

A Kernel-Based Identification Approach to LPV Feedforward: With Application to Motion Systems

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Abstract: The increasing demands for motion control result in a situation where Linear Parameter-Varying (LPV) dynamics have to be taken into account. Inverse-model feedforward control for LPV motion systems is challenging, since the inverse of an LPV system is often dynamically dependent on the scheduling sequence. The aim of this paper is to develop an identification approach that directly identifies dynamically scheduled feedforward controllers for LPV motion systems from data. In this paper, the feedforward controller is parameterized in basis functions, similar to, e.g., mass-acceleration feedforward, and is identified by a kernel-based approach such that the parameter dependency for LPV motion systems is addressed. The resulting feedforward includes dynamic dependence and is learned accurately. The developed framework is validated on an example.

Keywords: Mechatronics, Motion control systems, Linear parameter-varying systems, Bayesian methods, data-driven control

1. INTRODUCTION

Feedforward control can compensate for known disturbances in motion control, such as a reference trajectory. Typically, a feedforward controller is based on the inverse of a system (Hunt et al., 1996; Butterworth et al., 2012), where the control performance is determined by the accuracy of the inverse model (Devasia, 2002). The increasing demands for motion control leads to a situations where Linear Parameter-Varying (LPV) dynamics have to be explicitly taken into account (Groot Wassink et al., 2005).

For Linear Time-Invariant (LTI) systems, polynomial feedforward, where the feedforward signal is a linear combination of basis functions, results in good control performance. Often, the basis functions are chosen such that they relate to physical quantities, such as acceleration feedforward for the inertia (Lambrechts et al., 2005; Oomen, 2020), and snap feedforward for the compliance of a system (Boerlage et al., 2003). Several approaches have been developed to tune the feedforward parameters based on data, such as iterative learning control (Van de Wijdeven and Bosgra, 2010) and instrumental variable identification (Boeren et al., 2015) approaches. However, LTI feedforward leads to suboptimal performance when applied to LPV systems.

A key challenge in feedforward for LPV systems is modeling the dependency on the scheduling sequence. Additionally, the inversion of LPV systems generate terms that are often dynamically dependent on the scheduling sequence, i.e., dependency on the derivatives of the scheduling sequence (Sato, 2003). Hence, the dependence on the scheduling, including dynamic dependence, should be taken into account for feedforward of LPV systems and directly determines the achievable performance limit.

Several developments have been made in feedforward for LPV systems, and are directed at 1) identification of static LPV feedforward and 2) feedforward techniques based on forward LPV models. Inverse LPV system design is investigated in Balas (2002); Sato (2008), but rely on the forward LPV model and do not take dynamic dependence into account. In Van Haren et al. (2022), position-dependent snap feedforward is developed, that compensates for the static contribution of the position-dependent compliance. Data-driven feedforward approaches are developed in Butcher and Karimi (2009); De Rozario et al. (2018), but do not include dynamic dependence. In Theis et al. (2015); De Rozario et al. (2017); Bloemers et al. (2018), state-space models of LPV systems are used to create inverse systems and in Kontaras et al. (2016) a compliance compensation is developed, that do include dynamic dependence, but all heavily rely on the quality of the model, which is not addressed in these papers. Hence, current feedforward approaches for LPV systems either do not take dynamic dependence into account, or heavily depend on models, which directly limits the achievable performance and imposes a large burden on modeling effort.

^{*} This work is part of the research programme VIDI with project number 15698, which is (partly) financed by the Netherlands Organisation for Scientific Research (NWO). In addition, this research has received funding from the ECSEL Joint Undertaking under grant agreement 101007311 (IMOCO4.E). The Joint Undertaking receives support from the European Union Horizon 2020 research and innovation programme.

