Summing Up the Feynman Diagrams: Toward Quantum Gluonodynamics

Borros Arneth[*,1,2], Justus Liebig University Giessen, Giessen, Germany, and Philipps University Marburg, Marburg, Germany

[1]Philipps University Marburg, Baldingerstr. 1, 35033 Marburg, Germany

e-mail: borros.arneth@staff.uni-marburg.de

[2]Justus Liebig University Giessen, Feulgenstr 12, 35392 Giessen, Germany

e-mail: borros.arneth@uni-giessen.de

[*]Corresponding Author Dr. Borros Arneth, borros.arneth@staff.uni-marburg.de

Abstract

Summing up all *Feynman diagrams* describing an elementary particle can provide a measure of the energy and, with it, the mass of that particle. Moreover, a single mass quantum can be used to convert the *Feynman sum* into the particle mass. In the following, a mass formula for the calculation of the baryon and meson masses is introduced and explained. This formula involves calculating the number of possible Feynman diagrams and multiplying it by an elementary mass quantum. The mass formula results from a generalization of the connection between the electromagnetic coupling constant alpha (*Sommerfelds* constant) and the *Rydberg constant*. This mass formula adds an energy parametrization to the Standard Model, an important component that has been missing to date. Afterward, this mass formula is interpreted, leading to an interpretation of the elementary particles that is similar to the way in which molecules are interpreted. In this interpretation, gluons take the place of electrons in the case of elementary particles.

Résumé

La synthèse de tous les *diagrammes de Feynman* décrivant une particule élémentaire peut fournir une mesure de l'énergie et, avec elle, de la masse de cette particule. De plus, un seul quantum de masse peut être utilisé pour convertir la *somme de Feynman* en masse de particules. Dans ce qui suit, une formule de masse pour le calcul des masses du baryon et du méson est introduite et expliquée. Cette formule consiste à calculer le nombre de diagrammes de Feynman possibles et à le multiplier par un quantum de masse élémentaire. La formule de masse résulte d'une généralisation de la connexion entre la constante de couplage électromagnétique alpha (*constante de* Sommerfelds) et la *constante de Rydberg*. Cette formule de masse ajoute une paramétrisation énergétique au modèle standard, une composante importante qui a été manquante à ce jour. Cette formule de masse est ensuite interprétée, conduisant à une interprétation des particules élémentaires similaire à la manière dont les molécules sont interprétées. Dans cette interprétation, les gluons prennent la place des électrons dans le cas des particules élémentaires.

Keywords: partition function; origin of masses; particle interactions; particle vertices; energy masses and/or energies

The *Standard Model* of Particle Physics

The *Standard Model* is a theory that emerged in the 1970s to describe all known elementary particle interactions. This model incorporates quantum chromodynamics *(QCD)*, quantum electrodynamics *(QED)*, and the quantum well *(QW)* theory of electroweak processes and offers the most valid information to date for all known microscopic-world phenomena[1,2,3]. The outstanding success of the *Standard Model* is that it offers a complete understanding of electroweak interactions[4,5,6]. By describing the electromagnetic, strong, and weak nuclear forces and how certain fundamental particles mediate them, the *Standard Model* offers an adequate understanding of how matter particles interact with one another on a microscopic scale.

The *Standard Model* has a long history. In 1949, *Richard P. Feynman*, an American physicist, introduced a pictorial representation of particle interactions, now called Feynman diagrams[7]. As a result, *Feynman* was able to build a powerful mathematical theory called *QED* that describes the interactions between photons and other particles[2,7]. Today, *Feynman diagrams* are used to provide insight into the predictive power of the current *Standard Model* of particle physics[7,8].

Murray Gell-Mann also played a significant role in introducing concepts that led to the development of the *Standard Model*. He proposed that protons and neutrons are divisible and comprise smaller particles called quarks[9]. In 1956, *Gell-Mann* confirmed the findings of *Kazuhiko Nishijima* regarding the strange behaviors of cosmic rays and their products, such as sigma baryons, mesons, and lambda baryons[9,10]. *Nishijima* and *Gell-Mann* independently proposed that these strange behaviors could be understood if they had a quantum number known as “strangeness.” This explanation led to the emergence of the *Gell-Mann-Nishijima* formula, which came to form an integral part of particle physics[10,11].

Given Q=electric charge, I_3=3rd component of isospin, B=baryon number, S=strangeness number, C=charm number, B'=bottomness number, and T=topness number, the Gell-Mann-Nishijima formula becomes:

$$\mathrm{Q} = I_3 + \frac{1}{2}(B + S + C + B' + T) = I_3 + \frac{Y}{2} \qquad (1)$$

Equation[11,12]

Today, physicists describe the Standard Model mathematically using the notation of group theory. Given that $SU(3)$=gauge group of strong interactions and $SU(2) \times U(1)$=gauge group of electroweak interactions, the group theory notation becomes:

$$SU(3) \times SU(2) \times U(1) \qquad (2)$$

Equation[13]

Here, $SU(3)$, $SU(2)$, and $U(1)$ represent the sets of all 3×3, 2×2, and 1×1 unitary matrices, respectively, all with unit determinants.

The *Standard Model* has also been described mathematically using the *Standard Model Lagrangian*, as follows:

$$\mathcal{L} = -\frac{1}{4}\mathrm{F}_{\mu\nu}F^{\mu\upsilon} + i\bar{\psi}\not{D}\psi + \mathrm{h.c.} + \psi_i y_{ij}\psi_j\phi + \mathrm{h.c.} + \left|D_\mu\phi\right|^2 - V(\phi) \qquad (3)$$

Equation[14,15]

Here, $\mathcal{L}$ stands for the *Lagrangian density*; $-\frac{1}{4}\mathrm{F}_{\mu\nu}F^{\mu\upsilon}$ is the scalar product of the field strength tensor $F_{\mu\upsilon}$, which contains the mathematical encoding of all interaction particles apart from the Higgs boson; μ and υ are *Lorentz* indices representing the spacetime components; $i\bar{\psi}\not{D}\psi$ is a term describing the interactions within matter particles; ψ and $\bar{\psi}$ are fields describing antiquarks and antileptons respectively; $\not{D}$ is a covariant derivative that features all particles except the Higgs; $\mathrm{h.c.}$ denotes the *Hermitian* conjugate of term 2; $\psi_i y_{ij} \psi_j \phi$ is a term describing how matter particles couple to the *Brout-Englert-Higgs* (BEH) field ϕ; y_{ij} is the *Yukawa* matrix representing the coupling parameters to the BEH field; $|D_\mu \phi|^2$ is a term describing how the interaction particles couple to the BEH field; and $-V(\phi)$ is a term describing the potential of the BEH field[14,15].

The current *Standard Model* is based on earlier discoveries about quarks and gluons and the color interaction. These ideas may be best introduced by using the concept of a *quark-gluon plasma (QGP)* as theoretical motivation[16].

This theory asserts that nuclear matter exists at extremely high temperatures and densities due to composite states called hadrons losing their identity and dissolving into a 'soup' of their constituents, namely, quarks and gluons[16].

