Hedge Fund Barrier Option Analytics

A hedge fund barrier call option is a note whose payoff is based on a basket of hedge funds. The deals are structured so that once the barrier (usually set at 95% of the notional) is hit, the funds in the basket are sold off, with the realized fund value depending on the redemption period of each fund.

The difference between the redeemed value of the basket of funds and the strike price of the call (usually set at 75% of the notional) is forwarded to the investor. When the redeemed basket value falls below the strike price the investor loses money on the deal.

By construction, the probability of this happening is very small: no fund may make up more than _ 5% of the basket by value, so for the basket to fall below the strike price requires that at least 5 funds experience a serious downturn.

Since the funds in a basket are generally spread over a variety of strategies, the probability of experiencing a loss under to an individual deal under normal market conditions is small enough that traditional VAR measurements result in zero economic capital.

Nevertheless, when the entire portfolio is considered, there should be a quantifiable level of market risk. We have performed a detailed analysis of the expected losses to a small number of deals, and scaled the results to estimate the economic capital of the portfolio.

The market risk associated with an individual deal is small–the economic capital typically vanishes since the probability of a loss is small enough (< 0.03%) that there are no losses at the 99.93rd percentile. This will not be the case when considering the entire portfolio, since while the probability of a loss to each contract is small there are 130 contracts. (If the average loss probability for all individual deals is larger than _ 0.0005%, then ignoring correlations between deals, there should be capital on the portfolio.)

The goal here is to estimate the market risk of the entire portfolio of such deals through analysis of a small representative sample of the portfolio and scaling up to the entire portfolio. While simulating the entire portfolio would result in a more accurate determination of the capital, the result is small enough that the dominant risk factors likely arise from sources other than market risk, and an order-of magnitude determination is likely sufficient.

The methodology is straightforward: Each representative deal is simulated with a given number N of Monte-Carlo simulation paths to determine the loss distribution as a fraction of the initial basket level. Each deal in a class is assumed to have the same number of losses, identically distributed as a fraction of the starting basket level of the deal, from which dollar values of the losses for each deal are determined.

The aggregate of the dollar-value losses from all deals then represents the loss distribution of the entire portfolio in N simulations, and the 99.93rd percentile loss to the portfolio is then determined.

For example, assume that a simulation of a representative deal of 5, 000 Monte-Carlo paths resulted in 2 losses of 10% and 20% of its starting basket value. The total loss probability is 2/5, 000 = 0.04%, and there is therefore zero loss at the 99.93rd percentile and no capital on the individual deal.

If the class that is represented consists of three deals with starting basket levels of $10M, $15M and $20M, then they would be assumed to have losses of {$1M, $2M}, {$1.5M, $3M} and {$2M, $4M} respectively. This results in 6 losses to the class: {$1M, $1.5M, $2M, $2M, $3M, $4M}, for a loss probability of 6/5, 000 = 0.12%, and the 99.93rd percentile would fall in the $2M loss bin, for an aggregate capital of $2M/$45M = 400bp.

In practice the portfolio was divided into 5 slices by looking at the cash-adjusted leverage ratio of each deal:

$$\mathrm{CAL} = \frac{\text{Basket Value} - \text{Cash Holdings}}{\text{Option Value}}.$$

This should give a measure of the relative risk of each deal–the smaller the leverage-ratio, the smaller the risk. (This gives the ratio of the volatile portion of the basket to the current option value.) What this does *not* capture is the relative volatilities of each deal, which are perhaps as important for assessing risk, but are less readily available.

In generating the loss distributions for the representative deals, we assumed that the fund returns were correlated and log-normally distributed. The funds were simulated with a relatively short time interval (roughly 1 week) until the barrier was hit or the one year horizon expired.

If the barrier was hit the basket is to be sold off, and the funds were simulated in monthly intervals and the final values of each fund picked off of the resulting flows based on their redemption periods. If the sum of the final fund values (and any cash holdings) is less than the strike price of the deal, then there is a loss to the investor.

These are similar to the barrier options, except that the leverage ratio may be readjusted (‘releveraging’) if the basket value crosses a barrier–either upwards or downwards. These deals should therefore be less risky than ordinary barrier options, and we have shown the effect of including them in the loss aggregation as if they were ordinary barrier options.

In performing these simulations we made the following parameter assumptions:

• Only funds which made up at least 2% of the basket were modelled, with all funds in a given strategy scaled up so that the strategy weighting matched that of the actual deal. If no funds in a given strategy were above this 2% cutoff, then the largest was taken and scaled up in the same way.

• The historical fund volatilities provided by the AAG were doubled.

• Within each strategy fund correlations were assumed to be 0.75, with strategy to strategy correlations. (Typically the resulting correlations had to reduced by $75 - 90\%$ to have a positive-definite correlation matrix.)

• In a market (or sector) downturn it is likely that the given redemption periods for the funds would be optimistic. To account for this, the minimum redemption period allowed for each fund was set at either 6 or 12 months.

References:

https://finpricing.com/product.html