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A heuristic for the multi-period petrol station replenishment problem

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Abstract

In the *Multi-Period Petrol Station Replenishment Problem* (MPSRP) the aim is to optimize the delivery of several petroleum products to a set of petrol stations over a given planning horizon. One must determine, for each day of the planning horizon, how much of each product should be delivered to each station, how to load these products into vehicle compartments, and how to plan vehicle routes. The objective is to maximize the total profit equal to the revenue, minus the sum of routing costs and of regular and overtime costs. This article describes a heuristic for the MPSRP. It contains a route construction and truck loading procedures, a route packing procedure, and two procedures enabling the anticipation or the postponement of deliveries. The heuristic was extensively tested on randomly generated data and compared to a previously published algorithm. Computational results confirm the efficiency of the proposed methodology.

Keywords: fleet management, fuel delivery, replenishment, routing and scheduling

1. Introduction

The purpose of this article is to develop a heuristic for the *Multi-Period Petrol Station Replenishment Problem* (MPSRP) which consists of optimizing the delivery of several petroleum products to a set of petrol stations over a given planning horizon. More specifically, one must determine, for each day of the planning horizon, how much of each product should be delivered to each station, how to load these products into vehicle compartments, and how to plan vehicle routes.

The MPSRP can be formally defined as follows. Let $G = (V, A)$ be a directed graph where $V = \{0, \dots, n\}$ is the vertex set and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the arc set. Vertex 0 corresponds to the terminal while the remaining vertices represent petrol stations. Denote by t_{ij} and c_{ij} the travel time and the travel cost associated with arc (i, j) . A heterogeneous fleet of m tank trucks

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is based at the terminal. All trucks are assumed to travel at the same constant speed and have the same fixed and variable costs. Each truck is subdivided into several *compartments* of varying capacities. Each petrol station uses a number of underground *tanks* of standard capacities. These stations require the delivery of several petroleum products which must be stored in separate tanks. If m_{ip} denotes the underground tank capacity of product p at station i , s_{ipt} denotes the stock level of product p at station i at the beginning of period t , and v_{ipt} denotes the sales of product p at station i during period t ($m_{ip} \geq v_{ipt}$ for all t), then station i requires a delivery during period t if there exists a product p for which the minimal delivery quantity $v_{ipt} - s_{ipt}$ is positive. In practice, sales are stochastic but we consider them as deterministic in this study. Also, stock levels decrease continuously during the day and delivery amounts should be a function of trucks arrival time at a station. Our model and algorithm implicitly assume the existence of safety stocks so that it is feasible to deliver at any time during the day. When a truck delivers a product to a station, it completely empties the compartment containing that product. This restriction is a consequence of the fact that in the application that gave rise this study, trucks are not equipped with flow meters. Avella et al. [1] also applied this rule in their case study. However, contrary to what was done in Avella et al. [1], we do not impose that delivery quantities should be equal to the total capacity of one or several compartments. In other words, any given truck compartment can be partially filled when the truck leaves the terminal. The revenue per litre delivered is given. The MPSRP contains a temporal and a spatial dimension. Since it is not necessary to visit each station during each period of the planning horizon, one must determine the ideal periods to visit stations, taking into account stock levels, fleet capacity, and routing efficiency. This problem can therefore be viewed as an inventory routing problem [2] with additional constraints induced by varying tank sizes and truck compartments. Contrary to the problem treated by Avella et al. [1], stations do not specify fixed visit dates and delivery amounts. These decisions are optimized centrally by the distributor. In other words, the considered operating mode can be described as a *push* system.

The number of stations on any given route is limited. In our application, this limit is equal to two [3]. The two-stop limit per route is a common practice in North America. It is explained by the fact that most tank trucks have from four to six compartments and stations typically order two or three products, one of which often requiring two compartments. Irrespective of the quantity delivered at a station, the service time is assumed to be constant and equal to s . Several routes can be assigned to the same vehicle in any given period. The normal duration of a working day is equal to H ; however, this duration can be extended to H' by using overtime. A regular wage rate applies until time H and an overtime rate applies between H and H' . We solve the problem on a planning horizon of T periods. Note that the same methodology could be applied on a rolling horizon basis.

The MPSRP consists of determining, for each period t of the planning horizon:

- the set of stations to which deliveries should be made,
- the quantity of each product p to be delivered to each of these stations,
- the loading of these products into vehicle compartments,
- feasible delivery routes to these stations,
- the assignment of routes to available trucks.

The objective is to maximize the total profit equal to the revenue, minus the sum of routing costs and of regular and overtime costs.

Our objective is to develop a multi-phase heuristic for the MPSRP. The remainder of this paper is organized as follows. A literature review is provided in Section 2. The problem is modeled in Section 3. The heuristic is described in Section 4, and computational results are presented in Section 5. Conclusions follow in Section 6.

