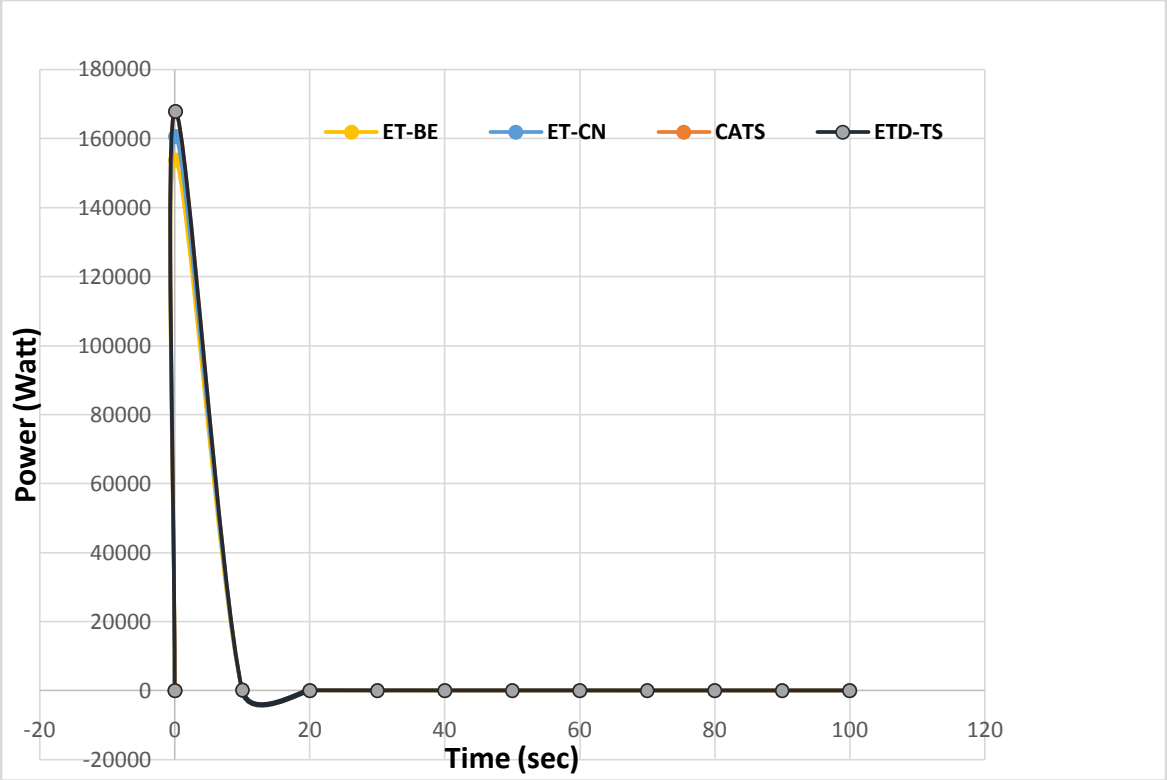

Fig.5:- Comparison of Transient Power (Watt) with initial step reactivity (2.0 $).

