Combinatorial Geometric Series: Infinite Series with Binomial Coefficients

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Abstract: This paper discusses the finite and infinite geometric series with binomial coefficients. This idea can enable the scientific researchers to solve the real-world problems.

MSC Classification codes: 05A10, 40A05 (65B10)

Keywords: computation**,** binomial coefficient, geometric series

1. Introduction

When the author of this article was trying to compute the multiple summations of a geometric series [1-12], a new idea stimulated his mind to create a new type of geometric series. As a result, a combinatorial geometric series [11-20] was developed with new idea of binomial coefficients.

2. Geometric Series with Binomial Coefficients

The combinatorial geometric series is derived from the multiple summations of a geometric series. The coefficient of each term in the combinatorial geometric series [17-33] refers to the binomial coefficient [29-42].

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \text{ and } V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r)}{r!},$$

where r is a positive integer, n is a non-negative integer, and V_n^r is a binomial coefficient.

Note that $V_n^0 = V_0^0 = 0! = 1$ and $V_0^r = \dfrac{(0+1)(0+2)(0+3)\cdots(0+r)}{r!} = \dfrac{r!}{r!} = 1.$
So, $V_n^0 = V_0^r = V_0^0 = 1.$

$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r}$ is the $(r+1)$summations of geometric series for $r = 0, 1, 2, 3, \cdots$

and $\sum_{i=0}^{n} V_i^r x^i$ is the finite geometric series with binomial coefficients.

$\lim_{n\to\infty}\sum_{i=0}^{n} V_i^r x^i = \sum_{i=0}^{\infty} V_i^r x^i$ is an infinte geometric series with binomial coefficients.

If $|x| < 1$, then $\lim_{n\to\infty}\sum_{i=0}^{n} x^i = \sum_{i=0}^{\infty} x^i = \dfrac{1}{1-x}$ and $\lim_{n\to\infty}\sum_{i=0}^{n} V_i^r x^i = \sum_{i=0}^{\infty} V_i^r x^i = \dfrac{1}{(1-x)^{r+1}}.$

3. Conclusion

In this article, the finite and infinite geometric series with binomial coefficients have been discussed and this information can enable the scientific researchers to solve the real life problems.

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