2. Literature review

The literature of the MPSRP is rather limited. The single period problem has been solved exactly by Cornillier et al. [3]. A related problem with single customer trips and time windows has been investigated by Brown and Graves [4], while Brown et al. [5] have developed a computerized assisted dispatch system for the problem. Several greedy heuristics followed by simple improvement procedures for the multi-period problem have been proposed by Taqa Allah et al. [6] and Malépart et al. [7].

Van der Bruggen et al. [8] solve the single period version of the problem as part of a broader study aimed at optimizing the distribution network of a large oil company operating in the Netherlands. More recently, Avella et al. [1] have proposed heuristic and exact algorithms for a different version of the single period problem. In particular, these authors assume that trucks compartments cannot be partially filled and their algorithm works on the basis that the set of stations to be delivered at any given period, as well as the delivery amounts, are known. In contrast, our model and algorithm consider the set of stations and the delivery quantities to be decision variables. In particular, deliveries can be anticipated if this proves to be profitable.

From a more operational point of view, Ronen [9] presents a number of different operational environments associated with the dispatching petroleum products, as well as some analytical tools used by oil companies. Bausch et al. [10] describe a decision system applied to a maritime transportation problem consisting of scheduling a fleet of coastal tankers scheduling carrying oil bulk products. More recently, Ben Abdelaziz et al. [11] have presented a real-life routing problem in which a variable neighbourhood search heuristic was applied to solve a single period petroleum products delivery problem using a heterogeneous fleet of compartmented tank trucks. Finally, Rizzoli et al. [12] has described a software tool, based on an ant colony heuristic, which assists dispatchers during the different stages of fuel distribution.

3. Mathematical model

The problem can be formulated as a large scale mixed integer program. While this formulation is too large to be used to solve the problem optimally, we believe it is useful to formulate it in order to remove any ambiguity regarding its precise definition. We first define the following parameters:

- R : revenue per delivered litre;
- C : unit regular time cost;
- C' : unit overtime cost;
- q_{wk} : capacity of compartment w of truck k ;
- m_{ip} : capacity of underground tank for product p of station i ;
- τ_{ij} : travel time from station i to station j , including loading time l at the terminal

and unloading time s at station j : $\tau_{ij} = t_{ij} + l + s$ when $i = 0$, $\tau_{ij} = t_{ij} + s$ when $i, j \neq 0$, and $\tau_{ij} = t_{ij}$ when $j = 0$.

The decision variables are:

- h_{kt} : regular time for truck k at period t ;
- h'_{kt} : overtime for truck k at period t ;
- z_{ijvkt} : binary variable equal to 1 if truck k travels from station i (the terminal if $i = 0$) to station j (the terminal if $j = 0$) within trip v at period t ;
- x_{ipwvkt} : quantity of product p of station i loaded in compartment w of truck k within trip v at period t ;
- y_{ipwvkt} : binary variable equal to 1 if and only if product p of station i is loaded in compartment w of truck k within trip v at period t .

The model is then as follows.

$$\text{Maximize } \sum_{(i,p,w,v,k,t)} R x_{ipwvkt} - \sum_{(k,t)} (C h_{kt} + C' h'_{kt}) - \sum_{(i,j,v,k,t)} c_{ij} z_{ijvkt} \quad (1)$$

subject to:

$$s_{ip,t+1} = s_{ipt} - v_{ipt} + \sum_{(w,v,k)} x_{ipwvkt} \quad \forall(i, p, t) \quad (2)$$

$$x_{ipwvkt} \leq q_{wk} y_{ipwvkt} \quad \forall(i, p, w, v, k, t) \quad (3)$$

$$s_{ipt} + \sum_{(w,v,k)} x_{ipwvkt} \leq m_{ip} \quad \forall(i, p, t) \quad (4)$$

$$\sum_{(i,p)} y_{ipwvkt} \leq 1 \quad \forall(w, v, k, t) \quad (5)$$

$$\sum_i z_{ijvkt} \geq y_{jpwvkt} \quad \forall(j, p, w, v, k, t), j \neq 0 \quad (6)$$

$$\sum_i z_{ijvkt} = \sum_i z_{jivkt} \quad \forall(j, v, k, t) \quad (7)$$

$$\sum_j z_{0jvkt} \leq 1 \quad \forall(v, k, t) \quad (8)$$

$$z_{ijvkt} + z_{jivkt} \leq 1 \quad \forall(i, j, v, k, t), i, j \neq 0 \quad (9)$$