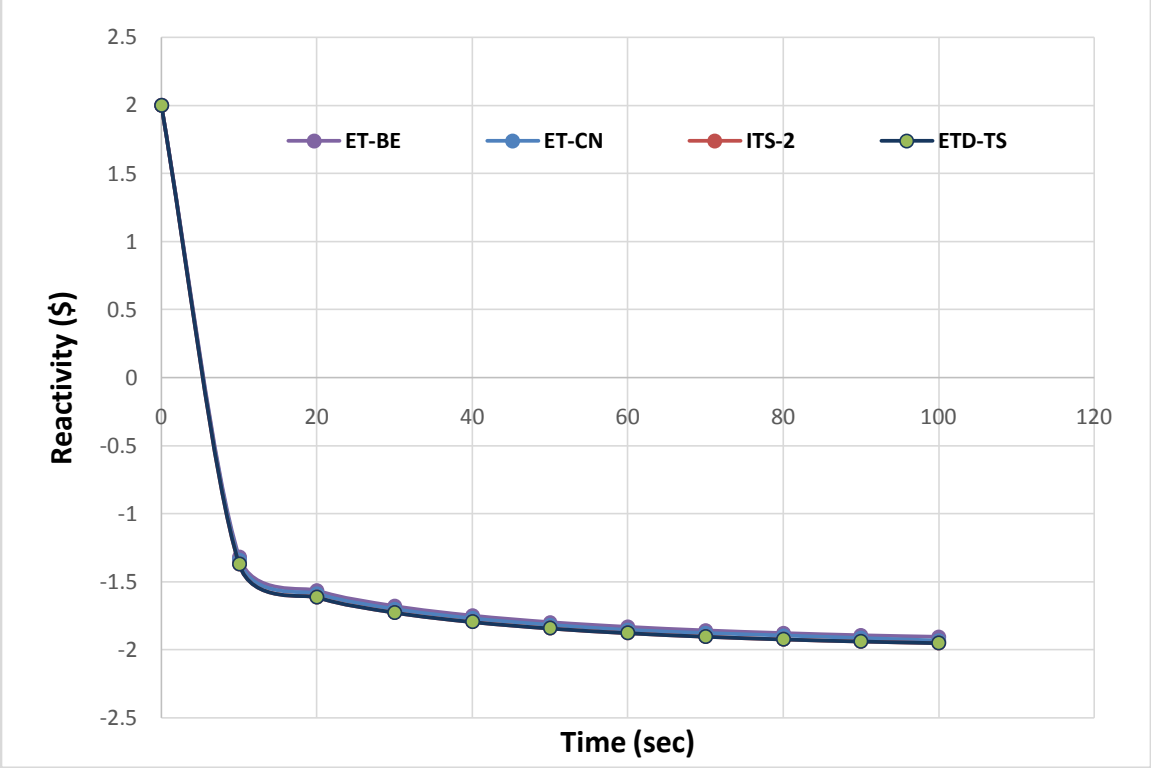

Fig.6:- Comparison of Transient Reactivity values with initial step reactivity (2.0 $).

Table 4. shows transient power values corresponding to different 'h' values using ETD-TS methods and results are compared with ITS2 method values for the transient power estimated up to 100.0 sec. Results are in excellent agreement with reference values.

Table 4:- Comparison of transient values of power and reactivity initiated due to step reactivity (2.0$) with different 'h' values.

Time (sec)	**Power (Watt)**			
	ETD-TS Method (Observed)			**ITS2–Method (Reference)**
	(h=0.1 sec) n=5	**(h=0.05 sec) n=5**	**(h=0.01 sec) n=5**	
0.0	1.0000	1.0000	1.0000	1.0000
20.0	38.8049	39.0941	39.1389	39.1389
50.0	10.2598	10.3090	10.3186	10.3186
80.0	4.4771	4.4951	4.4994	4.4994
100.0	2.7415	2.7524	2.7551	2.7551

Similarly, Table 5 shows the comparison of ETD-TS results with NAM (New Analytical Method) (Nahla, 2011), PCA (Piecewise Continuous Approximation method) (Kinard, 2004) and SCS methods. Comparison shows that transient power values estimated with ETD-TS method is quite accurate when compared with NAM and PCA method values. Also, ETD-TS method uses the time step size of (h=0.01 sec), which is larger than the time steps used in NAM and PCA methods (h=0.001 sec).This indicates that present method is efficient and accurate even with finer time steps and the values are in good agreement with most accurate ITS2 method values.

Table 5:- Comparison of ETD – TS power values with differentnumerical methods (with step reactivity of 2.0 $).

Time (sec)	**Power (Watt)**		
	NAM Method h=0.001 sec	**PCAMethod h=0.001 sec**	**ETD-TS Method (Observed) (h=0.01 sec) n=5**
0.0	1.0000	1.0000	1.0000
20.0	39.1793	39.0022	39.1389
50.0	10.3277	10.2872	10.3186
80.0	4.4855	4.4994	4.4994
100.0	2.7575	2.7463	2.7551

Table.6, Shows comparison of Peak transient power values and corresponding time to reach peak power at the initial step reactivity values i.e,1.0, 1.5 and 2.0 $ with other numerical methods. ETD-TS Method estimated values are in good agreement with ITS2 method values. From the comparison, it is observed that ETD-TS method is quite accurate and similar values when compared with highly accurate method ITS2 method.

Table 6:- Comparison of ETD – TS power peak values with different numerical methods with different initial reactivity values.

Initial Reactivity ρ_0($)	**Time to Peak (sec)**		**Peak Neutron Density**	
	ITS2	**ETD-TS**	**ITS2**	**ETD-TS**
1.0	0.953478	0.953476	8.078681E+02	8.078687E+02

1.5	0.168288		0.168289	4.302461E+04		4.302461E+04
2.0	0.098390		0.098399	1.678457E+05		1.678456E+05

* Numbers in the parenthesis denotes powers of 10.

Compensated ramp reactivity Insertion with neutron density feedback and its numerical solution

Compensated ramp functions which are defined previously, as in (Aboanber and Hamada,2002). The reactivity at time 't' can be described by the following equation.