Hadrons are classified into baryons and mesons. Every baryon comprises three quarks, whereas every meson consists of a quark and an antiquark[17]. Unlike atoms and molecules, which may be ionized to determine their constituents, quarks and gluons are confined inside hadrons and never exist freely[18]. *Torassa* [2018] compares this situation with the attempt to decompose a magnet into two parts. An attempt to isolate its poles, north and south, by surrounding the two magnetic poles with a superconducting medium confines the magnetic field into a ‘thin tube’[19,20,21]. Similarly, a hadronic string comprising quarks and antiquarks at its endpoints has a one-dimensional field confined by vacuum.[22]

According to *Xiong et al.* [2019], quarks are held together by a fundamental force called the strong force[23]. Similar to the electromagnetic interaction, which is based on electric charge, the strong interaction is purely based on color charge. This interaction may be described by applying the local gauge theory of *QCD*[22]. According to *QCD*, the symmetry in quarks is color, a conserved quantity, meaning that color cannot be created or destroyed[23]. In *QCD*, there exist imaginary field lines between quarks, which comprise numerous gluons[22]. Since all gluons constituting the imaginary field lines have a color charge, they attract each other. Unlike in the case of electrons, however, for which the force between them decreases with increasing distance, the color force binding quarks increases with increasing distance[24]. In other words, the color force behaves the same way as stretching a rubber band. The more a rubber band is stretched, the more force is needed to extend it further. Additionally, the color force appears to exert little force at short distances[23]. This property suggests that the quarks behave as free particles within the confining boundaries of the color force. Quarks only experience a solid confining force as they start to become too far apart.

Two or more quarks in close proximity to each other rapidly exchange gluons and, hence, create a strong field of color force that binds the quarks together[22]. There are three color charges *(red, green, and blue)* and three corresponding anticolor charges *(antired, antigreen, and antiblue)* [22].

Experiments show that as quarks exchange *gluons*, they also constantly exchange their *color charge*[22,23]. Although it is expected that there should be nine possible gluons with different *color-anticolor* combinations, one of these combinations is symmetrically eliminated such that gluons can carry only eight possible *color-anticolor* combinations. Since color-charged particles do not exist freely, the color-charged quarks are confined in hadrons with other quarks[24], and the resulting composites are color neutral. This discussion highlights the importance of the *Standard Model* in explaining why quarks combine only into baryons and mesons and not into four-quark objects.

Although the *Standard Model* was initially based on theoretical concepts such as the *Big Bang* theory, numerous experiments have been developed to offer strong evidence that this model is the correct model for particle physics. These experiments include the Positron-Elektron-Tandem-Ring-Anlage *(PETRA)*, Large Electron-Positron Collider *(LEP)*, and Large Hadron Collider *(LHC)* experiments[21,25]. Together, these experiments have led to the discovery of quarks and gluons and the color interaction, revealing how the protons are huddled together in the nucleus and how the electrons occupy an atom's energy levels. A recent discovery for which these experiments paved the way was the experimental discovery of the Higgs boson in 2012[25]. This discovery is expected to help unify all fundamental forces except gravity[25]. The observation of the Higgs mechanism confirms the existence of quarks and gluons and the color interaction and how their interactions keep matter particles bonded together.

Overall, the *Standard Model* provides the most valid explanation to date for all known microscopic-world phenomena. The model uses the *Big Bang theory* to describe matter particles and the forces binding them together. The *Standard Model* has led to the discovery of new models such as the *Higgs boson* and *QGP,* which have opened new directions for physics. Through these new models, physicists have discovered the existence of quarks and gluons and the color interaction, which explain how the protons are huddled together in the nucleus and how the electrons occupy an atom's energy levels. Future studies should focus on investigating the Higgs boson to establish whether it conforms to the paradigms of the *Standard Model* and whether new physics exists that complements the *Higgs boson*. However, an energetic parametrization of the *Standard Model* is still missing at present, meaning that it is still unclear and unknown how the elementary particles obtain the energies and masses they possess. On the other hand, it is clear that the masses of the elementary particles derive to a large extent from their intraparticle interactions. Therefore, the masses of the elementary particles and, in turn, their energy must largely result from their *quark-quark* and *gluon-gluon* interactions.

<u>Assignment here:</u> To find an *energy parametrization* completing the *Standard Model*

In the following, it is assumed that there exists a smallest energy (or mass) quantum that corresponds to the *Rydberg energy* (or the *Rydberg mass*).

This mass quantum (the *Rydberg energy*) is here thought to be the minimum energy equivalent and the minimal amount of energy needed for the realization of each single *Feynman diagram*. Thus, if each *Feynman diagram* needs one *Rydberg* to be realized, then the number of possible *Feynman diagrams* defines the energy and/or mass needed to realize the existence of a given particle.

Half of the square of the coupling constant of any interaction then corresponds to the ratio between the *Rydberg energy* and rest energy (i.e., mass) of the elementary particle belonging to this specific interaction (see formula 4).

$$E_{Ryd} = m_e c^2 \bullet \frac{\alpha^2}{2} = E_e \frac{\alpha^2}{2} \qquad \Leftrightarrow \qquad \alpha = \sqrt{2E_{Ryd}} \cdot \frac{1}{\sqrt{E}} \tag{4}$$

Equation 4: This equation gives the relationship between Rydbergs energy (ERyd), the mass of the electron (me); the light-speed constant (c); and the fine structure constant (FSC). This universal relationship is of fundamental nature. Background is the well-known relationship between Sommerfelds and Rydberg’s constants.

This behavior can explain the energy dependency of the different coupling constants for the strong, electromagnetic and weak interactions (see figure 1).

Energy-Dependency of Coupling Constants

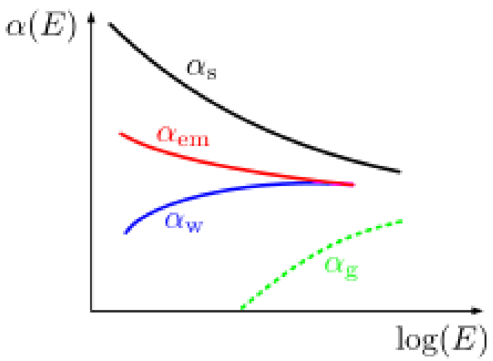


Figure 1: This figure gives the energy dependency of the coupling constants (identical to the fine structure constant, FSC in the electromagnetic case em).

Any coupling constant can now be expressed in the form of a power term of the circle number π. This is first done for the electromagnetic coupling constant in equations of formula 5 and 6.

$$\alpha_{EM} = \frac{1}{137} = \frac{\sqrt[4]{1-\frac{1}{3\pi}}}{(3\pi)^3} \cdot 2\pi \qquad (5)$$

Equation 5: This formula gives the relation between the fine structure constant and the circle number pi. It is possible to describe the FSC by using a term of pi. This relation is of fundamental nature.

$$\frac{1}{\alpha_{EM}} = \frac{27\pi^2}{2\sqrt[4]{1-\frac{1}{3\pi}}} = 137.028744 \qquad (6)$$

Equation 6: This equation gives the relation between the inverse of the fine structure constant and its description using a pi-term. Again this relation is of fundamental nature.

Later, this is generalized in the form of equation formula 7.

Increasing-factor for the electron: $$\frac{m_e}{m_{Ryd}} = \frac{E_e}{E_{Ryd}} = \frac{2}{\alpha_{EM}{}^2} = 2^{-1}3^2(3\pi)^4\left(\frac{1}{\sqrt[8]{1-\frac{1}{3\pi}}}\right)^4 \qquad (7)$$

Equation 7: This formula gives the increasing-factor for the electron. The mass of the electron can be described as a multiple of Rydberg's energy. Furthermore, the increasing-factor is given in a generalized form valid for each particle. Each particles mass can be described as a multiple of Rydberg's energy. In this sense the Rydberg energy is the smallest quantum of mass.

There are two motivations for using the circle number π here:

1.) The number 2π plays a central role in Fermi's golden rule in quantum dynamics.