$$\sum_{(i,j)} z_{ijvkt} \leq 3 \quad \forall(v, k, t) \quad (10)$$

$$\sum_{(i,j,v)} \tau_{ij} z_{ijvkt} = h_{kt} + h'_{kt} \quad \forall(k, t) \quad (11)$$

$$0 \leq h_{kt} \leq H \quad \forall(k, t) \quad (12)$$

$$0 \leq h'_{kt} \leq H' - H \quad \forall(k, t) \quad (13)$$

$$z_{ijvkt} \in \{0, 1\} \quad \forall(i, j, v, k, t) \quad (14)$$

$$y_{ipwvkt} \in \{0, 1\} \quad \forall(i, p, w, v, k, t) \quad (15)$$

$$x_{ipwvkt} \in \mathbb{R}^+ \quad \forall(i, p, w, v, k, t) \quad (16)$$

$$h_{kt} \in \mathbb{R}^+ \quad \forall(k, t) \quad (17)$$

$$h'_{kt} \in \mathbb{R}^+ \quad \forall(k, t). \quad (18)$$

In this formulation, the objective function (1) maximizes the total profit which can be decomposed into revenue per delivered litre, minus regular working time and overtime costs, and travel costs c_{ij} . Equations (2) ensure stock equilibrium between two consecutive periods. Constraints (3) specify that the quantity loaded does not exceed the allotted compartment capacity q_{wk} . Constraints (4) specify that stock level after a delivery should not exceed the tank capacity. Constraints (5) ensure that at most one product demand is assigned to a compartment. By constraint (6) a product quantity can be delivered to station j only if a truck travels from some station i to station j . Equation (7) specifies that the number of arrivals at a given station must be equal to the number of departure from this station. Constraints (8) impose for each trip at most one departure from the depot. Constraints (9) eliminate round trips between two stations, and the number of visited stations is limited to two per trip by constraints (10). Equations (11) decompose for each truck the workload into regular working time and overtime, with a regular time limited to H hours (12) and a maximum overtime limited to $H' - H$ hours (13).

4. Multi-phase heuristic

We propose the following multi-phase heuristic for the MPSRP. Starting from the first period $t = 1$ of the planning horizon, the heuristic iteratively constructs delivery plans for each period. Denote by X_t the solution for period t which gives the stations to be visited, products and quantities to deliver, trucks loading and truck routes. Also denote by M_t the duration of the maximum working time of a vehicle in X_t . Three cases are possible:

- (1). $H < M_t \leq H'$,
- (2). $M_t > H'$,
- (3). $M_t \leq H$.

In the first case, a feasible solution in which at least one vehicle uses overtime has been identified at period t and the algorithm proceeds to period $t + 1$. In the second case, the solution is infeasible because at least one vehicle exceeds the maximum working time H' , and a *look-back procedure* is applied in an attempt to regain feasibility: the algorithm tries to move some deliveries backward to period $t - 1$ in order to make X_t feasible. If this cannot be done without making X_{t-1} infeasible, this process is recursively reapplied starting from $t - 1$ until X_t, \dots, X_1 are feasible, in which case the process moves to period $t + 1$ to construct X_{t+1} , or until it is discovered that no feasible solution can be identified, in which case the algorithm terminates. In the third case, a feasible solution without overtime is found and a *look-ahead procedure* is applied in an attempt to increase the number of stations visited at period t by anticipating future deliveries. The process then moves to period $t + 1$. A flowchart of the heuristic is provided in Figure 1.

In terms of the mathematical model, the heuristic first assigns a value of 0 or 1 to each variable $\bar{y}_{it} = \sum_{(p,w,v,k)} y_{ipwvkt}$ for all i and t , under constraints (5). For each t , it then jointly determines the values of y_{ipwvkt} , x_{ipwvkt} and z_{ijvkt} for all i, j, p, w, v and k through the exact algorithm described in Cornillier et al. [3]. During this process, constraints (2) to (10) are satisfied while constraints (11) to (13) are relaxed. Since the resulting solution is not necessarily feasible with respect to constraints (11) to (13), new values may have to be assigned to the \bar{y}_{it} variables, which implies recomputing the y_{ipwvkt} , x_{ipwvkt} and z_{ijvkt} variables. This process is iteratively reapplied until a termination criterion is met.

We now explain this heuristic in more detail.