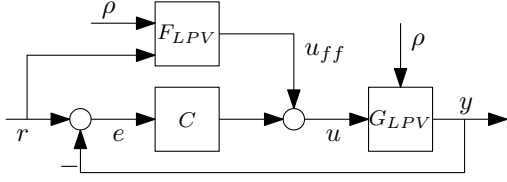


Fig. 1. Feedforward structure considered

Although feedforward approaches for LPV systems have been substantially developed, techniques for direct and accurate identification of LPV feedforward controllers that include dynamic scheduling dependence, which is required for high-performance motion control, are currently lacking. In this paper, feedforward parameters for a class of LPV motion systems are directly identified using data with kernel-based approaches, see, e.g., (Pillonetto et al., 2014; Blanken and Oomen, 2020), which results in a feedforward strategy that includes dynamic dependence on the scheduling sequence, and retains the polynomial feedforward structure, which is often desirable in motion control (Lambrechts et al., 2005; Oomen, 2020). This relates to the Bayesian approaches in (Golabi et al., 2017; Darwish et al., 2018), yet identifies inverse models for feedforward control and enables dynamic dependence on the scheduling sequence. The contributions include

- (C1) Development of a feedforward parameterization for LPV motion systems that includes dynamic dependency on the scheduling sequence.
- (C2) Identification of feedforward parameters of the developed parameterization by kernel regularized methods.
- (C3) Validation of the framework in a benchmark example.

The outline in this paper is as follows. In Section 2, the feedforward problem for LPV motion systems is shown. In Section 3, the developed feedforward parameterization is introduced. In Section 4, the identification of LPV feedforward parameters using input-output data is presented. In Section 5, a benchmark example is shown, validating the framework. Finally, in Section 6, a summary and recommendations are given.

2. PROBLEM FORMULATION

In this section, the problem related to feedforward control for LPV motion systems is formulated. First, the control setting and feedforward goal for LPV motion systems is described. Second, polynomial feedforward for LTI systems is investigated. Third, challenges in designing feedforward controllers for LPV motion systems are shown, that motivates the problem definition in Section 2.4.

2.1 Control Setting

The control goal is to develop LPV feedforward controller F_{LPV} to reduce the tracking error $e = r - y$ for single-input single-output LPV system G_{LPV} , where perfect tracking is achieved by $F_{LPV} = G_{LPV}^{-1}$. The control structure can be seen in Fig. 1, where C is a stabilizing feedback controller. The reference trajectory r is a smooth reference, which can be differentiated at least four times, as in Lambrechts et al. (2005). The considered class of LPV systems G_{LPV} can be represented in Continuous-Time (CT) by input-output representations and are shown in Definition 1.

Definition 1. (CT-IO-LPV system). The considered class of LPV motion systems are static dependent on the scheduling and given by

$$G_{LPV} : \sum_{i=-2}^{n_a} a_i(\rho(t)) y^{(i)}(t) = \sum_{j=0}^{n_b} b_j(\rho(t)) \frac{d^j}{dt^j} \iint u(t) dt^2 \quad (1)$$

with scheduling sequence $\rho \in \mathbb{R}^{n_\rho}$ and $y^{(i)}(t)$ is the i th time derivative of $y(t)$ if $i \geq 0$, and the i th integral if $i < 0$. The following is assumed of the considered class of LPV systems.

Assumption 2. The following two assumptions are made for the considered LPV systems.

- (1) Only the body that is of interest is connected to the fixed world, hence the double integral of the input signal u in (1).
- (2) The contributions of the zero dynamics to the system, i.e., $\sum_{j=1}^{n_b} b_j(\rho(t)) \frac{d^j}{dt^j} \iint u(t) dt^2$, are negligible.

The LPV coefficients $a_i(\rho(t))$ and $b_j(\rho(t))$ have a static dependency on $\rho(t)$, e.g., $a_i(\rho(t)) = \rho(t)$ or $a_i(\rho(t)) = \rho^3(t)$. For ease of notation, the dependence of signals on (t) is from now on omitted.

The considered class of LPV motion systems in Definition 1 shows a double integral of the input signal, which is generally the case for motion systems with masses.

Remark 3. The LPV coefficients $a_i(\rho)$ and $b_j(\rho)$ have a static dependency on the scheduling sequence ρ , but can be extended to include dynamic dependency, i.e., $a_i(\rho, \dot{\rho}, \dots)$ and $b_j(\rho, \dot{\rho}, \dots)$, and is part of ongoing research.