$\rho(t) = a(t - t_0) - b\int_{t0}^{t} n(t')dt$ …… (13)

After applying the procedure in (2002,Cox) the above equation can be rewritten as shown below:

$\rho(t_{n+1}) = \rho(t_n)e^{-\frac{h}{\Lambda}} + e^{-\frac{h}{\Lambda}}\int_0^{r}(a - b.n(t_n+r)+\frac{1.0}{\Lambda})dr$ …… (14)

After expanding $n(t_n+r)$ using Taylor series expansion, we can estimate the reactivity values at different time steps 't_{n+1}'using Eq.(14).

Power Transients of thermal reactor caused by compensated ramp reactivity with Power Feedback

We next consider response to insertion of ramp reactivity as shown in Eq (13)in the thermal reactor.The power transients caused by compensated ramp reactivity is considered.Six group kinetic parameters are $\lambda_1 = 0.0124sec^{-1}$, $\lambda_2 = 0.0305sec^{-1}$, $\lambda_3 = 0.111\ sec^{-1}$, $\lambda_4 = 0.301\ sec^{-1}$, $\lambda_5 = 1.13\ sec^{-1}$,. $\lambda_6 = 3.00\ sec^{-1}$, $\beta_1 = 0.00021$, $\beta_2 = 0.00141$, $\beta_3 = 0.00127$, $\beta_4 = 0.00255$, $\beta_5 = 0.00074$, $\beta_6 = 0.00027$.$\beta_{eff} = 0.0065$, $\Lambda = 0.00005$ (sec). Here, time step chosen for all the calculations using ETD-TS method is h = 0.000025 sec.Neutron Density values are estimated up to 10.0 seconds, with fixed shutdown coefficient $b = 10^{-11}\ cm^3$ /sec and 'a' values are changed from 0.003-0.1 sec^{-1}. Transient Neutron Density values are in excellent agreement with ITS2method values at all the time steps and results are shown in Table 7-8. Transient Neutron density and reactivity values are plotted against ITS2 and ETD-TS values corresponding to fixed shutdown coefficient ($b=10^{-11}\ cm^3$ /sec) up to 10.0 sec with $a=0.003sec^{-1}$ and up to 1.0 sec with $a=0.1sec^{-1}$as shown in Fig.6-9. In Most of the cases, Neutron density and Reactivity values are well agreement with most accurate ITS2 method values up to 4decimal places.

Table 7:- Comparison of Compensated response to a ramp change of reactivity witha = 0.003 sec^{-1} and b = $10^{-11}\ cm^3$ /sec.

Time (sec)	**Neutron Density (/cm^3)**		**Reactivity ($)**	
	ITS-2	**ETD-TS n=3 ;h=0.000125 s**	**ITS-2**	**ETD-TS n=3 ;h=0.000125 s**
0	1.0000	1.000	0.0	0.0
0.5	1.3247E+00	1.3247E+00	2.3256E-01	2.3255E-01
1.0	2.0532E+00	2.0532E+00	4.6512E-01	4.6511E-01
1.5	4.3472E+00	4.3472E+00	6.9767E-01	6.9767E-01
2.0	2.3921E+01	2.3922E+01	9.3023E-01	9.3024E-01
2.5	1.4390E+04	1.4397E+04	1.1628E+00	1.1628E+00
3.0	8.0571E+08	8.0525E+08	6.4673E-01	6.4669E-01
4.0	3.4133E+08	3.4134E+08	5.0440E-01	5.0439E-01
5.0	3.2933E+08	3.2934E+08	4.5283E-01	4.5283E-01
10.0	3.1459E+08	3.1460E+08	2.7934E-01	2.7934E-01

* Numbers in the parenthesis denotes powers of 10.