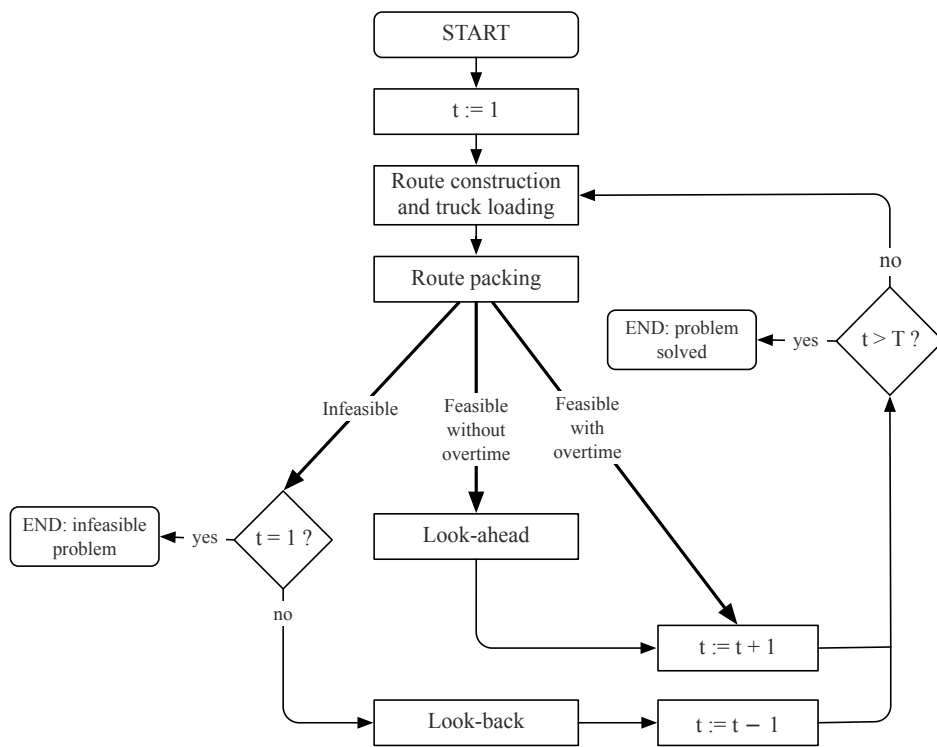


Figure 1: Flowchart of the multi-phase heuristic

4.1. Route construction and truck loading

Denote by S_t the set of stations to be delivered at period t . This set is composed of all stations that would run out of stock for at least one product if they were not visited in period t , and of any station selected by the *look-back* and the *look-ahead* procedures. A routing plan minimizing travel time is computed for S_t . The two-stop limit per route considerably simplifies the problem. Indeed, since each truck can visit one or at most two stations within the same trip, determining an optimal set of routes can be achieved by solving a series of matching problems [3]. The matching costs are the travel times needed to visit the two stations if the route is feasible. However, this may not be the case. It may indeed be impossible to deliver the demand of some routes selected by the matching algorithm if none of the available trucks has sufficient capacity to deliver the corresponding minimal required quantities. The matching cost of any infeasible route is set to infinity and the resulting matching problem is solved. This procedure is repeated until a feasible loading can be found for all selected routes. Checking whether a route is feasible or not is achieved by solving exactly the *Tank Truck Loading Problem* (TTLP) [3]. The proposed solution algorithm maximizes the delivered quantity of route r and assigns products to the compartments of truck k . The optimal quantity x_{ipt} of product p to be delivered to station i at period t depends on the station's sales and underground tanks capacity. It must lie between $\max\{0, v_{ipt} - s_{ipt}\}$ and $(m_{ip} - s_{ipt})$. Other constraints on x_{ipt} relate to the compartment capacities of the truck delivering to station i .

4.2. Route packing

Having determined a set of feasible routes and, for each of these routes, the set of associated trucks, we compute for each feasible pair (r, k) the revenue R_{rk} corresponding to the optimal quantities x_{ipt} determined by the TTLP algorithm. The next step is to construct truck working days by assigning routes to trucks. The objective is to maximize the total revenue, minus the overtime cost. However, because a feasible solution does not always exist, i.e. some truck working days may exceed H' hours, we subtract from the objective a term proportional to the total time in excess of H' in the solution. Formally, the following mixed integer program is solved heuristically. Let y_{rk} be a binary variable equal to 1 if and only if route r is assigned to truck k , let T_r be the duration of the route r , let w'_k be the amount of overtime of truck k , and let z_k be the time worked by truck k in excess of H' . If $z_k > 0$ in the optimal solution, this means that it is impossible to pack all routes into a feasible working day. Let C' be the unit overtime cost and M , an arbitrary large constant. The *Route Packing Problem* (RPP) is then:

$$(RPP) \quad \text{Maximize} \quad \sum_r \sum_k R_{rk} y_{rk} - C' \sum_k w'_k - M \sum_k z_k \quad (19)$$

subject to:

$$\sum_r T_r y_{rk} \leq H + w'_k + z_k \quad \forall k \quad (20)$$

$$\sum_k y_{rk} = 1 \quad \forall r \quad (21)$$

$$0 \leq w'_k \leq H' - H \quad \forall k \quad (22)$$

$$y_{rk} \in \{0, 1\} \quad \forall (r, k) \quad (23)$$

$$w'_k \in \mathbb{R}^+ \quad \forall k \quad (24)$$

$$z_k \in \mathbb{R}^+ \quad \forall k. \quad (25)$$

In this model, the left-hand side of (20) defines the working time of truck k , while the right-hand side is equal to H if no overtime is performed by truck k , equal to $H + w'_k$ if the amount of overtime is w'_k , and equal to $H + w'_k + z_k$ if overtime exceeds $H' - H$. Constraints (21) specify that each route is assigned to exactly one truck. Note that the RPP without the w_k and z_k variables is a *Generalized Assignment Problem* which is NP-hard. In practice, we solve this problem by means of a greedy heuristic followed by a simple improvement phase. Initially, all routes are sorted in the ascending order of the number of trucks to which they could be feasibly assigned. Ties are broken by first considering the longest route. In other words, the routes that are most difficult to assign appears earlier in the list. The routes are then sequentially assigned to trucks in order to maximize the additional profit defined by (19). Whenever assigning a route to a truck causes H' to be exceeded, an attempt to reassign one of the routes of that truck is made in order to regain feasibility. All such reassignment moves are considered and the best one is implemented. After all routes have been assigned to a truck, pairwise exchanges are made to improve the solution.

4.3. Look-back procedure

The look-back procedure is applied whenever the maximum working time M_t of at least one vehicle exceeds the maximum allowed working time duration H' . The procedure works with a matrix (b_{it}) . The counter b_{it} takes the value 0 if station i is not included in the set S_t of stations to replenish at period t ; it takes the value 1 if i must be replenished at period t in order to avoid stockout; it takes the value $u - t + 1$ if period u is the first period at which station i will run out of stock for at least one product if it is not replenished before that period. The value of b_{it} must be updated whenever the delivery period of i is modified. Initially $b_{it} = 1$ if station $i \in S_t$ and 0 otherwise. In order to make X_t feasible, the procedure iteratively moves the visit of some stations from period t to period $t - 1$. The procedure first selects a station $i \in S_t, t > 1$ and moves it to period $t - 1$. It sets $b_{i,t-1} := b_{it} + 1$ and $b_{it} := 0$. It then solves the route construction, truck loading and route packing problems for period t without station i . This operation is repeated until X_t is feasible, in which case the look-back procedure terminates and the process moves to period $t - 1$.

We have developed three variants of this procedure. They differ in the criterion λ used to select which station $i \in S_t$ should be delivered in period $t - 1$. Each variant first computes for period t the set $I_t = \arg \min_i \{b_{it} \mid b_{it} > 0 \text{ and } b_{i,t-1} = 0\}$. This set contains stations to be delivered at period t ($b_{it} > 0$) and for which the number of days by which their delivery has been advanced is minimal. For each $i \in I_t$, a pair $(b_{it}, \alpha_{it}^\lambda)$ is also computed, where α_{it}^λ is defined below. The $(b_{it}, \alpha_{it}^\lambda)$ pairs are then ordered lexicographically and stations i are removed one by one from S_t until X_t is feasible.

The definitions of α_{it}^λ are as follows. In the first variant, $\alpha_{it}^1 = \max_p \{v_{ipt}/m_{ip}\}$, i.e. α_{it}^1 computes the maximum, over all products of station i , of the demand to tank capacity ratio. The procedure will therefore favour a station having a tank that empties slowly. In the second variant, $\alpha_{it}^2 = \min_p \{s_{ipt}/v_{ipt}\}$, i.e. α_{it}^2 computes the minimum over all products, of the stock level to sales ratio. The procedure will therefore favour a station having a product that will reach zero inventory the soonest. In the third variant, $\alpha_{it}^3 = \min_{j \in S_{t-1}} \{t_{ij}\}$. The procedure will therefore favour the closest station to one of those that should be visited at period $t - 1$.