2.) Cauchy distributions play a very important role in quantum dynamics.

The area under a Cauchy distribution corresponds exactly to π. With three Cauchy distributions for the three important *color-anticolor* interactions, *red-antired*, *green-antigreen*, and *blue-antiblue*, this situation corresponds to exactly the number 3π.

The *mass formula*

The *mass formula* that I have developed contains several different terms or interactions, each of which influences the mass of a particle. The mass formula is given in formulas/equations 8 and 9.

Increasing-factor for a particle:
$$\frac{m_p}{m_{Ryd}} = \frac{E_p}{E_{Ryd}} = \frac{2}{\alpha^2} = 2^a \cdot 3^b \cdot (3\pi)^c \cdot \left(\frac{1}{\sqrt[8]{1 - \frac{1}{3\pi}}} \right)^d \qquad (8)$$

Equation 8: This formula gives a generalized mass formula. This generalized mass formula is composed of several factors. One factor for each interaction-form present in a given particle. In turn the different factors form the partition function. The product of the partition function and the Rydberg energy leads us to the mass and/or energy content of the particle.

Mass- formula: The rest- mass and/or rest- energy of a given particle is given by

$$2^{a} \cdot \quad 3^{b} \cdot \quad (3\pi)^{c} \cdot \quad \left(\frac{1}{\sqrt[8]{1-\frac{1}{3\pi}}} \right)^{d} \cdot E_{Ryd} \tag{9}$$

qq qqq 3 gluon vertices 4-gluon vertices

quark-quark gluon-self-coupling 2-gluon interactions

I interpret the terms of equation 9 as *follows*:

The formula (eq 9) seems to be the result of a weighted summing-up of all possible *Feynman diagrams* that describe the particle in question. The different parts of the formula describe the different interacting constellations, and their product describes the number of all possible interacting constellations. The energy and, with it, the mass of a particle is given by the total number of all possible *Feynman diagrams* for that particle.

Regarding the different potencies (a,b,c,d) in the formula eq(9)

a- Number of *two*-quark interactions (or color-anticolor interactions) *qq*

Two to the power of a describes the number of *two-quark* interactions realized in the elementary particle, considering, in particular, the number of *color-anticolor* (or, more specifically, in most cases *quark-antiquark*) interactions.

b- Number of *three*-quark interactions (or color interactions) *qqq*

Analogously, three to the power of b describes the number of color interactions, in particular the number of *three-quark* (*three-color*) interactions realized in the particle of interest.

c- Number of *gluon three*-vertex interactions G^3

Three π to the power of c describes the number of quantum-mechanical gluon self-interactions. The number of these gluon self-interactions is given by the number of three-vertex gluon interactions.

In particular, this term describes the number of three-vertex interactions corresponding to *gluon-antigluon* particle formation. As *gluon-antigluon* particle formation is necessary for this form of interaction, the corresponding energy (3π) is quite high.

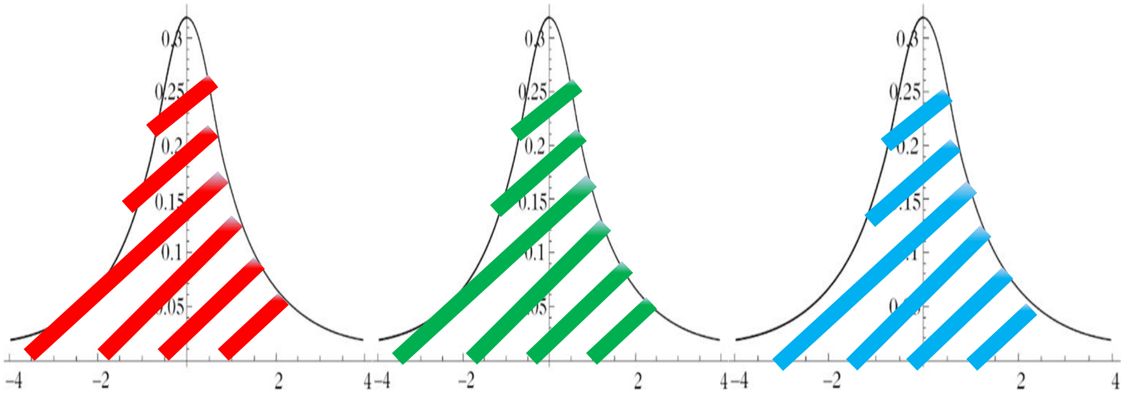
If we identify the energy associated with such interactions with the number of *Feynman diagrams* and propose a relation between the number of *Feynman diagrams* and the system energy of a certain particle, then we can also go a step further and additionally identify the arrow length with the base value of a certain partial binding energy (here, the base value is 3π). For example, a three-gluon vertex can be written in the form of a *Dyson-Schwinger equation* (DSE). From this DSE of the 3-gluon vertex, three so-called *swordfish equations* result. The arrows in the *Feynman diagrams* of these swordfish equations form circles. If we now look at two opposite points lying on a circle, the distance between these points is equal to π. For three *DSE swordfish* equations, the distance is then equal to

3π. Hence, this could be a graphical explanation for the base energy of 3π emerging here for the three-gluon vertex. However, all other terms of the *DSE* should then sum up to zero for the three-gluon vertex.

d- Number of *gluon four*-vertex interactions G^4

Analogously, one minus one divided by 3π describes the probability of the counter-events to a three-vertex event. The 8th root of this quantity then corresponds to the conditional probability of a *two-gluon 4-vertex* event. The reciprocal of this corresponds to the corresponding sum of states (i.e., to the corresponding *Cauchy* distribution again). This interpretation of the mass formula is also illustrated in the equations of figure 2 and equations 10 -13.

Figure 2 Three Cauchy-distributions 3π partition function / state-sum of micro-conditions



π **rr** + π **gg** + π b**b**

=3π (3 gluon vertex, gluon-self-interaction)

$$\frac{1}{3\pi}\longrightarrow$$ Probability for a r**r**, g**g**, b**b**- event (3-vertex- event) (10)

$$1-\frac{1}{3\pi}\longrightarrow$$ Complement probability for not a r**r**, g**g**, b**b**- event (11)

$$\sqrt[8]{1-\frac{1}{3\pi}} \longrightarrow$$ Probability for conditional event / conditional probability for 4 gluon vertex (12)

$$\frac{1}{\sqrt[8]{1-\frac{1}{3\pi}}} \longrightarrow$$ Partition function / State sum for the 4 gluon-vertex- event , (13)

this event is favored by a quark asymmetry

= 4-gluon vertex, 2nd way of gluon-self-interaction

Equations 10, 11, 12, 13: These formulas 10-13 visualize the factor 3pi with three Cauchy distributions. One distribution for each color interaction rr,gg, and/or bb. Furthermore the probablitiy of the 3-gluon vertex and the probability for the 4-gluon vertex are calculated.

This term represents the number and energy of *4-gluon-vertex (G^4)* events in the particle of interest, which correspond to gluon-gluon interactions. Because no additional particles or gluons need to be formed for a *4-gluon-vertex* event, this energy is relatively small.

Based on these considerations, an elementary particle can be interpreted in a completely analogous way to a molecule in chemistry, with the electrons corresponding to the gluons.

The gluons hold a particle together, analogous to the electrons that hold a molecule together. Moreover, similar to electrons, gluons can exist in different gluon orbitals, and the orbitals themselves can hybridize with one another. Therefore, the symmetry of a particle depends on how many and which gluon states there are and how the gluons can be distributed and hybridized with one another in that particle.