2.2 LTI Feedforward

LTI polynomial feedforward, see e.g. Lambrechts et al. (2005), approximates an inverse system to reduce the tracking error e . LTI polynomial feedforward applied to the LPV system in (1) parameterizes the feedforward by evaluating (1) at $\rho = \bar{\rho}$, assumes that $b_j = 0 \forall j > 0$ according to Assumption. 2, and differentiates both sides twice, resulting in

$$F_{LTI} : u_{ff} = \sum_{i=-2}^{n_a} \frac{a_i(\bar{\rho})}{b_0(\bar{\rho})} \frac{d^{i+2}}{dt^{i+2}} r = \sum_{i=1}^{n_\theta} \theta_i \psi_i \left(\frac{d}{dt} \right) r, \quad (2)$$

where $\psi_i(\frac{d}{dt})$ contain differentiators, e.g., $\psi_i(\frac{d}{dt}) = \frac{d^2}{dt^2}$ for acceleration feedforward. The parameters can be tuned manually (Lambrechts et al., 2005) or estimated by using data (Boeren et al., 2018). The resulting feedforward controller is an interpretable, simple and effective strategy for LTI systems, but does not take LPV dynamics into account.

2.3 Feedforward Problem for LPV Systems

Developing an inverse model for polynomial feedforward for LPV systems, similar to polynomial feedforward for LTI systems, is challenging due to the dynamic scheduling dependency introduced by deriving an inverse model. The dynamic dependence is observed when inverting (1), i.e., by differentiating both sides twice, derivatives of the scheduling sequence ρ directly appear. A fixed structure

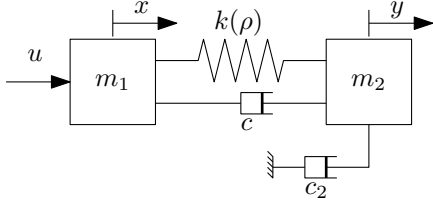


Fig. 2. LPV mass-spring-damper with parameter-dependent spring $k(\rho)$ that is considered in this paper.

for identifying the inverse model could be used, e.g., a polynomial of ρ , $\dot{\rho}$ and $\ddot{\rho}$, however, it is unclear how to choose the structure and order. Example 4 illustrates the complexity of feedforward control for LPV systems.

Example 4. (Feedforward problem for LPV system) Consider the two-mass-spring-damper system with parameter-dependent spring in Fig. 2, with input u , that is the force on the first mass, and output y , that is the position of the second mass. The input-output behavior in the form of (1) is given by

$$\left(m_2 m_1 \frac{d^2}{dt^2} + (c(m_1 + m_2) + c_2 m_1) \frac{d}{dt} + (k(\rho)(m_1 + m_2) + cc_2) \right) y + k(\rho)c_2 \int y dt = \left(c \frac{d}{dt} + k(\rho) \right) \iint u dt^2, \quad (3)$$

which directly shows the double integral of the input signal seen in (1). LTI polynomial feedforward from (2) is

$$u_{ff} = \frac{m_2 m_1}{k} \ddot{r} + \frac{c_2 m_1 + c(m_1 + m_2)}{k} \ddot{r} + \left((m_1 + m_2) + \frac{cc_2}{k} \right) \ddot{r} + c_2 \frac{d}{dt} \dot{r}, \quad (4)$$

that consists of the well-known snap, jerk, acceleration and velocity feedforward. However, the true inverse dynamics of (3), when neglecting the zero dynamics of the system, are given by

$$u \approx \frac{m_2 m_1}{k(\rho)} \ddot{y} + \frac{c(m_1 + m_2) + c_2 m_1}{k(\rho)} \ddot{y} + (m_1 + m_2) \ddot{y} + c_2 \dot{y} + \left(\frac{2\dot{\rho}^2 k'(\rho)}{k^2(\rho)} - \frac{\dot{\rho}^2 k''(\rho)}{k^2(\rho)} - \frac{\ddot{\rho} k'(\rho)}{k^2(\rho)} - \frac{2\dot{\rho} k'(\rho)}{k^2(\rho)} \right) f(y, \dot{y}, \ddot{y}), \quad (5)$$