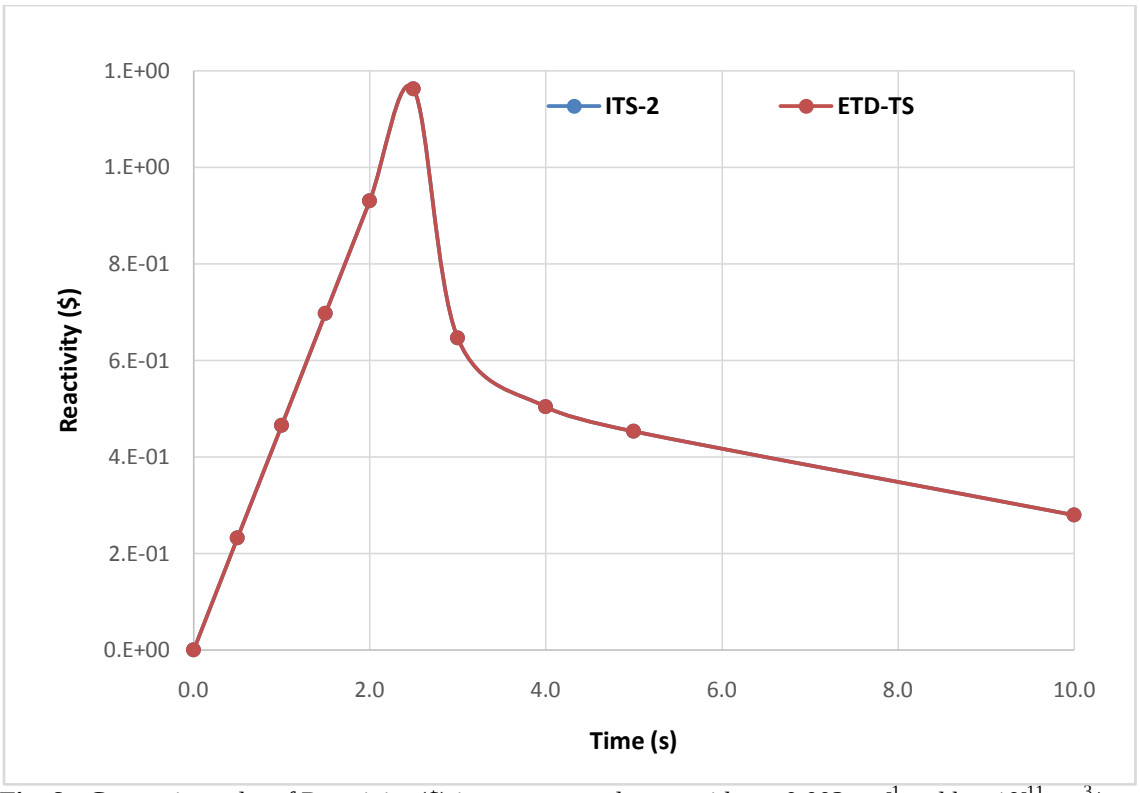

Fig. 6:- Comparison plot of Reactivity ($) in compensated ramp with a = 0.003 sec^{-1} and b = 10^{-11} cm^3/sec.

