Variable Maturity CDO Valuation

The model serves the purpose of pricing a non-vanilla synthetic CDO trade, where the maturities of the underlying synthetic assets in the collateral pool can be different from the trade maturity.

There are three changes made to the model, namely:

1. The asset substitution option is implemented. If an asset matures before both the trade maturity and the substitution termination date, it is substituted by an appropriate asset.
2. The model is implemented upon the Oscar/Fritz credit library platform, which is more computational efficient.
3. The sensitivity computation via Weighted Monte Carlo (WMC) approximation is replaced by a direct full Monte Carlo simulation of the perturbed scenarios. The new method is in line with the outstanding one for vanilla CDO trades.

To our best knowledge to date, there is no generally accepted methodology of modelling the asset substitution option. The choice of substitution asset has to follow the contractual portfolio management guideline, which is contingent upon the outcome of various portfolio statistics performed at the maturity of the substituted asset. From a mathematical modelling viewpoint, the value of this option cannot be fully determined unless a dynamic process of the portfolio is modelled. Furthermore, it is almost impossible to get the pertinent market information to calibrate such a process.

Instead of attempting to solve this complicated modelling problem, a simple and flexible approach is adopted in the submitted model. Although it is unpredictable how the portfolio will evolve over time and what the substitution asset would be, it is expected that the substitution asset should have certain probability of default before the trade maturity. In a MC scenario in which the substituted asset survives beyond its maturity, a default time of the substitution asset is then simulated by assuming that it is forward starting at the time of substitution. The uncertainty in modelling the asset substitution is cushioned by an appropriate model risk reserve, which has been set up by a joint effort of GRMMR London and GRMMR QA. The model risk reserve policy, drafted by GRMMR London, is attached as Appendix II.

An upgraded version of variable maturity synthetic CDO (VMS-CDO) trade valuation is presented. A VMS-CDO trade has a non-vanilla structure, in which the maturities of the underlying obligors could be different from that of the CDO trade.

Compared with the initially approved version, there are three changes to the model, which are:

1. The asset substitution option is implemented. If an asset matures before both the trade maturity and the substitution termination date, it is replaced with an appropriate asset. The selection of the substitution asset is dependent on a full analysis of the portfolio at the time of the substitution.

2. The submitted model is a new implementation built upon the Oscar/Fritz credit library platform. Compared with the old pricing template, which is based on old credit library SH3, the new model is computationally more efficient.

3. The sensitivity computation via Weighted Monte Carlo (WMC) approximation is replaced by a direct full Monte Carlo simulation of the perturbed scenario. For the sensitivities, such as credit spread sensitivity, default sensitivity, and recovery rate sensitivity, the credit of each obligor is perturbed. Using the MC valuation model (see https://finpricing.com/lib/EqCallable.html), it is very time consuming to revalue the perturbed scenarios when there is a large number of obligors in the collateral pool. In SH3, a WMC approximation is adopted so that a MC re-valuation for each perturbed scenario is not necessary. Only the path weight of each MC scenario is re-calculated with the perturbed risk factors [3]. In the new Oscar/Fritz library, the MC simulation is so efficient that a full MC simulation re-valuation with the perturbed risk factors becomes possible [4]. Hence the WMC approximation is no longer necessary.

Assume the underlying collateral pool is a set of obligors, $N = \{1, 2, \cdots, n\}$, in which each obligor has maturity T_i, a recovery rate R_i, and a notional amount $Notl_i$. A VMS-CDO with this collateral pool has a maturity T and a substitution termination date $T_s (< T)$.

The upgraded modelling framework for the VMS-CDO involves the following three procedures:

1. The correlated defaulted events are generated by the Monte Carlo (MC) simulation via either normal copula model or Poisson model. At the present time, the approved default correlation model is the normal copula model.

2. The losses associated with the defaulted obligors are allocated to each tranche according to its subordination. In the VMS-CDO trade, three non-vanilla features are implemented, which are defined as recovery pay down, principal pay down, and asset substitution. For example, in a simulated scenario for the i^{th} obligor in the collateral pool, the default time is τ_i. Dependent on the simulated default time, the pay off function associated with the three non-vanilla features are as follows.

    - **Recovery Pay Down**
      If $\tau_i < T_i < T$, a loss with the amount $Notl_i \times (1 - R_i)$ is claimed from the surviving junior tranche and a pay down with the amount $Notl_i \times R_i$ is removed from the outstanding most senior piece. Note that in the vanilla CDO trade, this default pay down is usually ignored.

    - **Principal Pay Down**
      If $\tau_i > T_i (> T_s)$, the full notional amount $Notl_i$ is deducted from the most senior tranche at time T_i. This is associated with the scenario of the early retirement of the obligor.

    - **Asset Substitution**
      If $\tau_i > T_i (< T_s)$, the asset is replaced by another appropriate asset with the same notional amount $Notl_i$. The substitution happens at the asset maturity time T_i.

Following the usual procedure of the MC simulation, the mark-to-market (MTM) of the trade can then be calculated. Note that two non-vanilla features, recovery pay down and principal pay down, had been implemented in the initial version of the model. The details of these two features and the associated tests can be found in Ref. [2].