with $f(y, \dot{y}, \ddot{y}) = m_1 m_2 \ddot{y} + (c(m_1 + m_2) + c_2 m_1) \dot{y} + (k(\rho)(m_1 + m_2) + cc_2)y$, where $k' = \frac{dk(\rho)}{d\rho}$ and $k'' = \frac{d^2 k(\rho)}{d\rho^2}$. When comparing LTI feedforward in (4) with the approximate inverse in (5), it is observed that LTI feedforward lacks both static and dynamic dependency on ρ .

2.4 Problem Definition

A technique for manual tuning or direct data-driven identification of feedforward controllers for LPV motion systems, including dynamic dependency on the scheduling sequence, is currently lacking. The problem addressed in this paper is the direct identification of LPV polynomial feedforward controller of the form

$$\tilde{F}_{LPV} : u_{ff} = \sum_{i=1}^{n_\theta} \theta_i(\rho, \dot{\rho}, k'(\rho), \dots) \psi_i \left(\frac{d}{dt} \right) r, \quad (6)$$

based on input-output data $\{u, y\}$, for the class of LPV motion systems in (1), that includes dynamic dependence

on the scheduling sequence ρ , where the structure and order of the model is not fixed a priori, and minimizes the tracking error e in the configuration of Fig. 1.

3. LINEARLY PARAMETERIZED FEEDFORWARD FOR LPV MOTION SYSTEMS

In this section, a polynomial feedforward strategy for LPV systems as in (6) is developed, by posing an alternative parameterization for the class of LPV motion systems in (1), that includes dynamic dependency on the scheduling sequence, but simplifies the identification problem significantly.

The key idea is to rewrite the system dynamics in (1) as

$$\sum_{i=-n_i}^{n_a} a_i(\rho) y^{(i)} = \sum_{j=0}^{n_b} b_j(\rho) \frac{d^j}{dt^j} w, \quad (7)$$

where a change of variables is used as $w = \iint u dt^2$. Similarly to LTI polynomial feedforward in (2), the inverse model for LPV systems is parameterized by setting the values of $b_j \forall j \geq 1$ in (7) to zero, resulting in

$$F_{LPV} : \begin{cases} w_{ff} = \sum_{i=1}^{n_\theta} \theta_i(\rho) \psi_i \left(\frac{d}{dt}, I \right) r, \\ u_{ff} = \frac{d^2}{dt^2} w_{ff}, \end{cases} \quad (8a) \quad (8b)$$

where ψ contains differentiators $\frac{d}{dt}$ or integrals I , e.g. $\psi_i \left(\frac{d}{dt}, I \right) r = \frac{d^2}{dt^2} r = \ddot{r}$ or $\psi_i \left(\frac{d}{dt}, I \right) r = I r = \int r dt$.

Note that from (8a), the second integral of the input w_{ff} is composed out of basis functions, in contrast to the input u_{ff} for LTI polynomial feedforward. The dynamic dependence on the scheduling sequence seen in (5) is introduced by the second derivative with respect to time in (8b), which introduces time derivatives of $\theta(\rho)$. In Example 5, an example is shown for the two-mass system.

Example 5. (LPV feedforward). Consider the two mass-spring-damper system from Example 4, with input-output behavior in (3). The polynomial feedforward strategy is then defined, by neglecting the zero in the right-hand side of (3) according to Assumption. 2, i.e., $\frac{c}{k(\rho)} \approx 0$, as

$$w_{ff} = \underbrace{c_2}_{\theta_1(\rho)} \underbrace{\int r dt}_{\psi_1} + \underbrace{\left(m_1 + m_2 + \frac{cc_2}{k(\rho)} \right)}_{\theta_2(\rho)} \underbrace{1}_{\psi_2} r + \underbrace{\frac{c(m_1 + m_2) + c_2 m_1}{k(\rho)}}_{\theta_3(\rho)} \underbrace{\frac{d}{dt}}_{\psi_3} r + \underbrace{\frac{m_2 m_1}{k(\rho)}}_{\theta_4(\rho)} \underbrace{\frac{d^2}{dt^2}}_{\psi_4} r, \quad (9)$$

