Ratio Tunnel Option Model

A ratio tunnel option offers the contract holder the right either long or short, at a contract maturity, a preset underlying collar. Notional principals and strikes in the collar may be different. In the system, the ratio tunnel option and the underlying collar are strictly European type. The ratio tunnel option model applies analytical close form pricing formulae for vanilla compound options.

A ratio tunnel option is defined as a European vanilla option upon a European collar. It is another special case of a general compound option. We can get a close form pricing formula for the option on collar, or the ratio tunnel option (see https://finpricing.com/knowledge.html). Most of results presented in the report (see footnote 1) can be applied here. Therefore, in the following, we just provided some specified information which is necessary in this case.

Our test cases cover currency pairs under European and American quoting conventions, and different call-put principal ratios. Risk numbers, including spot rate Delta's, forward point Delta's, Gamma's, Vega's, and Rho's, as well as P/L's have also been tested.

For an underlying collar, let us define Kc and Kp be the strikes of a call and a put in the collar, respectively,
and Tclr be the maturity of the collar. Let T be the maturity of the ratio tunnel option where T < Tclr and PN be the notional principal of the option measured in the unit of an underlyer. Now we have

$$\Gamma = \{(\alpha_{\rm c}, K_{\rm c}, T_{\rm clr}, 1),\ (-\alpha_{\rm p}, K_{\rm p}, T_{\rm clr}, -1)\}\ ,$$

where

$$\alpha_{\rm c},\ \alpha_{\rm p} > 0\ .$$

Then the matured payoff of the ratio tunnel call option at T can be given by

$$P_{\mathrm{N}} \times \{[\alpha_{\mathrm{c}} \cdot p(T, S_T; K_{\mathrm{c}}, T_{\mathrm{clr}}, 1) - \alpha_{\mathrm{p}} \cdot p(T, S_T; K_{\mathrm{p}}, T_{\mathrm{clr}}, -1)] - K\}^{+} \ ,$$

where K is the unit strike of the ratio tunnel option. Clearly, the ratio tunnel put option can be defined by the following matured payoff

$$P_{\mathrm{N}} \times \{K - [\alpha_{\mathrm{c}} \cdot p(T, S_T; K_{\mathrm{c}}, T_{\mathrm{clr}}, 1) - \alpha_{\mathrm{p}} \cdot p(T, S_T; K_{\mathrm{p}}, T_{\mathrm{clr}}, -1)]\}^{+} \ .$$

Let us consider the following non-linear equation

$$g(S) \equiv [\alpha_{\mathrm{c}} \cdot p(T, S; K_{\mathrm{c}}, T_{\mathrm{clr}}, 1) - \alpha_{\mathrm{p}} \cdot p(T, S; K_{\mathrm{p}}, T_{\mathrm{clr}}, -1)] - K = 0 \ .$$

The function *g*(¢) is strictly increasing with respect to *S* and with the co-domain including [0*;1*).
Therefore,
for any *K* ¸ 0, there is a unique strictly positive solution of (4), which is denoted by *S*clr. Further, we have

$$\frac{\partial g}{\partial S} = \frac{F(t, T_{\mathrm{clr}})}{F(t, T)} \cdot \frac{\mathrm{df}(t, T_{\mathrm{clr}})}{\mathrm{df}(t, T)} \times [\alpha_{\mathrm{c}} \cdot \Phi_1(D_T^{\mathrm{c}}) - \alpha_{\mathrm{p}} \cdot \Phi_1(D_T^{\mathrm{p}}) + \alpha_{\mathrm{p}}] > 0$$

where

$$D_T^{\mathrm{c}} = \ln\left(\frac{F(T, T_{\mathrm{clr}})}{K_{\mathrm{c}}}\right) \frac{1}{\tilde{\sigma}(T, T_{\mathrm{clr}})} + \frac{1}{2}\tilde{\sigma}(T, T_{\mathrm{clr}}) \ , \quad D_T^{\mathrm{p}} = \ln\left(\frac{F(T, T_{\mathrm{clr}})}{K_{\mathrm{p}}}\right) \frac{1}{\tilde{\sigma}(T, T_{\mathrm{clr}})} + \frac{1}{2}\tilde{\sigma}(T, T_{\mathrm{clr}}) \ .$$

The solution *S*clr can be obtained by using Newton-Raphson method.

Let *t* · *T* be a generic valuation time and *p*cClr(*t; St*;*K;* ¡) be the *t*-value of a ratio tunnel call per unit notional principal. Then we have

$$
\begin{aligned}
p_{\mathrm{cClr}}(t,S_t;K,\Gamma) &= \mathrm{df}(t,T)\times \mathrm{E}_t\left[\mathrm{df}(T,T_{\mathrm{clr}})\cdot\alpha_{\mathrm{c}}\cdot \mathrm{E}_T\left[(S_{T_{\mathrm{clr}}}-K_{\mathrm{c}})^+\right]\right.\\
&\qquad\qquad \left. -\ \mathrm{df}(T,T_{\mathrm{clr}})\cdot\alpha_{\mathrm{p}}\cdot \mathrm{E}_T\left[(K_{\mathrm{c}}-S_{T_{\mathrm{clr}}})^+\right]-K\ ;\ \{S_T\geq S_{\mathrm{clr}}\}\right]\\
&= \alpha_{\mathrm{c}}\cdot \mathrm{df}(t,T_{\mathrm{clr}})\cdot \mathrm{E}_t\left[\mathrm{E}_T\left[(S_{T_{\mathrm{clr}}}-K_{\mathrm{c}})^+\right]\ ;\ \{S_T\geq S_{\mathrm{clr}}\}\right]\\
&\quad -\alpha_{\mathrm{p}}\cdot \mathrm{df}(t,T_{\mathrm{clr}})\cdot \mathrm{E}_t\left[\mathrm{E}_T\left[(K_{\mathrm{c}}-S_{T_{\mathrm{clr}}})^+\right]\ ;\ \{S_T\geq S_{\mathrm{clr}}\}\right]\\
&\quad -\mathrm{df}(t,T)\cdot \mathrm{E}_t\left[K\ ;\ \{S_T\geq S_{\mathrm{clr}}\}\right]\ .
\end{aligned}
$$

The expectation in the second term on the right side of (7), denoted by *I*2, can be re-written as

$$
\begin{aligned}
I_2 &= \mathrm{E}_t\left[\mathrm{E}_T\left[(K_{\mathrm{p}}-S_{T_{\mathrm{clr}}})^+\right]\ ;\ \{S_T\geq S_{\mathrm{clr}}\}\right]\\
&= \mathrm{E}_t\left[(K_{\mathrm{p}}-S_{T_{\mathrm{clr}}})^+\right]-\mathrm{E}_t\left[\mathrm{E}_T\left[(K_{\mathrm{p}}-S_{T_{\mathrm{clr}}})^+\right]\ ;\ \{S_T\leq S_{\mathrm{clr}}\}\right]\ .
\end{aligned}
$$

The expectation in the third term on the right side of (7), denoted by *I*3, is given by

$$
\begin{aligned}
I_3 &= K\cdot\Phi_1(D_{\mathrm{clr}}-\tilde{\sigma}(t,T))\ ,\\
D_{\mathrm{clr}} &= \frac{1}{\tilde{\sigma}(t,T)}\ln\left(\frac{F(t,T)}{S_{\mathrm{clr}}}\right)+\frac{1}{2}\tilde{\sigma}(t,T)\ .
\end{aligned}
$$