4.4. Look-ahead procedure

The look-ahead procedure is applied whenever the maximum working time M_t does not exceed the normal duration H of a working day. Its purpose is to increase the workload of period t without exceeding H . It operates with a parameter θ equal to the number of periods considered ahead of t . The procedure computes for each station $i \notin S_t$, the period $\pi_i + t$ at which the first stockout will occur at that station. If $\pi_i \leq \theta$, station i is included in S_t and b_{it} is set equal to $\pi_i + 1$; otherwise this station is not considered for an anticipated visit and b_{it} remains equal to zero. The route construction procedure is then applied to S_t . If the solution contains routes visiting a station or pair of stations whose delivery has been anticipated, then these stations are removed from S_t and the corresponding b_{it} values are set back to zero. In other words, if at period t a route contains only stations that can feasibly be delivered later than t , then this route can be postponed and is therefore eliminated from the delivery plan of period t . The truck loading procedure is then applied to the remaining routes of period t . If the maximum truck workload does not exceed H , the algorithm proceeds to the next period. Otherwise, some stations of S_t are moved back to their original positions. This is achieved in a manner similar to what was done in the look-back procedure. Three variants λ are considered, and for each of them a coefficient β_{it}^λ is defined: $\beta_{it}^1 = \alpha_{it}^1$, $\beta_{it}^2 = \alpha_{it}^2$, and $\beta_{it}^3 = \min_{j \in S_t} \{t_{ij}\}$. The $(b_{it}, \beta_{it}^\lambda)$ pairs are then ranked in reverse lexicographical order and stations i are removed one by one from S_t until stations of S_t can be visited within time H . This process ensures that the stations benefitting the least from the visited period t are removed first from S_t .

5. Computational results

The algorithm described was coded in Objective-C and run on a PowerPC G4 1.33 GHz processor. It was tested on 100 randomly generated problem instances with 200 stations over a planning horizon of 28 days. Because the problem under study is highly complicated, no optimal solutions or even good lower bounds are available. As a result, we can only make indirect or limited comparisons. Note that the route construction and truck loading procedures of the algorithm are optimal [3]. The route packing problem is similar to that of Boctor et al. [13] which has proved superior to alternative heuristics including simulated annealing. For the look-back and look-ahead procedures, we have conducted an in-depth sensitivity analysis of the parameters (Section 5.2). To our knowledge, only one paper [6] has treated the same problem as this one and we will provide a full comparison in Section 5.3.

5.1. Test instances

We have generated test instances having similarities with real-life problems. Thus, using a set of real data [14], we have determined six categories of petrol stations in function of their total daily sales. Table 1 gives the lower and upper bounds on daily sales, and the percentage of stations belonging to each category. Historical daily consumption data indicate that on average the sales of regular, intermediate and super petrol grades are 76%, 7% and 17% of the total, respectively. Because sales vary from one day to another, Table 2 gives the percentage of weekly sales for each weekday and the ratio of daily sales with respect to the weekly average. Our data also indicate that underground tank capacities and daily sales are related. Table 3 gives the most commonly observed tank capacities, respectively for the three petrol grades, in function of total daily sales. Finally, we consider three tank truck configurations among those commonly used in practice. Table 4 provides the number of compartments and their capacity for each truck type.

Table 1: Daily sales distribution

Daily sales bounds (litres)	Percentage (%)
0 – 1 350	21.7
1 350 – 2 700	22.6
2 700 – 5 400	29.8
5 400 – 8 100	13.6
8 100 – 10 800	6.2
10 800 – 16 200	6.1

Table 2: Weekday adjustment coefficient

Weekday	Sales (%)	Adjustment coefficient
1	13	0.91
2	13	0.91
3	13	0.91
4	18	1.26
5	18	1.26
6	12.5	0.875
7	12.5	0.875

Table 3: Underground tanks configurations in function of daily sales

Daily sales (litres)	Tank	Tank size (litres)
0 – 2 700	1	25 000
	2	15 000
	3	15 000
2 700 – 8 100	1	35 000
	2	22 700
	3	25 000
8 100 – 16 200	1	50 000
	2	25 000
	3	35 000

Table 4: Configurations of the tank trucks

Type	Total capacity (1000 litres)	Number of compartments	Capacities (1000 litres)
1	60	6	17, 6, 10, 10, 7, 10
2	54	5	16, 6, 6, 10, 16
3	50	4	16, 8, 12, 14

To generate sales data for a given station, we start by generating an average daily sales for the whole planning horizon (Table 1). This is done by randomly selecting a size category according to the daily sales distribution and by then randomly determining the average daily sales within the lower and upper limits of the selected size category. This daily sales average is then multiplied by the weekday adjustment coefficient (Table 2) to determine the total daily sales for every day of the planning horizon, and by the product sales coefficient to determine daily sales for each product type. For each station, the underground tanks configuration is randomly selected among the three configurations presented in Table 3. We select the configuration corresponding to the daily sales with a probability of 80%, and one of the two other configurations with probability of 10% for each one.

Depot coordinates are (50, 150) for all instances, while stations coordinates are integers randomly drawn from a uniform distribution in a 100km×300km Euclidean space. Distances are Euclidean and are not truncated.