where the applied feedforward force is calculated using (8b). The applied feedforward force contains both static and dynamic scheduling dependence when substituting (9) into (8b), i.e.,

$$u_{ff} = \frac{m_2 m_1}{k(\rho)} \ddot{r} + \frac{c(m_1 + m_2) + c_2 m_1}{k(\rho)} \ddot{r} + (m_1 + m_2) \ddot{r} + c_2 \dot{r} + \left(\frac{2\dot{\rho}^2 k'(\rho)}{k^2(\rho)} - \frac{\dot{\rho}^2 k''(\rho)}{k^2(\rho)} - \frac{\ddot{\rho} k'(\rho)}{k^2(\rho)} - \frac{2\dot{\rho} k'(\rho)}{k^2(\rho)} \right) f(r, \dot{r}, \ddot{r}), \quad (10)$$

which is equal to (5) when substituting y for r .

The applied feedforward force u_{ff} in (8b) includes the dynamic dependency on the scheduling signal, e.g. shown in (5), while the modeled w_{ff} in (8a) is only statically dependent on the scheduling sequence.

4. KERNEL REGULARIZED LEARNING OF LPV FEEDFORWARD PARAMETERS

In this section, the functions $\theta_i(\rho)$ in (8a) are identified using kernel regularization, which models the functions without a specified structure or order, since the solution is in the infinite-dimensional Reproducing Kernel Hilbert Space (RKHS). Second, kernel design for LPV feedforward parameters is described. Finally, the developed approach is summarized in a procedure.

4.1 Kernel Regularized Identification

A cost function is defined using input-output data as (Pillonetto et al., 2014; Blanken and Oomen, 2020)

$$\hat{\Theta} = \arg \min_{\Theta} \|\bar{w} - \Phi\Theta\|^2 + \gamma\|\Theta\|_{\mathcal{H}}^2, \quad (11)$$

with Euclidean norm $\|\cdot\|$, $\Phi\Theta$ equal to w_{ff} in (8a), and measurement data vector \bar{w} , that is constructed as

$$\bar{w} = [w(0T_s) \ w(1T_s) \ \dots \ w((N-1)T_s)]^\top. \quad (12)$$

The squared induced norm on the RKHS \mathcal{H} is denoted as $\|\Theta\|_{\mathcal{H}}^2$, that is given by (Pillonetto et al., 2014),

$$\|\Theta\|_{\mathcal{H}}^2 = \Theta^\top K^{-1}\Theta, \quad (13)$$

with kernel K . The parameter vector Θ and basis function matrix Φ are built up as

$$\Theta = [\bar{\theta}_1^\top \ \bar{\theta}_2^\top \ \dots \ \bar{\theta}_{n_\theta}^\top]^\top, \quad \Phi = [\bar{\phi}_1 \ \bar{\phi}_2 \ \dots \ \bar{\phi}_{n_\theta}], \quad (14)$$

where the individual parameter vector $\bar{\theta}_i$ and $\bar{\phi}$ are constructed by gathering the values over a training period as

$$\bar{\theta}_i = \begin{bmatrix} \theta_i(\rho)(0T_s) \\ \theta_i(\rho)(1T_s) \\ \vdots \\ \theta_i(\rho)((N-1)T_s) \end{bmatrix} \quad (15)$$

$$\bar{\phi}_i = \begin{bmatrix} (\psi_i y)(0T_s) & 0 & \dots & 0 \\ 0 & (\psi_i y)(1T_s) & \dots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & (\psi_i y)((N-1)T_s) \end{bmatrix}$$

where $(\frac{d}{dt}, I)$ has been left out for brevity.

The solution to the cost function in (11) is given by (Pillonetto et al., 2014; Blanken and Oomen, 2020)

$$\hat{\Theta} = K\Phi^\top (\Phi K\Phi^\top + \gamma I_N)^{-1} \bar{w}, \quad (16)$$

where parameters θ are estimated at any ρ^* using the representer theorem (Pillonetto et al., 2014, Section 9.2).