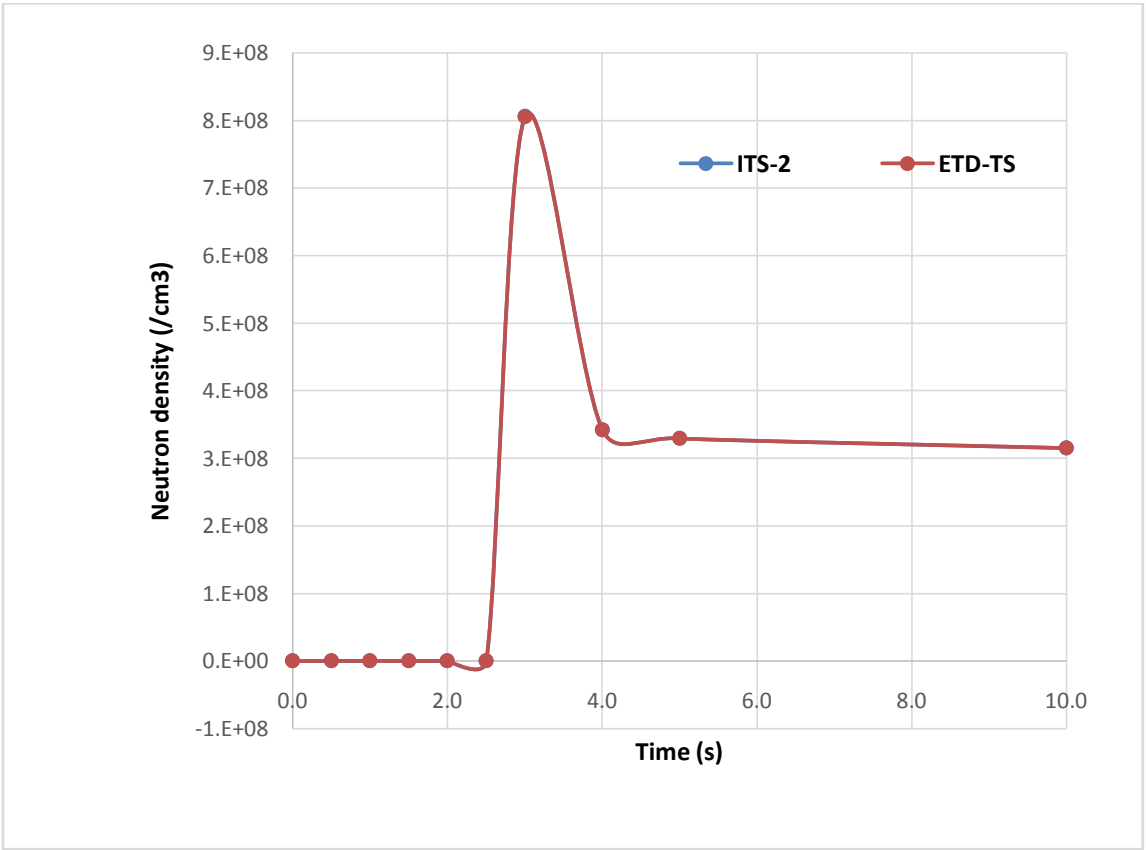

Fig. 7:- Comparison plot of Neutron density in compensated rampwith a = 0.003 sec^{-1} and b = 10^{-11} cm^3/sec.

Table 8:- Comparison of Compensated response to a ramp change of reactivity a = 0.1 sec^{-1} and b = 10^{-11} cm^3 /sec.

Time (sec)	Neutron Density (/cm^3)		Reactivity ($)	
	ITS-2	ETD-TS n=3 ;h=0.000125 s	ITS-2	ETD-TS n=3 ;h=0.000125 s
0.00	1.00	1.00	0.00	0.00
0.10	2.4734E+01	2.4730E+01	1.5504E+00	1.5504E+00
0.20	7.2379E+08	7.2371E+08	3.0965E+00	3.0964E+00
0.30	8.1620E+08	8.1619E+08	-3.8677E-01	-3.8682E-01
0.40	3.7509E+09	3.7504E+09	9.0487E-01	9.0481E-01
0.50	9.9500E+09	9.9503E+09	7.2456E-01	7.2456E-01
0.60	1.1466E+10	1.1466E+10	8.7409E-01	8.7411E-01
0.70	9.7644E+09	9.7643E+09	8.2555E-01	8.2555E-01
0.80	1.0086E+10	1.0086E+10	7.9324E-01	7.9324E-01
0.90	1.0153E+10	1.0153E+10	7.8014E-01	7.8014E-01
1.00	1.0104E+10	1.0104E+10	7.6207E-01	7.6207E-01

* Numbers in the parenthesis denotes powers of 10.