Through analysis, different exponent series *(a, b, c, d)* can be found for the different elementary particles. These different exponent series *(a, b, c, d)* are given in table 1. In the following, these exponent series or *n*-tuples are interpreted in relation to their respective particles.

Table 1 Exponent- series of the mass- formula calculated for the different particles in MeV

a	b	c	d	mass [MeV]	particle	composition
leptons						
-1	2	4	4	0.5109	electron	TTT
-3	2	7	3	105.44	muon	TTT*
1	2	7	7	1784.49	tauon	TTT**
quarks						
1	2	4	8	2.16	up	u, TTV, TVT, VTT
2	2	4	8	4.67	down	d, TVV, VTV, VVT
2	3	6	15	1270	charm	c
3	2	5	17	92	strange	s
1	2	9	13	172421	top	t
2	3	6	100	4184	bottom	b
vector bosons						
0	0	10	5	80700.0	W	
0	0	10	14	91554.0	Z	
0	0	10	36	124634.0	H	
mesons						
-1	1	7	0	134.8	pi 0	(uu-dd)/2
-1	0	8	11	494.128	K+	us
-1	-2	9	15	547.299	eta 0	uu+dd-2ss
0	0	8	9	960.9	eta dash	uu+dd+ss

1	0	8	7	1868.71	D+	cd
1	0	8	11	1976.5	Ds+	cs
-1	0	9	20	5283.0	B meson	ub
-1	0	9	22	5433.0	strange meson	sb
-1	0	9	33	6339.0	charm meson	cb
vector mesons						
3	0	7	5	771.0	rho 0	(uu-dd)/2
0	0	8	4	895.8720	K*0	ds
3	0	7	6	782.0	omega	(uu+dd)/2
0	0	8	13	1016	phi	ss
1	0	8	12	2004.0	D*0	cu
1	0	8	43	3095.6	J/psi	cc
baryons						
0	-2	9	4	938.15	proton	uud
-3	2	8	11	1111.0	lambda	uds
-3	2	8	16	1192.5	sigma 0	uds
-3	2	8	18	1226.4	delta	ddd
-3	2	8	23	1315.5	xi 0	uss
-1	-1	9	3	1387.6	sigma-*	dds
-3	2	8	29	1430.95	N(1440)	udd
-1	-1	9	10	1530.7	xi 0 reson	uss
1	0	8	1	1670.0	Omega -	sss
-3	2	8	63	2304.0	lambda-c	udc
-3	2	8	68	2472.0	c-sigma	ddc
			129	5815.0	cascade B	usb
-3	2	8	130	5897.0	bottom sigma	ddb

-3	2	8	72	2614.9	charmed xi prime	usc
-3	2	8	96	3661.0	double charmed xi	ucc

Table 1: Table 1 gives the exponent series ((a,b,c,d)-tupels) for the different particles and the total energies of the particles (masses of the particles) calculated from these data.

Regarding the meaning of the ***algebraic pre-signs*** **of the exponents a and b**

When looking at table 1, it is evident that the exponents *a* and *b* can have both positive and negative algebraic signs. Thus, the question arises as to what meaning the algebraic signs of these exponents have. Both *2-quark* and *3-quark* color interactions can exist either in an attractive, stabilizing, energy-releasing form or in a nonattractive, nonstabilizing, energy-consuming form. It is therefore obvious that the negative sign should describe the energy-releasing form and that the positive sign should describe the nonbinding, energy-consuming form. As corresponding examples, the proton and the lambda particle can be considered here. The proton possesses two binding *3-color 3-quark* interactions and no destabilizing *2-color 2-quark* interactions at all. On the other hand, stabilizing *color-anticolor* quark interactions occur in the lambda particle. Theoretically, three such constellations exist, with three possible *color-anticolor* interacting constellations. The question then arises of what is left for the third quark. The third quark can have two further possible color constellations for each of these three pre-existing color constellations. Because of this, *b* is then equal to two, and since the *3-color* constellation in this case cannot be binding and energy-releasing in nature, this value of 2 is positive.

This leads us to the ***energy-defining quotient***

The result of these considerations is a quotient that defines the mass or energy of a particle. This quotient consists of the product or sum of the energy-consuming *Feynman diagrams* divided by the number of *Feynman diagrams* of the energy-releasing constellations.

Interpretation of the *energy-defining quotient*

The question arises of how this energy-defining potency series and quotient can be interpreted and why a product of powers emerges here rather than a simple plus-minus calculation. Usually, energies are calculated through a series of plus and minus calculations of the different partial energies. If energy is released, then this energy needs to be regarded as negative when considered in the calculation. Here, however, the situation is different. Mathematically, everything is, in a sense, elevated by one calculation step: where we usually have a plus, we instead have a multiplication, and where we usually have a minus, we instead have a division, or exponentiation by minus one. Why is it that way here?

There are two possible interpretations of the potency series

1.) The potency series is a form of ***weighted combinatorial counting***. The **weighted number** of all possible constellations is calculated. Hence, in graphical terms, the weight is equal to the number of arrows and/or ***arrow equivalents* in the *Feynman diagrams***. A *quark-antiquark* interaction, for example, has an arrow equivalent of 2; a three-quark interaction has an arrow equivalent of 3; and a 3-gluon vertex has an arrow equivalent of 3π. The arrow equivalent is the basis of the potency, and the exponent gives the number of such interactions present in the particle. Together, the potency gives the number of possible constellations times the number of arrows per constellation, and thus, the number of arrows of the whole interaction form of that particular particle. If we look at *the 3-*

quark interaction in the nucleon, for example, the interaction number is *b = - 2*. Accordingly, we have the constellation possibilities *c**c**, **cc**, and cc*, as two of these interactions are present in the particle. Three different constellations times three Feynman arrows per constellation, this means that we have nine possible arrows in total for this particular interaction in this particular particle, and so on.

2.) The potency series can also be seen as the ***partition function*** of this particular interaction. If we look at the *3-gluon vertex*, for example, the value of 3π can also be interpreted as the area under three Cauchy distributions (one for each color), being the state sum of this particular partition function. If we use the logarithms of the potency series, we can handle these logarithms of single potencies in almost the same way as conventional partial functions—more specifically, as partial energies, or perhaps it is better to say that they are treated like energy coefficients and/or like logarithms of partial partition functions, and we can add them instead of multiplying them. However, the *logarithmic energy* (*ln(E)*) is then obtained as the result, which is equal to the sum of these energy coefficients. In other words, the *logarithmic energy* (*ln(E)*) appears to be equal to the sum of these partition functions (for partial sums please see equation formula 14) .