Remark 6. Note that (11) is an open-loop solution, while a closed-loop control structure is assumed as shown in Fig. 1, hence measurement noise introduces bias (Blanken and Oomen, 2020). The addition of instrumental variables is capable of removing this bias, and is reported elsewhere.

The kernel K can be designed to incorporate prior knowledge on the feedforward parameters, such as smoothness or periodicity, and will be discussed in the next section.

4.2 Kernels for LPV Feedforward Parameters

The kernel incorporates prior knowledge on the feedforward parameters, hence is important to design carefully. The optimal kernel for solving (11), when treating feedforward parameters as random variables, is equal to

$$\Pi = \mathbb{E}(\Theta\Theta^\top) = \begin{bmatrix} \mathbb{E}(\bar{\theta}_1\bar{\theta}_1^\top) & \mathbb{E}(\bar{\theta}_1\bar{\theta}_2^\top) & \dots & \mathbb{E}(\bar{\theta}_1\bar{\theta}_{n_\theta}^\top) \\ \mathbb{E}(\bar{\theta}_2\bar{\theta}_1^\top) & \mathbb{E}(\bar{\theta}_2\bar{\theta}_2^\top) & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}(\bar{\theta}_{n_\theta}\bar{\theta}_1^\top) & \dots & \dots & \mathbb{E}(\bar{\theta}_{n_\theta}\bar{\theta}_{n_\theta}^\top) \end{bmatrix}. \quad (17)$$

For LPV motion systems, parameters may correlate, i.e., $\mathbb{E}(\bar{\theta}_i\bar{\theta}_j^\top) \neq 0 \forall j \neq i$. For example, when looking at (9), parameters θ_3 and θ_4 are scaled versions of each other. Hence, the framework is capable of incorporating correlation between feedforward parameters.

The optimal kernel is approximated by a kernel matrix,

$$\mathbb{E}(\bar{\theta}_i\bar{\theta}_j^\top) = K_{ij}(\bar{\rho}, \bar{\rho}), \quad (18)$$

which only has a static dependency on ρ , while the framework produces feedforward which is dynamically dependent on the scheduling sequence as shown in Section 3.

The kernel matrix K_{ij} is determined by evaluating a kernel function, such as the squared exponential kernel function

$$k_{ij,SE}(\rho, \rho') = \sigma_{ij}^2 \exp\left(-\frac{(\rho - \rho')^2}{2\ell_{ij}^2}\right). \quad (19)$$

The hyperparameters of the kernel, i.e., for the squared exponential kernel in (19) the output variances σ_{ij}^2 and length scales ℓ_{ij} can be tuned using marginal-likelihood optimization. The kernel choice provides the user to apply prior knowledge on the feedforward parameters.

4.3 Developed Procedure

The developed procedure is summarized in Procedure 1.

Procedure 1. (Kernel regularized LPV feedforward identification)

- (1) Apply reference r to closed-loop system in Fig. 1 and record y , ρ and u .
- (2) Construct kernel matrices by evaluating (18).
- (3) Calculate matrix Φ using (14) and (15).
- (4) Compute \bar{w} from (12) using the second integral of the input $w = \int \int u \, dt^2$.
- (5) Estimate the feedforward parameters $\hat{\Theta}$ using (16).

To conclude, kernel regularized identification is capable of identifying LPV feedforward parameters with input-output data of a system, without specifying a structure or order. In the following section, an example is shown that validates the developed framework.

5. EXAMPLE

In this section, the developed approach of feedforward for LPV systems is validated on an example.

5.1 Example Setup

The two-mass-spring-damper in Fig. 2 is considered. The feedback controller C is a lead filter. The system is seen in

(3) and Fig. 2, with damper constants $c = 1$ and $c_2 = 10^{-4}$ Ns/m, masses $m_1 = 1$ and $m_2 = 0.5$ kg. Stiffness $k(\rho)$ is

$$k(\rho) = \frac{EA}{\rho(L - \rho)}, \quad (20)$$

with length $L = 1$ m, Young's modulus $E = 0.24 \cdot 10^9$ Pa and area $A = 1 \cdot 10^{-5}$ m². The reference r is chosen as a fourth order point-to-point motion, as in Lambrechts et al. (2005), consisting of 1810 samples. The scheduling sequence ρ used is the reference itself, and ranges from 0.2 m to 0.8 m. The feedforward parameters $\theta_i(\rho)$ in (8a) are identified according to Procedure 1.

