Understanding Riemann Hypothesis by Matrix-Graph Relation

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December 6, 2022

Abstract

Understanding Riemann Hypothesis could be possible by in extension of understanding Fermat's Last Theorem.

1 Understanding

By continuing preceding discussion in Fermat's Last Theorem [1]:

$$\begin{aligned}&\text{Assuming that, } a^n + b^n = c^n \\ &\Rightarrow \text{(In matrix form of) } ((1 \times p) \cdot (p \times 1))^n + ((1 \times q) \cdot (q \times 1))^n = ((1 \times r) \cdot (r \times 1))^n \\ &\Rightarrow n^{(\max p)+(\max q)} = r^n \ (n \in \mathbf{N}^+) \Rightarrow n = 1 \ or \ 2\end{aligned}$$

This equation implies if any r $(r < c, r \in \mathbf{N}^+)$ is given, there is always solution that fits a and b. In extension:

$$\begin{aligned}&((1 \times p) \cdot (p \times 1))^n + ((1 \times q) \cdot (q \times 1))^n = ((1 \times r) \cdot (r \times 1))^n \\ &\Rightarrow 2^{(\max p)+(\max q)} = r^2\end{aligned}$$

By giving condition of $1 \le p \le a$, $1 \le q \le b$ in extension to $\mathbf{C}$:

$$\begin{aligned}&a^n + b^n = c^n \Rightarrow \exists (r \le c) \implies 2^2 \le \left(2^{(\max p)+(\max q)} = r^2\right) \le 2^{a+b} \\ &\implies a^n + b^n = c^n \Rightarrow 2^2 \le c^2 \le 2^{a+b} \Rightarrow 2^2 \le (a^2 + b^2) \le 2^{a+b} \\ &\implies 2 \le \frac{a^2 + b^2}{2} \le 2^{a+b-1}\end{aligned}$$

And definition of Riemann zeta function follows:

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

Riemann hypothesis statement asserts all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2 [2].

$$s = \frac{1}{2} + ki \ (k \in \mathbf{R}) \Rightarrow 0 = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

By analytic approach, assuming sum in sequence of $x_{y+1} = x_y^{\log_y(y+1)}$

$$\frac{1}{2^{a+b}} \leq \frac{1}{a^2+b^2} \leq \frac{1}{4}$$

$$\left(\frac{1}{2^{a+b}}\right)^{\log_2 3} \leq \left(\frac{1}{a^2+b^2}\right)^{\log_2 3} \leq \left(\frac{1}{4}\right)^{\log_2 3}$$

$$\cdots$$

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{2^{a+b}}+\frac{1}{3^{a+b}}+\cdots \leq 1+\frac{1}{2}+\left(\frac{1}{a^2+b^2}\right)^{\log_2 3}+\left(\left(\frac{1}{a^2+b^2}\right)^{\log_2 3}\right)^{\log_3 4}+\cdots$$

$$\leq 1+\frac{1}{2}+\frac{1}{4}+\left(\frac{1}{4}\right)^{\log_2 3}+\left(\frac{1}{9}\right)^{\log_3 4}+\cdots \ (\because a+b>1)$$

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{2^{a+b}}+\frac{1}{3^{a+b}}+\cdots \leq \frac{1}{2}+\frac{\pi^2}{6}=\frac{1}{2}+\frac{(-i\log(-1))^2}{6}$$

$$\Rightarrow \zeta(a+b) \leq \frac{1}{2}+\frac{(-i\log(-1))^2}{6}$$

$$\Rightarrow 1+\frac{1}{2}+\left(\frac{1}{c^2}\right)^{\log_2 3}+\left(\frac{1}{c^2}\right)^{\log_2 4}+\left(\frac{1}{c^2}\right)^{\log_2 5}+\cdots \leq \frac{1}{2}+\frac{(-i\log(-1))^2}{6}$$

$$\Rightarrow 1+\frac{1}{2}+\left(\frac{3}{4}\right)^{\log_2 c}+\left(\frac{1}{1}\right)^{\log_2 c}+\left(\frac{5}{4}\right)^{\log_2 c}+\cdots \leq \frac{1}{2}+\frac{(-i\log(-1))^2}{6}$$

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{4^{log_2 c}}\left(3^{\log_2 c}+4^{\log_2 c}+5^{\log_2 c}+\cdots\right) \ (\because a+b>1)$$

$$=\zeta(-\log_2 c) \leq \frac{1}{2}+\frac{(-i\log(-1))^2}{6}$$

$$\Longrightarrow a+b \leq \zeta^{-1}\left(\frac{1}{2}+\frac{(-i\log(-1))^2}{6}\right) \leq \frac{1}{a^2+b^2}$$

$$\Longrightarrow \zeta\left(\frac{1}{a^2+b^2}\right) \leq \frac{1}{2}+\frac{(-i\log(-1))^2}{6} \leq \zeta(a+b)$$

$$\Longrightarrow \text{As keeping symmetry in } \zeta \text{ function} \Rightarrow \Re(a+b)=\frac{1}{2}$$

References

[1] J. Yoon, "Understanding Fermat's Last Theorem through Matrix-Graph Relation," Zenodo, Tech. Rep., Dec. 2022. [Online]. Available: https://zenodo.org/record/7390191

[2] E. Bombieri, "Problems of the millennium: The Riemann hypothesis," *Clay Mathematics Institute*, 2000.