Logarithmic Interpretation as four partition functions

$$E = 2^a \cdot \ 3^b \cdot \ (3\pi)^c \cdot \left(\frac{1}{\sqrt[8]{1 - \frac{1}{3\pi}}} \right)^d \bullet E_{Ryd} \tag{14}$$

,is equivalent to the formulation

$$\Leftrightarrow E = Z_2 \cdot \ Z_3 \cdot \ Z_{3-vertex} \cdot \ Z_{4-vertex} \cdot E_{Ryd} \tag{15}$$

$$\Leftrightarrow E = Z_{0,2}^a \cdot \ Z_{0,3}^b \cdot \ Z_{0.3-vertex}^c \cdot \ Z_{0,4-vertex}^{d/8} \cdot E_{Ryd} \tag{16}$$

with the logarithmic energy ln(E)

$$\Rightarrow \ln(E) = \; a\cdot\ln(2) + \; b\cdot\ln(3) + \; c\cdot\ln(3\pi) - \frac{d}{8}\ln\left(1-\frac{1}{3\pi}\right) + \; \ln(E_{Ryd}) \tag{17}$$

with the partition functions Z

$$\Rightarrow \ln(E) = \; \ln(Z_2) + \; \ln(Z_3) + \; \ln(Z_{3-vertex}) + \; \ln\left(Z_{4-vertex}\right) + \; \ln(E_{Ryd}) \tag{18}$$

with the basic partition functions Z_0

$$\Rightarrow \ln(E) = a\cdot\ln(Z_{0,2}) + \; b\cdot\ln(Z_{0,3}) + c\cdot\ln(Z_{0,3-vertex}) + \frac{d}{8}\cdot\ln\left(Z_{0,4-vertex}\right) + \ln(E_{Ryd}) \tag{19}$$

partition function **q**q, qqq, 3- gluon 4-gluon Rydberg-Energy

or with probabilities $Z_i = \dfrac{1}{P_i}$ (20)

$$\Rightarrow E = \frac{1}{P_2}\cdot\; \frac{1}{P_3}\cdot\; \frac{1}{P_{3-vertex}}\cdot\; \frac{1}{P_{4-vertex}}\cdot E_{Ryd} \tag{21}$$

$$\Rightarrow \ln(E) = \; -\ln(P_2) - \; \ln(P_3) - \; \ln(P_{3-vertex}) - \; \ln\left(P_{4-vertex}\right) + \; \ln(E_{Ryd}) \tag{22}$$

or with basic probabilities $Z_{0,i} = \dfrac{1}{P_{0,i}}$ (23)

$$\Rightarrow E = \left(\frac{1}{P_{0,2}}\right)^a\cdot\; \left(\frac{1}{P_{0,3}}\right)^b\cdot\; \left(\frac{1}{P_{0,3-vertex}}\right)^c\cdot\; \left(\frac{1}{P_{0,4-vertex}}\right)^{d/8}\cdot E_{Ryd} \tag{24}$$

$$\Rightarrow \ln(E) = -a \cdot \ln(P_{0,2}) \quad -b \cdot \ln(P_{0,3}) - c \cdot \ln(P_{0,3-vertex}) - \frac{d}{8} \cdot \ln\left(P_{0,4-vertex}\right) + \ln(E_{Ryd}) \qquad (25)$$

$$P_{0,3} = \frac{1}{3\pi} \qquad (26)$$

$$P_{0,4} = 1 - \frac{1}{3\pi} = 1 - P_{0,3} \qquad (27)$$

Equations 14-27: These equations 14 to 27 give the partition function for a particle. The partition-function can also be written in a logarithmic form. When we write the partition function in a logarithmic form, the logarithm of the energy results as the sum of the partial partition functions. The probability thereby is the inverse of a partition function.

It is believed here that the above two different interpretations of the energy-defining quotients <u>are equivalent to each other</u> and can both be used interchangeably. However, in the end, we have a number of possible interactions and/or *Feynman arrows* and/or ways for the particle to exist, and this total number of *Feynman arrow equivalents* then needs to be multiplied by the universal mass quantum in order to be transformed into the energy needed.

On the other hand, if one were to think about *partition functions* at a very fundamental level from an ontological point of view, it is questionable whether anything like such sum-able partial energies could exist at this level.

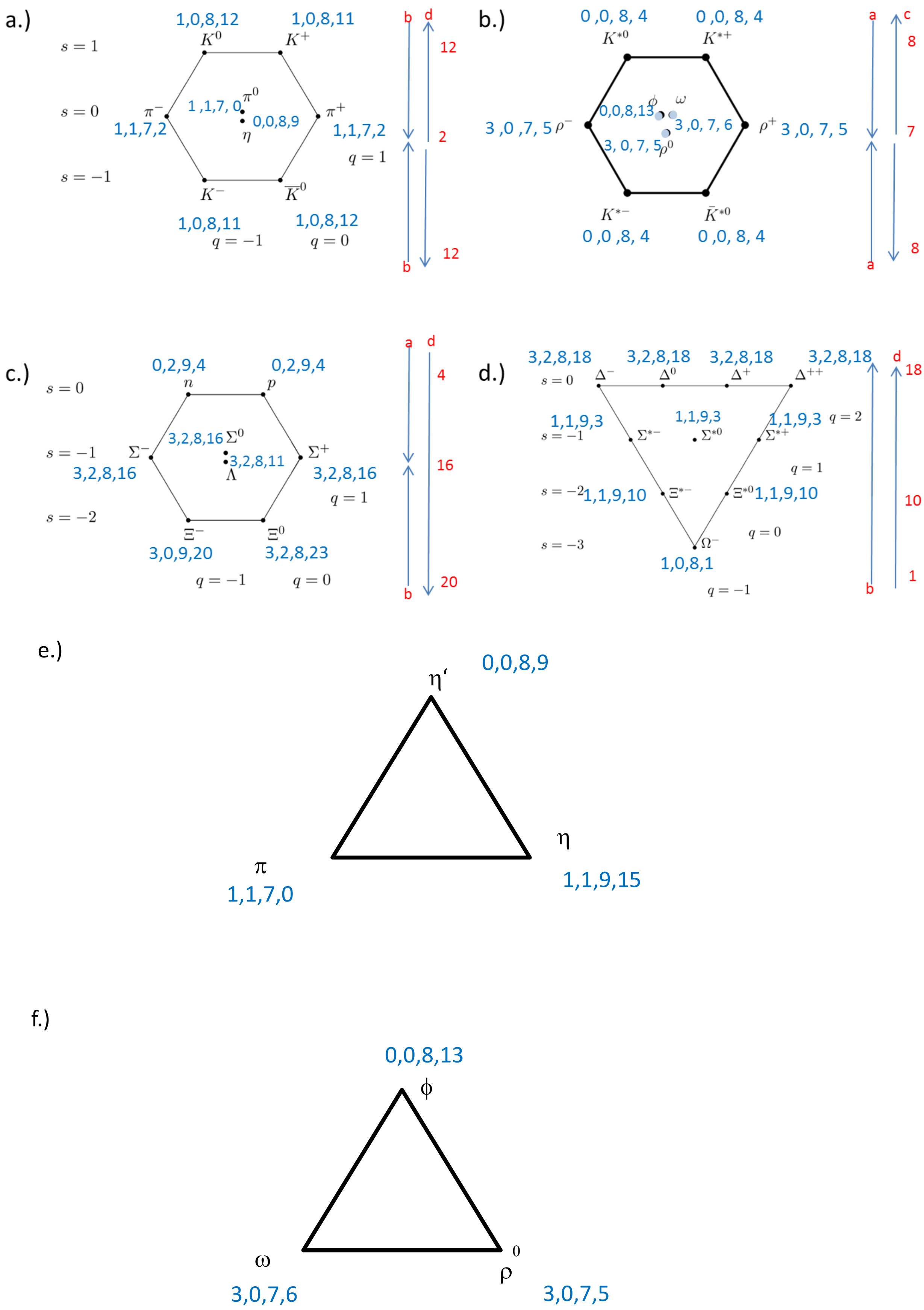
In more detail, one might claim that <u>*at the very bottom level*</u>, the fundamental energy cannot be split into any such summable partial energy. Therefore, one would reason that the formula for the fundamental energy of a particle must not contain any *additive relation*, nor, indeed, any *logarithmic relation*, because this would equivalently correspond to an additive component. Therefore, the partition function <u>*at the very bottom*</u> should be related to multiplication only and, therefore, to the potency series only,- and should be, in the simplest way, given by the potency series itself. Furthermore, the particle energy should then be, in the simplest case, proportional to the partition function of that particle, and the proportionality factor between the particle's partition function and the particle's energy could be given directly by the Rydberg energy, again.