For all problems, the fleet is composed of three trucks of type 1, three of type 2 and two of type 3 for a total of eight tank trucks. Also we used the following data for all instances:

revenue per delivered litre:	\$0.004;
regular working time hourly cost per regular working hour:	\$15.00;
overtime hourly cost:	\$30.00;
travel variable cost per kilometer:	\$0.67;
average travel speed (km/h):	70.0;
truck loading time (minutes):	30;
station delivery time (minutes):	45;
daily regular working hours:	8;
daily maximum overtime hours:	4.

5.2. Performance of the proposed the look-back and look-ahead procedures

In this section, we analyze the performance of the proposed look-back and look-ahead procedures in function of its different parameters and embedded rules. Normalized average results over 100 instances are reported in Table 5. The first two lines (in bold) correspond to the case base to which all remaining scenarios are compared.

We can observe that for a given λ value, larger average profits are obtained twice with $\theta = 4$ and once with $\theta = 3$. This can be explained by the success of the look-ahead procedure. As an example, for $\lambda = 1$ and $\theta = 4$, we obtain a profit of 1.0919 (9.19% larger than with $\lambda = 1$ and $\theta = 0$) which comes from a reduction of overtime (0.6970) and from a higher delivered quantity (1.0010). In this case the improvement in the quantity delivered and in overtime is larger than the cost increase (larger distance traveled and higher total working time), which

yields a higher profit. The same behaviour is observed for each λ criterion, meaning that the impact of the number of periods considered in the look-ahead procedure is independent of λ .

For a given θ , no λ value dominates the others. Note that the maximal profit (1.0919) is obtained with $\lambda = 1$ and $\theta = 4$.

Computing times range from 3.9 seconds (for $\lambda = 1$ and $\theta = 0$) to 15.6 seconds (for $\lambda = 3$ and $\theta = 5$). As expected, computing times are closely related to the number of period ahead considered in the look-ahead procedure. Using $\lambda = 3$ seems to require slightly more computing time.

5.3. Comparison with the Taqa Allah et al. algorithms

Taqa Allah et al. [6] have proposed several solution procedures and have shown that their A4 algorithm was the best available. In order to further assess the quality of the solution procedure proposed in this paper, we compare it to the A4 heuristic which can be decomposed into eight steps:

1. identify stations that must be delivered at period t , determine products and quantities to deliver, assign a vehicle to each station, and load their compartments;
2. if possible, assign the least satisfied products to free compartments;
3. if possible, assign products that do not need to be delivered at period t to free compartments;
4. attempt to match stations requiring one compartment and others requiring two;
5. attempt to group together stations requiring one compartment and assign them to the same vehicle;
6. assign routes to vehicles (if needed, determine the number of vehicles to rent);
7. if there are unused compartments, try to insert stations that need to be delivered at period $t + 1$ into existing routes;
8. if there are under-used vehicles, try to construct routes with stations that need to be delivered at period $t + 1$.

We observe that the A4 heuristic was designed to solve a slightly different version of the MPSRP where trucks are identical and composed of three compartments (one for each product), demands are constant over the planning horizon, and where it is possible to rent extra trucks if needed. In addition, the A4 heuristic uses truck fixed costs. Consequently, we had to generate another set of test instances to fit these extra features in order to perform our comparative analysis.

We randomly generated 30 test instances with 200 stations and used a 28 days horizon. In these instances, we used a homogeneous fleet composed of three-compartment trucks with capacities 25 000, 15 000 and 9 000 litres for a total of 49 000 litres. Daily sales are constant over time. In addition, we set daily fixed costs per truck equal to \$250 and a daily rental cost per truck equal to \$750. In order to allow each method to produce its best possible results, we tried different fleet sizes. The proposed algorithm was run using $\theta = 4$ and $\lambda = 1$.

We have carried out comparisons with the Taqa Allah et al. heuristic on the total cost basis. The original A4 algorithm was coded in Turbo Pascal but run on the same computer. Results presented in Table 6 show that the solution procedure developed in this paper outperforms the Taqa Allah et al. heuristic for all fleet sizes. Our procedure produced its best results with a fleet of five trucks (cost: \$109 462) while the A4 algorithm requires seven trucks to produce its best results (cost: \$169 053). If we compare the best results of both procedures, we see that

Table 5: Normalized average results as a function of the parameters.

