

Output feedback control of a UAV for vision-based target tracking

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I. INTRODUCTION

In recent years, small unmanned aerial vehicles (UAVs) such as quadrotors have become a popular platform for control and robotics research with application to mapping, delivery, and surveillance problems, to name a few [2]. The research activity in autonomous navigation has increased dramatically and the proposed solutions are rapidly approaching the goal of near-complete autonomy [1]. To achieve this ambitious goal, it is also necessary to provide the vehicle with autonomous maneuvering capabilities. In this respect, the incorporation of visual cues in the flight control system is seen as a key element for designing robust and reactive control laws [4].

In this paper, a model-based control approach is proposed for autonomous vision-based tracking of a moving target by a quadrotor vehicle. To solve this problem, we rely on an output feedback perspective within a robust control framework.

II. CONTROL PROBLEM FORMULATION

A. Quadrotor dynamic model

Let $p \in \mathbb{R}^3$ and $v \in \mathbb{R}^3$ be the quadrotor inertial position and velocity vectors, respectively. Moreover, let R be the rotation from the quadrotor body frame to the inertial frame, and $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$ be the angular velocity of the body frame relative to the inertial frame, expressed in the body frame. The quadrotor dynamics are described by the reduced-order model [3]

$$\begin{aligned} \dot{p} &= v \\ \dot{v} &= \frac{f}{m} R_3 + g \\ \dot{R} &= R[\omega]_{\times} \end{aligned} \quad (1)$$

where $f \in \mathbb{R}_{\geq 0}$ is the total thrust, $[\omega]_{\times}$ is the skew symmetric matrix associated to the vector ω , $m \in \mathbb{R}_{> 0}$ is the mass of the quadrotor, g is the gravitational acceleration vector, and R_j denotes the j -th column of R . In (1), the total thrust f and the angular velocity ω are regarded as a control inputs. In practical applications, these terms are fed as reference signals to a low-level rotor controller.

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B. Target tracking problem

The control objective for the quadrotor is to track a moving target whose dynamics are unknown. The target inertial position at time t is denoted by $p_o(t)$. In order to perform autonomous relative navigation, the quadrotor is equipped with an optical camera providing line-of-sight and apparent angular size measurements of the target. The line-of-sight l corresponds to the unit vector joining the quadrotor and the target, expressed in the camera frame. Formally, $l = R_c^T R^T \frac{p_e}{\|p_e\|}$ where $p_e = p_o - p$ and R_c is the rotation from the camera frame to the body frame. The apparent angular size is modeled by using the small-angle approximation $\alpha = c_s \|p_e\|^{-1}$ where c_s is the diameter the target, which for simplicity is assumed to be spherical. Notice that, in general, the parameter c_s is not known. In addition to the optical camera, the quadrotor is equipped with an inertial measurement unit (IMU) providing an estimate of the attitude matrix R . By combining optical and IMU data, one can estimate the scaled relative position vector $y_p = p_e c_s^{-1}$.

Without loss of generality, let us assume that camera image plane is aligned to the $Y_c Z_c$ -plane of the camera frame. The considered tracking problem consists in regulating the composite output vector

$$z = l \alpha^{-1} = R_c^T R^T y_p \quad (2)$$

towards the reference vector $z_r = [\alpha_r^{-1} \ 0 \ 0]^T$, where $\alpha_r > 0$ is a predefined constant. This corresponds to keeping the target at the center of the image plane, while driving its apparent angular size α to the desired value α_r . A major issue with this task is that the quadrotor tilt dynamics cannot be controlled independently from the relative motion dynamics. An effective way to address this issue is to employ a gimbal camera. Herein, it is assumed that the camera is free to rotate about the pitch axis of the body frame. In particular, $R_c = R_Y(y_c)$, where

$$R_Y(y_c) = \begin{bmatrix} \cos(y_c) & 0 & \sin(y_c) \\ 0 & 1 & 0 \\ -\sin(y_c) & 0 & \cos(y_c) \end{bmatrix}$$