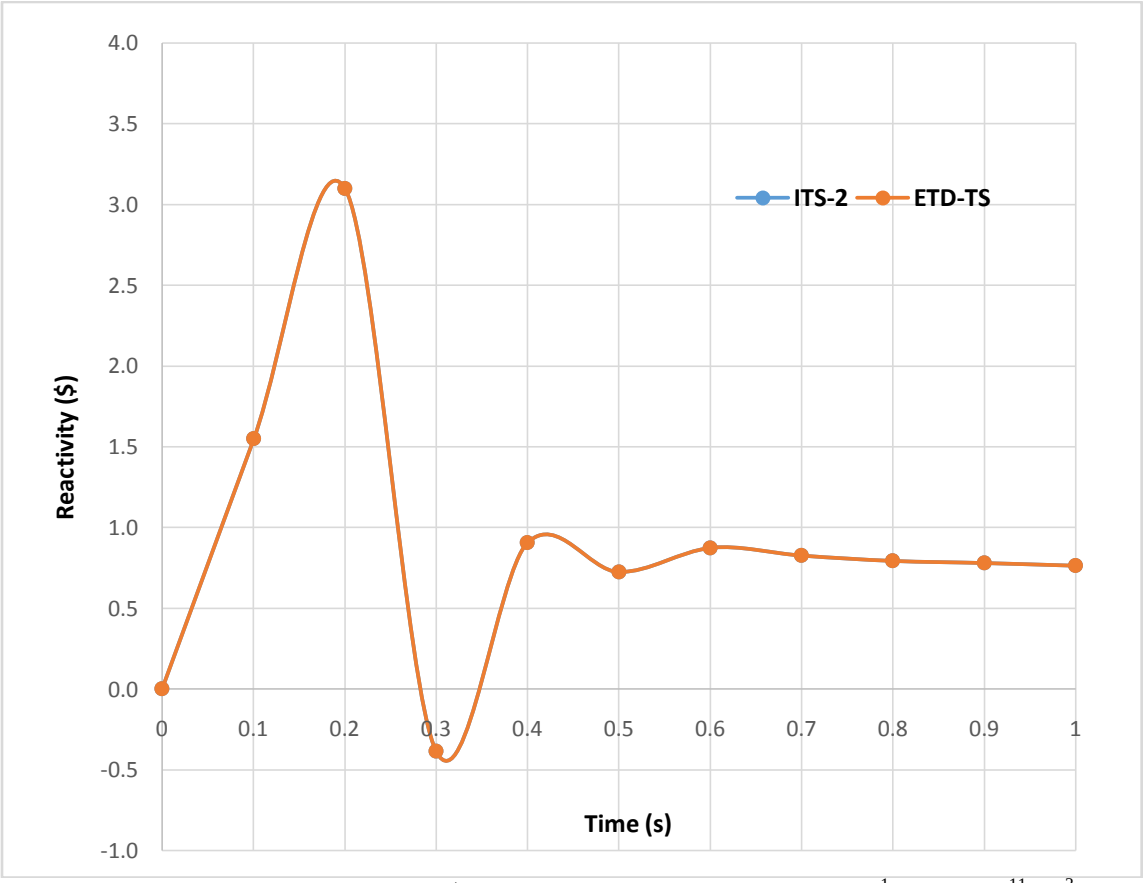

Fig. 8:- Comparison plot of Reactivity ($) in compensated rampwith a = 0.1 $sec^{-1}$and b = 10^{-11} cm^3/sec.

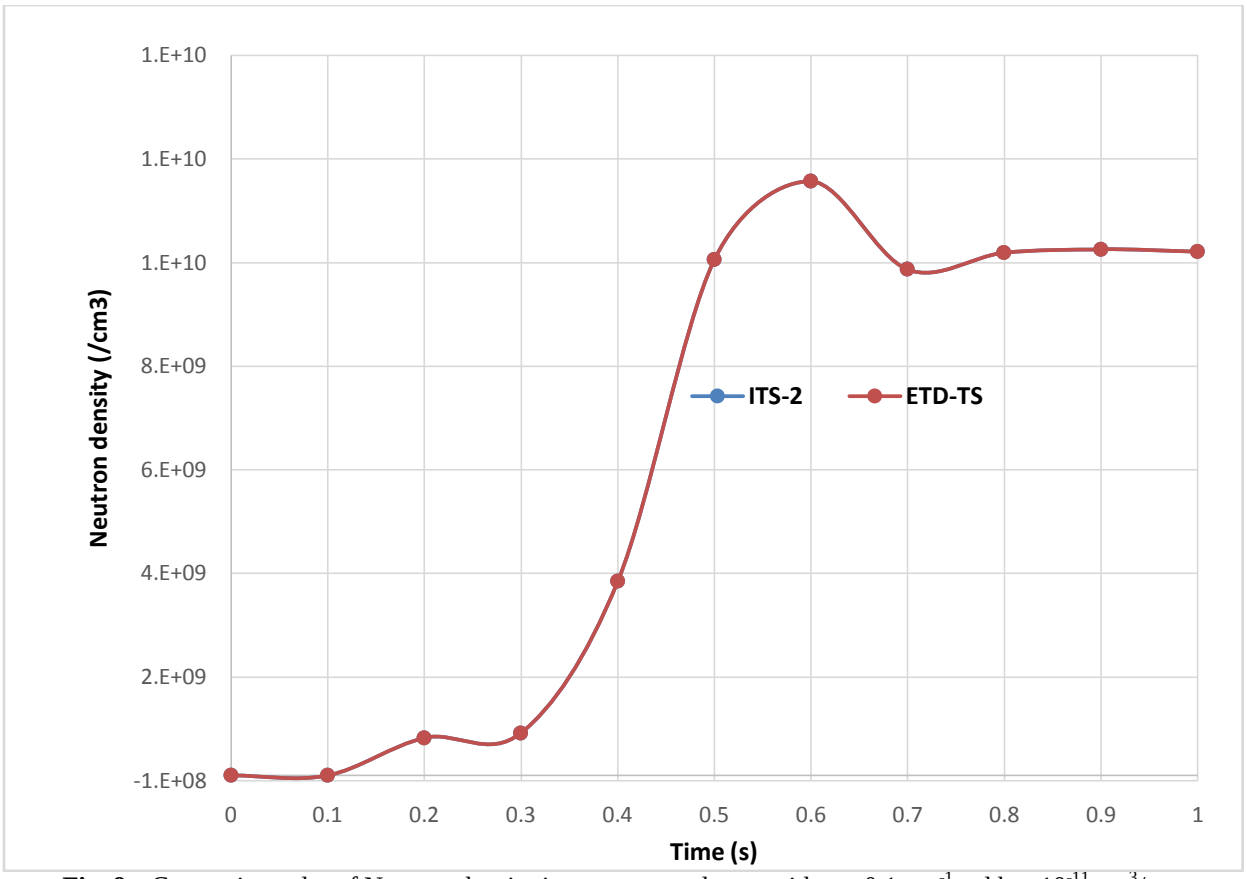

Fig. 9:- Comparison plot of Neutron density in compensated rampwith a = 0.1 $sec^{-1}$and b = 10^{-11} cm^3/sec.