λ	θ	Dist	TT	OT	Vis	Tps	Qty	Profit	Sec
1	0	74 308 km	1 857 h	264 h	795	405	22 350 489 1	\$7 792	3.93
		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0057	1.0075	0.8191	1.0099	1.0105	0.9978	1.0035	1.1908
	2	1.0065	1.0106	0.7042	1.0158	1.0168	0.9996	1.0664	1.9312
	3	1.0070	1.0113	0.7004	1.0168	1.0180	1.0010	1.0777	2.6284
	4	1.0057	1.0104	0.6970	1.0164	1.0180	1.0010	1.0919	3.2086
	5	1.0054	1.0104	0.7087	1.0164	1.0187	1.0013	1.0915	3.7404
2	0	0.9996	0.9997	1.0008	0.9998	0.9998	0.9998	1.0009	1.0610
	1	1.0056	1.0076	0.8192	1.0102	1.0108	0.9979	1.0051	1.2951
	2	1.0076	1.0122	0.7167	1.0180	1.0195	1.0020	1.0748	1.9083
	3	1.0070	1.0117	0.7150	1.0176	1.0195	1.0025	1.0862	2.5776
	4	1.0083	1.0130	0.7150	1.0188	1.0202	1.0029	1.0785	3.2442
	5	1.0076	1.0123	0.7223	1.0183	1.0199	1.0032	1.0850	3.7888
3	0	1.0004	1.0004	1.0017	1.0004	1.0004	1.0003	0.9990	1.1170
	1	1.0071	1.0088	0.8218	1.0109	1.0117	0.9985	0.9969	1.4427
	2	1.0072	1.0120	0.7130	1.0180	1.0190	1.0016	1.0751	2.0254
	3	1.0070	1.0118	0.7024	1.0178	1.0196	1.0021	1.0881	2.7303
	4	1.0078	1.0129	0.6998	1.0193	1.0213	1.0028	1.0888	3.5115
	5	1.0067	1.0114	0.7110	1.0173	1.0191	1.0018	1.0839	3.9745

λ criterion used in the look-back and look-ahead procedures;
 θ number of periods ahead considered in the look-ahead procedure;
 Dist total distance travelled;
 TT total working time needed (including overtime);
 OT total overtime used ;
 Vis total number of visits;
 Tps total number of trips;
 Qty total delivered quantity;
 Profit total profit realized;
 Sec average computing time in seconds.

Table 6: Comparison with the Taqa allah et al. A4 algorithm.

	Tr	RTr	TT	OT	Dist	Vis	Tps	Qty	Cost	Sec
Taqa allah	5	36	2 050	437	123 030	756	732	25 684 954	181 718	0.1
	6	16	2 031	340	121 930	754	731	25 680 605	171 436	0.1
	7	7	2 024	221	121 479	753	730	25 677 446	169 053	0.1
	8	2	2 019	126	121 221	752	729	25 672 442	170 968	0.1
Proposed in this paper	5	–	1 298	262	77 937	831	423	21 280 112	109 462	15.9
	6	–	1 352	104	81 156	869	442	22 193 457	118 226	15.0
	7	–	1 354	10	81 283	886	450	22 340 271	123 936	21.0
	8	–	1 379	0	82 779	913	463	22 580 225	132 164	16.1
Deviation	5		–36.7%	–40.1%	–36.7%	+10.0%	–42.2%	–17.1%	–39.8%	
	6		–33.5%	–69.5%	–33.4%	+15.3%	–39.5%	–13.6%	–31.0%	
	7		–33.1%	–95.4%	–33.1%	+17.6%	–38.4%	–13.0%	–26.7%	
	8		–31.7%	–99.6%	–31.7%	+21.4%	–36.5%	–12.0%	–22.7%	

Tr number of trucks;
RTr number of rented trucks over the 28 days horizon;
TT total working time (including overtime);
OT total overtime hours used ;
Dist total distance travelled;
Vis total number of visits;
Tps total number of trips;
Qty total delivered quantity in litres;
Cost total cost;
Sec average computing time in seconds.

the proposed solution procedure yields a total cost 35% lower than that of the Taqa allah et al. algorithm. If we examine the ratio Vis/Tps, we can also see that the proposed procedure mainly produces routes visiting two stations (with an average of 1.93), whereas the Taqa allah et al. algorithm mainly uses routes that visit only one station (with an average of 1.03). Our algorithm therefore reduces the number of trips by 42% and the total travelled distance by 35%. This yields a total working time reduction of 35%. With our algorithm, the delivered quantities are reduced by 17%. This means it attempts to deliver smaller amounts without incurring stockouts. The cost per delivered litre is 22% lower with our algorithm, and the number of litres per kilometers is 29% higher. These results are hardly surprising since the Taqa allah algorithm is a simple greedy procedure which is very quick but fails to perform an in-depth search.

6. Conclusion

We have developed a heuristic solution procedure for the Multi-Period Petrol Stations Replenishment Problem. This iterative heuristic is composed of several procedures and embeds the solution of a Route Packing Problem. The proposed heuristic was extensively tested on randomly generated problems and compared to a previously published algorithm. Our comparative analysis shows that the proposed heuristic is significantly superior to that algorithm. Avenues for future research include the possibility of considering routes with more than two stations, and partial unloadings of compartments.

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