5.2 Compared Approaches

The following three approaches are compared in feedforward to evaluate the developed framework.

LTI Completely ignoring the LPV dynamics of the system and using static feedforward parameters as in (2), where the feedforward parameters are taken as the true parameters for $\rho = 0.5$ m, i.e.

$$F_{LTI} : u_{ff} = \theta_1 \dot{r} + \theta_2 \ddot{r} + \theta_3 \dddot{r}. \quad (21)$$

Static LPV Including the LPV effects in the standard polynomial snap feedforward, but ignoring the additional terms which arise due to the chain and product rule of integration (Van Haren et al., 2022), i.e.,

$$F_{SLPV} : u_{ff} = \theta_1 \dot{r} + \theta_2 \ddot{r} + \theta_3(\rho) \dddot{r}. \quad (22)$$

Dynamic LPV Application of the developed feedforward approach in (8a) and (8b) with dynamic snap feedforward, i.e.,

$$F_{LPV} : \begin{cases} w_{ff} = \theta_1 \int r dt + \theta_2 r + \theta_3(\rho) \ddot{r}, & (23a) \\ u_{ff} = \theta_1 \dot{r} + \theta_2 \ddot{r} + \theta_3(\rho) \dddot{r} + u_{dyn}, & (23b) \end{cases}$$

with

$$u_{dyn} = \ddot{\rho} \theta'_3(\rho) \ddot{r} + \dot{\rho}^2 \theta''_3(\rho) \ddot{r} + 2\dot{\rho} \theta'_3(\rho) \ddot{r}. \quad (24)$$

The parameters $\theta_i(\rho)$ are identified using the developed framework, where, for simplicity, the kernel is chosen block-diagonal, i.e., $K_{ij} = 0 \forall i \neq j$, meaning different feedforward parameters θ_i and $\theta_j \forall i \neq j$ do not correlate, but individual feedforward parameters θ_i on different values of ρ do correlate. The kernels K_{11} and K_{22} are chosen to be identity matrices of appropriate size, i.e., parameters θ_1 and θ_2 are constant. The kernel K_{33} is chosen as the squared exponential kernel from (19), where the σ_{33}^2 and ℓ_{33} are optimized using marginal likelihood optimization.

5.3 Results

In this section, the results of the example are shown. First, in Fig. 3, an error plot is shown for the three feedforward approaches. Second, in Fig. 4, the contribution of the developed feedforward approach due to the dynamic dependency is shown. Third, the contribution of snap feedforward for both static LPV and the developed feedforward approach is shown in Fig. 5. Finally, a surface plot of the true and estimated dynamic dependent snap feedforward is shown in Fig. 6. The following observations are made:

- Fig. 3 shows that the best tracking performance is achieved by the developed approach, while the static LPV feedforward performs better than LTI

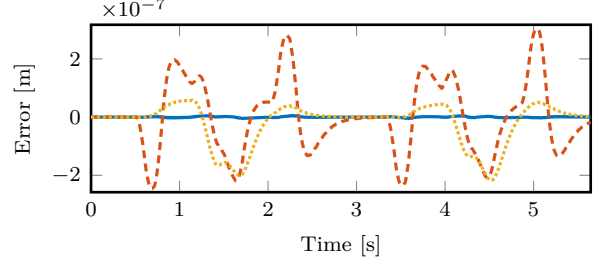


Fig. 3. Time-domain error for LTI (---), static LPV (....) and developed (—) feedforward approaches.