"*Zero-point analyses*" of the *exponent series* of table 1 in relation to their respective particles

The charged and uncharged *pions*

The first zero-point analysis concerns the *pion* as a back (symmetrical) zero point. It is described by the exponents *1, 1, 7, and 2* (charged *pion*) or *1, 1, 7, and* ***0*** (uncharged *pion*; see figure 3 for comparison). The neutral pion is a superposition of two *quarkonium states*, namely, the *uu*- and *dd* quarkonia. This superposition is highly symmetrical and does not contain any asymmetry. To generate a *2-gluon (4-vertex)* event, however, quark asymmetry is required. This is missing in the case of the neutral pion; therefore, no *2-gluon* event can occur. Due to the high symmetry, there is a trailing back zero for a *2-gluon (4-vertex)* event.

Figure 3



a.)
1,0,8,12
1,0,8,11
1 ,1,7, 0
0,0,8,9
1,1,7,2
1,1,7,2
1,0,8,11
1,0,8,12
12
2
12
b.)
0 ,0, 8, 4
0 ,0, 8, 4
3 ,0 ,7, 5
0,0,8,13
3 ,0, 7, 6
3, 0, 7, 5
3 ,0, 7, 5
0 ,0 ,8, 4
0 ,0, 8, 4
8
7
8
c.)
0,2,9,4
0,2,9,4
3,2,8,16
3,2,8,11
3,2,8,16
3,2,8,16
3,0,9,20
3,2,8,23
4
16
20
d.)
3,2,8,18
3,2,8,18
3,2,8,18
3,2,8,18
1,1,9,3
1,1,9,3
1,1,9,3
1,1,9,10
1,1,9,10
1,0,8,1
18
10
1
e.)
0,0,8,9
η'
η
1,1,9,15
π
1,1,7,0
f.)
0,0,8,13
ϕ
ω
ρ
0
3,0,7,6
3,0,7,5

Figure 3: Figure 3 gives the exponent series (a,b,c,d)-tupels for each particle

The *proton* and *neutron*

In contrast, the *proton* and *neutron*, described by the exponents ***0**, 2, 9, and 4,* have a leading zero. These particles contain no antimatter. For this reason, they cannot participate in any quark-antiquark or quark-driven *color-anticolor* interaction. Only the much stronger and more stable *three-color* interaction remains to hold the particle together. As a maximum of nine symmetrical *3-gluon self-interactions* and four asymmetrical *four-vertex two-gluon* interactions are present in the proton or neutron, the result is a very stable particle.

The *eta* particles

The *eta* particle exists in two variants, namely, the *eta* and *eta-dash* particles. The *eta* particle is an asymmetric superposition of *u**u**, d**d*** and minus 2 *s**s***. As such, it has exponents of *1, 1, 9,* and *15*. In contrast, the eta-dash particle corresponds to a symmetrical superposition of *u**u**, d**d**,* and *s**s***.

The *eta-dash* particle has the exponents *0, 0, 8,* and *9*. Therefore, it cannot undergo any gluon-driven color-anticolor interactions nor any three-color interactions. The eta-dash particle is very similar to the nonexistent 9th gluon, which is a strongly analogous symmetrical superposition of *r**r**, b**b*** and *g**g***. As such, the 9th gluon is genuinely colorless and cannot interact via the strong interaction or any color interaction. Likewise, the eta-dash particle, with the exponents *0, 0, 8, and 9*, cannot participate in any quark-antiquark, three-quark, or quark-driven two-color or three-color interactions. Only symmetrical gluon-self interactions *(3-vertex interactions)* and antisymmetric *2-gluon* interactions *(4-vertex interactions)* remain in which the eta-dash particle can participate. For this reason, the eta-dash particle can be described as a real double zero with regard to *quark-quark* interactions.

The various *Kaons*

The *kaons* have a zero in the second position, with exponents of *1, **0**, 8, and 11* (K+, us) and *1, **0**, 8, and 12* (K0, ds). The *kaon* can occur as a K+ particle, with the quark composition us, or as a K0 particle, with the composition *ds*. The phenomenon of neutral particle oscillation describes the oscillation of a neutral *kaon* particle into the corresponding antiparticle, i.e., an *antikaon*. In any case, it seems as if the strange quark has a tendency to oscillate in its antiparticle. The strange quark, at least, seems to have the ability to participate in quark-driven *color-anticolor* interactions. The second zero in the kaon indicates that due to its *s*-quark, the kaon also has the ability to interact in the form of quark-antiquark interactions and/or in the form of quark-driven *color-anticolor* interactions. On the other hand, there are no possible three-quark or quark-generated three-color interactions (hence, the second number is zero). Furthermore, the kaons have eight gluon self-interactions and various (11 or 12) *four-vertex 2-gluon* interactions.

The different particles and their corresponding exponent series can also be arranged in several diagrams with the form of the *eightfold way*. This has been done in figure 3.

***Quarkonium* states**

The two quarkonium particles cc (*psion)* and *bb (Y)* have the exponent series *(0, 1, 9, 11) (psion, cc)* and *(0, 0, 9, 12) (Y, bb).* At first glance, the lack of quark-antiquark interactions in both particles is surprising. However, this is consistent with the *Okubo-Zweig-Iizuka* (OZI) rule. Strong decays are a suppressed branch for these particles. According to the *OZI* rule, no direct annihilation, i.e., reaction between the two quarks of the quarkonium, is possible. Rather, this direct reaction is suppressed. On the other hand, three-quark or three-color reactions are possible within the cc quarkonium. In the case of the *bb (Y)* particle, however, even this reaction is suppressed and/or impossible.

The omega-minus particle consists of three strange quarks (*sss*) and has the exponent series *(1, **0**, 8, 1)*. Consequently, it cannot participate in a *3-quark* particle reaction and has only a single *2-gluon* reaction that is possible. However, this reaction corresponds to the extremely high degree of symmetry in this particle.

Analyses of individual special and special mesons

The *omega-minus* particle

The omega-minus particle contains three strange quarks, with the configuration sss. Its exponent series is *(1, **0**, 8, 1)*. It does have one color-anticolor interaction. The matter s-quark has the ability to participate in color-anticolor interactions. Therefore, two s quarks are bound by a *color-anticolor* interaction, and one s quark is unbound. Because of this constellation, it cannot participate in any *3-color 3-quark* interaction. All 8 gluons participate in *gluon-gluon self-interactions*, and a single two-*gluon 4-vertex* interaction results. This agrees quite well with the high symmetry, leading to equity and identity of all gluons in the omega-minus particle.

An explanation for the *2-color* interaction (*quark-antiquark*) can again be found in the tendency of the strange quark to oscillate and participate in quark-antiquark interactions, as seen, for example, in an oscillation of the form *sss* → ***s**cc*.

The *lambda* and s*igma* baryons

The lambda and sigma baryons, with exponents *3, 2, 8, and 11* (lambda) and *3, 2, 8, and 16* (sigma), also seem to be special. These two particles seem to differ only in their numbers of *4-vertex* gluon-gluon interactions. Another particle in this series is the *Xi-zero* baryon, with the exponents *3, 2, 8, and 23*. Similar to the lambda and sigma baryons, the *Xi-zero* particle differs from them only in its number of *4-vertex gluon-gluon* interactions.

