

Figured Magic Squares of Order 22 Using Bordered Magic Rectangles: A Systematic Procedure

Inder J. Taneja¹

Abstract

Recently, author constructed even order magic squares from orders 6 to 20 with different styles and models, for examples the order 20 is with 1616 magic squares, order 18 with 810 magic squares, etc. These can be seen at [31, 32, 33, 33, 34, 35, 36, 37]. The aim is to proceed for the further orders of magic squares. In this work there are few examples of magic squares given as figures of order 20. A systematic procedure to construct these magic squares is given. It is based on the magic squares and bordered magic rectangles (BMR) of orders 4, 6, 8 etc forming external borders. Then the internal borders are filled with previous known magic squares. The presentations is in figures instead of numbers. The readers can find replies in numbers from references given above. For the orders multiples of 4, we can always write magic squares with equal sums blocks of magic squares of order 4. This procedure is very helpful for the orders of type $2p$, where p is a prime number, for examples, 14, 22, 26, 34, 38, etc. For the orders like 18, 30, etc. we can make good external blocks with order 4, and for orders like 16, 20, 28, 32, etc. we can make good external borders of order 6, and so on.

¹Formerly, Professor of Mathematics, Federal University of Santa Catarina, Florianópolis, SC, Brazil (1978-2012).
Also worked at Delhi University, India (1976-1978).
E-mail: ijthaneja@gmail.com; Web-sites: <http://inderjtaneja.com>; <http://numbers-magic.com>;
Twitter: @IJTANEJA; Instagram: @crazynumbers.

Contents

1	Introduction	3
2	Magic Squares of Order 22	4
2.1	Bordered Square of Order 22	4
2.2	Magic Squares of Order 20 With BMR	5
2.3	Cornered Magic Squares of Order 4	8
2.4	Closed Border of Order 4	13
2.5	Cornered Magic Squares of Order 6	18
2.6	Closed Border of Order 6	23
2.7	Cornered Magic Squares of Order 8	28
2.8	Closed Border of Order 8	40
2.9	Blocks of Orderw 10, 12 and BMR's of Order 10 × 12	43
3	Appendix	53
4	Author's Contribution to Magic Squares and Recreation Numbers	55

1 Introduction

The magic sums of order n of consecutive numbers from 1 to n^2 is given by

$$S_{n \times n} := \frac{n \times (1 + n^2)}{2}, n \geq 3. \quad (1)$$

Recently, the author [31, 32, 33, 34, 35, 36, 37] constructed magic squares of even orders from 8 to 20 using **bordered magic rectangles**. This construction is based on two aspects:

- (i) Using **magic rectangles** or **bordered magic rectangles**.
- (ii) Using algebraic formula like $(\mathbf{a} + \mathbf{b})^2$, $\mathbf{a} \neq \mathbf{b}$.

For the above magic squares no construction procedure is explained. The aim is to proceed further orders of magic squares. In this work, a systematic procedure to construct these magic squares is given. It is based on the magic squares and bordered magic rectangles (BMR) of orders 4, 6, 8 etc forming external borders. Then the internal borders are filled with previous known magic squares. For the orders multiples of 4, we can always write magic squares with equal sums blocks of magic squares of order 4. This procedure is very helpful for the orders of type $2\mathbf{p}$, where \mathbf{p} is a prime number, for examples, 14, 22, 26, 34, 38, etc. For the orders like 18, 30, etc., we can make good external blocks with order 4, and for orders like 16, 20, 28, 32, etc. we can make good external borders of order 6, and so on. There is no explanations for the orders 6, 8, 10 and 12. The real construction starts from the order 14.

The whole the work is done manually, without use of any programming language, except for the constructions of small blocks of **bordered magic rectangles**. This construction is based on the software due to H. While. Later, these BRM's are readopted according to distribution of each magic square. The distribution of magic squares or bordered magic rectangles is based on **half-sequential** numbers. By **half-sequential** numbers we understand that the total numbers

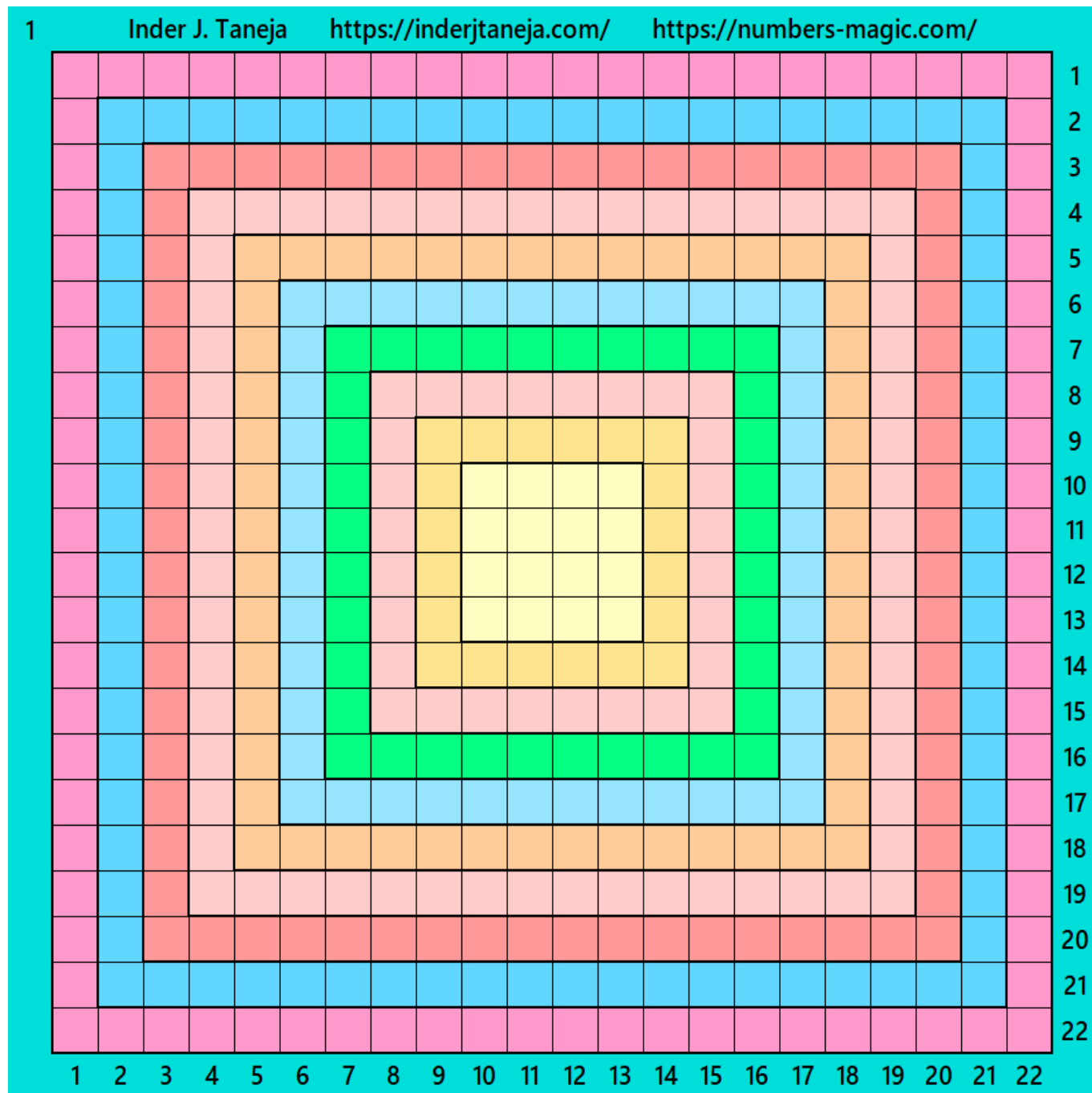
in each case are divided in two equal parts. First part is one sequence and second part is another sequence. Due to **half-sequential** numbers, it is not possible to construct all orders **bordered magic rectangles**. In Appendix 3, there are tables showing the existence of these **bordered magic rectangles** for **half-sequential**. For simplicity, we shall write **BMR** as **bordered magic rectangle**.

2 Magic Squares of Order 22

This section brings in figures (without numbers) magic squares of order 22. In some case its construction is explained.

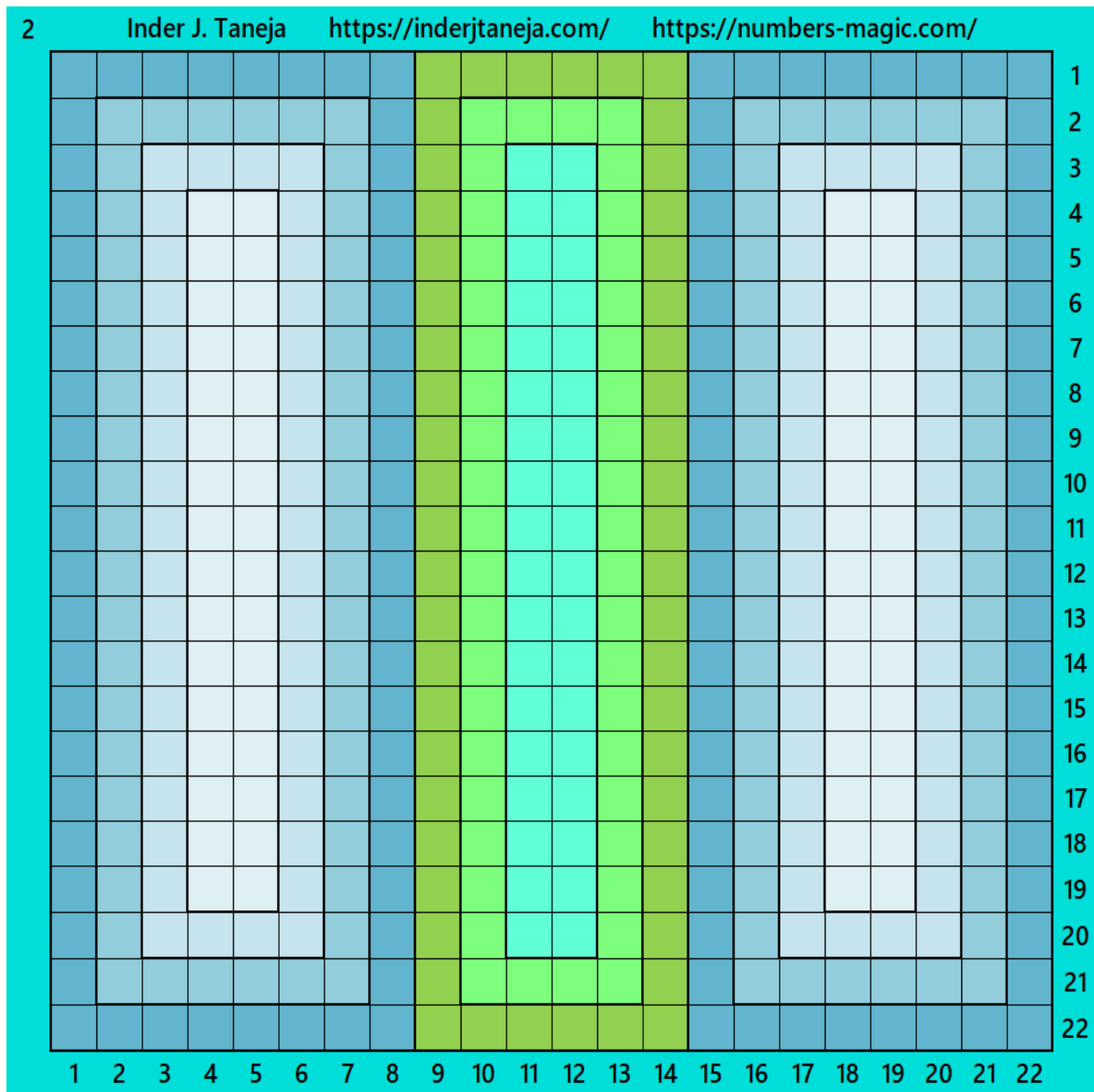
2.1 Bordered Square of Order 22

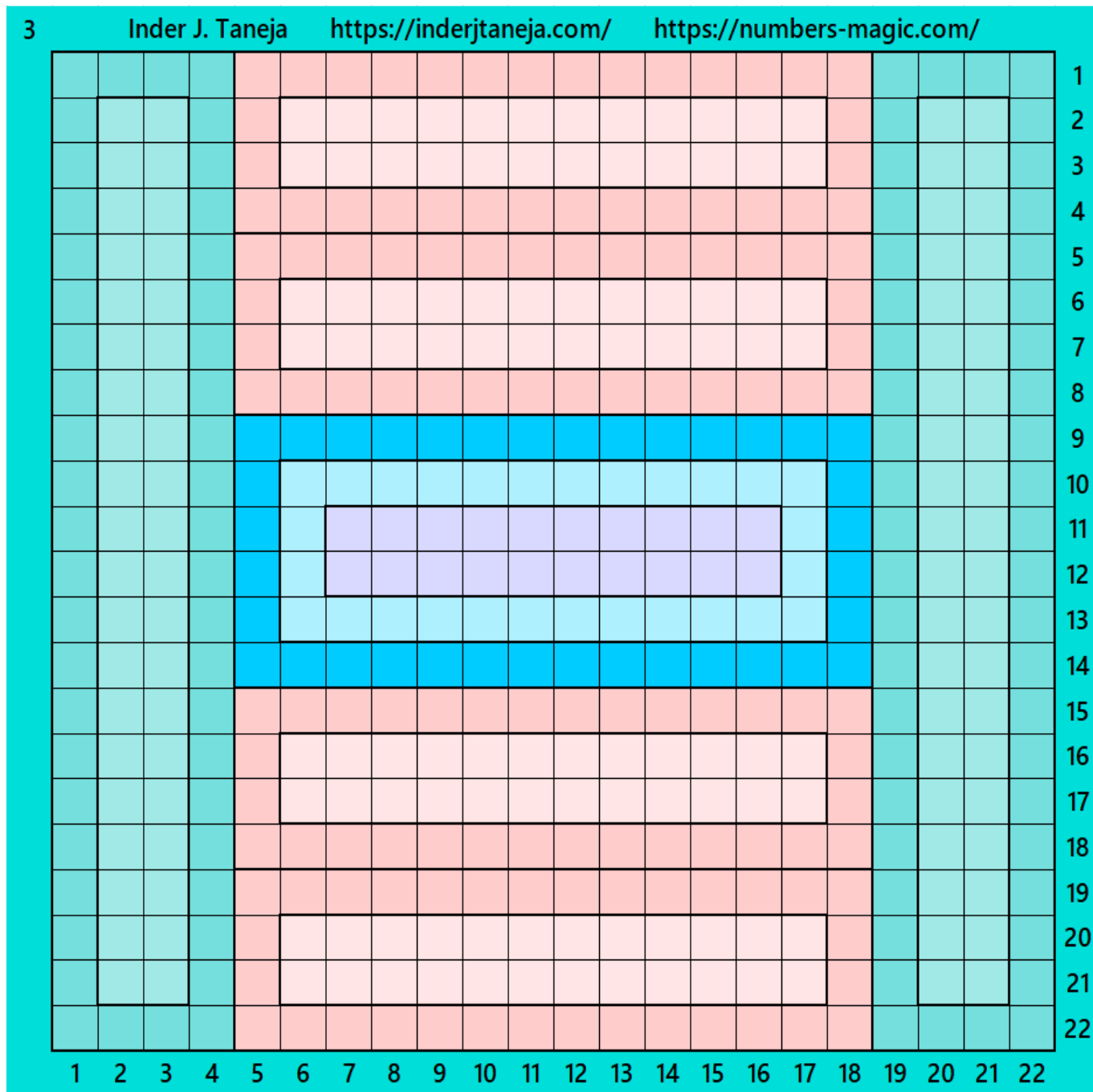
Below are four magic squares of order 22 already known in the literature. For more details refer author's work [22, 24].

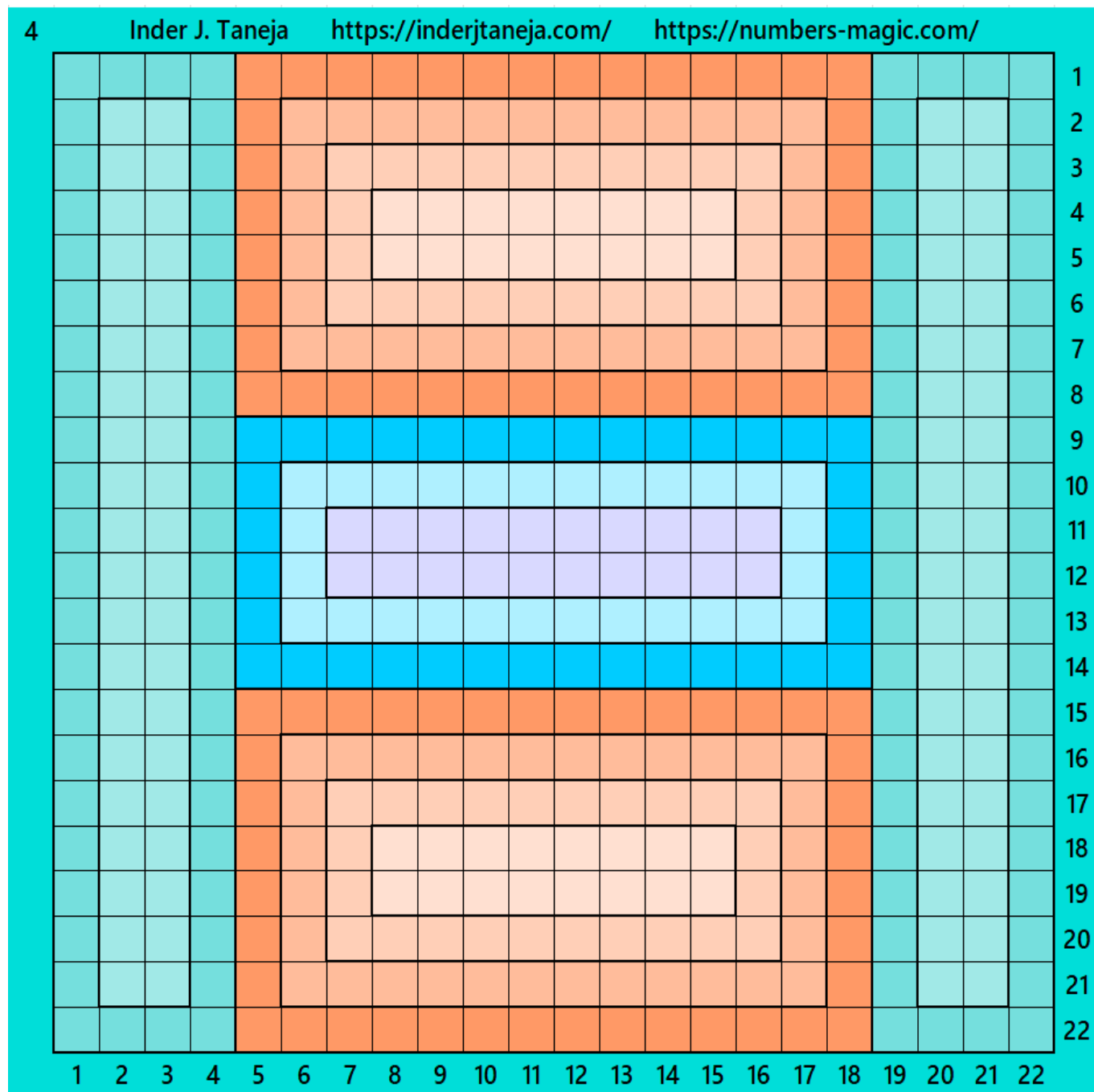


2.2 Magic Squares of Order 20 With BMR

Below are three examples of magic squares of order 22 made with BMR. These are constructed separately. For more details with numbers refer [37].