Similarly, Neutron Density values are estimated up to 10.0 seconds, with fixed shutdown coefficient b = 10-13 cm3 /sec and for three different 'a' values 0.003,0.01 and 0.1 sec^{-1}. Transient Neutron Density values are in excellent agreement with other accurate methods likeEPCA and CATS at all the time steps and results are shown in Table 9. Transient Neutron density and reactivity values are plotted against CATS and ETD-TS values corresponding to fixed shutdown coefficient (10^{-13} cm^3 /sec) up to 10.0 sec as shown in Fig.10. In Most of the cases, Neutron density and Reactivity values are well agreement with most accurate CATS method.

Table 9:- Comparison of Neutron densities in compensated ramp changes of reactivity with b =10^{-13} cm^3/secfor different 'a' values.

Time (sec)	**EPCA (Reference)**	**ITS-2 (Reference)**	**ETD-TS (Observed) n=3 (h=0.000125 sec)**
Case 1. a =0.1 (sec^{-1})			
0.1	2.4734E+01	2.4734E+01	2.4730E+01
0.5	1.5434E+12	1.5434E+12	1.5434E+12
5.0	1.0030E+12	1.0030E+12	1.0030E+12
7.5	1.0018E+12	1.0018E+12	1.0018E+12
10.0	1.0012E+12	1.0012E+12	1.0012E+12
Case 2. a =0.01 (sec^{-1})			
0.1	1.1672E+00	1.1672E+00	1.1672E+00
0.5	4.2699E+00	4.2700E+00	4.2697E+00
5.0	1.0339E+11	1.0339E+11	1.0339E+11
7.5	1.0195E+11	1.0195E+11	1.0195E+11
10.0	1.0124E+11	1.0124E+11	1.0124E+11
Case 3. a =0.003 (sec-1)			

0.1	1.0453E+00	1.0453E+00	1.0453E+00
0.5	1.3246E+00	1.3246E+00	1.3246E+00
5.0	3.2156E+10	3.2156E+10	3.2156E+10
7.5	3.2102E+10	3.2102E+10	3.2102E+10
10.0	3.1456E+10	3.1456E+10	3.1456E+10

