

Logical necessity of Quantum Mechanics

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Abstract

From classical mechanics, in particular the motion in a straight line, together set theory and ordinal number theory, we prove a not-classical behaviour, a discontinuous motion and emission.

1 Introduction

Quantum mechanics axioms [1] imply the not-classical behaviour; mathematically, from Hilbert space and linear operators, only discrete spectrum of eigenvalues has a physical meaning. They are justified by specific physical observations such as the well-known black body radiation and wave behaviour of matter, in contrast with classical physic.

Zeno, with his paradoxes [3], stated the impossibility to reach all the infinite parts of a segment, but not in a clear way, getting at the conclusion about the impossibility of motion. We define more precisely the problem and with the help of the concept of the first transfinite ordinal number ω (see Cantor [2]) we get the existence of discontinuous motion and emission, instead of stating the impossibility of motion.

2 Discontinuous motion and emission

Suppose to divide a finite segment into infinite N parts dx , that is $\frac{1}{N} = dx$ is an infinitesimal part, with N an infinite hypernatural number. We can imagine a part dx delimited by two points. We observe an object going from the first point to the second, third, fourth and so on. But it is not possible to observe the object crossing all infinite points. Inductively it is not possible

to reach infinity from the finite. In fact with a proof by induction (using the Dedekind definition of infinity and finite [4] for proving the induction step) the well known fact that each natural number is finite can be shown. If n is finite, then $n + 1$ is finite; because there aren't infinite subsets of $n + 1$. Then the first transfinite ordinal number ω is not a successor of a natural number (the successor of a natural number is a natural number), that is $\nexists n(\omega = S(n))$; so, starting from the finite, one by one, the object doesn't reach all infinite points.

But the object runs across the entire finite segment in a finite time! So the object has necessarily to cross a finite number of points and the motion of the object along the segment is discontinuous. The same consideration is valid along a time interval; so a flow of energy or matter is observed discontinuous; it is not possible to count infinite portions of energy or matter to go out a surface.

This consideration is in concordance with Schrödinger equation, just think of a particle in a box; considering knots and maximums of probability it doesn't have a continuous motion.

But how many finite parts are there and, above all, how small is a part? They may be so small to become infinitesimally; there is no limit in reducing a single part, except the previous consideration about finite number of points in a segment. It is necessary a law to avoid a contradiction (Δx cannot be infinitesimally and Δx can be infinitesimally). A solution is the Heisenberg uncertainty principle, in particular $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$ and $\Delta t \cdot \Delta E \geq \frac{\hbar}{2}$. So $\Delta x = dx$ and $\Delta t = dt$ only when $\Delta p = \infty$ and $\Delta E = \infty$, but, at these conditions, p and E are not observable, being $\Delta p \geq p$ and $\Delta E \geq E$; we cannot describe the motion of an object and then we don't observe a continuous motion.

3 Conclusions

Zeno didn't define rigorously why it is not possible reach all infinite part of a segment, and over the centuries efforts were focused on proving reachability of infinite totalities, utilising series and integrals. It was believed that the paradoxes were solved, without to consider them in the general context (the entire path and motion).

Now we have obtained that the not-classical behaviour is essentially due to $\nexists n(\omega = S(n))$. But incredibly this could have been discovered about 140 years ago, before Planck theory. So quantum behaviour looks very natural

and classical mechanics has to be rejected. The classical limit $\hbar = 0$ cannot exist.

References

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