The pure quarkonium particles cc (psion, charmonium) and Υ (bottonium) are also very special. With exponents of ***0**, 1, 9, and 11* (psion, cc) and ***0**, 0, 9, and 12* (Υ, bb), they show no quark-antiquark interactions at all, although they possess quark-antiquark structures. This agrees well with the OZI rule (branch rule), which prohibits or strongly suppresses such a quark-antiquark interaction. Otherwise, these particles are likely to be extremely instable, such that they cannot be observed.

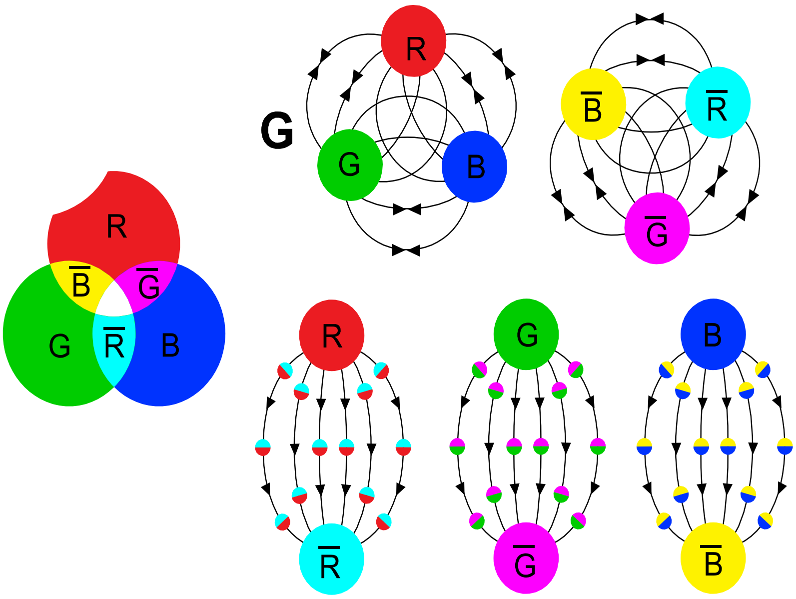
W **and** ***Z*** **bosons**

Other easy-to-understand particles are the W and Z bosons. These particles do not contain any quarks. Therefore, their a and b values are zero.

Conclusion

From the above analyses, one can conclude that the numbers (*a, b, c, and d)* can characterize an elementary particle in great detail. These numbers describe the interactions of the quarks and gluons as well as gluon self-interactions (*3-vertex interactions*) and gluon-gluon interactions (*4-vertex interactions*) in the particle of interest. Accordingly, the gluons can undergo self-interactions, interactions and hybridizations very similar to those of the electrons in a molecule. Therefore, the gluons in an elementary particle behave very much like the electrons in a chemical molecule.

Figure 4



Up to two three- quark (qqq) Interactions --> b=1,2

Up to three two-quark (qq) Interactions -> a=1,2,3

Figure 4 This figure gives the number of possible combinations for color-anticolor interactions (qq). Up to three different combinations are possible due to the three possible colors. And this figure gives the number of possible combinations for color-interactions (qqq). Up to two different combination of color interactions (qqq) are possible.

Figure 5

G3 and G4 self-couplings of Gluons

G^3

BG--> BR + RG	Insertion of RR	→	BR + RG--> BG+RR
RB--> RG + GB	Insertion of GG	→	RG + GB --> RB+GG
GR--> GB + BR	Insertion of BB	→	GB + BR --> GR+BB

G^3

BG--> BR + RG	Insertion of RR	→	BR + RG-->BG+RR
RB--> RG + GB	Insertion of GG	→	RG + GB--> RB+GG
GR--> GB + BR	Insertion of BB	→	GB + BR--> GR+BB

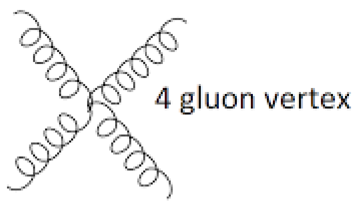
½ (GG-RR)

1/6 (RR+GG-2BB)

Up to 9 G3 Gluon self-couplings -> c=1,..,9

G^4

	1	2	3	4	5	6	7	8
1								
2	1							
3	2	8						
4	3	9	14					
5	4	10	15	19				
6	5	11	16	20	23			
7	6	12	17	21	24	26		
8	7	13	18	22	25	27	28	



Up to 28 Two-Gluon G^4 Interactions (4-gluon vertices)-> d=1,2,3,...,28,...100

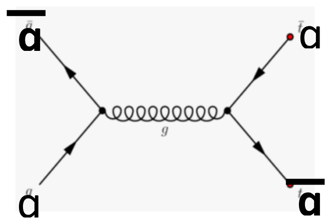
Figure 5: Figure 5 explains the meaning of the (a,b,c,d)-tupel and the maximal possible interactions. The tupel represents the number of interactions present in a given particle. It gives the number of possible three-quark interactions, which is defining the b-value of the n-tupel, the number of two-quark interactions, which is defining the a-value of the n-tupel, the number of three-gluon vertices, which is defining the c-value of the n-tupel, and the number of four-gluon vertices, which is defining the d-value of the n-tupel. The (a,b,c,d)-tupel product represents the sum of all possible combinations of all interactions.

Figures 4 and 5 show that a total of a = 3 *two-quark* interactions, b = 2 *three-quark* interactions, c = 9 *gluon self-interactions*, and d = 28 *two-gluon 4-vertex* interactions can be formulated. This is consistent with the exponent series found for the different particles (see *table 1*). Another important thing that we learn here is that in *quantum- gluonodynamics*, we must multiply the different partial energy states (and/or constellation possibilities), while in other areas of physics, partial energies are summed-up to form the total energy.

Figure 6

Feynman diagrams

Color-anticolor interaction (qq)

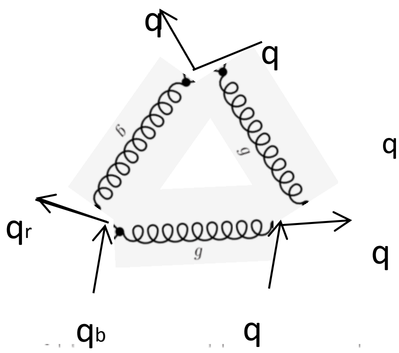


$$q^a = 2^a$$

q: number of arrows / arrow length per diagram

a: number of 2-quark vertex Feynmans

Color interaction (qqq)

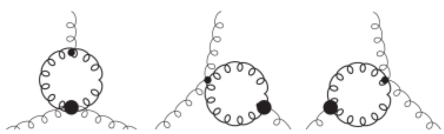


$$q^b = 3^b$$

q: number of arrows per diagram

b: number of 3-quark Feynmans diagrams

3-gluon vertex (DSE)

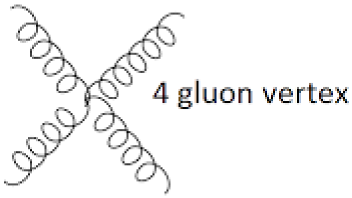


$$q^c = (3\pi)^c$$

q: arrow length and number of arrows per diagram

c: number of 3-gluon Feynmans diagrams

4-gluon vertex



$$q^d = \left(\frac{1}{\sqrt[8]{1 - \frac{1}{3\pi}}} \right)^d$$

Figure 6: This figure gives the interactions present in a particle and how to calculate the partition function which in turn gives the number of all possible interactions present in a given particle. The figure visualizes the meaning of the potencies a,b,c, and d.

