Multiple-Underlying Capped Accumulated Return Call Option

A pricing model is presented for capped-accumulated-return-call (CARC) with multiple underlyings. At each reset date, a weighted stock price is calculated. These weighted stock prices are used to compute returns.

Let $S_1, \cdots, S_N$ be *N* stocks in a given basket, $S_j(t)$ be the price process of the *j*th stock and $0 \le j \le N$, and $\{t_0 < t_1 < \cdots < t_n\}$ be a set of reset dates and $T \ge t_n$ be a payoff settlement date. The CARC with the multiple underlyings $S_1, \cdots, S_N$ is a European type derivative security whose matured payoff at the settlement date is given by

$$N + N \times \max\{R_c, R_f\} \tag{1}$$

where R_f is the global floor (strike) of the return rate, N is the notional principal, and R_c is capped-accumulated-return and defined as

$$R_c = \prod_{i=1}^{n}(1 + R_{cap}^{(i)}) - 1 \tag{2}$$

where $R_{cap}^{(i)}$ is the capped return-rate for each period explained as follows. Define the actual period return-rate as

$$R_i = \frac{\overline{S}(t_i) - \overline{S}(t_{i-1})}{\overline{S}(t_{i-1})}, i = 1, \cdots, n, \tag{3}$$

Where

$$\overline{S}(t_i) = \sum_{j=1}^{N} w_j S_j(t_i). \tag{4}$$

Here $w_j, j = 1, \cdots, N$ are the weights and

$$\sum_{j=1}^{N} w_j = 1. \tag{5}$$

Then we define

$$R_{cap}^{(i)} = \min\{c, R_i\}, \tag{6}$$

where *c* is the cap.

Let t be the current value date, then the current value of this CARC can be written as

$$df(t,T) \times N[1 + E_t[\max\{R_c, R_f\}]] \tag{7}$$

where $df(t,T)$ is the discounting factor at the value date. The above formula is in a world that is forward risk-neutral with respect to a specific currency C_p.

As a result, the notional principal N is measured in the currency C_p, and the discounting factor should be calculated by a C_p zero curve (ref. https://finpricing.com/lib/IrCurveIntroduction.html) given at the value date. If the underlying asset is measured in another currency C_U, assuming the option is a Quanto type transaction, the governing price dynamics of the underlying asset in the risk-neutral world of C_p should be written as

$$dS_t = (r^U - q - \rho\sigma_x\sigma_s)S_t dt + \sigma_s S_t dW_t \tag{8}$$

where r^U is the short rate of C_U, q is the dividend yield of the asset, σ_s is the volatility of the asset price, σ_x is the volatility of the exchange rate between C_p and C_U, ρ is correlation coefficient between the asset price and the exchange rate, and W_t is the Wiener process. All these parameters are assumed deterministic.