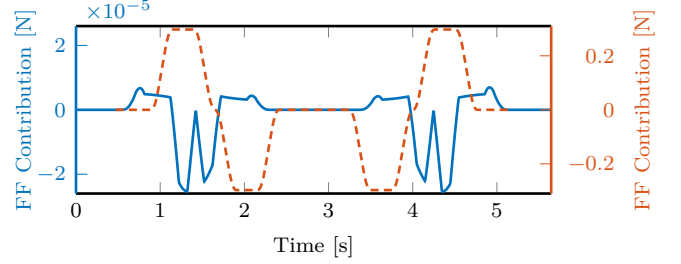


Fig. 4. Left axis: Dynamic dependent feedforward u_{dyn} from (24) (—). Right axis: Total feedforward (---).

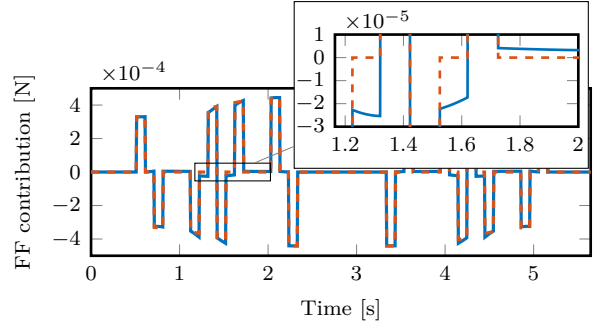


Fig. 5. Feedforward contribution of static LPV $\theta_3(\rho) \dddot{r}$ (---) and developed approach $\frac{d^2}{dt^2}(\theta_3(\rho) \ddot{r})$ (—). Note that the difference is solely caused by the dynamic scheduling dependency.

feedforward. The 2-norm of the errors are respectively $5.8 \cdot 10^{-8}$ m, $2.5 \cdot 10^{-6}$ m and $4.2 \cdot 10^{-6}$ m.

- The LPV snap parameter $\frac{m_1 m_2}{k(\rho)}$ is estimated accurately, with a root-mean-square error of $4 \cdot 10^{-5}$ kgs².
- Fig. 4 and Fig. 5 show that the contribution of dynamic feedforward is not zero, hence should be taken into account.
- For a high contribution of the dynamic feedforward in Fig. 4, the tracking error for static LPV in Fig. 3 increases, showing that the dynamic dependence has significant effect on the tracking error.
- Fig. 6 shows that, for the reference, the dynamic contribution to the feedforward is estimated accurately.

6. CONCLUSIONS

In this paper, a method is developed to directly identify feedforward controllers for LPV motions systems, including dynamic dependence on the scheduling sequence. A polynomial feedforward model is developed for LPV motion systems by using a change of variables, i.e., the

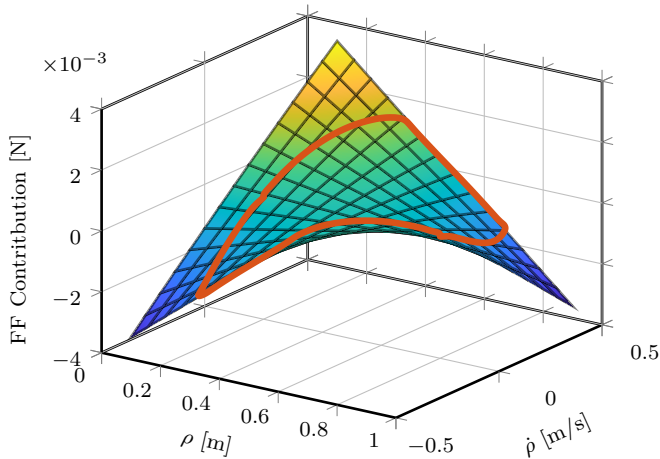


Fig. 6. True dynamic contribution u_{dyn} from (24) and estimated given the data (—) for $\dot{r} = 1$ and $\ddot{r} = 20$.

double integral of the input signal. The feedforward parameters are directly identified with input-output data using a kernel-regularized approach. An example shows that tracking performance is significantly improved compared to existing LTI or LPV approaches that do not take dynamic dependence into account.

Ongoing research is aimed at adding instrumental variables and directly learning LPV feedforward parameters without change of variables. Finally, experimental validation of the framework is part of ongoing work.

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