3. The sensitivities are calculated by perturbing the appropriate risk factors and revaluing. The current model is in line with that for vanilla CDO trades. Currently the model supports the computation of the credit spread sensitivity and the default sensitivity via "Curve Substitution Task".

From the viewpoint of modelling, it is very difficult to model the asset substitution feature. The asset substitution happens in the future and is determined by both quantitative and qualitative criteria set by the portfolio management guidelines. As an example, the substitution guideline for a real trade, called Logan CDO II, is attached in the Appendix I. We can see that, the criteria are dependent on the market condition and portfolio behaviour at the time of substitution. Unless we can model the dynamic process of all those factors, we won't be able to accurately model the asset substitution option hence the underlying option value. Furthermore, it is almost impossible to get the pertinent market information to calibrate such a process.

However, no matter which asset is substituted, it is expected that the asset should have a certain probability of default before the trade maturity or, in other words, a possible default event of the substitution asset which will be added to the expected loss of the trade. By employing the current market standard structure credit derivative method it is possible to simulate a default time for the substitution asset without a dynamic process, under the assumption of a hazard rate and some correlation for the substitution assets.

This is actually the approach employed in the model. Assume in an MC scenario, for the i^{th} asset the default time $\tau_i > T_i (< T_s)$, the asset is replaced by another asset, which is described by another hazard rate curve $\tilde{h}_i$. The default time of the substitution asset is simulated. Firstly an uncorrelated uniform random number $U_i \in [0,1]$ is drawn. The default time of the substitution asset would be

(1) $\tilde{\tau}_i = F^{-1}(U_i) + T_i$.

$\tilde{F}(t) = 1 - \exp(-\int_0^t \tilde{h}_i ds)$ is the cumulative default probability of the substitution asset seen at time zero. This default time is then processed in the same way as that in the MC simulation for a vanilla CDO trade.

Note that the current method bears the following assumptions:

1) The default time is forward starting, with survival probability being 1 at the substitution time, that is, $\tilde{S}(T_i) = 1$. This is the prerequisite of modelling the substitution asset, which has to unconditionally survive to the substitution date.
2) The substitution asset is uncorrelated.
3) The model is flexible such that the credit curve of the substitution asset can be chosen at the user's own discretion. For simplicity, normally a spot curve is used, which is exact if the hazard rate curve is flat over time. However, it is only an approximation if the hazard rate curve is non-flat. Given the fact that a flat curve will always be used for the underlying asset, the current approach is adequate.

Note that, even though a credit spread may be reasonably speculated, it is not possible to find a correlation value in the above method or even the definition of such correlation within the current modelling framework. Hence, instead of pricing the substitution option value, we would rather regard this simple and flexible approach as a tool to convert the substitution option into a quantitative measure.

In order to manage the uncertainty in the modelling, an appropriate model risk reserve has been set up by a joint effort of GRMMR London and GRMMR QA. The model is considered acceptable, only if the substitution option value is fully cushioned in the model risk reserve. The model risk reserve policy, drafted by GRMMR London, is attached as Appendix II. Note that in this appendix a guideline for selecting the credit spread of the substitution asset is also included.

For each test series, the mean and an error bound over random seeds are then calculated and compared. Because we are comparing results generated by MC simulation, we define an error bound, computed as

$$\text{(2)} \qquad \text{Error bound} = \sqrt{\Delta_{SH3}^2 + \Delta_{Oscar-Fritz}^2}$$

where

$$\text{(3)} \qquad \Delta = \frac{(\max - \min)}{2},$$

where max and min are the maximum and minimum prices in twenty simulations with different random seeds respectively.

The upgraded VMS-CDO valuation model has been investigated. Test results show that the submitted model is consistent with the model outlined in this report and implemented correctly. Therefore, it is approved for the purpose of marking-to-market and calculating sensitivities for such trades.

There are three changes, which are

1) The asset substitution option is implemented. If an asset matures before both the trade maturity and the substitution termination date, it is substituted by an appropriate asset.
2) The model is implemented upon the Oscar/Fritz credit library platform, which is computationally more efficient.
3) The sensitivity computation via Weighted Monte Carlo (WMC) approximation is replaced by a direct full Monte Carlo simulation of the perturbed scenario.

From the viewpoint of mathematical modelling, it is very difficult to model the asset substitution feature, which happens at some point of time in the future and is determined by both quantitative and qualitative portfolio guidelines. In the model, this feature is simply modelled by simulated forward starting default time of the substitution asset, in a MC scenario in which the substitution option is exercised.

In this modelling approach, even though a credit spread may be reasonably estimated, there is no way to speculate the correlation value or even the definition of such correlation within the current modelling framework. We would rather regard this simple and flexible approach as tool to convert the substitution option into a quantitative measure.

The model is considered acceptable, only if the asset substitution option value is fully cushioned by an appropriate model risk reserve. GRMMR London and GRMMR QA have jointly set up the model risk reserve, based on the analysis of the current market information and the historical data.

The model relies on the MC simulation. Hence the existing restriction and limitations, imposed on the MC simulation for vanilla CDO trades, also applies to the VMS-CDO trade.

GRMMR QA is fully aware that the current modelling is not ideal and better models will be developed as the market evolves and the research progresses. Hence the next re-review date is set to be one year later, which is June 29, 2007.