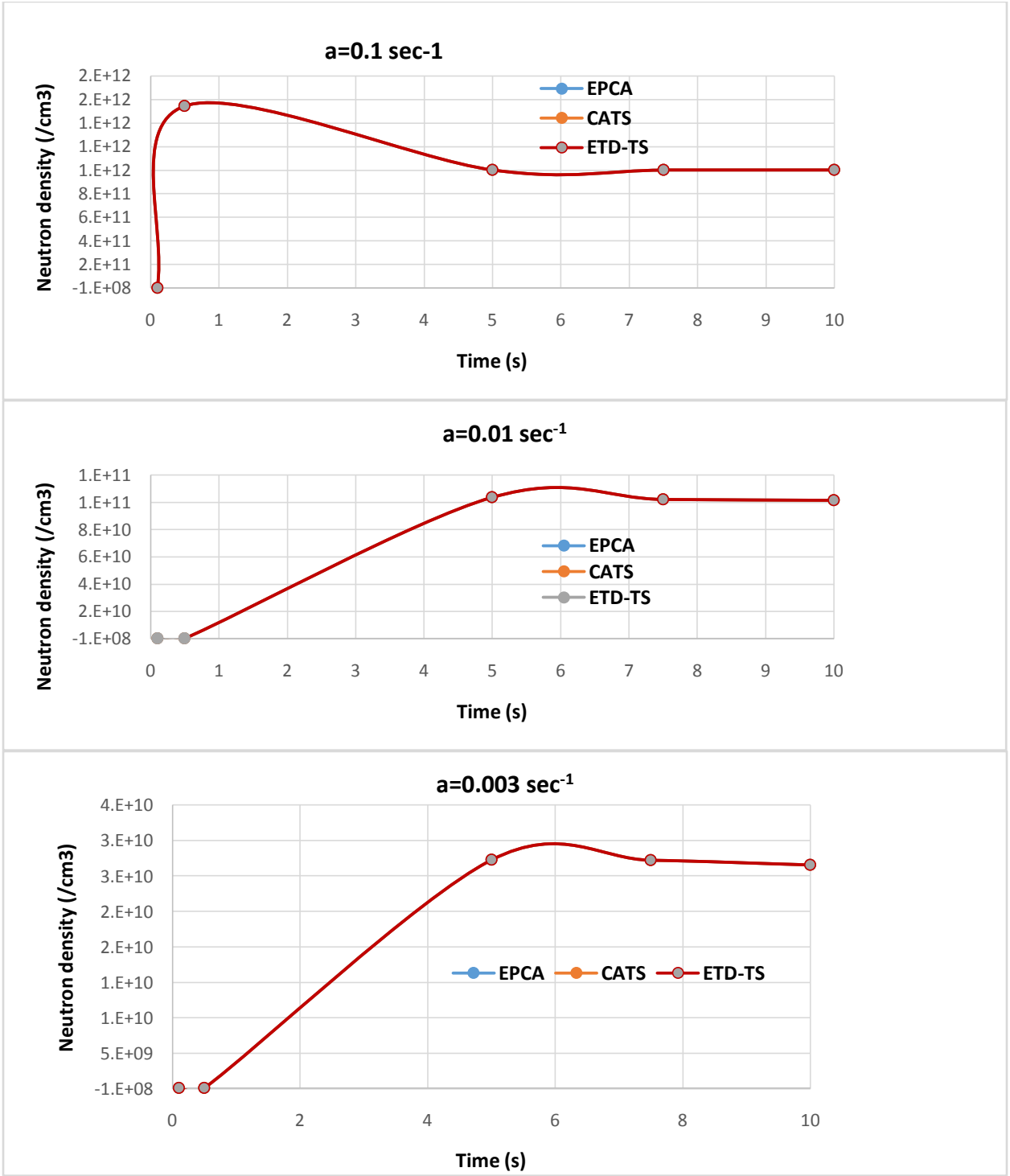

Fig. 10:- Comparison plot of Neutron density in compensated ramp change with b = 10^{-11} cm3/sec and different a (sec-1) values.

Conclusion:-

ETD-TS method is used for solving point kinetics equations with multi group of delayed neutrons in the presence of temperature and power feedback. Thermal reactor benchmark problems with stepand compensated ramp reactivity insertions are considered for validation of this method. In ETD-TS method integral form of PKEs are solved using Taylor Series expansion method with temperature and power feedbacks. Time constant '-1/Λ' is chosen as stiffness constant. PKEs are integrated using exponential factor. Smaller time step ~ 10^{-2}sec is sufficient for estimation of power transients with step reactivity insertion. But, for transients with compensated ramp reactivity insertion, time step size ~10^{-4}sec was used in ETD-TS method to estimate the power transients. Computational time is found to be relatively less without any loss of accuracy when compared with ITS2 method.It is observed that ETD-TS method estimates the power transients for longer duration of time with good accuracy. The method was found to give accurate results similar to PCA and NAM methods even with larger time steps in the case of compensated ramp reactivity insertion case. ETD-TS method can be further used for estimation of power transients in 3D space-time calculations.

References:-

1. Kinard, M., Allen, E.J., 2004. Efficient Numerical Solution of Point Kinetics Equations in NuclearReactor Dynamics, Annals of nuclear energy, vol. 31, page 1039-1051.
2. Nahla, A.A., 2011. An efficient technique for point reactor kinetics equations with Newtonian temperature feedback effects, Annals of nuclear energy, vol.38,2810.
3. Ganapol, B., Picca, P., Previti, A., Mostacci, D., 2012. The solution of the point kinetics equations via converged accelerated Taylor series (CATS). In: Advances in Reactor PhysicseLinking Research, Industry and Education (PHYSOR 2012), Knoxville, Tennessee, USA, April 15-20, 2012.
4. Picca, P., Furfaro, R., Ganapol, B.D., 2013. A highly accurate technique for the solution of the non-linear point kinetics equations. Ann. Nucl. Energy 58, 43.
5. Serigo Q.B.L.,Marco, T. D. V., 2016. Solution of the point reactor kinetics equations with temperature feedback by the ITS2 method, Progress in Nuclear Energy, Vol .91, pages 240-249.
6. Hamada, Y.M., 2018. A New Accurate Numerical Method Based on Shifted Chebyshev Series for Nuclear Reactor Dynamical Systems. Sci. Technol. Nucl. Install. 2018.
7. Cox, S., M., Mathews, P., C., 2002. Exponential time differencing method for stiff systems, J. Comput. Phys., vol. 176, pages 430-455.
8. Mohideen Abdul Razak, M., Devan, K., Shathiyasheela, T., 2015. The modified exponential time differencing (ETD) method for solving the reactor point kinetics equations, Annals of nuclear energy, vol. 76, pages 193-199.
9. Hetrick, D.L., 1971. Dynamics of Nuclear Reactors. University of Chicago Press, Chicago and London.
10. Stacey, W.M., 2001. Nuclear Reactor Physics. John Wiley and Sons Inc., USA
11. Aboanber, A.E., Hamada, Y.M., 2002. PWS: an efficient code system for solving space-independent nuclear reactor dynamics. Ann. Nucl. Energy 29, 2159.