Equity Futures Contract Valuation Model

The output of the model is the mark to market value of such a contract, that is, the Equity Futures price less the strike (if long position).

Let t be the current date, and let 0 be the trade date, T be the maturity date, $0 \le t \le T$. Let S_t be the underlying (equity) price at time t, $F_{t,T}$ be the Equity Futures price for maturity T, and K be the strike price (delivery price).

The payoff at time T of an Equity Futures contract (long position) is then $F_{T,T} - K = S_T - K$.
The mark to market value of this contract at time t is: $F_{t,T} - K$.

Let $0 \le t \le T_1 < T_2 < ... < T_n \le T$ be the dates when corresponding discrete dollar dividends, $D_1, D_2, ..., D_n$, are paid (if a payment lag is applicable, then it should be added to the above dates). The present value of these dividends is then:

$$D = \sum_{i=1}^{n} D_i \cdot \exp\left(-R\left(t, T_i\right) \cdot \tau\left(t, T_i\right)\right),$$

where $R(t,T)$ is the (continuously compounded) zero in interest rate applicable from t to T as seen at t, $\tau(t,T)$ is the distance between those dates in a given day count convention (usually, actual/365). Alternatively, let q be a (continuously compounded) dividend yield.

By no-arbitrage argument, the Equity Futures price is then:

$$F_{t,T} = \begin{cases} (S_t - D)\cdot \exp(R(t,T)\cdot \tau(t,T)), \text{ if discrete dividend,} \\ S_t \cdot \exp((R(t,T) - q)\cdot \tau(t,T)), \text{ if dividend yield.} \end{cases}.$$

Accordingly, the value of the Equity Futures contract (long position) at time *t* is:

$$V_t = \begin{cases} (S_t - D)\cdot \exp(R(t,T)\cdot \tau(t,T)) - K, \text{ if discrete dividend,} \\ S_t \cdot \exp((R(t,T) - q)\cdot \tau(t,T)) - K, \text{ if dividend yield.} \end{cases}$$

Delta, sensitivity to underlying, is going to be computed as follows:

$$\Delta_t = \frac{\partial V_t}{\partial S_t} = \begin{cases} \exp(R(t,T)\cdot \tau(t,T)), \text{ if discrete dividend,} \\ \exp((R(t,T) - q)\cdot \tau(t,T)), \text{ if dividend yield.} \end{cases}$$

In the following listings the symbol ”//” marks commented out text fragments containing either explanatory information or possible alternatives for the values of the corresponding attributes.

Deal 1

VALUATION_AS_OF_DATE 20050509

VALUATION_AS_OF_TIME 00:00

GREEKS DELTA

RANDOM_OBJECT_TYPE EQUITY

UNDERLYING SPX

EXCHANGE Z

UND_CURRENCY USD=

DRIFT_RATE_FILE USD_RATE_CURVE (see https://finpricing.com/lib/IrCurveIntroduction.html)

DIVIDEND_FILE spx_div

ASP 1172.08

STRIKE 0

OPTION_TYPE EQUITY_FUTURES

PAYOFF_CURRENCY USD=

MODEL CLOSED_FORM

CLASS EUROPEAN

EXPDATE 20050617

EXPTIME 17:00

DISCOUNT_RATE_FILE USD_RATE_CURVE

BEGIN_FILE_INFO spx_div

SPX Z 20050510 0.030443

SPX Z 20050511 0.641146

SPX Z 20050512 0.126079

SPX Z 20050513 0.131578

SPX	Z	20050516	0.123795
SPX	Z	20050517	0.124188
SPX	Z	20050518	0.175363
SPX	Z	20050519	0.010361
SPX	Z	20050520	0.036185
SPX	Z	20050523	0.009276
SPX	Z	20050524	0.008820
SPX	Z	20050525	0.113729
SPX	Z	20050526	0.116603
SPX	Z	20050527	0.122092
SPX	Z	20050601	0.388127
SPX	Z	20050602	0.006363
SPX	Z	20050603	0.009627
SPX	Z	20050606	0.067421
SPX	Z	20050607	0.038778
SPX	Z	20050608	0.232640
SPX	Z	20050609	0.000359
SPX	Z	20050610	0.031369
SPX	Z	20050613	0.290447
SPX	Z	20050614	0.056587
SPX	Z	20050615	0.031406

SPX	Z	20050616	0.009151
SPX	Z	20050617	0.013423