Quantum Zero Point Energy and Lorentz Invariants

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In previous notes we have argued that the notion of quantum period and wavelength follow from the Lorentz invariant A = - Et+px. In this note, we wish to investigate these ideas more closely. For example, hbar/E and hbar/p yield -Et+px = 0 so one may shift to, xo, but such a shift does not hold for another Lorentz invariant: xx-tt = constant. As a result, the notion of frequency and wavelength are governed by a constraint equation linked to p and E which are considered exact numbers calculated using mo (rest mass) and v velocity.

We argue that given a rest state with mo, x=0 and t=to there is actually decoupling of space and time and arbitrariness of their values. In other words, there is a dualism of determinism and indeterminism already in the rest frame with p=0, v=0 being deterministic, but x,t values being independent and arbitrary. This dualism should be preserved if one views the system from a frame moving at constant speed -v. Then the particle is seen to move at speed of v which is deterministic as are E and p. The arbitrariness of x,t in the rest frame and their independence have now been supposedly lost as one has a 'x/t'=v. In other words, "information" has been lost.

We argue that one should seek to regain this information. E',p' are precise values and are linked with x',t' through the Lorentz invariant A=-Et+px. This invariant allows for independence of x and t, and also for shifted values whose ratio "x"/ "t" do not equal v, yet keep A constant. Thus we argue that seeking independence of x and t and a notion of arbitrariness in x', t', both features which exist in the rest frame, leads to the notion of a quantum period and wavelength. This, however, begs the question: Why does a shift of period and wavelength leave -Et+px unchanged, but change the Lorentz invariant xx-tt?

Introduction

The notion of a quantum period and wavelength in a constant momentum photon or particle with rest mass seems to be unusual given that these objects do not form classical waves which have energy spread out over the wave. These quantum waves, rather, seem to be probability waves, i.e. there is a probability to find a momentum p at a point, but one receives the entire p impulse at that point, not a fraction of it (according to Einstein's photoelectric effect) if an interaction occurs. Thus, it seems important to find the reason for such a probabilistic scenario which at the same time is linked to determinism i.e. x=vt.

Particle in a Rest State

Special relativity allows for the notion of a particle in a rest state. For example, one may imagine a particle with rest mass mo (mocc=energy), x=0 and t=to. One may immediately note three important pieces of information:

(A) X and t are arbitrary numbers. X may be set to any value and t to any positive value.
(B) X and t are independent

(C) There is determinism as E=mocc, an exact value, p=0 and v=0. These may not be arbitrarily shifted, especially p.

We now consider, as special relativity does, what this particle would look like as seen from a frame moving with constant speed -v. The particle would seem to be moving with a constant velocity of v. The two conditions (A) and (B) should still hold, however, because one has not physically changed the state. It is simply being viewed in a different manner.

One may note, however, that the Lorentz transformation transforms (x=0, t=to=0) into (x',t')=(0,0). Thus the general statement dx/dt=v=constant is replaced by a more specific one, namely x'/t'=v. In principle, however, the fact that the particle is moving with speed v (as seen in the moving frame) should not give any information about its x' and t' values. In other words there should still be independence of x',t' (which is broken by x'/t'=v) and also arbitrariness of x',t'. Is it possible that these two conditions exist in special relativity even though the Lorentz transformation takes a specific 4-vector and transforms it into another specific one? It seems the answer is yes and follows from the (1,-1) nature of the metric. In other words, Lorentz invariants involve two terms separated by a minus sign so "cancellations" of shifted values may possibly subtract out.

Given that we are interested in x',t' there immediately exist two Lorentz invariants:

Constant = -Et+px ((1a)) constant 2 = -tt + xx (c=1) ((1b))

In both cases, t and x appear to be independent. Can a shift in x,t be created for each which leaves the Lorentz invariant unchanged? For ((1a)) a shift in t of hbar/E and in x of hbar/p leaves -Et+px unchanged. In ((1b)) t→t+b and x→x+a yields an extra term of -bb+aa -2tb+2xa which must be 0. Given that x/t=v one has: -bb+aa+t2(-b+va). In order to have a shift of 0 independent of t, -b+va=0, but then -bb+aa is not 0 and so the shift does not exist which leaves -tt+xx constant. Thus a shift leaving the Lorentz invariant constant exists only for ((1a)) which contains information not only of x,t, but of p,E which are also pieces of information which exist in the moving frame. Thus ((1a)) becomes a constraint equation on the arbitrariness of x,t allowed. In other words, in a moving frame one has:

A1) x'/t'=v

B1) x',t' independent which violates A1)

C1) shifts in x',t' which leave -Et+px unchanged.