is an elementary rotation about the Y -axis by the manipulated angle y_c . Now, let us define the heading output

$$y_h = \text{atan}_2(R_2^T y_p, R_1^T y_p) \quad (3)$$

From (2)-(3). It is not difficult to show that if the following conditions are satisfied

$$y_c = -\text{asin}(\|y_p\|^{-1} R_3^T y_p) \quad (4)$$

$$y_h = 0 \quad (5)$$

$$y_p = r_p \quad (6)$$

where r_p is a (possibly time-varying) reference signal such that $\|r_p\| = \alpha_r^{-1}$, then $z = z_r$. In this work, it is assumed that the camera control system is able to satisfy (4). In the following, two feedback controllers are presented whose objective is to meet conditions (5) and (6).

III. OUTPUT FEEDBACK CONTROL DESIGN

A. Position control

Let us assume that $p_o(t)$ is a thrice differentiable function of time. For the relative position control problem at hand, we find it convenient to define the state vector

$$x = (y_p, \dot{y}_p, \ddot{y}_p) \quad (7)$$

where $x_j \in \mathbb{R}^3$. Moreover, in order to meet the quadrotor thrust constraint $f \geq 0$, we define

$$f = e^\lambda \quad (8)$$

where λ is the new thrust control input. By differentiating (7) with respect to time, it can be shown that

$$\dot{x}_1 = x_2, \dot{x}_2 = x_3, \dot{x}_3 = b e^\lambda R q + d_p \quad (9)$$

where $b = (mc_s)^{-1}$ is an uncertain parameter (recall that c_s may not be known), $d_p = \ddot{p}_o/c_s$ is an unknown time-varying disturbance accounting for the dynamic behaviour of the target, and $q = (-\omega_2, \omega_1, -\dot{\lambda})$ is a vector of manipulated variables.

Similarly to what is observed in previous research, system (9) can be made linear by applying the feedback linearizing transformation $q = e^{-\lambda} R^T u$. However, perfect knowledge of R is not available in practice. To account for this limitation, in this work R is replaced by its estimate \hat{R} . Hence,

$$q = e^{-\lambda} \hat{R}^T u \quad (10)$$

By enforcing (10) in (9) and using the approximation $R\hat{R}^T \approx I + [\epsilon]_\times$, which holds when ϵ is small enough, one gets $\dot{x}_3 = b(I + [\epsilon]_\times)u + d_p$. Based on the dynamics in (9) and of x_3 above, and on

the definition of the output $y_p = x_1$, the following state-space model is obtained

$$\dot{x} = Ax + B_\Delta u + B_d d, \quad y_p = Cx \quad (11)$$

where A, B_Δ, B_d , and C are suitably defined matrices, with B_Δ being uncertain. The proposed position feedback controller addresses the following performance requirements: (i) robust stabilization of system (11); (ii) practical regulation of the output y_p towards the reference r_p ; (iii) rejection of the disturbance d_p over a suitable frequency band. Controller design is performed in a \mathcal{H}_∞ perspective by hinging upon the results in [5]. The uncertainties on the matrix B can be conveniently represented via a polytopic description. This enables to recast the design of a robust output feedback controller as the solution to a semidefinite program, i.e., an optimization problem over linear matrix inequalities constraints. This type of problems can be efficiently solved via off-the-shelf software, thereby making our approach constructive and systematic.

B. Heading control

In the previous design, the degree of freedom provided by ω_3 has not been used. Such control term can be exploited in order to regulate the quadrotor heading error y_h to zero. Taking the time derivative of (3), one obtains

$$\dot{y}_h = -\omega_3 + d_h \quad (12)$$

where the term d_h can be regarded as a bounded unknown disturbance. For (12), we employ the proportional controller $\omega_3 = k_h y_h$, where $k_h > 0$ is chosen to reduce the effect of d_h .

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