This operation is justified by the *Feynman diagrams* (see *figure 6*). The total energy or mass of an elementary particle is thought to be related to the *sum* **of all possible** *Feynman diagrams*. The total sum of the *Feynman diagrams* of a certain particle is given by the formula mentioned here and is calculated through a multiplication of the different *partial intraparticle interaction possibilities* and, with them, the numbers of *gluon interactions*.

Importantly, the base value for each of these interaction intensities seems to be related to the *path length* of the associated *main arrow* in the associated main *Feynman diagram* and/or to the number of *arrows per diagram*.

Thus, it seems to be in such a way that the **total energy** or mass of a certain particle is related to the ***length*** of a hypothetical summarized ***total Feynman arrow length***, which includes and summarizes all arrows in all possible *Feynman diagrams*.

The work presented here is deeply quantum mechanical in nature. The summation of the *Feynman diagrams* corresponds to the combinatorial calculation of the number of all possible constellations for an elementary particle. In turn, this *Feynman sum* then defines the energy or mass of the elementary particle in question.

This corresponds to *Schrödinger's cat*: it is not a single constellation that defines the mass but rather the sum of all theoretical possibilities. In this respect, all different possibilities are realized at the same time, although in any particular instance, we can draw down only one possibility with the help of the *Feynman diagrams*.

Weak Interaction Parameters (a,b,c,d) in case of the Leptons and the Quarks

However if we transfer this theory to the leptons as electrons and quarks we get for the beta decay, which is in most cases the decay of a neutron:

Beta-decay

quarks:

$$d^{-0.3} \rightarrow u^{+0.67} + W^{-} \rightarrow u^{+0.67} + e^{-} + \underline{v} \qquad \textbf{(28)}$$

Rishons:

$$VVT^{-0.3} \rightarrow ttv^{+0.67} + (TT)TVV(V)^{-} \rightarrow ttv^{+0.67} + TTT^{-} + VVV$$

Interactions:

$$\underline{(2;2;4;8)} \rightarrow (1;2;4;8) + \underline{(0;0;10;5)} \rightarrow (1;2;4;8) + \underline{(-1;2;4;4)} + \underline{(-1;-2;0;0)}$$

Through adding of TTV to d the a and b- interactions get somewhat lost. Only the c and d interaction are possible in the W-particle. Afterwards the W-particle forms two particles with a and b interactions again.

Myon-Decay

μ^- → W + v_μ → e^- + $\underline{v_e}$ +v_μ **(29)**

Rishon:

TTT^- → $(TT)TVV(V)^-$ + vvv → TTT^- $^+$ VVV + vvv

Interactions:

$\underline{(-3;2;7;3)}$ → $\underline{(0;0;10;5)}$ + (-1;-0;0;0) → $\underline{(-1;2;4;4)}$ + $\underline{(-1;-2;0;0)}$ + (-1;-0;0;0)

Pion-Decay

$\pi^-(\underline{u},d)$ → W^- → μ^- + $v_{\underline{\mu}}$ **(30)**

Rishon:

$TTV^{-0.67}$ + $VVT^{-0.3}$ → $(TT)TVV(V)^-$ → TTT^- $^+$ VVV

Interactions:

$\underline{(1;2;4;8)}$ + $\underline{(2;2;4;8)}$ → $\underline{(0;0;10;5)}$ → $\underline{(-3;2;7;3)}$ + $\underline{(-1;0;0;0)}$

It can be recognized that a much better and more exact description of the beta decay and other weak decays can be given by using the (a,b,c,d)- tuples in the way done above. These results leads us to the hypotheses that the (a,b,c,d)-tuples can not only be used in order to better describe the strong interaction (color interaction), but can also be used in order to better describe the weak interaction and all weak decays. Both interactions are similar and result from 4 interactions (a,b,c,d).

Both interactions (strong and weak) seem to be similar with regard to the (a,b,c,d)-tuples. But of cause the weak interaction is acting within a much smaller radius within single electrons and single quarks and can be better explained by using the Harari- model,- while the strong interaction is acting on a much larger radius between different quarks and is equivalent to the color interaction.

References

1. Narison S. Nuclear and Particle Physics Proceedings. Vol 312-317: 1-206. (2021)

2. Smailagic A, Spallucci E.*Phys. G: Nucl. Part. Phys.* 48. 125002 (2021) DOI 10.1088/1361-6471/abfd70

3. Ünsal Ö, Gönül B. Turkish Journal of Physics. 41:337-343 (2017) doi:10.3906/ z-1703-25

4. Cushman P. Physics. 14, 54 (2021) DOI: 10.1103/Physics.14.54

5. Grant A. Physics Today 70(9), 22 (2017); https://doi.org/10.1063/PT.3.3683

6. Vergados JD. The standard model and beyond. (World Scientific, New Jersey, 2018).

7. Feynman R. Physical Review. 76(6):769-789 (1949).

8. Basso B, Dixon L. Phys. Rev. Lett. 119, 071601 (2017) —DOI:10.1103/PhysRevLett.119.071601

9. Gell-Mann M. Nuovo Cimento. 4(S2):848-866 (1956).

10. Weltevrede P, Benetge B P Perera, Lina Levin Preston, Sotiris Sanidas, International Astronomical Union. Symposium. Pulsar astrophysics: the next fifty years: proceedings of the 337th Symposium of the International Astronomical Union held at Jodrell Bank Observatory, United Kingdom, September 4-8, 2017. (Cambridge University Press, Cambridge, United Kingdom; New York, NY, USA, 2018).

11. Ma H. A American Journal of Modern Physics. 9(6),84 (2020).

12. Nakatsugawa K, Ohaga M, Fujii T, Matsuyama T, Tanda *Symmetry 12*(10), 1603 (2020) https://doi.org/10.3390/sym12101603

13. Rui Wen and Wei-jie Fu 2021 *Chinese Phys. C* **45,** 044112 **DOI** 10.1088/1674-1137/abe199

14. Abdelgabar M, Yakubu A, Abdallah M. *Open Access Library Journal*, **8**, 1-8. (2021) doi: 10.4236/oalib.1107875.

15. Nagashima Y. Beyond the standard model of elementary particle physics. (Wiley-VCH, New Jersey, USA, 2014)

16. Tuchin K arXiv:1301.0099v2 [hep-ph] (2013)

17. Tanabashi M, Hagiwara K, Hikasa K, Nakamura K, Sumino Y, Takahashi F, et al. (Particle Data Group). Phys. Rev. D98, 030001. In this review, see "Structure functions." 318–333. (2018)

18. Braun J, Fist er L, Pawlowski J, Rennecke F. *Phys. Rev*. *D 94*, 034016 (*2016*). doi.org/10.1103/PhysRevD.94.034016

19. Torassa E. Progress in Particle and Nuclear Physics. 100:69-106.(2018) 10.1016/j.ppnp.2018.01.003

20. Kobayashi T. Proceedings of the Japan Academy, Series B. 97(5):211-235.(2021)

21. Acquaro P. Frontiers in Sensors (FS) Volume 3, (2015) doi: 10.14355/fs.2016.04.001

22. Skands P arXiv:1207.2389 [hep-ph] (2017)

23. Xiong Y, Tang X, Luo Z. Chinese Physics C. 43(11):113103.(2019).

24. Somogyi G, Tramontano F. *J. High Energ. Phys.* 142 (2020)doi: 10.1007/JHEP11(2020)142

25. Demirci M Turk. J Phys. 43(5),442-458. (2019) DOI: https://doi.org/10.390 6/fiz-1903-15