B1) and C1) violate A1) because if x'/t'=v then {x'+hbar/p} / {t'+hbar/E} not= v. In order for there to be independence of x' and t', one only considers the point x'+hbar/p and t'+hbar/E as being correlated. Within a length of hbar/p and a time of hbar/E, x' and t' are independent.

Thus special relativity is consistent with determinism i.e. A1) x'/t'=v, but a specific Lorentz invariant, namely -Et+px retains the notion of x,t independence and arbitrariness which are found in the rest state mo, x=0, t=to. In the rest state, the determinism is: p=0, v=0. Thus determinism and indeterminism (i.e. independence and arbitrariness of x,t) exist together. One may note that v=dx/dt so dx/dt=0 does not change the arbitrariness of x,t.

As noted, in the moving frame the determinism of velocity, but arbitrariness of x',t' need to be maintained and this we argue leads to the quantum ideas of period and wavelength.

Implementing The Quantum Wavelength/Period

We noted above that even though there is determinism of a particle as seen in moving frame i.e. E',p', v=x'/t', there is also independence and arbitrariness which violates x'/t'=v. In order to maintain -Et+px constant, there needs to be a correlation between x and t i.e. when x=xo+hbar/p then t=to + hbar/E, but between xo,to and xo+hbar/p, to+hbar/E there is no correlation. Thus one has a kind of periodicity which is captured by the eigenfunctions of id/dt and -id/dx namely exp(-iEt) and exp(ipx).

It seems the arbitrariness and independence of x,t lead to distributions cos(Et) and cos(px), but these do not indicate directions (especially in p case) and also do not treat all t and x points as having equal weights. It is the constant magnitude objects exp(-iEt) and exp(ipx) which allow for this.

It is important to know that the x and t distributions are linked to p and E. Thus independence of x,t and arbitrariness already exist in the rest frame, but they are quantified through a constraint equation which is best seen in the moving frame i.e. -Et+px. P represents an impulse, so one may think of the two x-probability distributions as being associated with such an impulse.

Zero Point Energy

In the physical world one is interested in regions not points. The above ideas seem to indicate that regions of length hbar/p are associated with momentum p through the unusual action-reaction type of function exp(ipx).This function is periodic and extends for all x in the positive and negative directions. Thus some kind of superposition or Fourier series would be needed to create the notion of fixed length and this leads, it seems, to the idea of zero point energy.

Conclusion

In conclusion, the main point we make is that given a particle at rest i.e. mo rest mass, x=0, t=to, there is determinism in that p=0 and v=0, but also indeterminism in the form of independence of x,t and arbitrariness of x,t. X may take on any value (not just x=0) any t any positive value. These deterministic-nondeterministic properties represent basic information in the rest frame and should not disappear if the particle is viewed from a frame moving at constant velocity -v. The deterministic features as seen from the moving frame are E',p' and v with v=x'/t' so that (x=0,t=to=0) transforms to (x',t'), but where are the indeterministic features i.e. independence of x,t and arbitrariness of x,t? We argue that these appear in the Lorentz invariant -Et+px, but not -tt+xx because the latter does allow for shifts which leave the invariant unchanged.

Thus for -Et+px, the shifts hbar/E in t and hbar/p in x leave the invariant unchanged. Within the time region to t—-- to+hbar/E and length region xo to xo+hbar/p, t and x are independent. Thus the independence and arbitrariness are preserved as is the determinism x=vt. Thus determinism seems to be associated with unperturbed motion in space, while indeterminism is associated with p which also represents impulse. Thus the indeterministic features should appear during interactions the size of the wavelength, which they do.