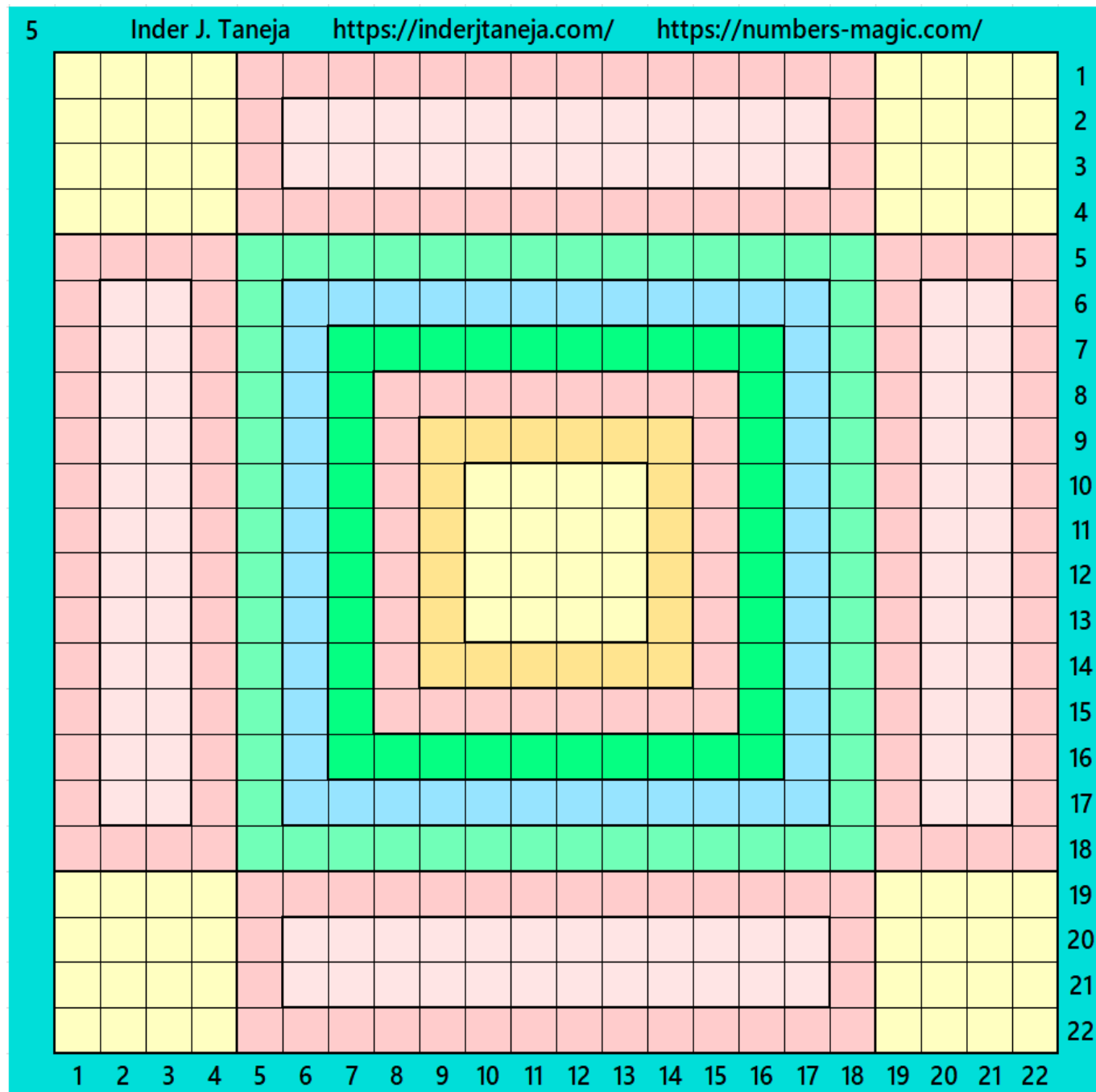


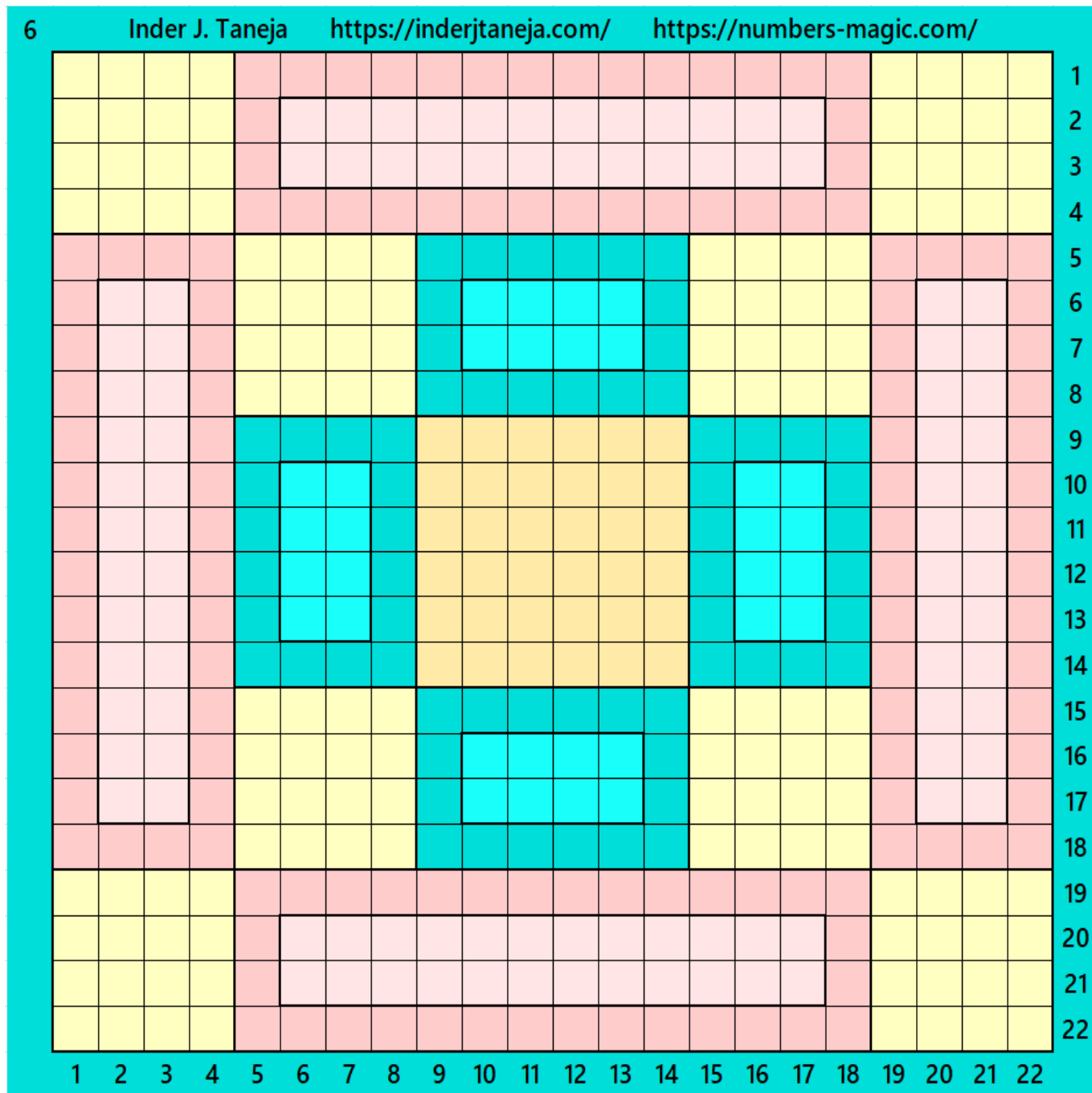


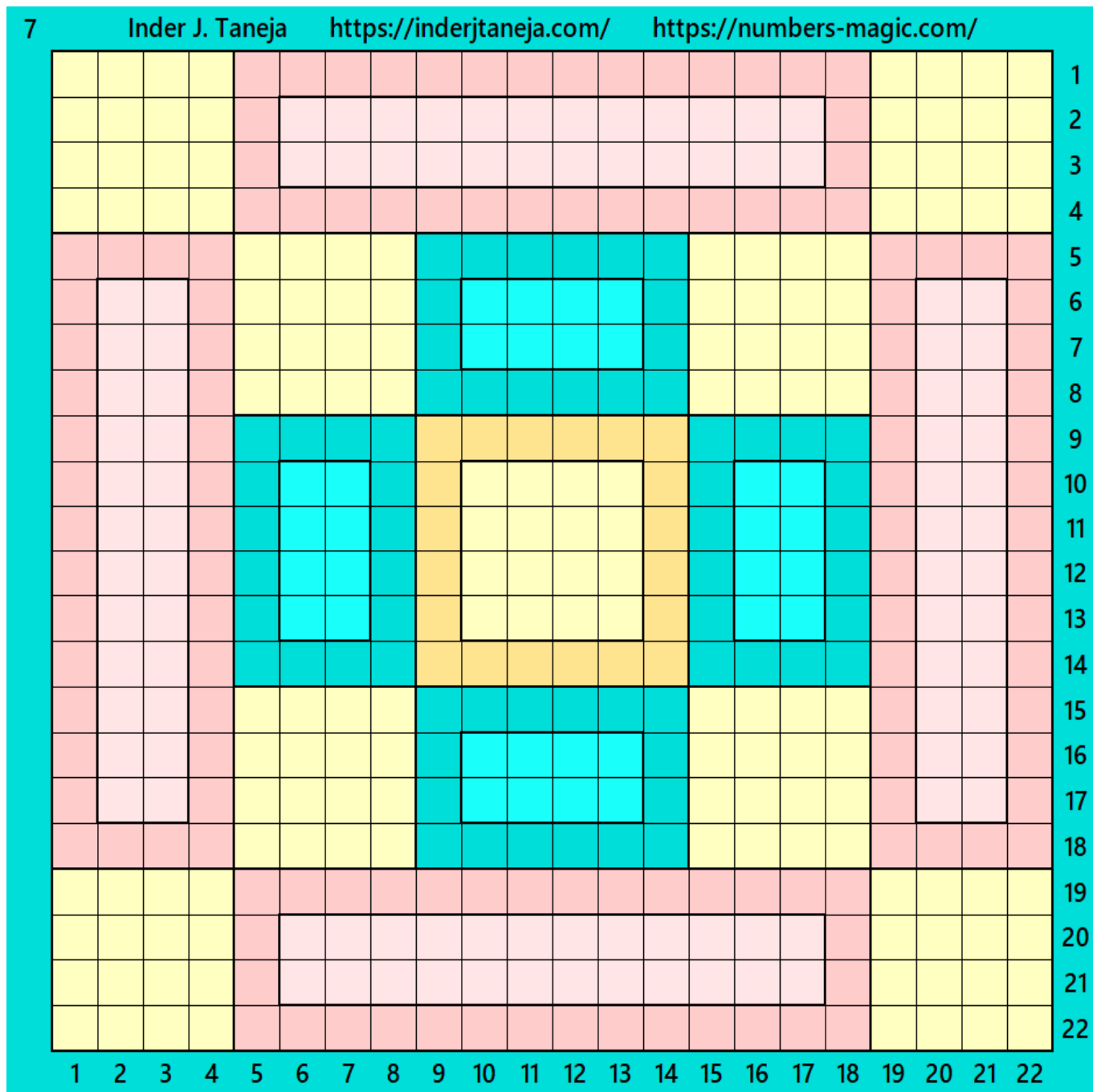
2.3 Cornered Magic Squares of Order 4

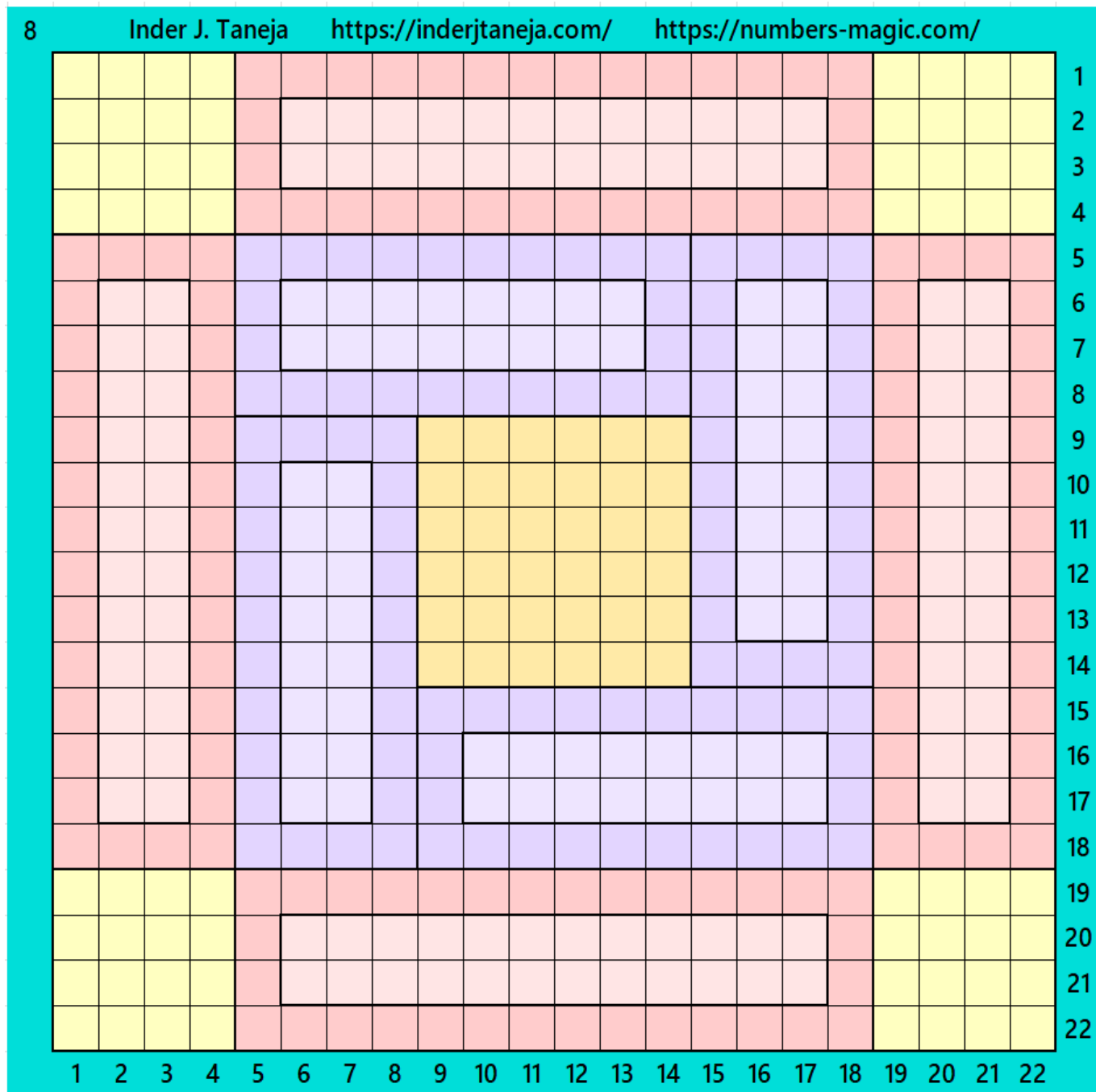
Let's consider an external border, where there are 4 magic squares of order 4 at the corners. Let's make an external border by putting BMR of order 4×14 in each row and column. In the

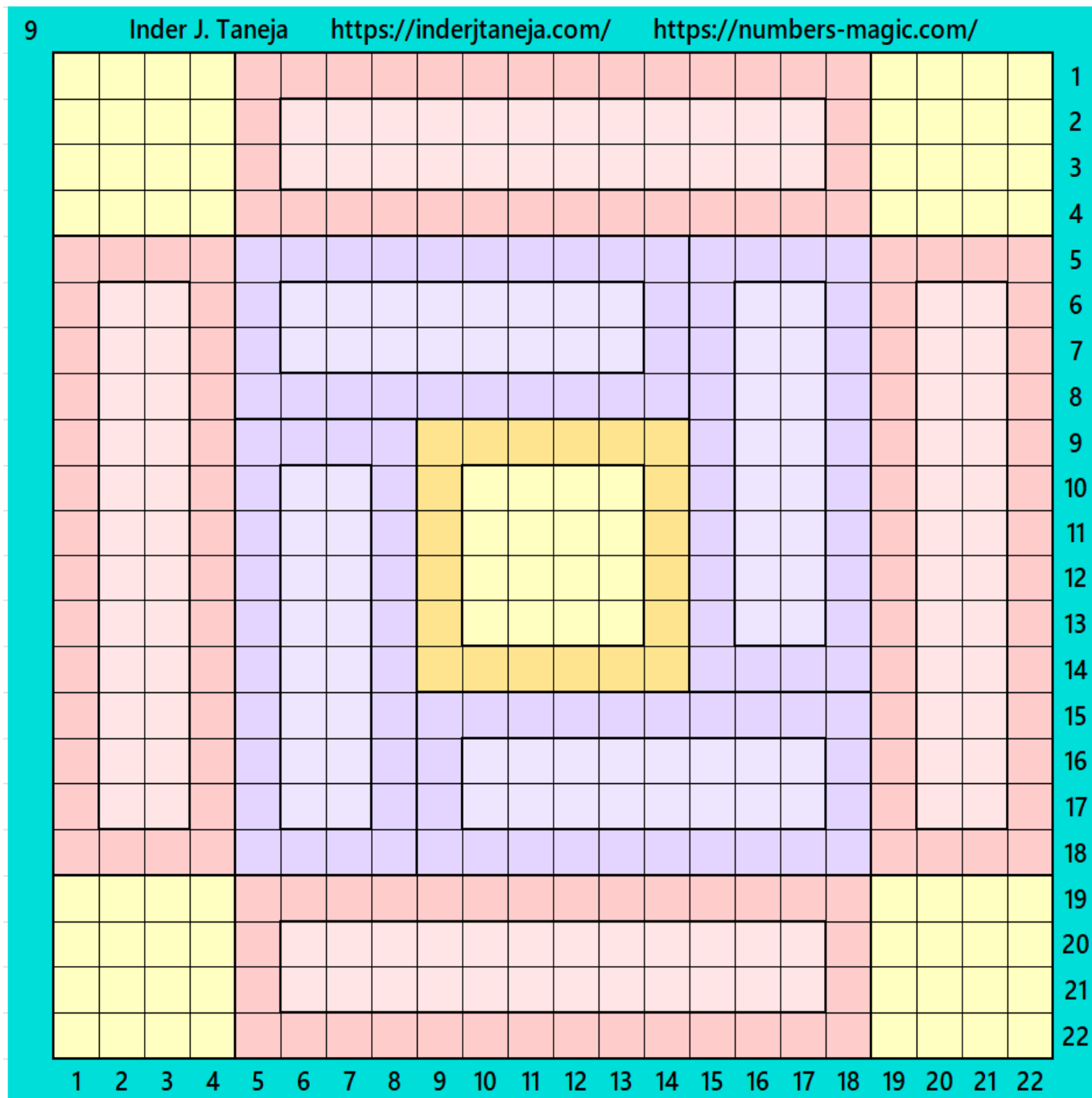
middle we are left with block of order 14. Writing middle block of order 14 with different types of magic squares, we get magic squares of order 22. See below few examples:







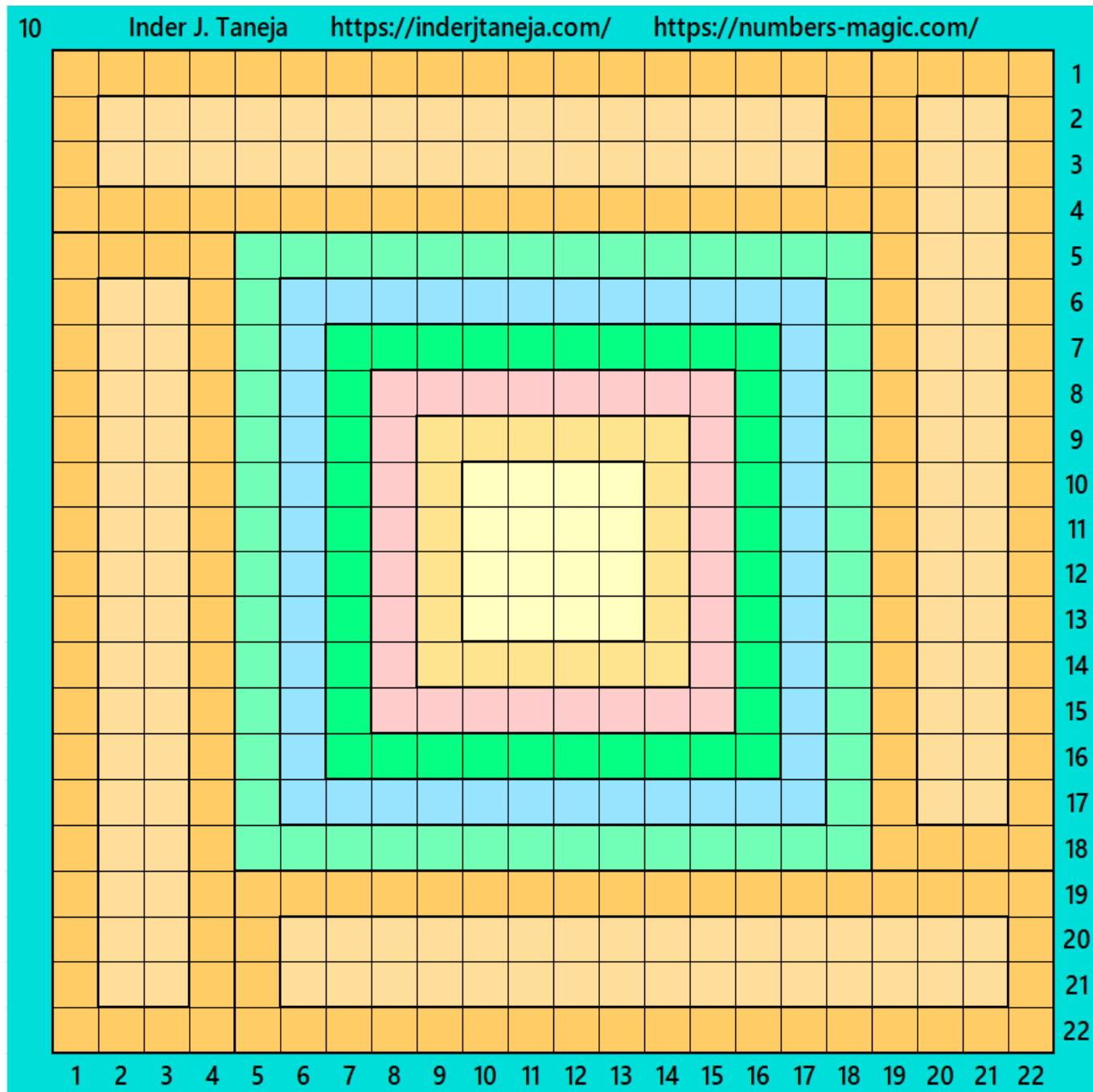


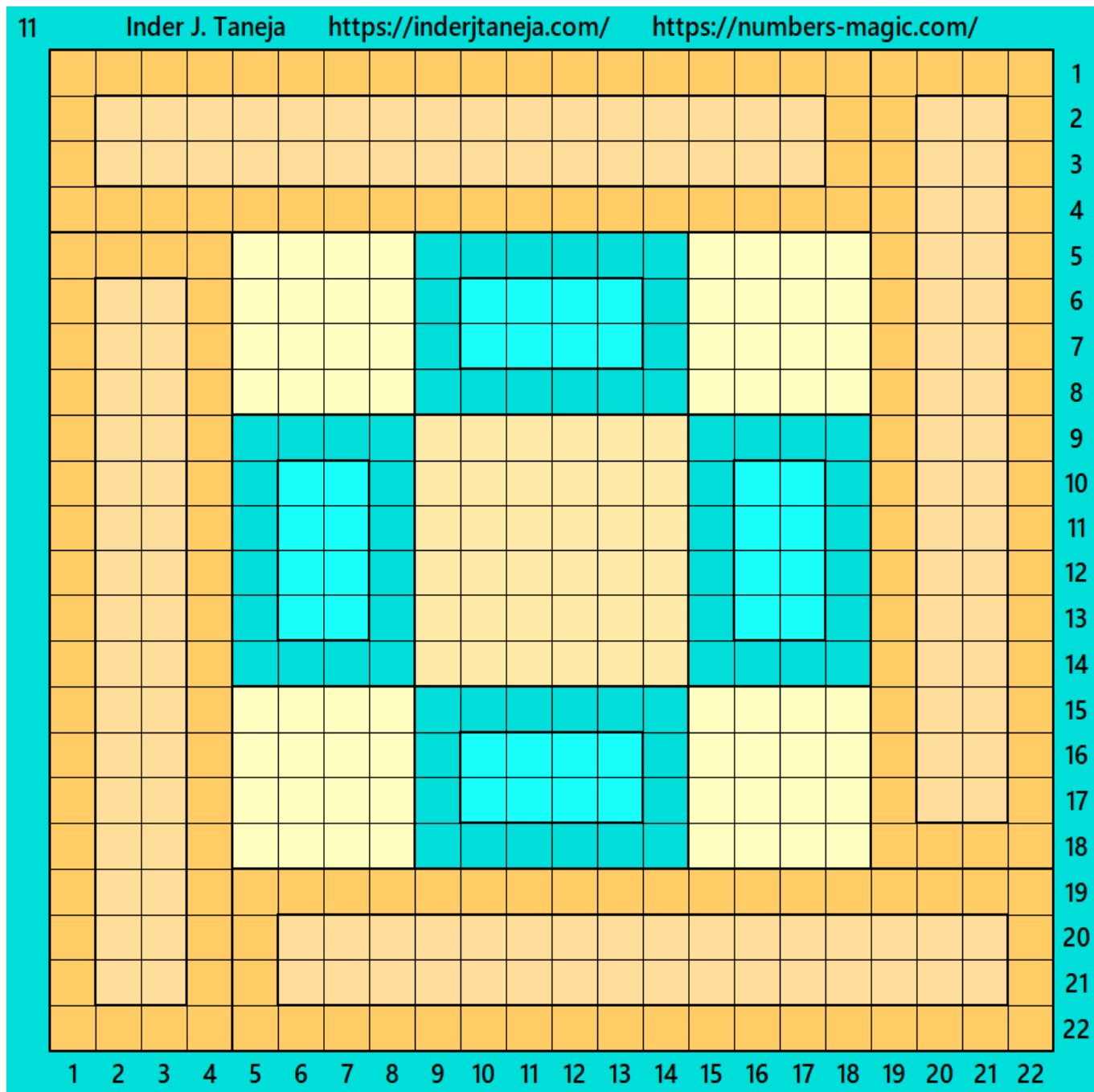


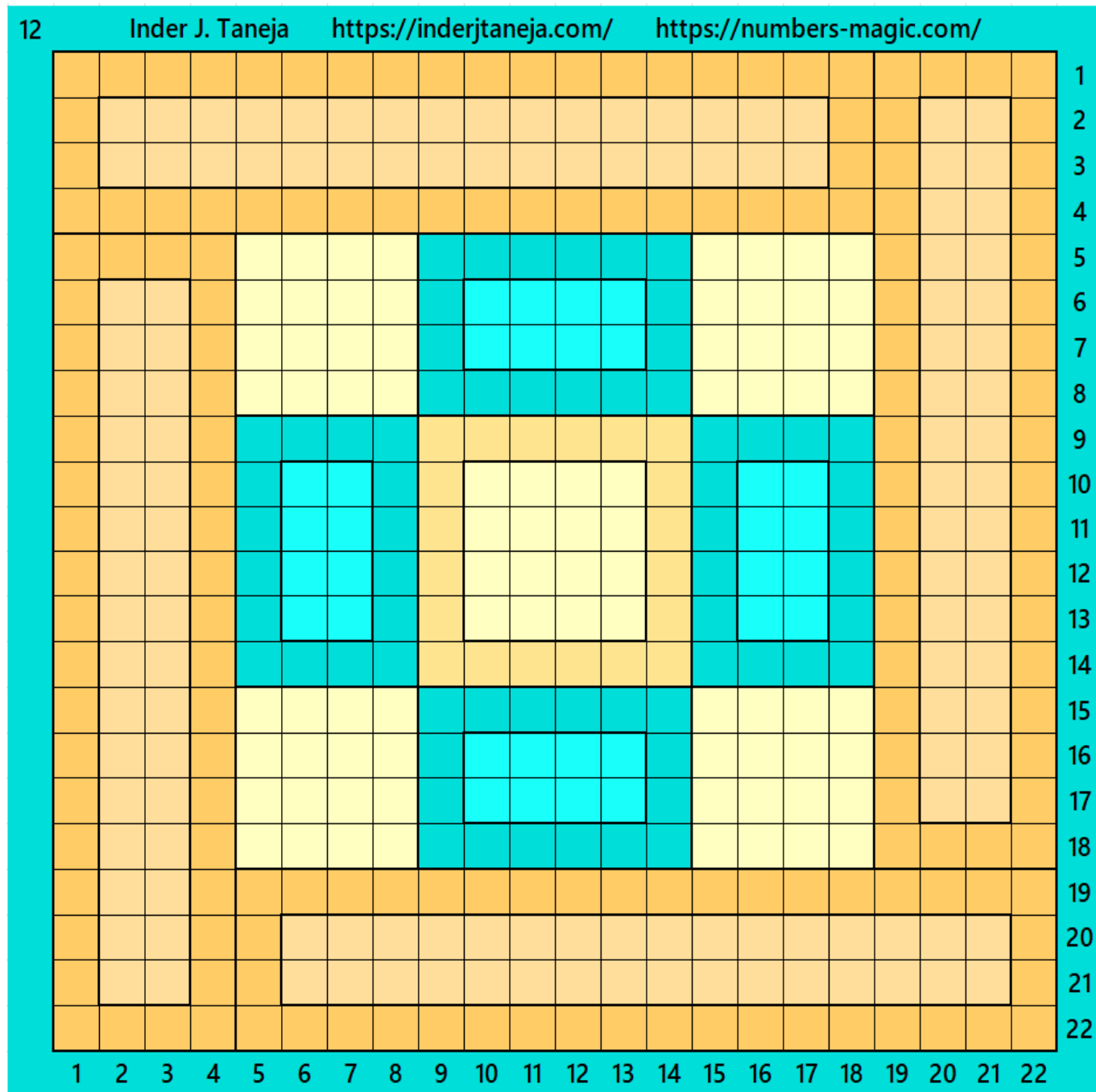
2.4 Closed Border of Order 4

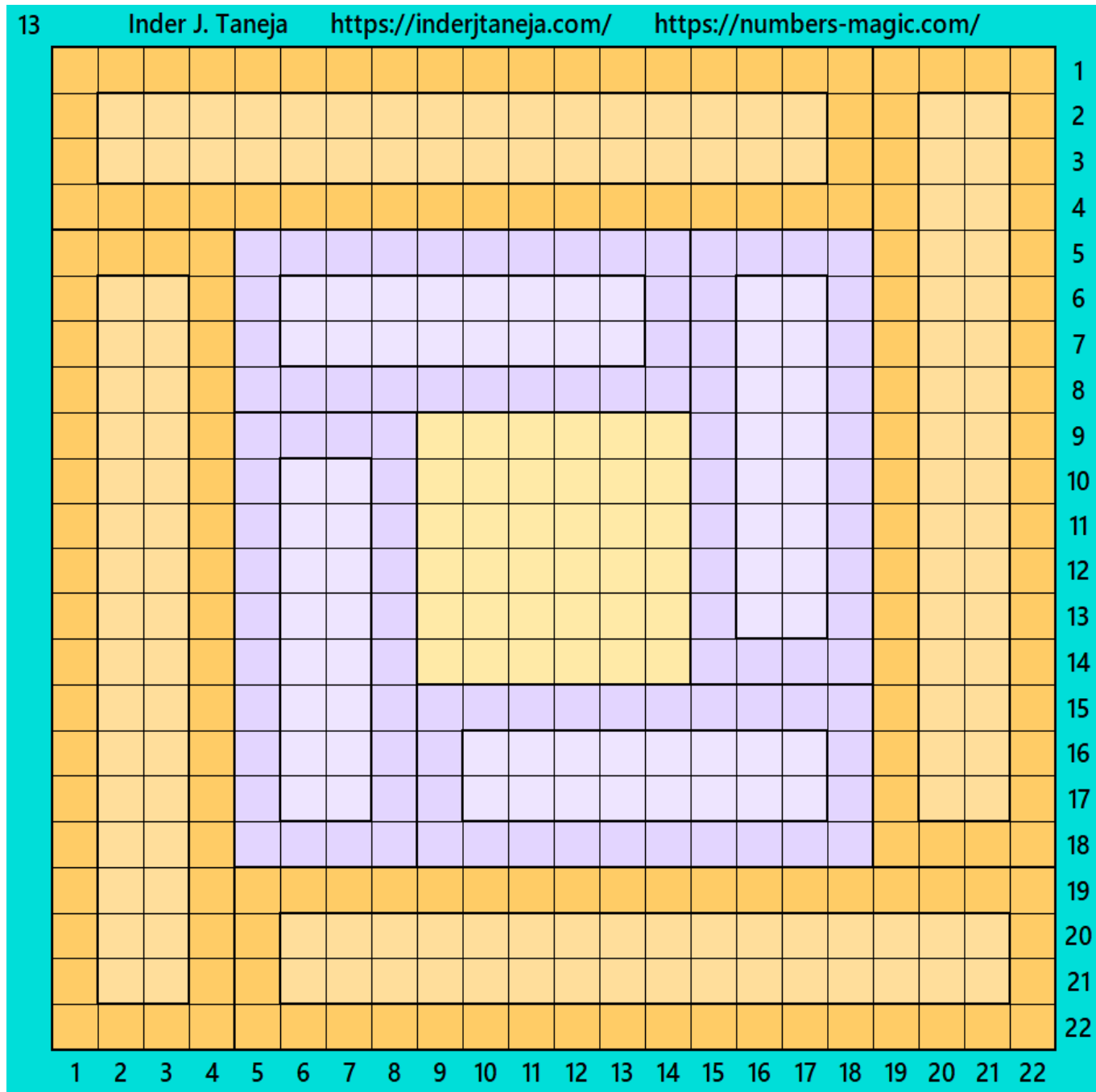
Let's consider 4 equal sums BMR of order 4×14 . Then put them in a symmetric way in external part of the magic squares. We are left with internal block of order 14. Writing middle block of

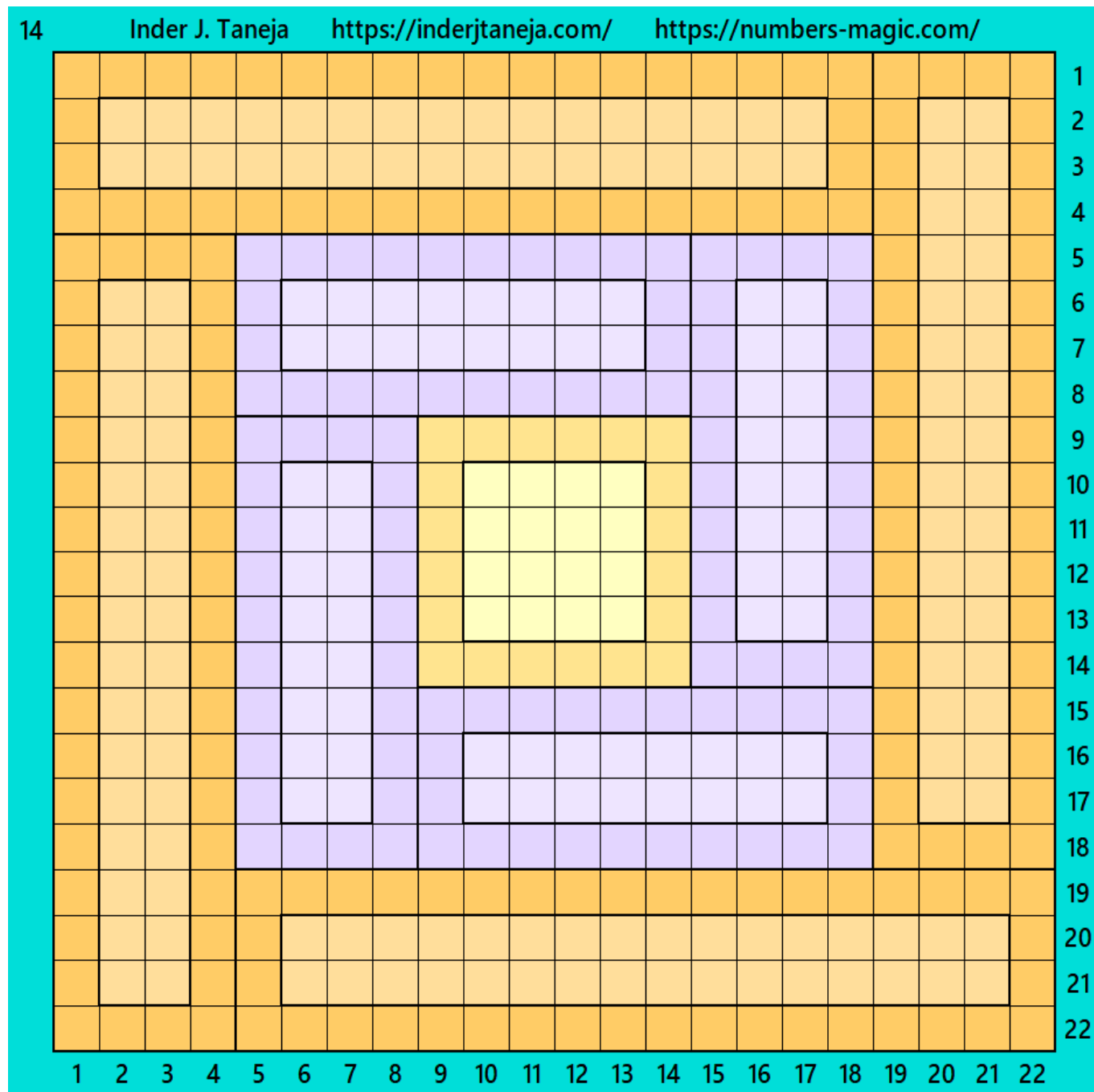
order 14 with different types of magic squares of order 14, we get magic squares of order 22. See below few examples:







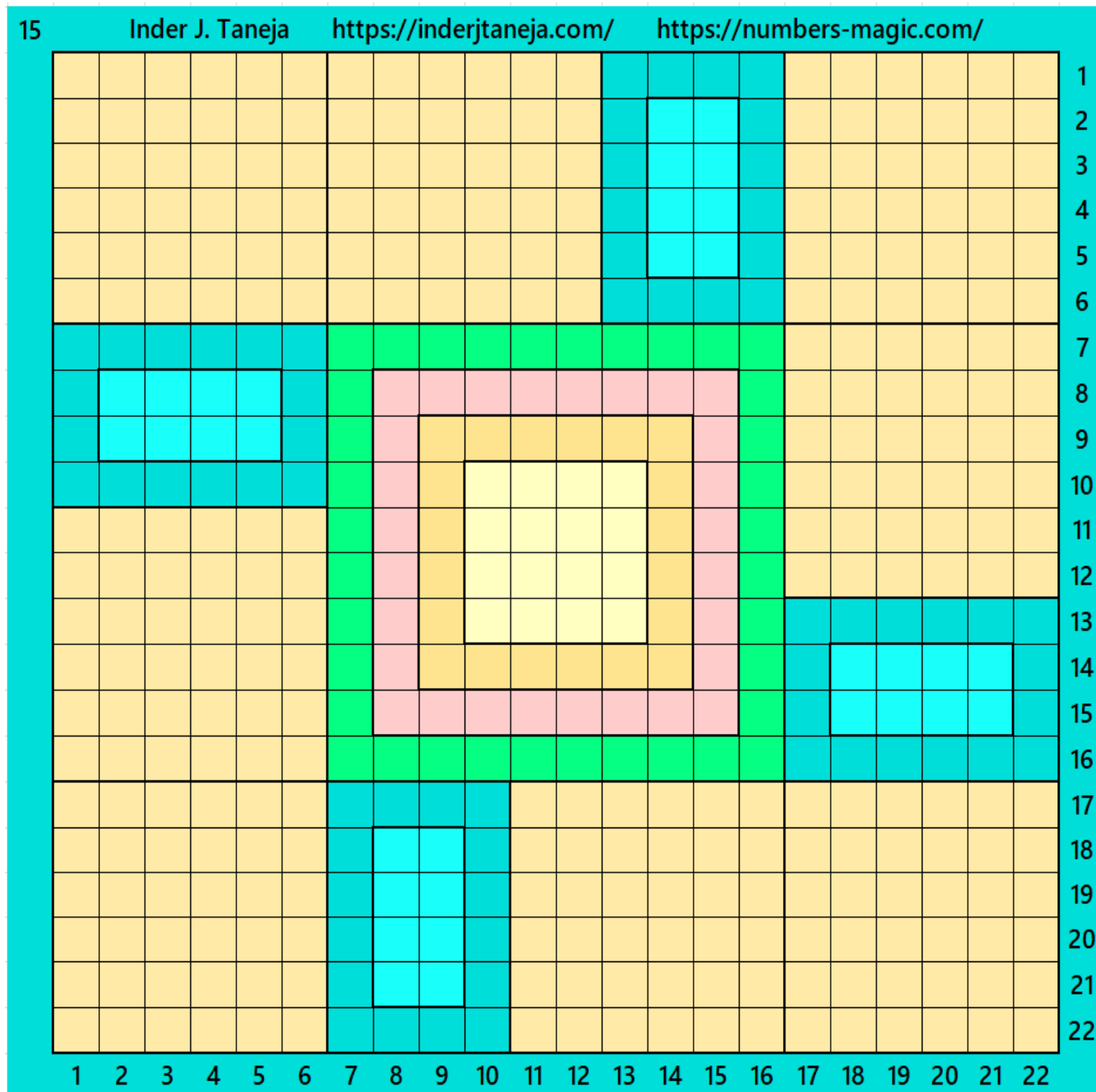


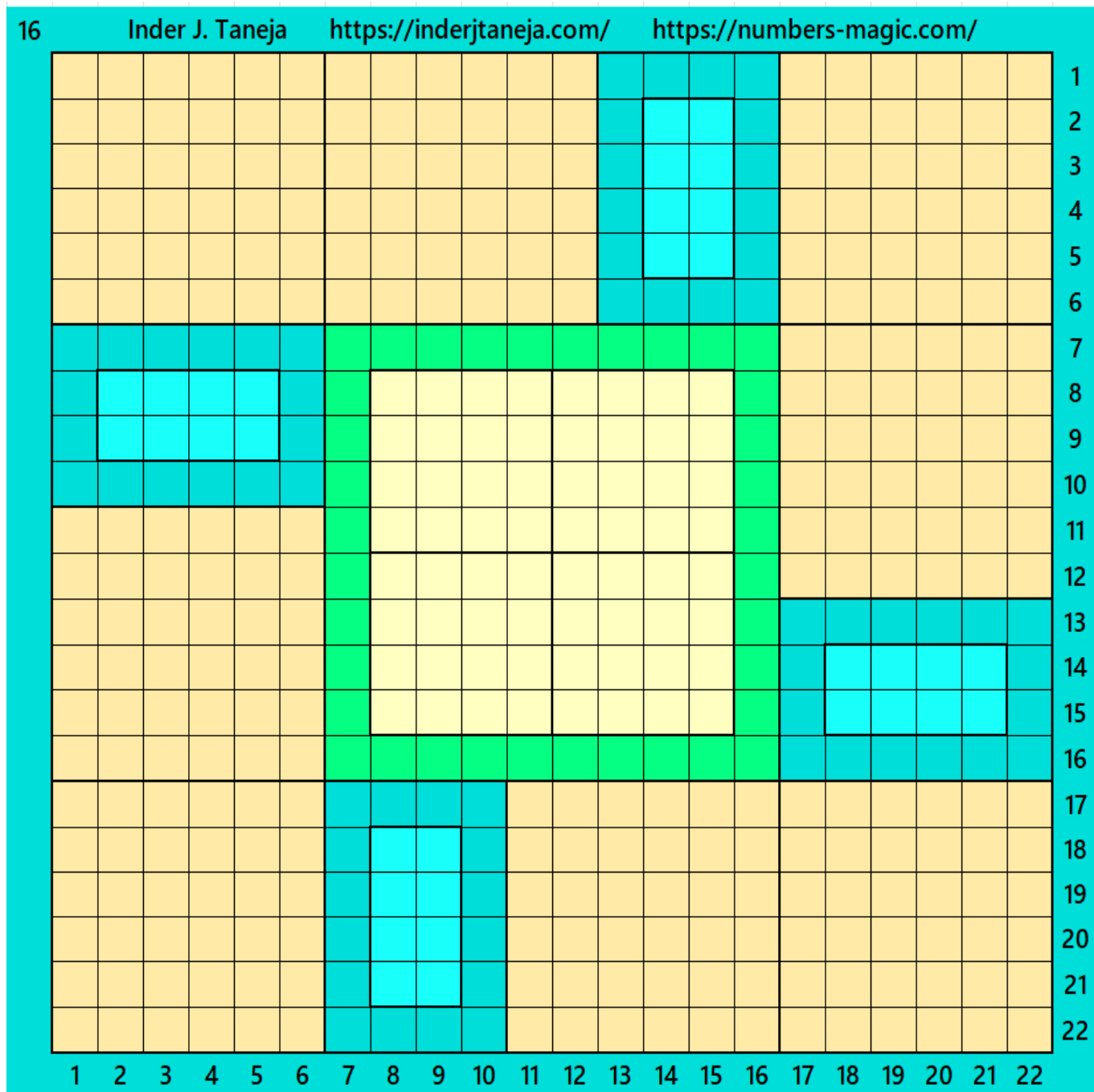


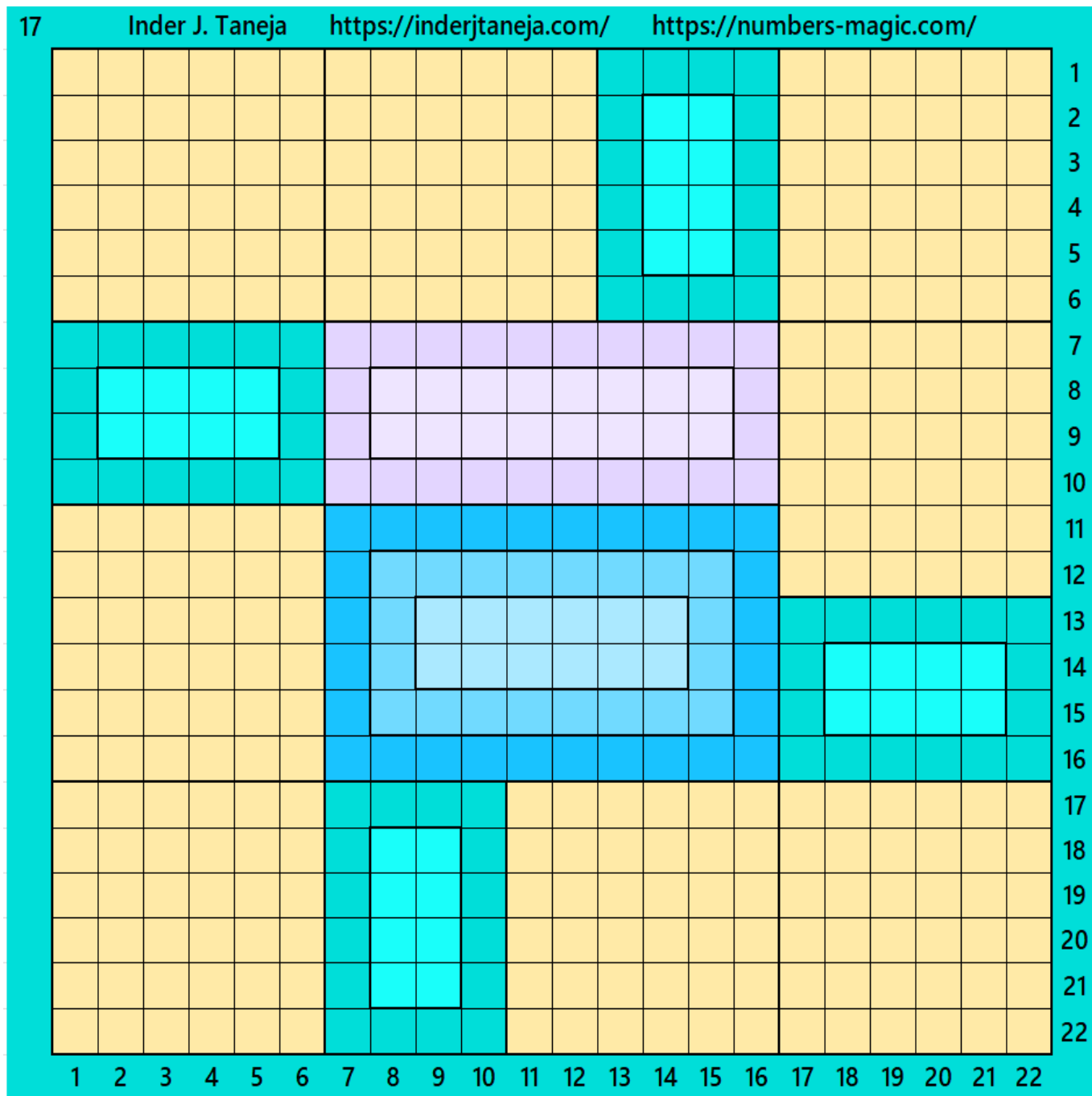
2.5 Cornered Magic Squares of Order 6

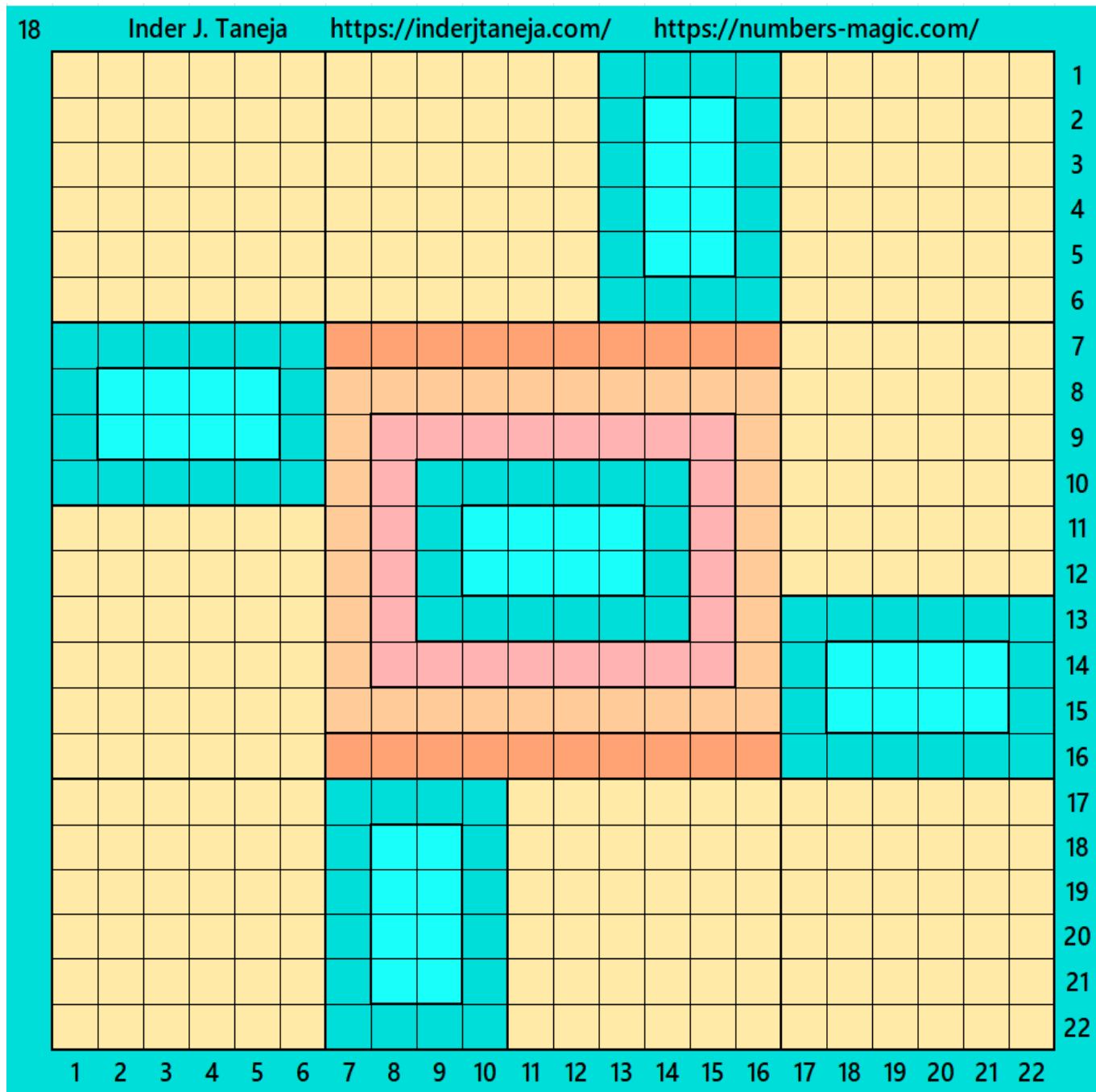
Let's consider an external border, where there are 8 magic squares of order 6 at the corners. Let's make an external border by putting BMR of order 4×6 in each row and column. In the

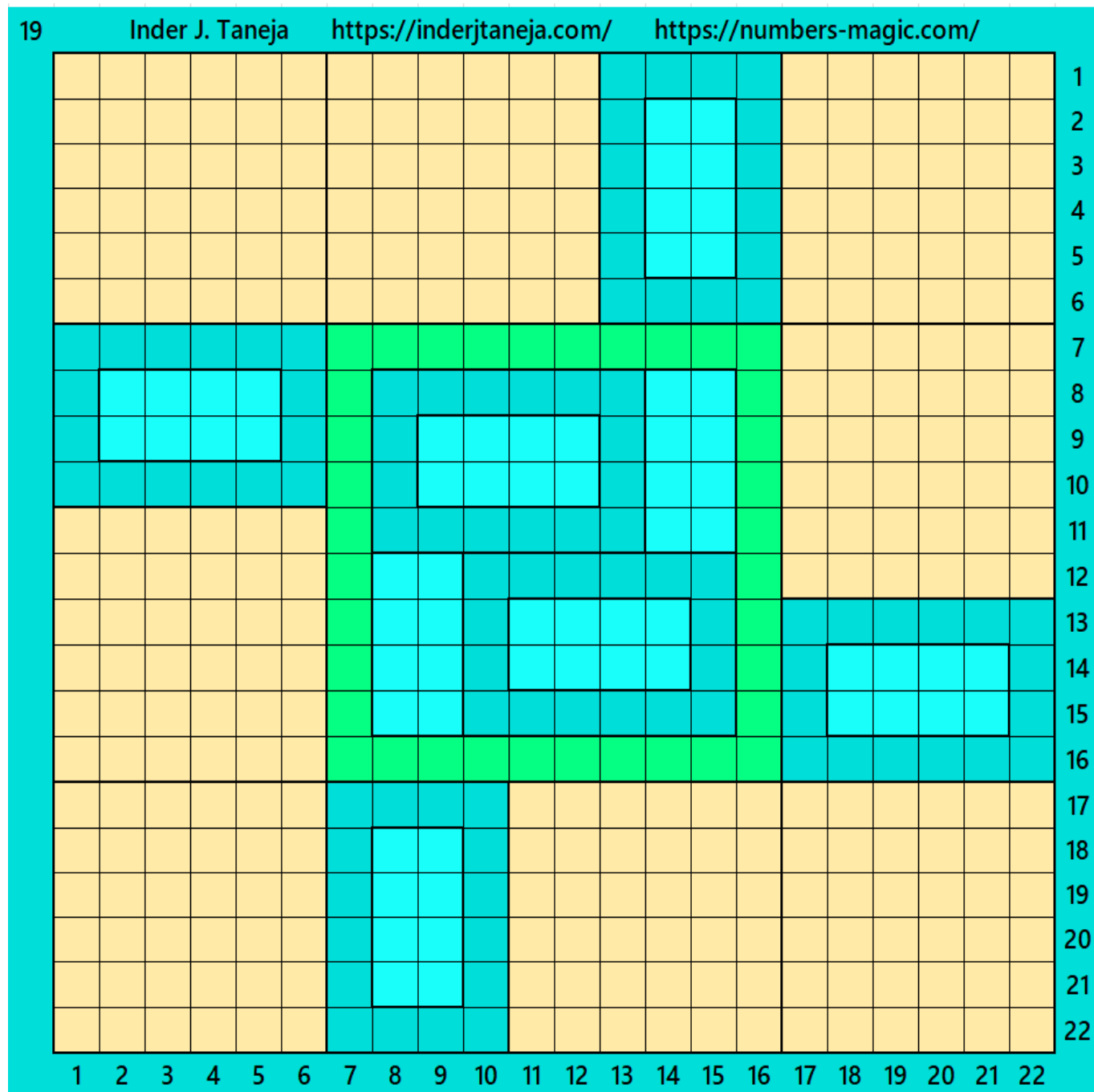
middle we are left with block of order 10. Writing middle block of order 10 with different types of magic squares, we get magic squares of order 22. See below few examples:







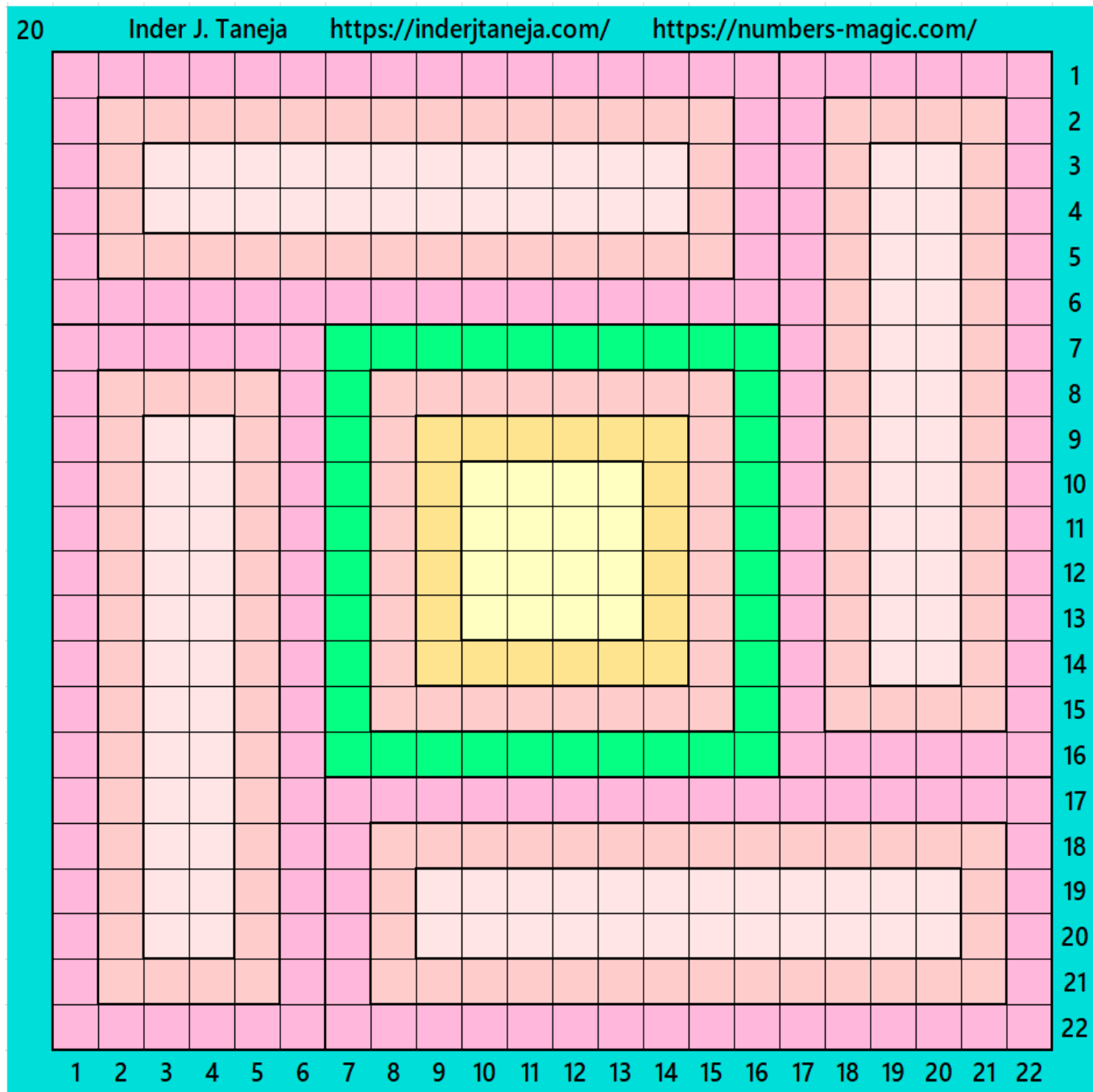


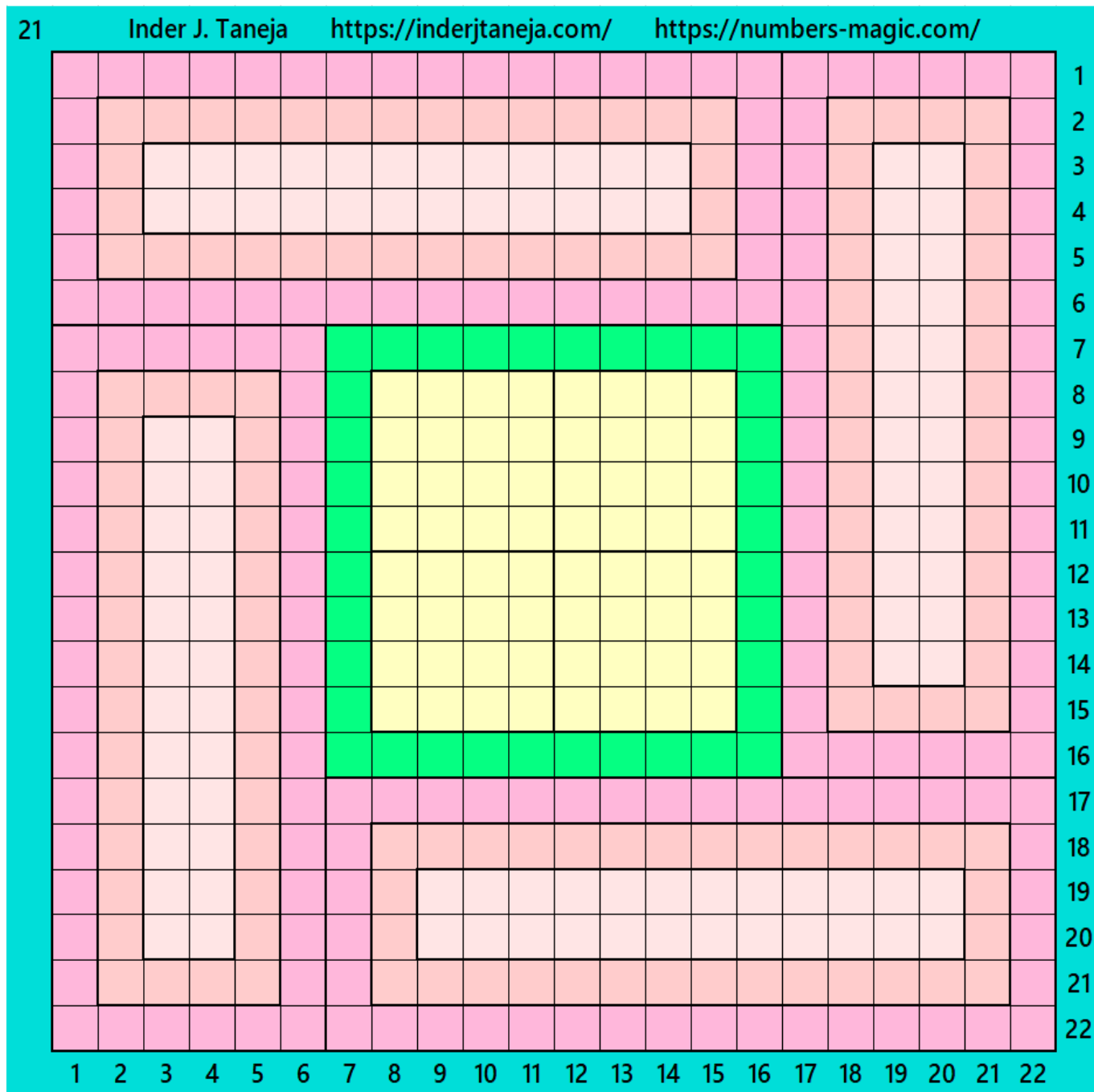


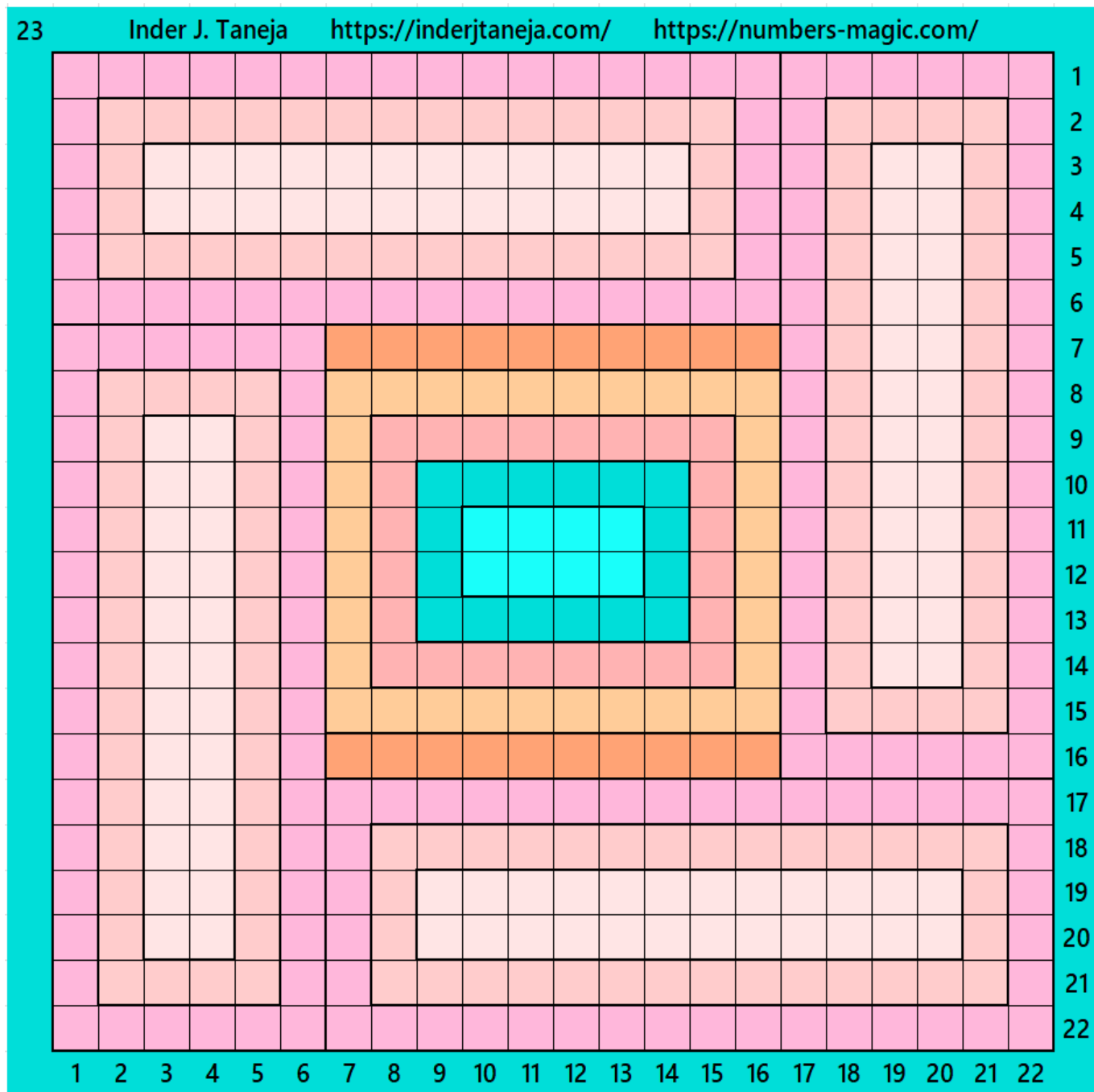
2.6 Closed Border of Order 6

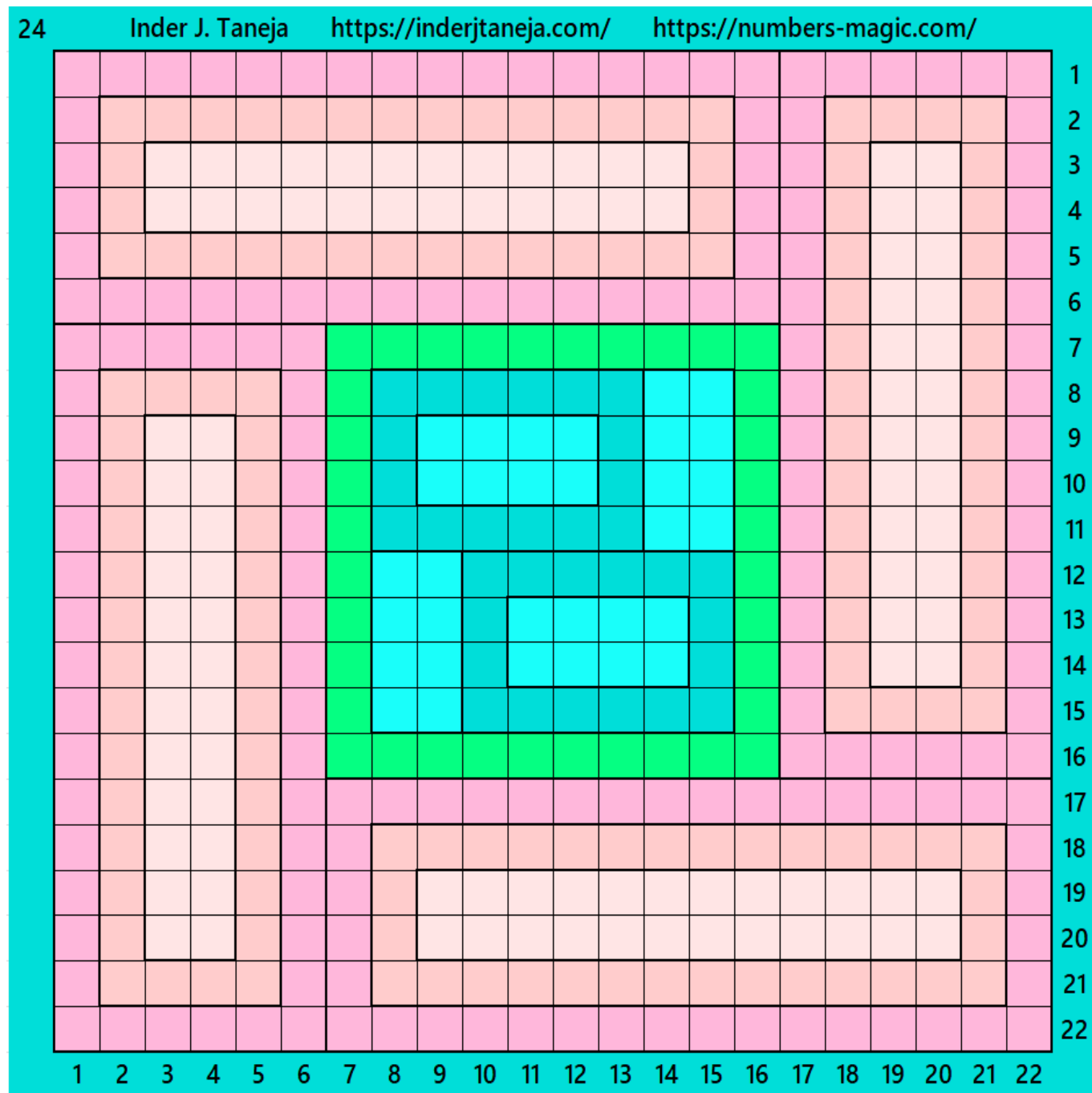
Let's write a closed external border formed by four equal sums BMR's of order 6×16 . In the middle we are left with block of order 10. Writing middle block of order 10 with different types

of magic squares, we get magic squares of order 22. See below few examples:





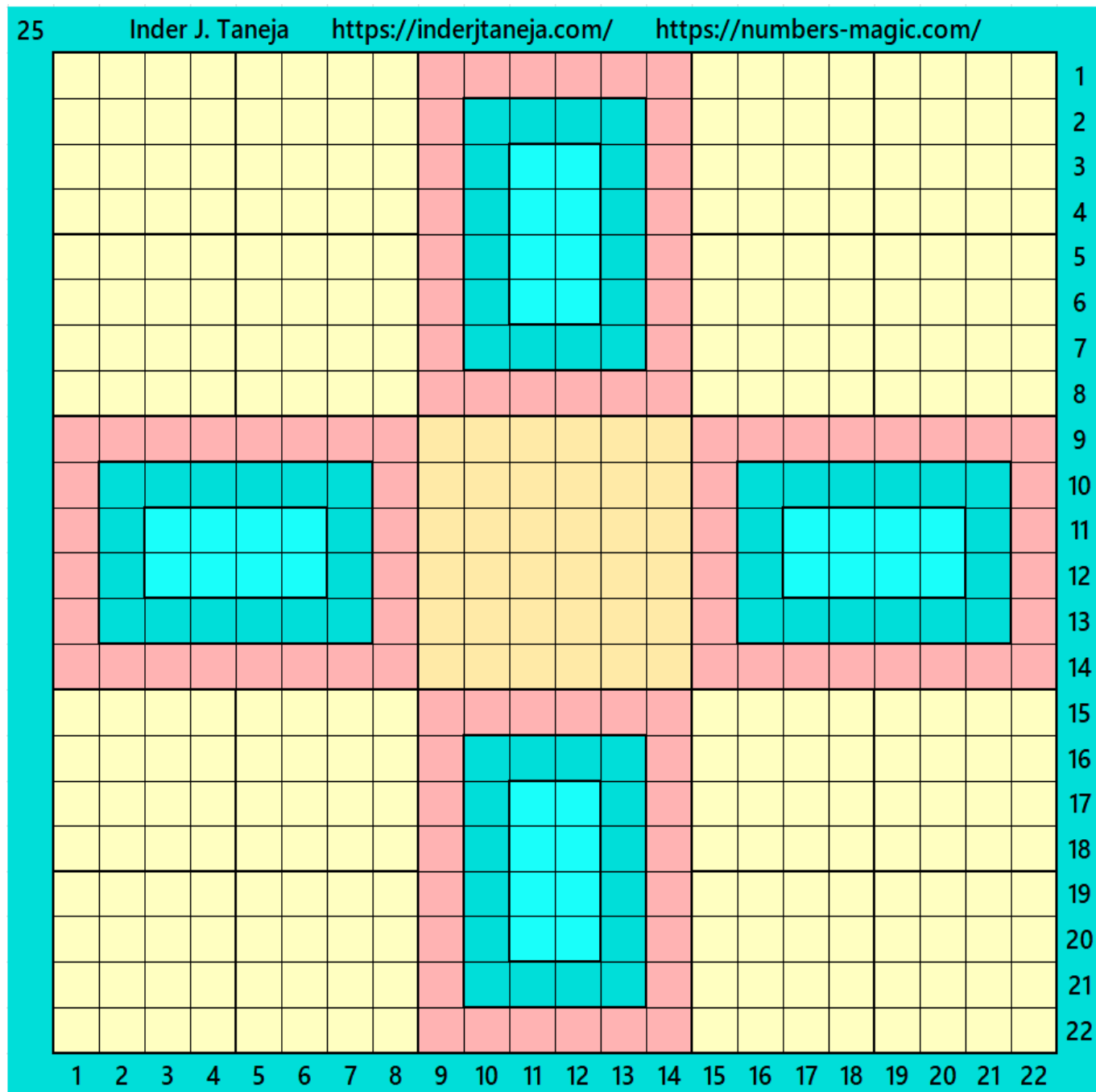


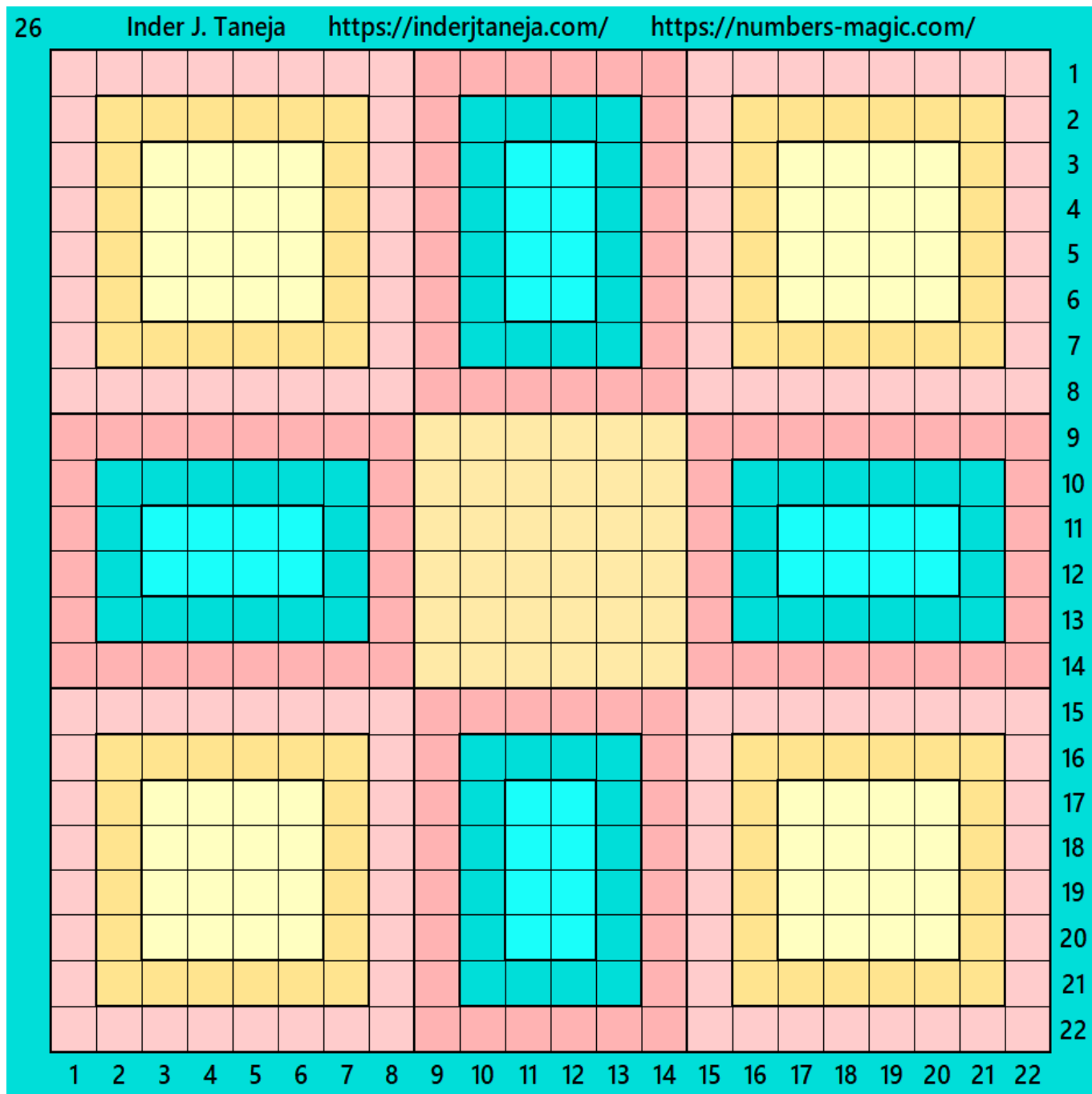


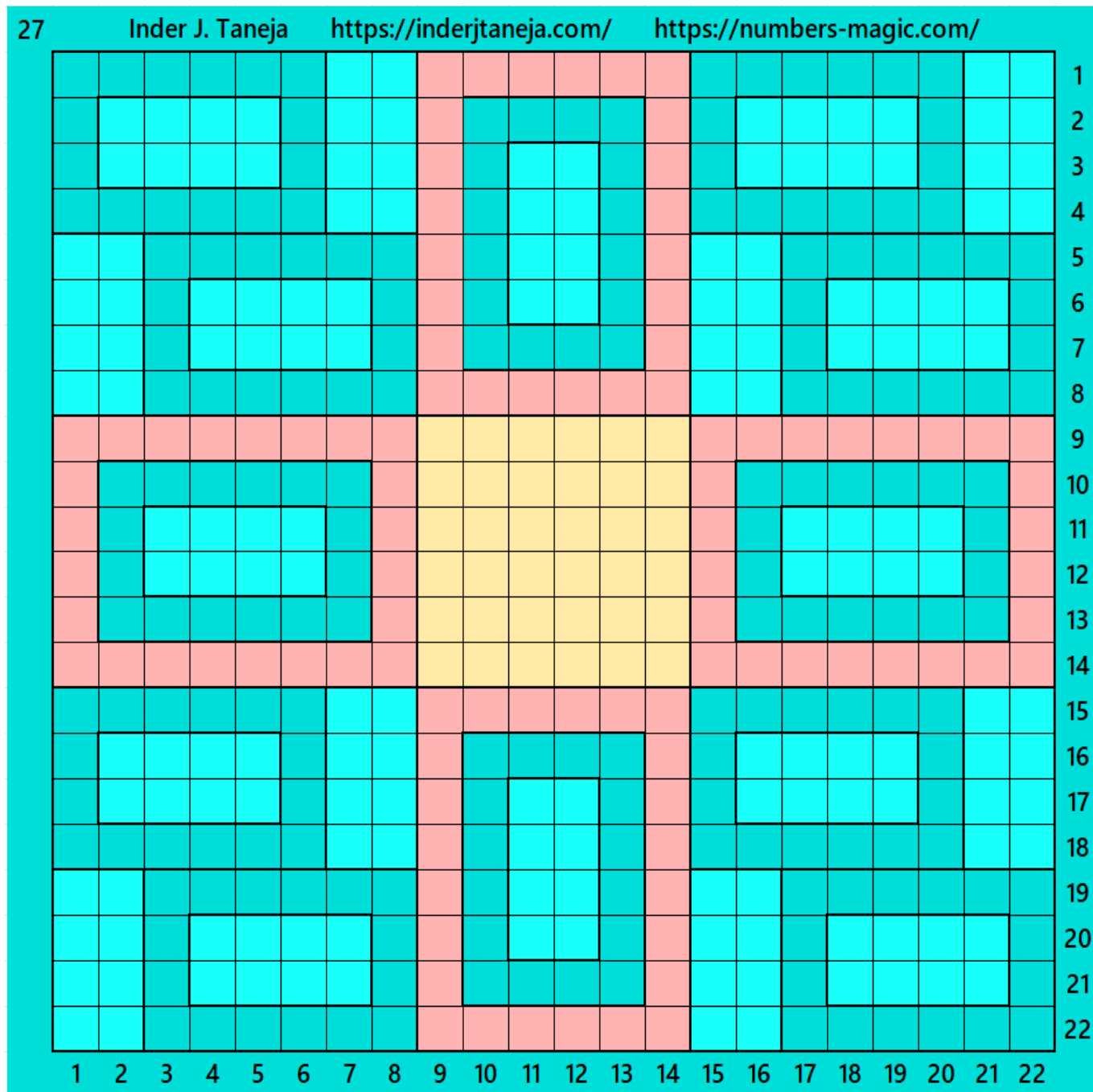
2.7 Cornered Magic Squares of Order 8

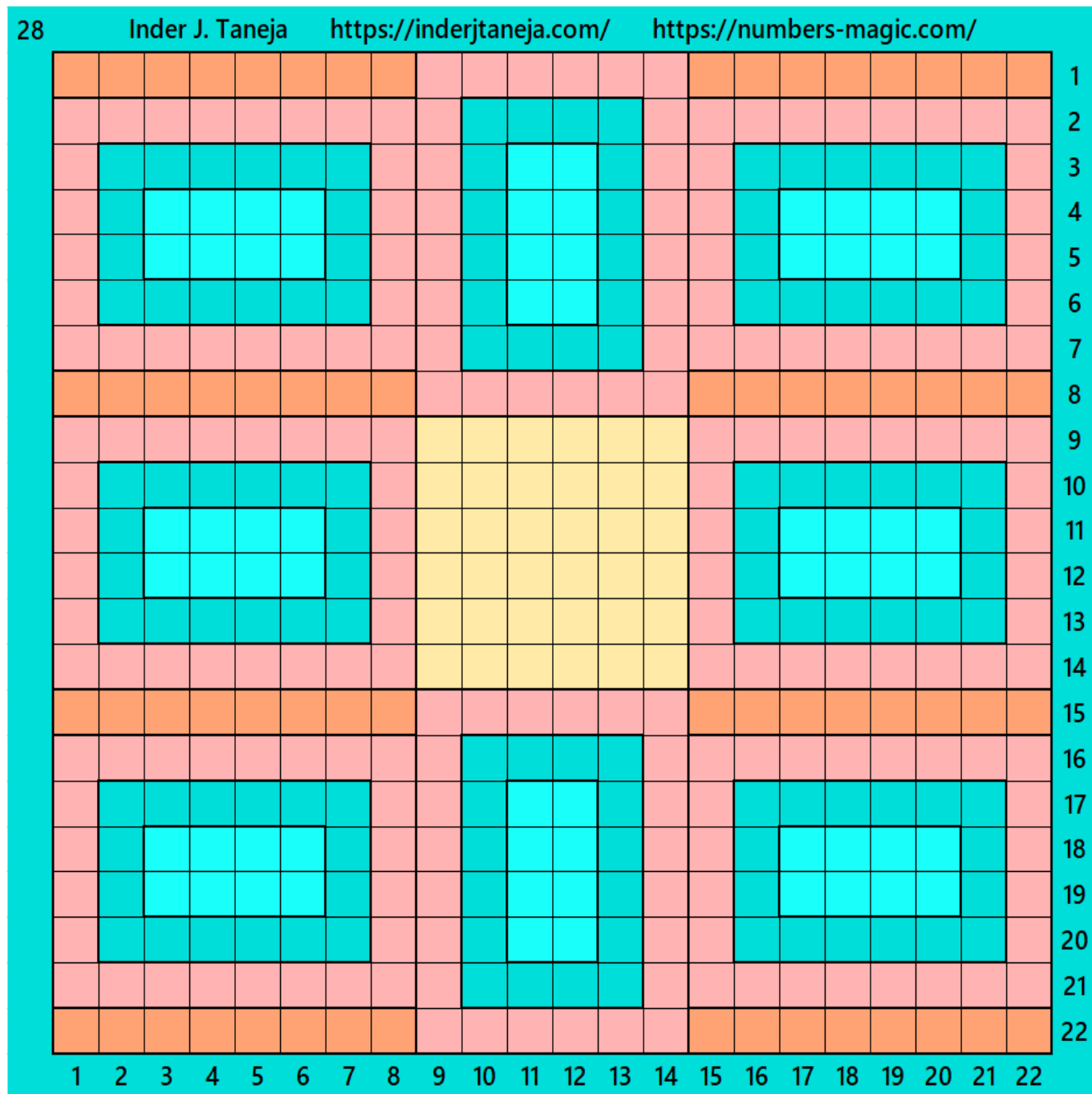
Let's consider an external border, where there are 4 magic squares of order 8 at the corners. Let's make an external border by putting BMR of order 6×8 in each row and column. In the

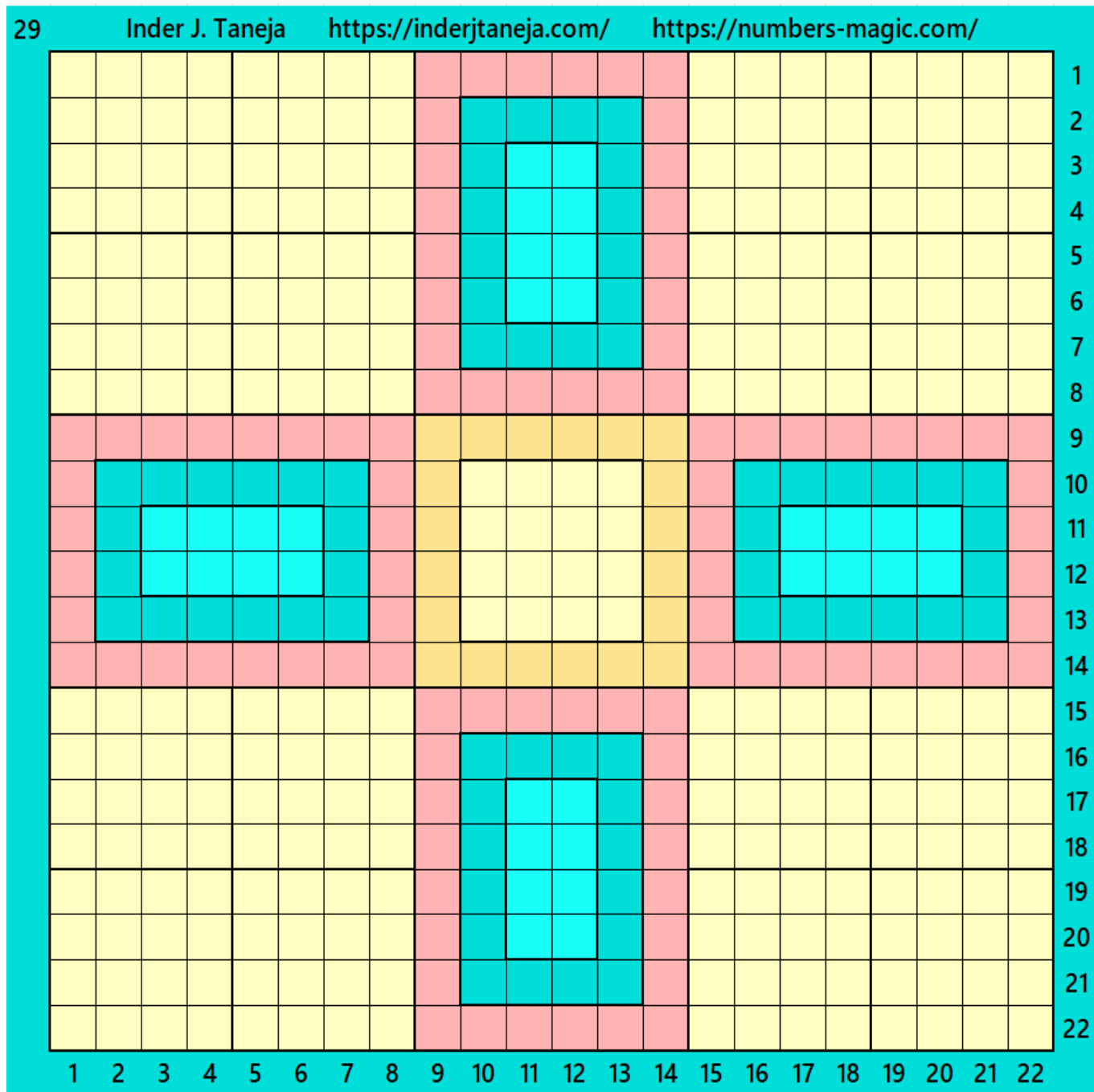
middle we are left with blocks of order 6. Writing middle block of order 6 and different types of magic squares of order 8, we get magic squares of order 22. See below few examples:

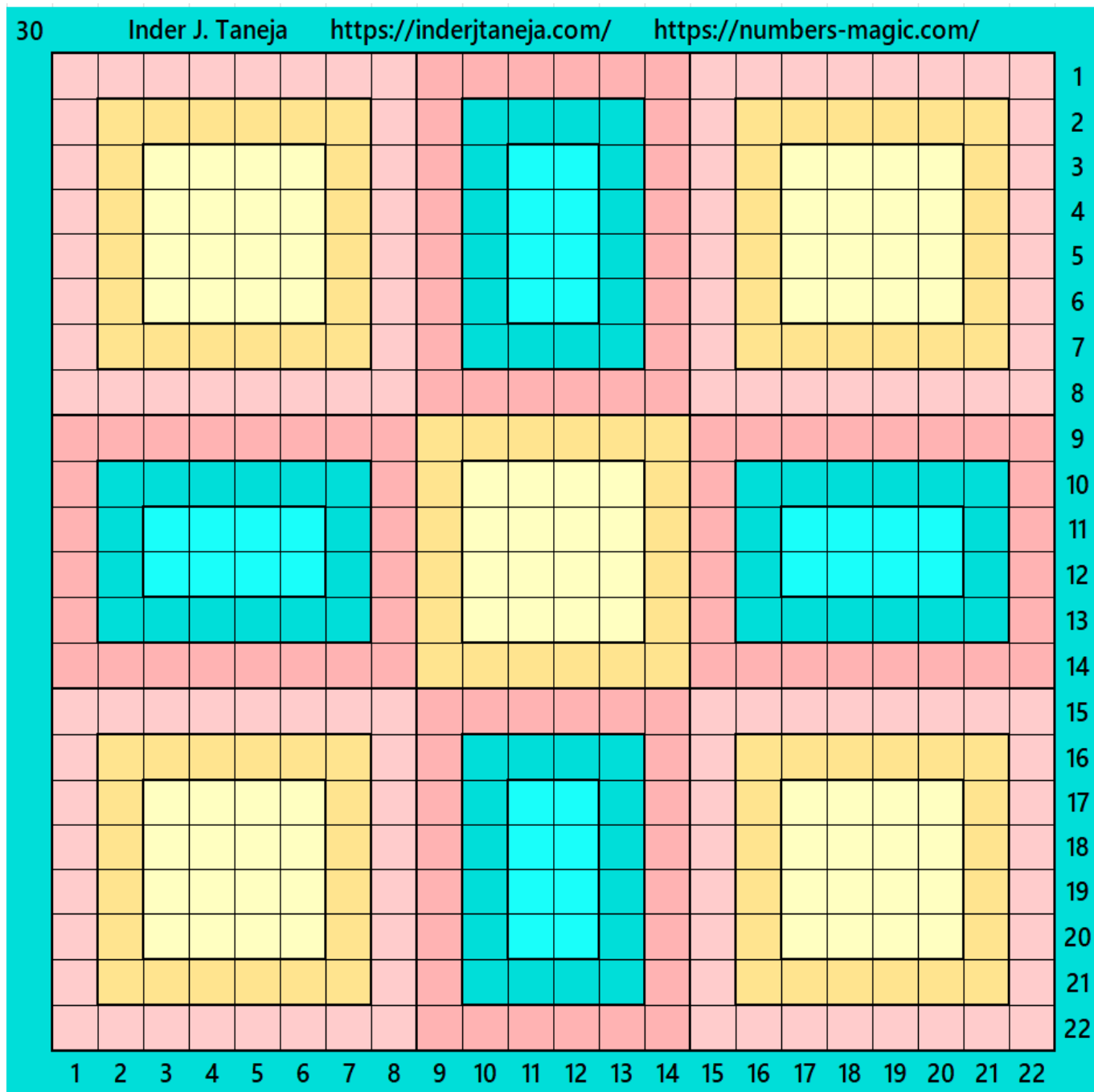


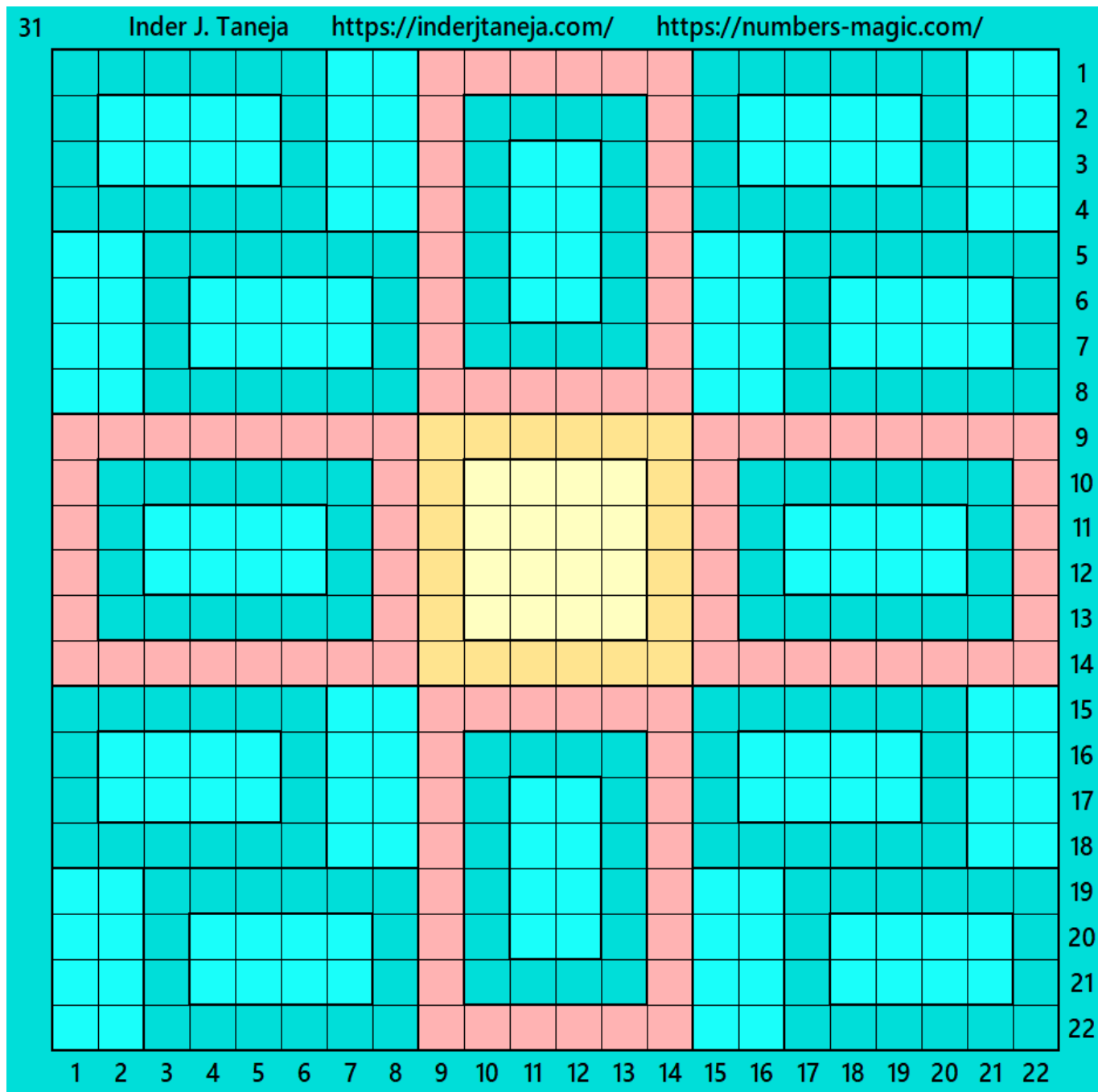


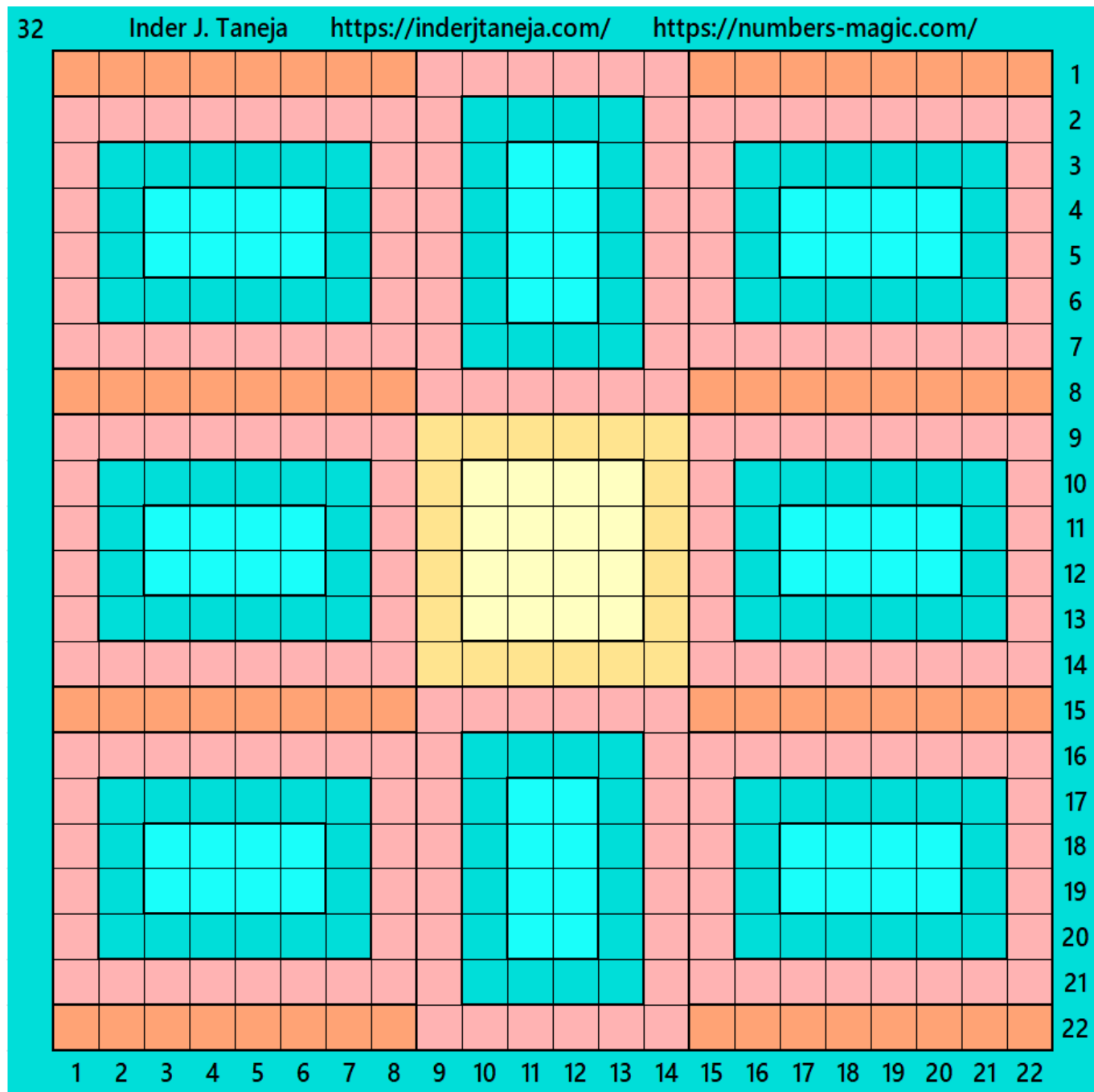


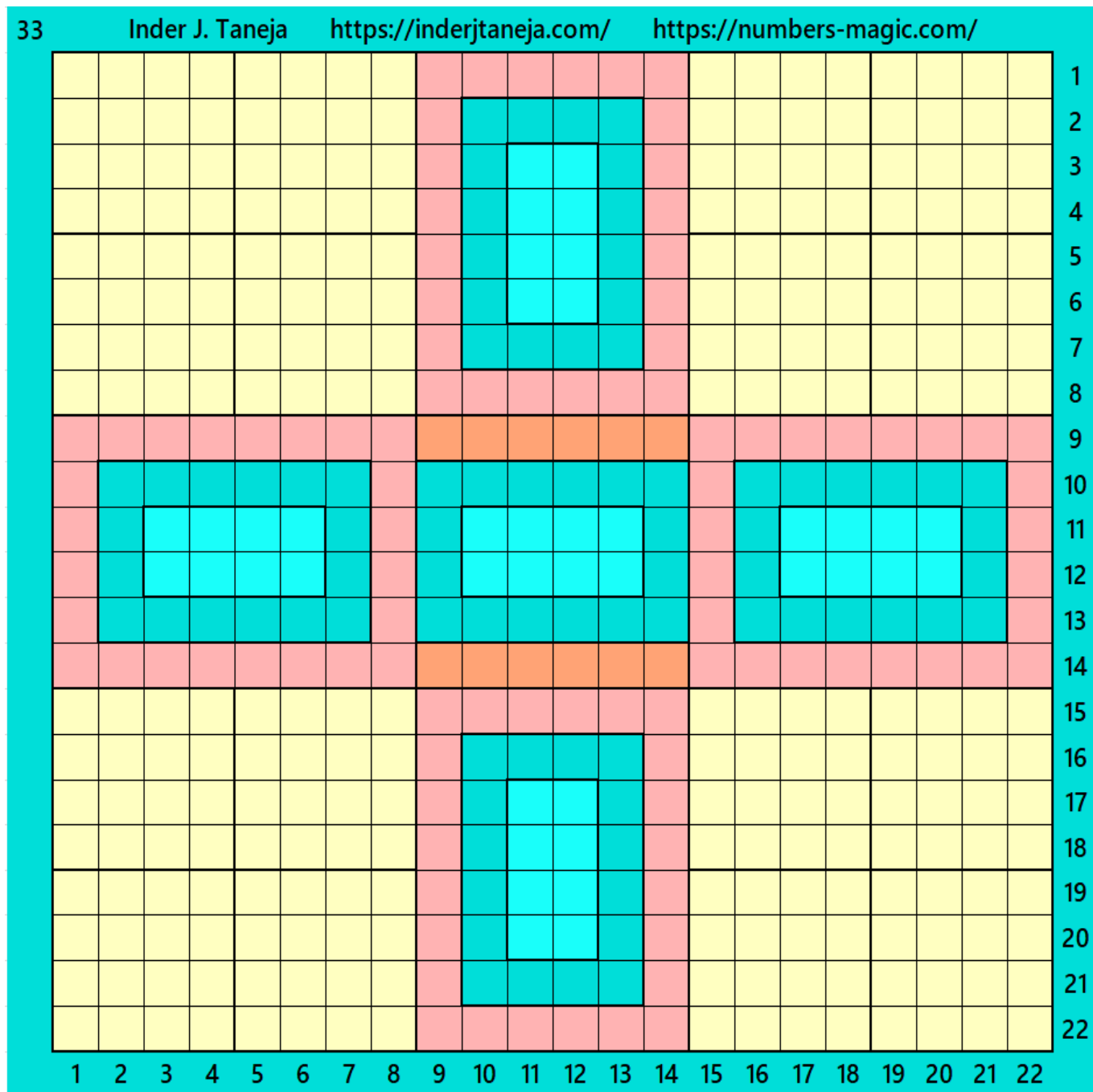


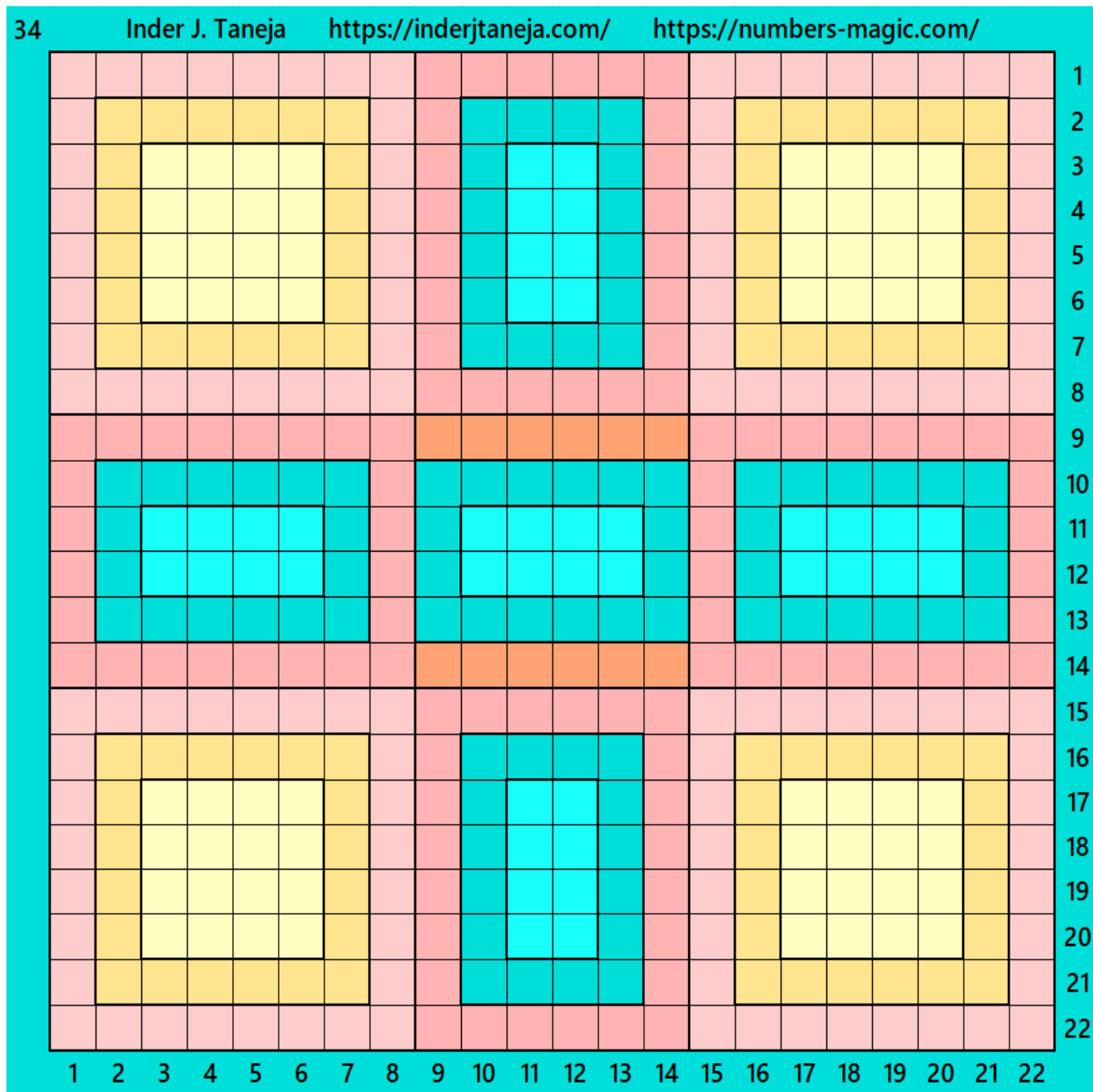


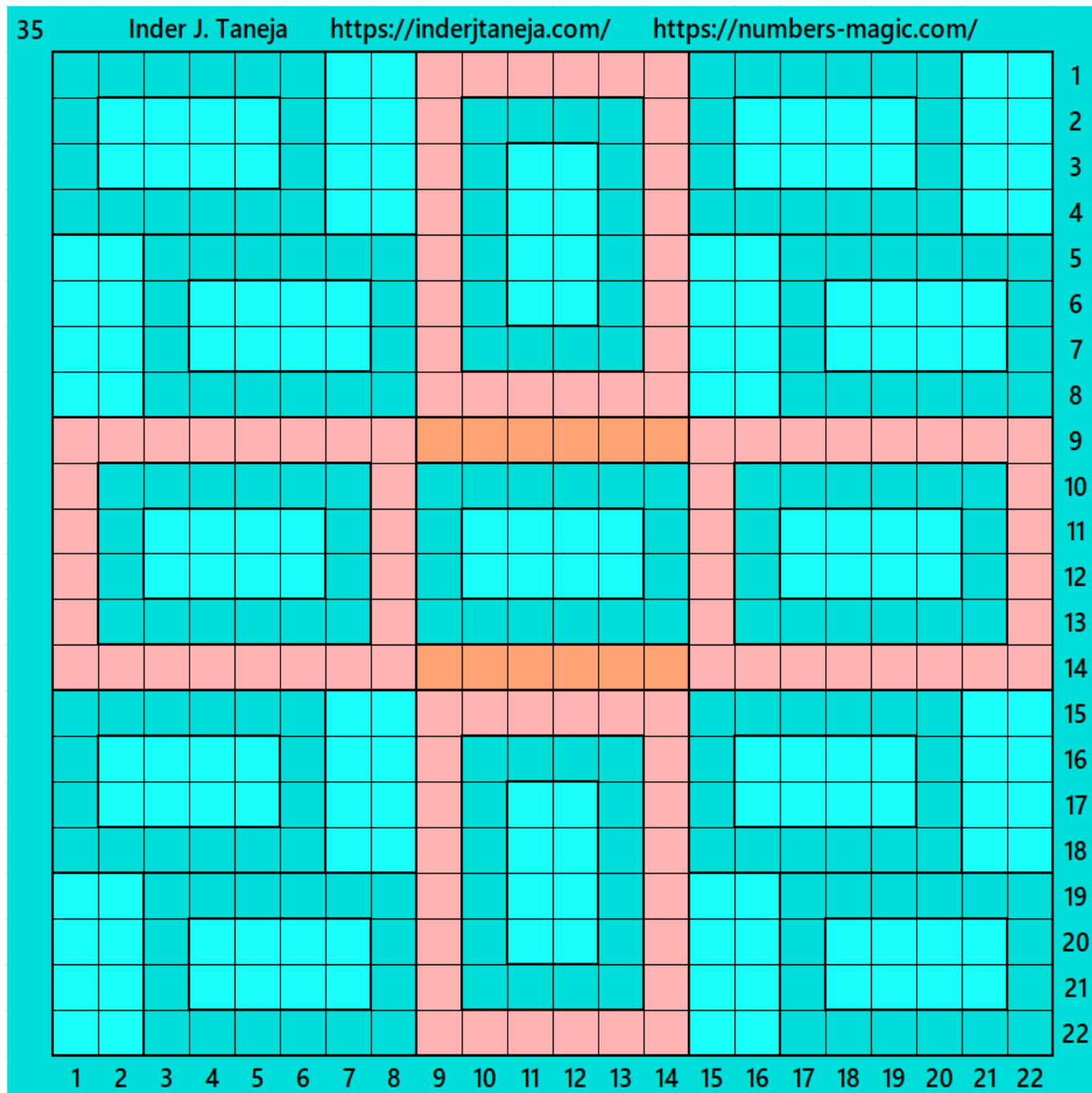


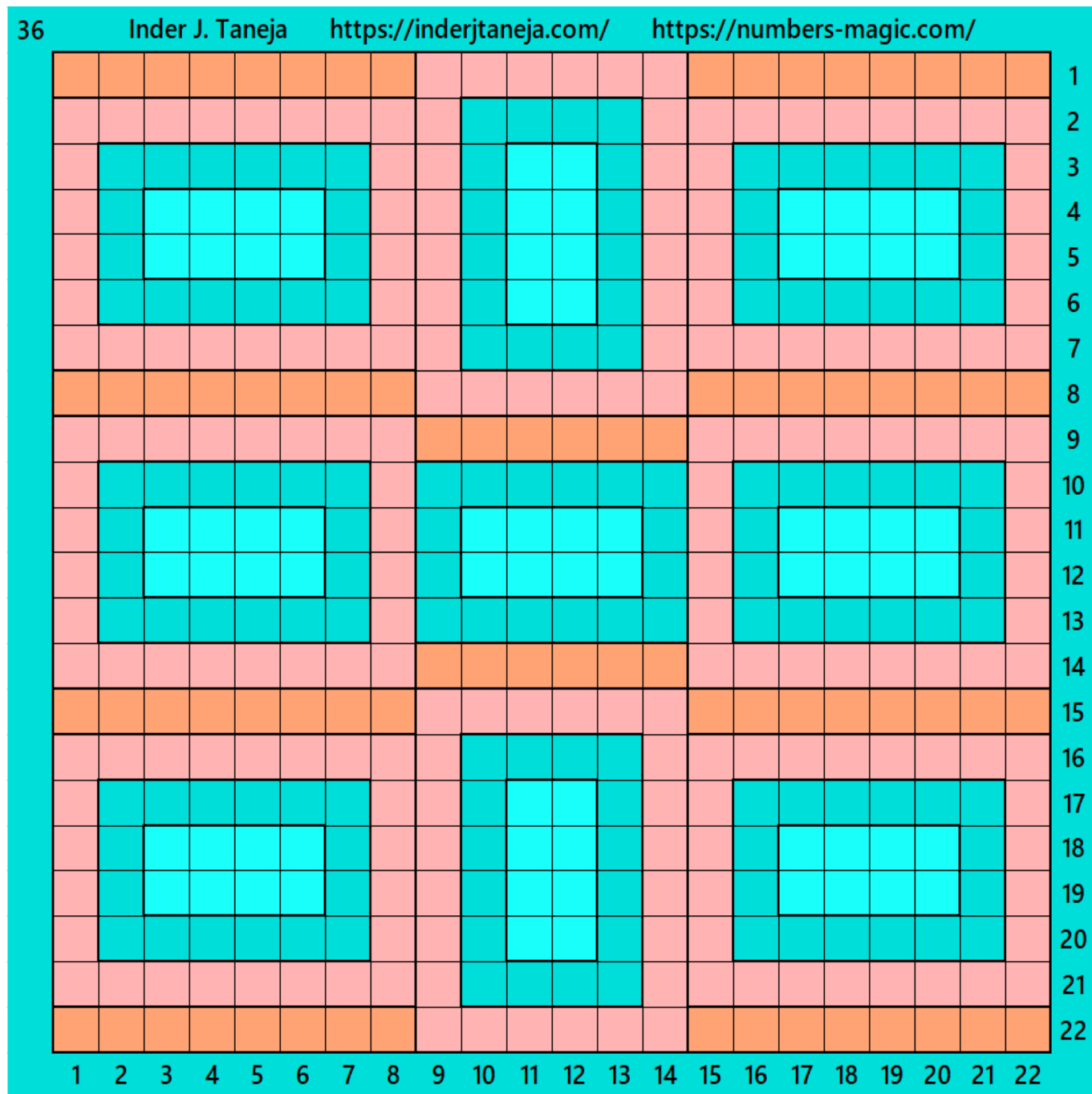








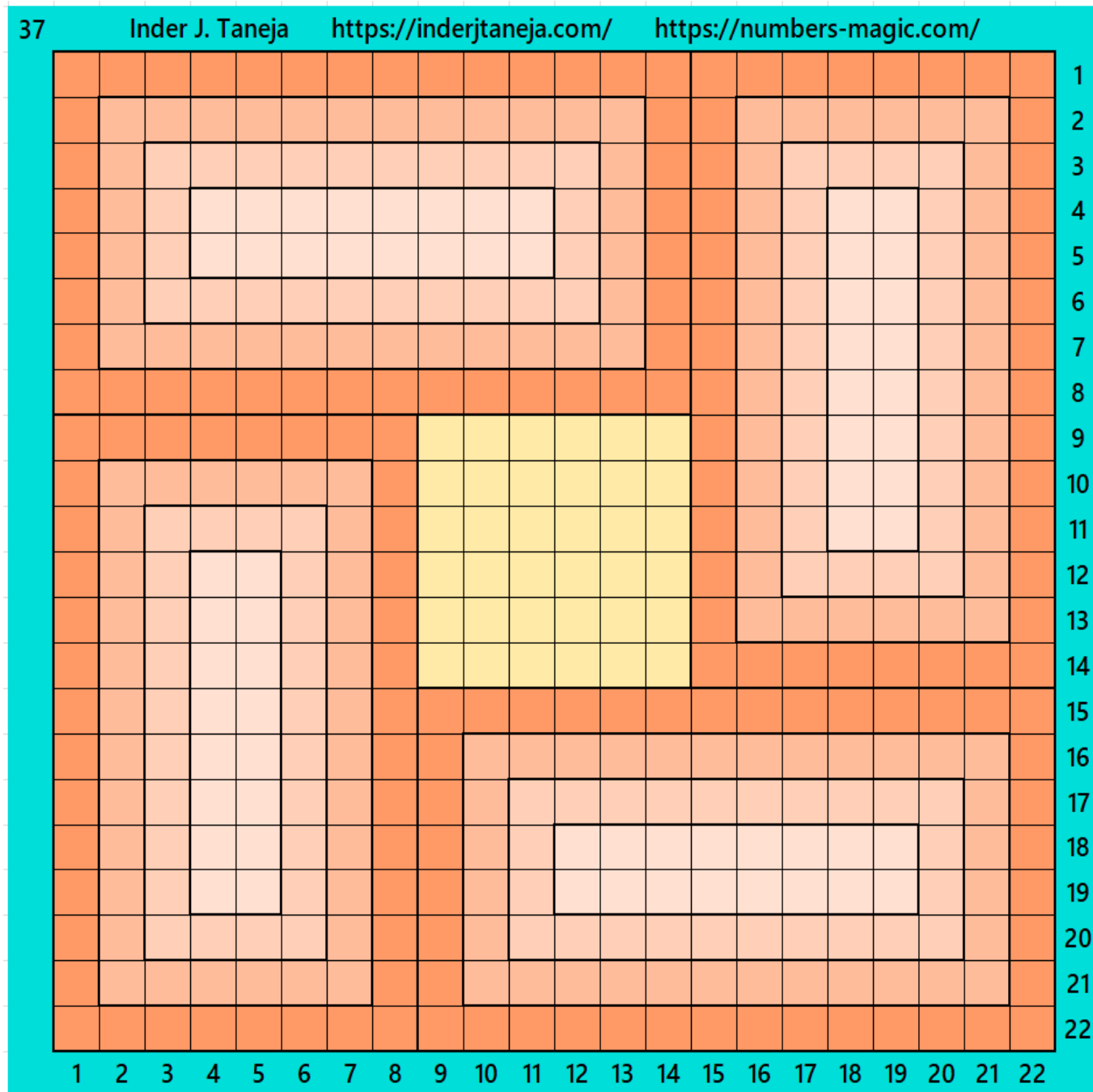


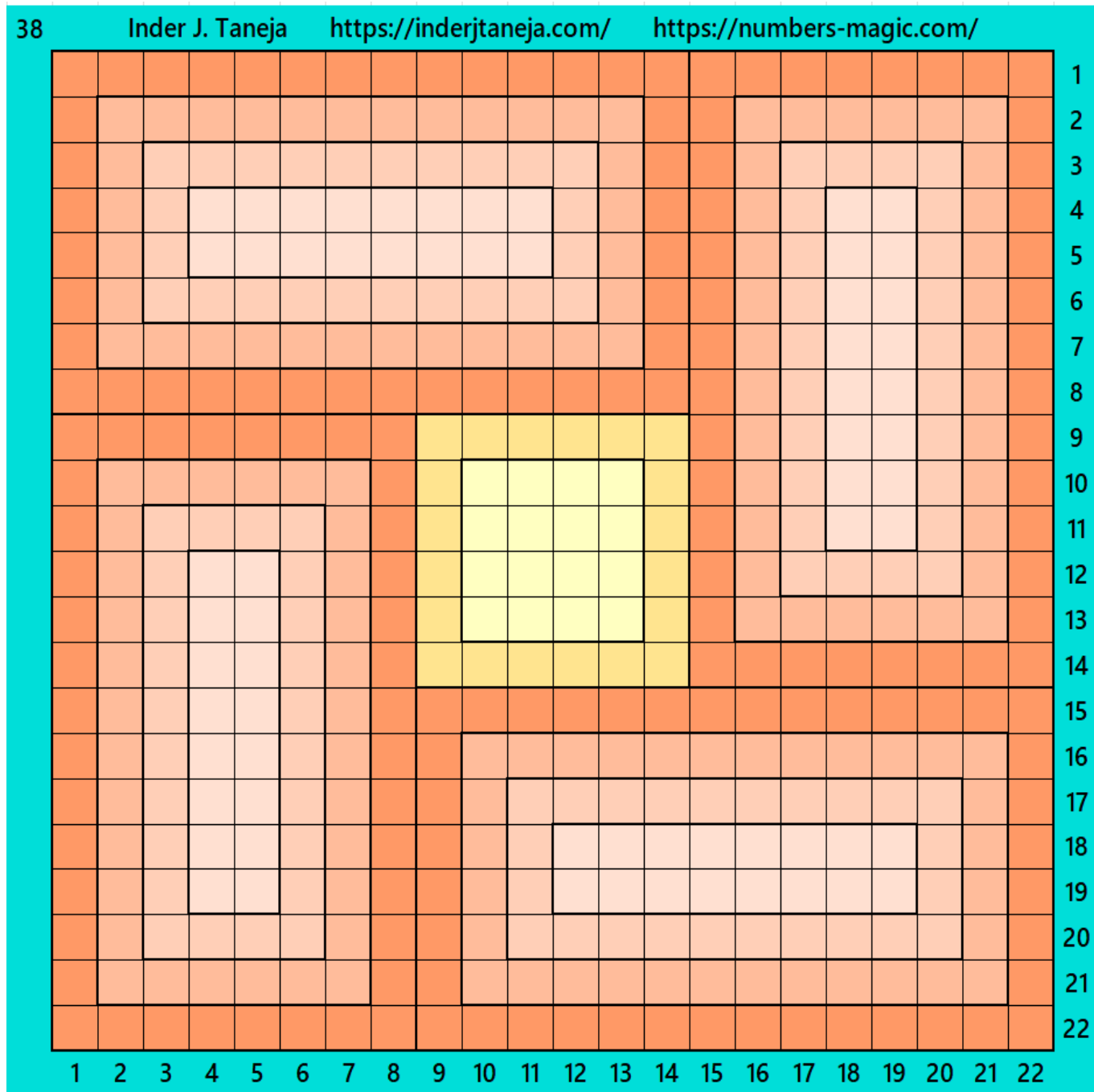


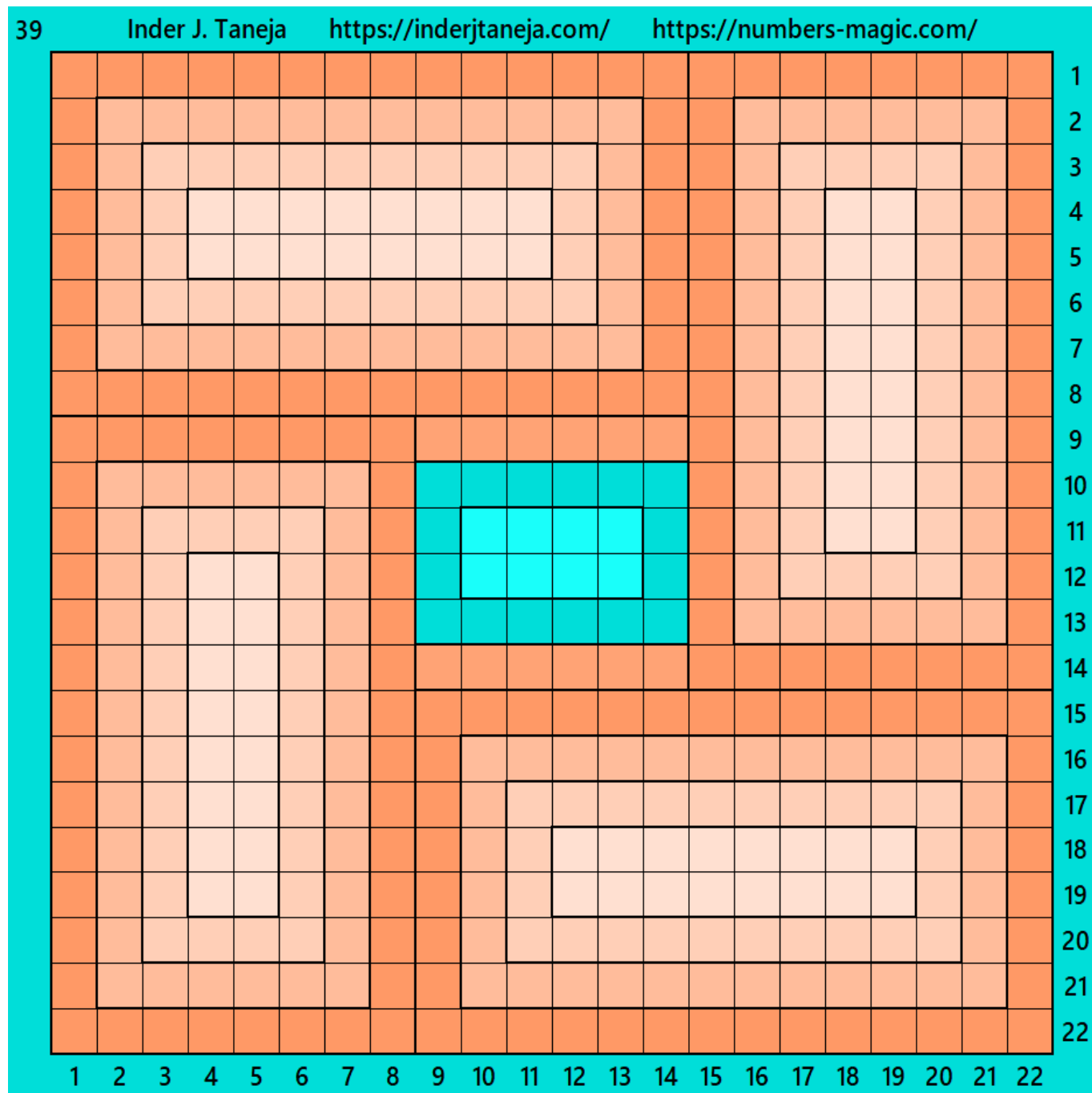
2.8 Closed Border of Order 8

Let's consider an external border, where there are 4 BMR's of order 8×14 . Putting them in rows and columns, we get a closed border of order 8. In the middle we are left with blocks of

order 6. Writing middle block of order 6 and different types of magic squares of order 8, we get magic squares of order 22. See below few examples:

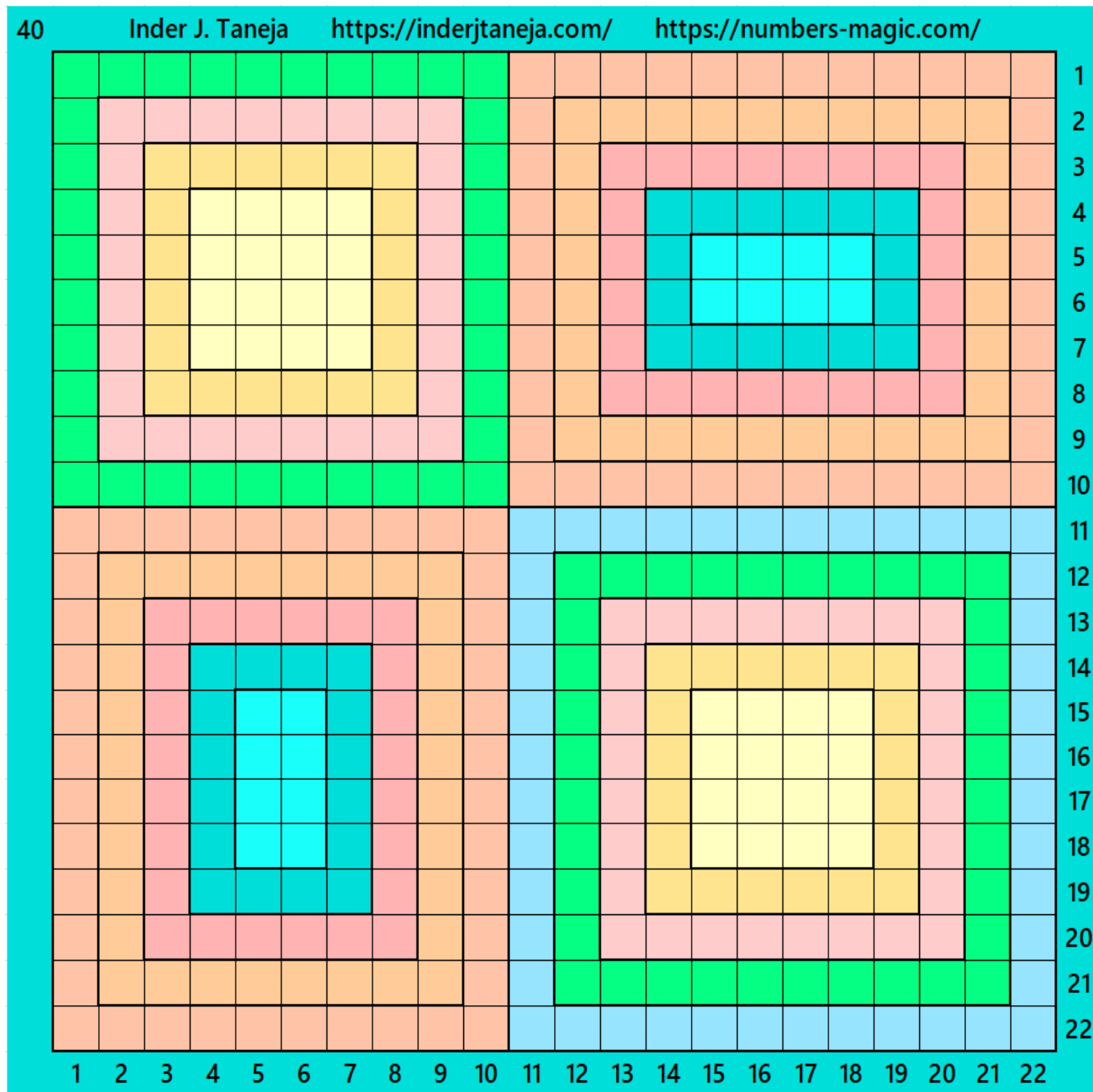


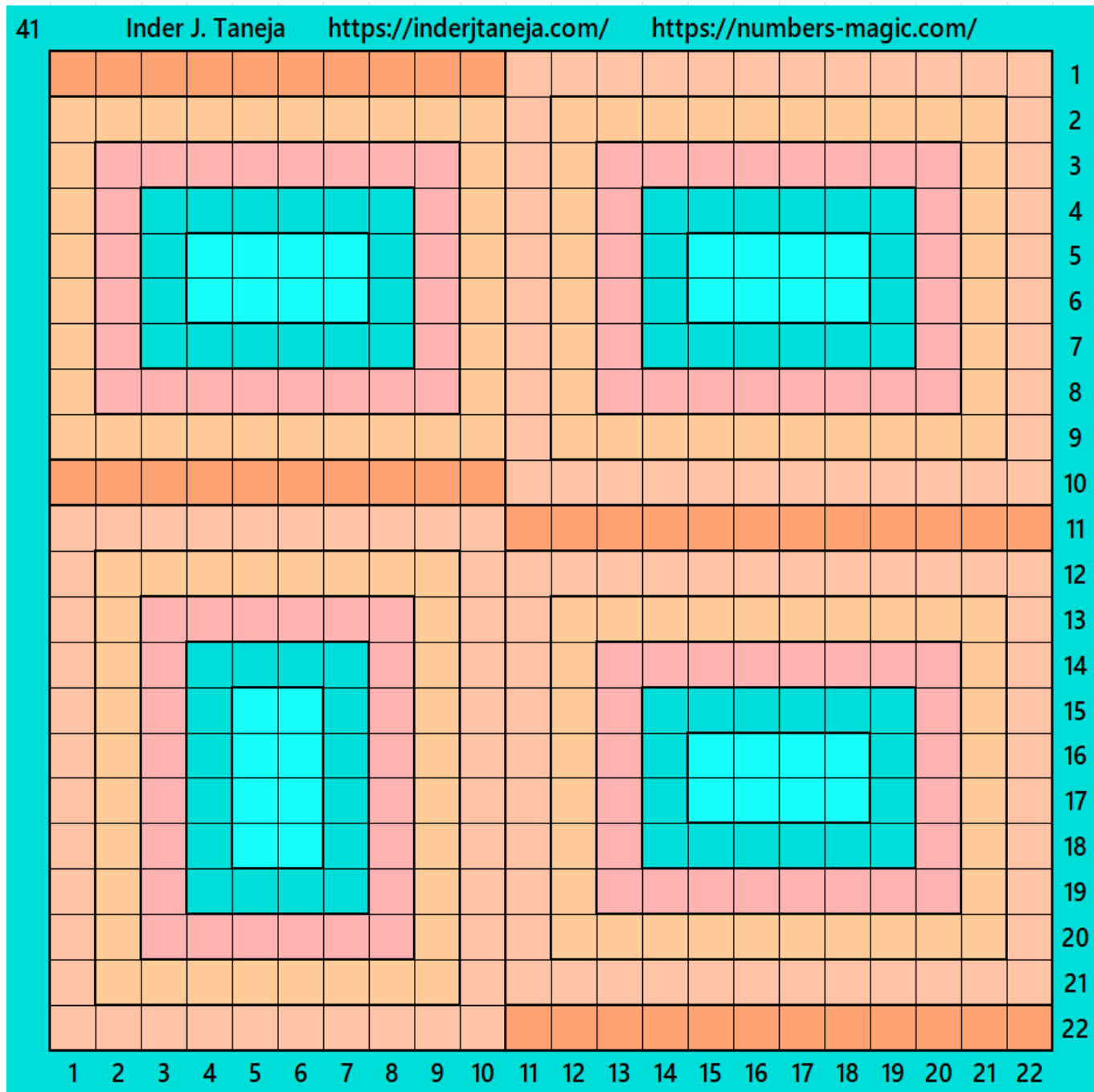


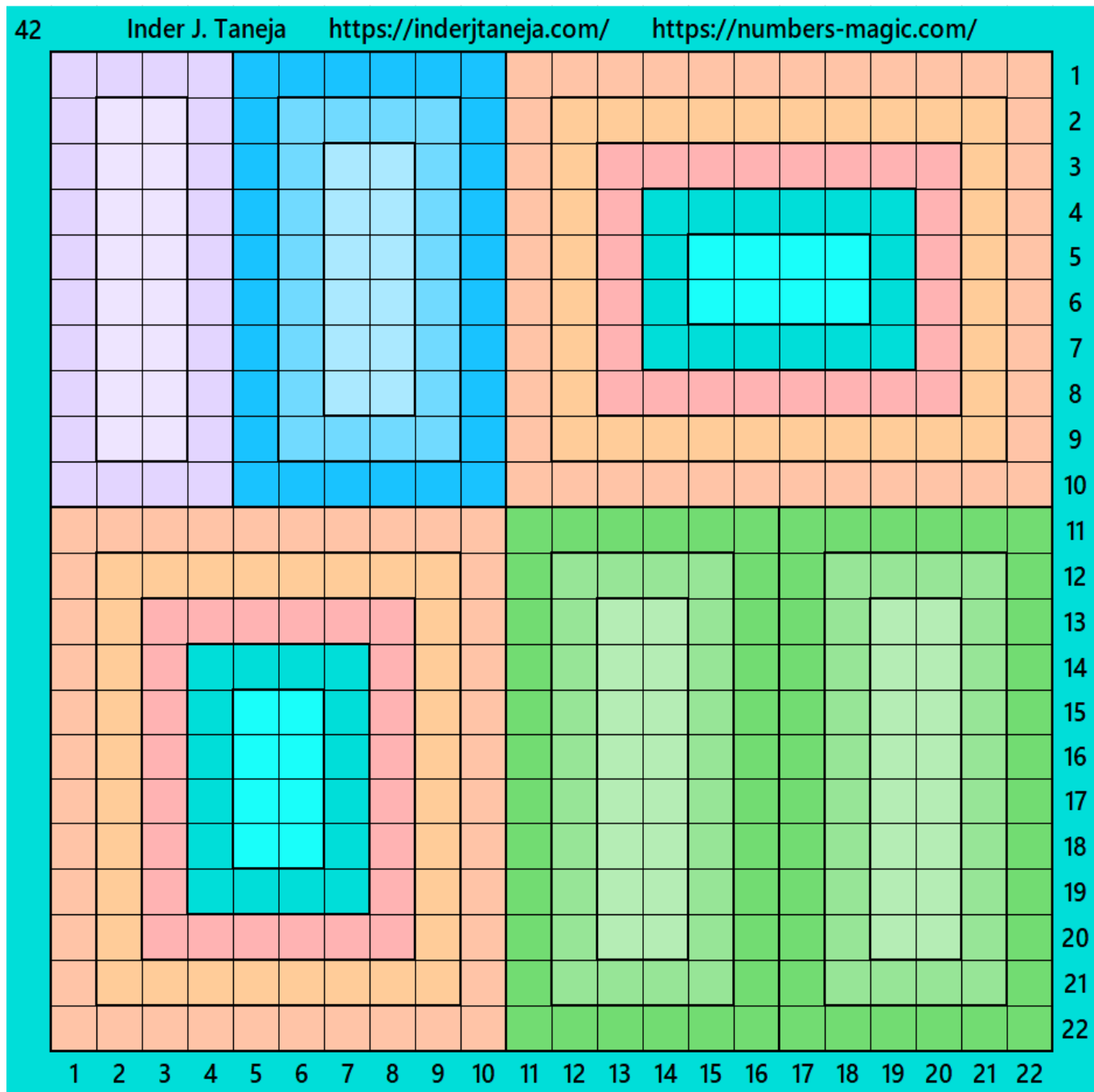


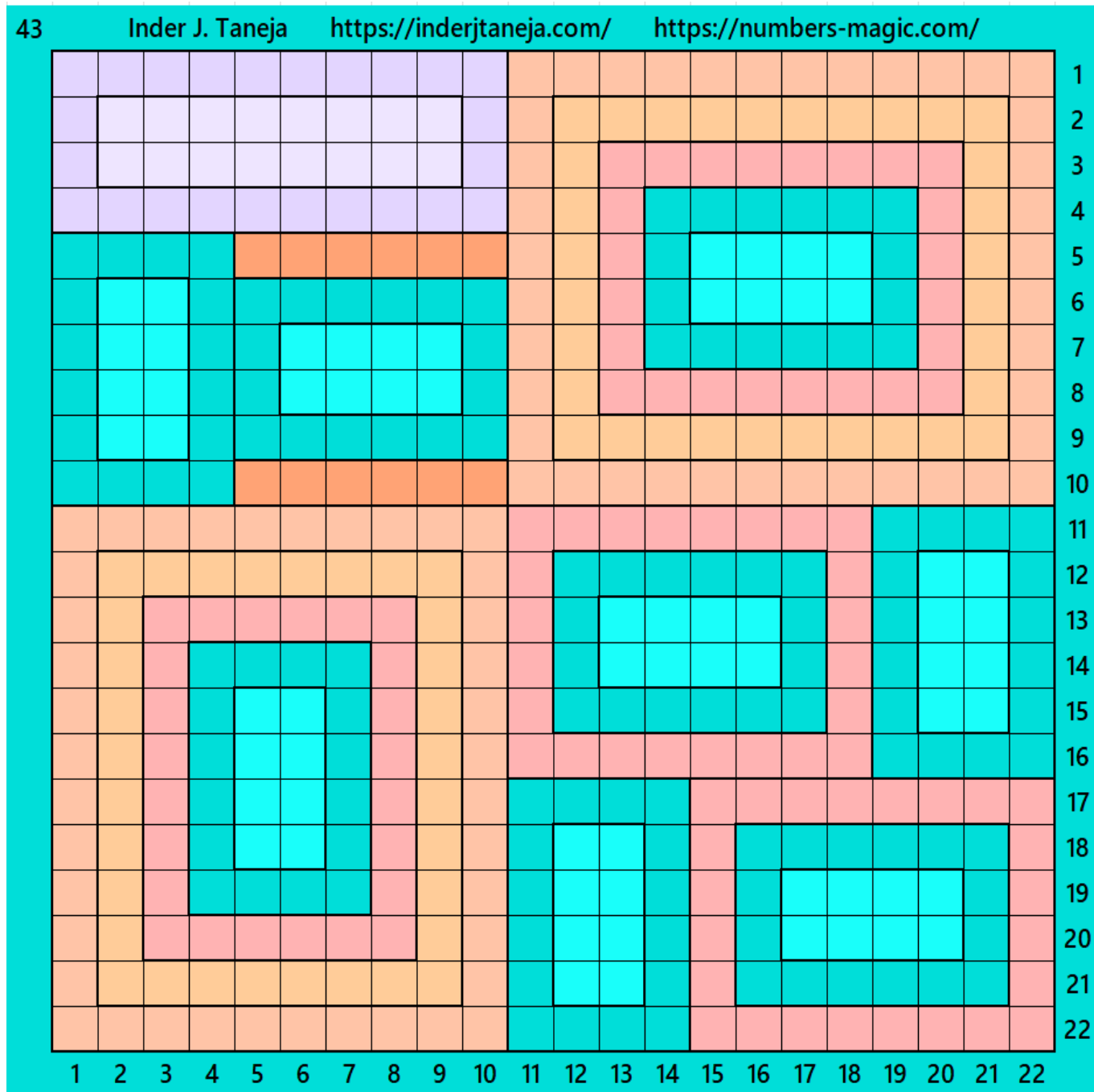
2.9 Blocks of Order w 10, 12 and BMR's of Order 10×12

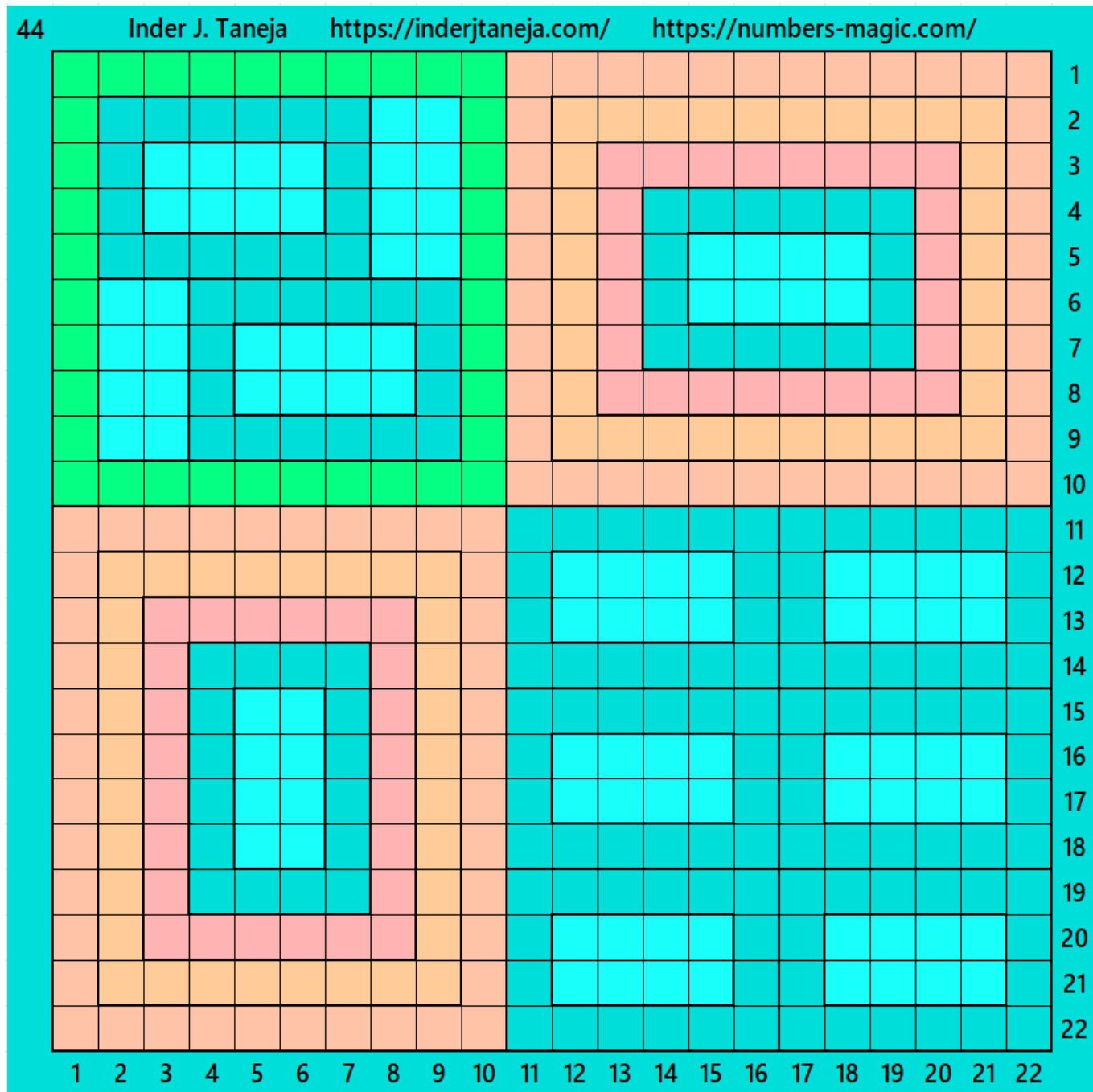
Magic squares of orders 10 and 12 with two BMR's of order 10×12 lead us to very interesting magic squares of order 22. See below few examples:

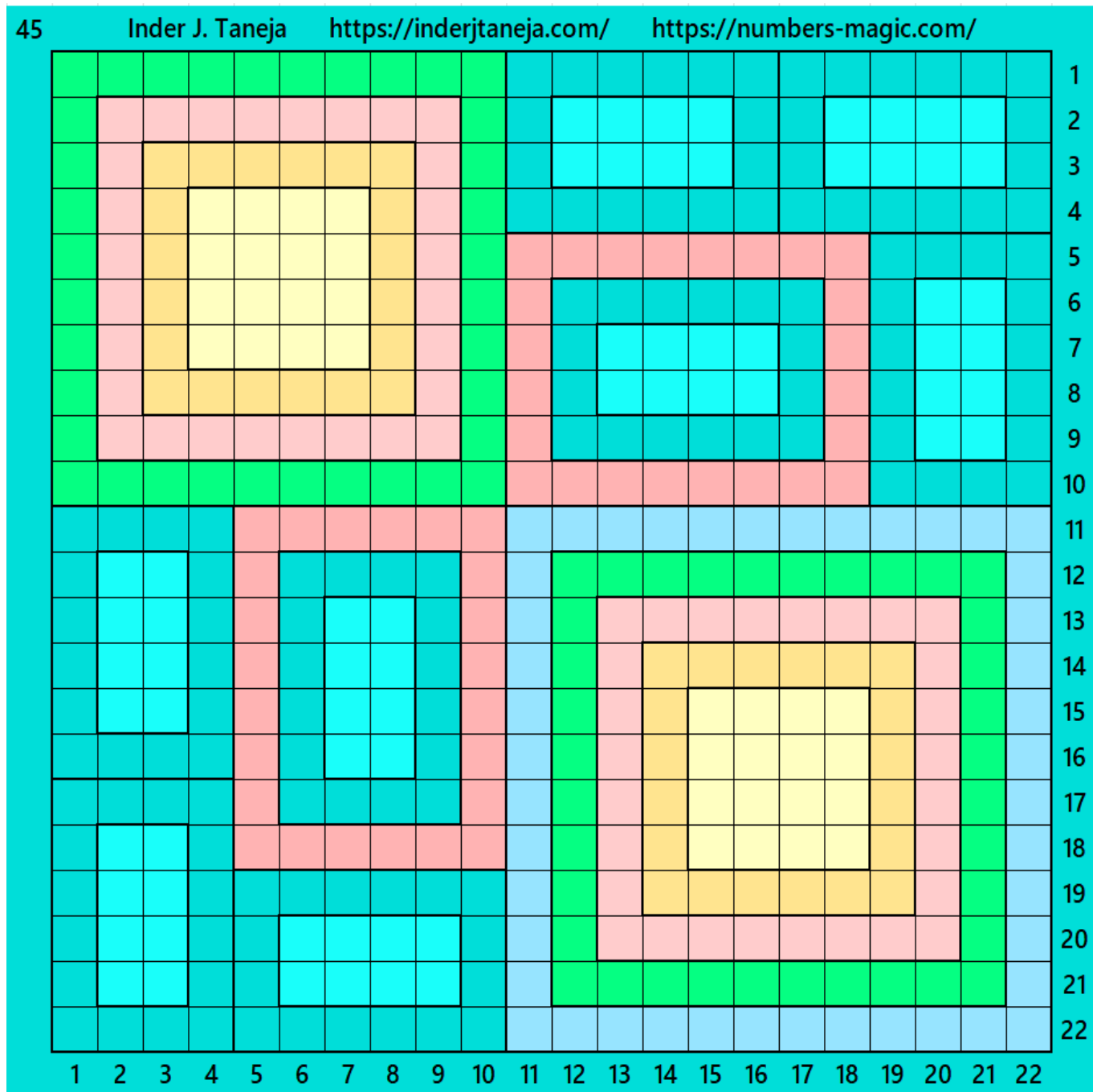


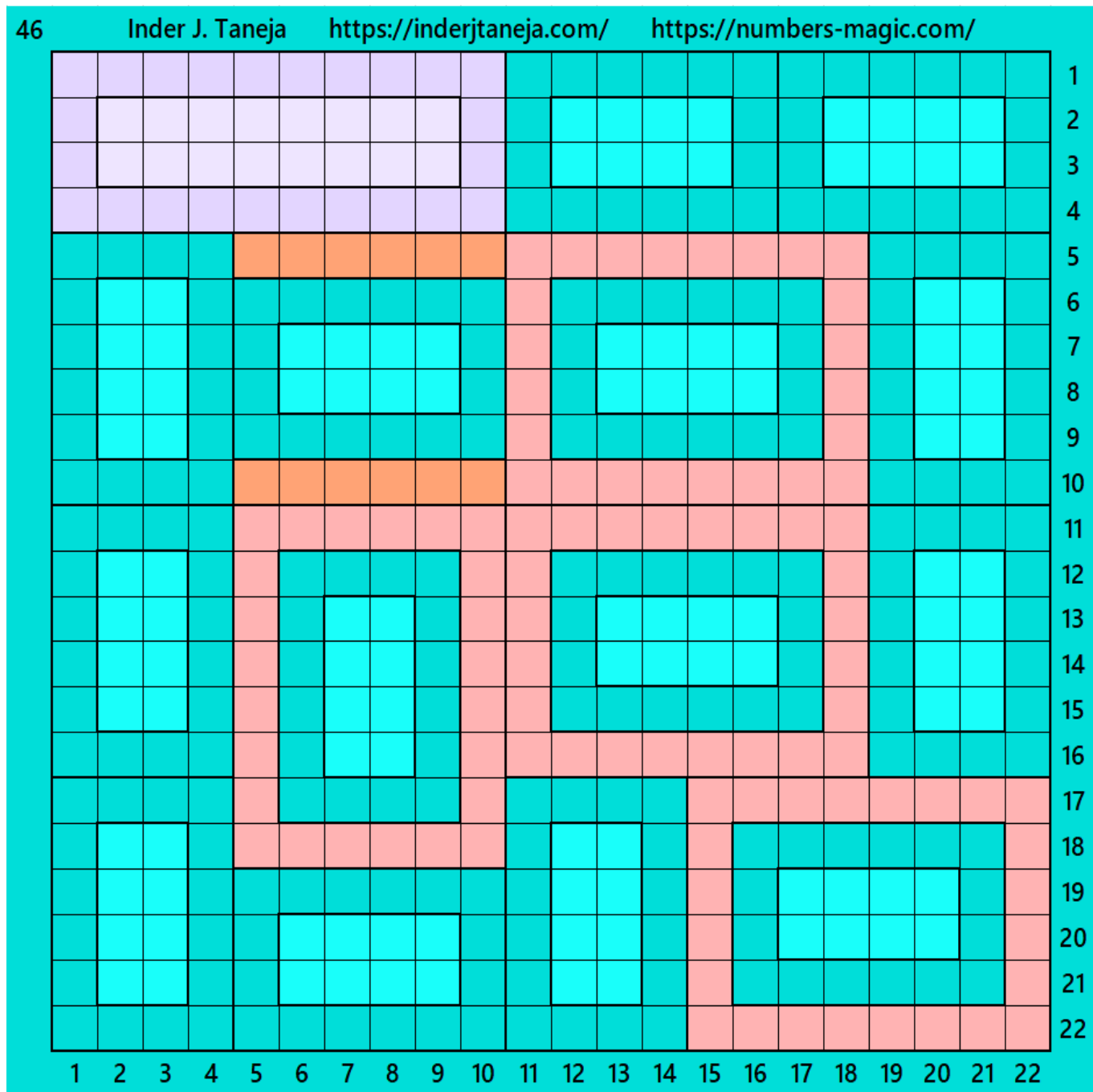


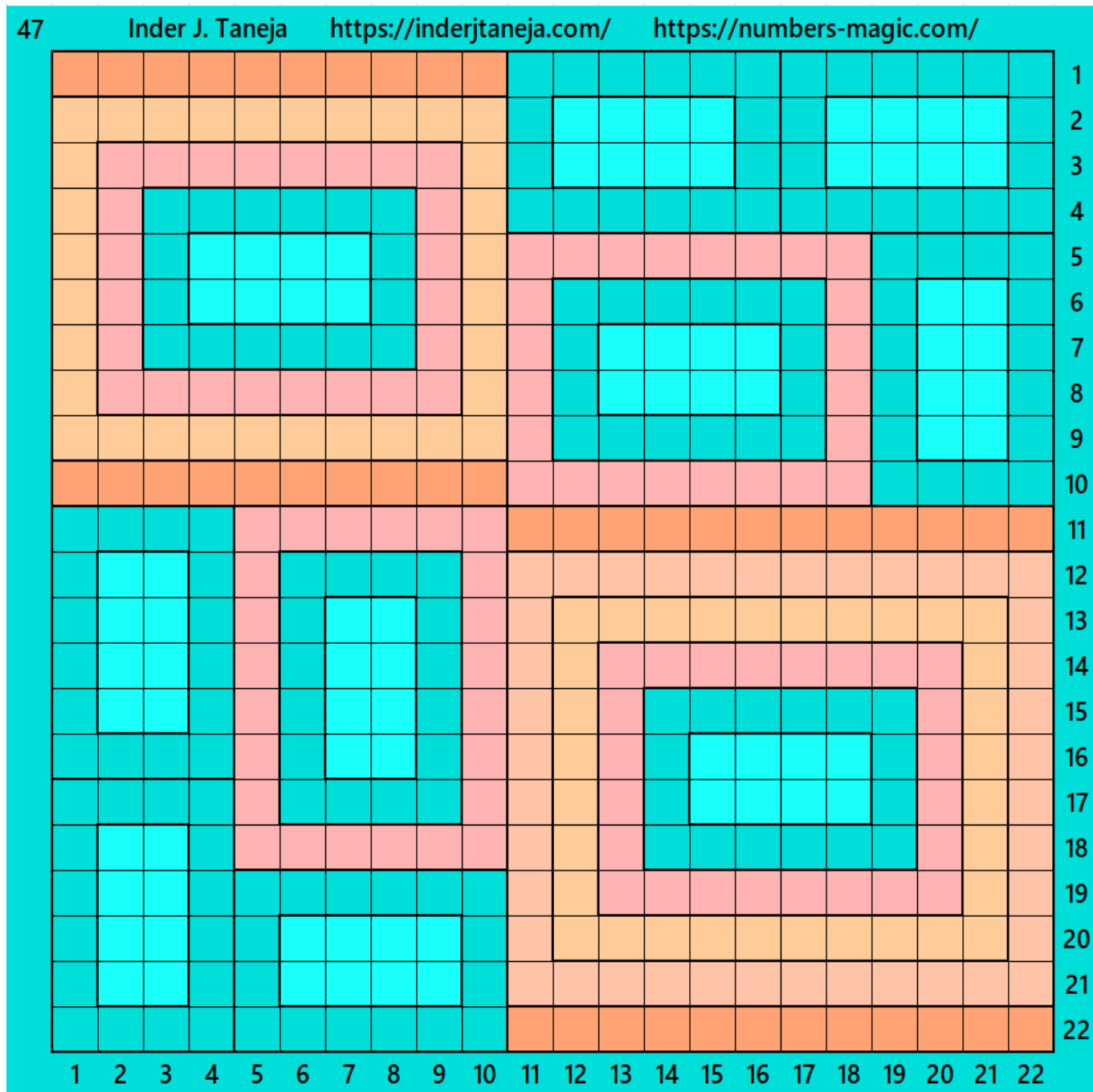


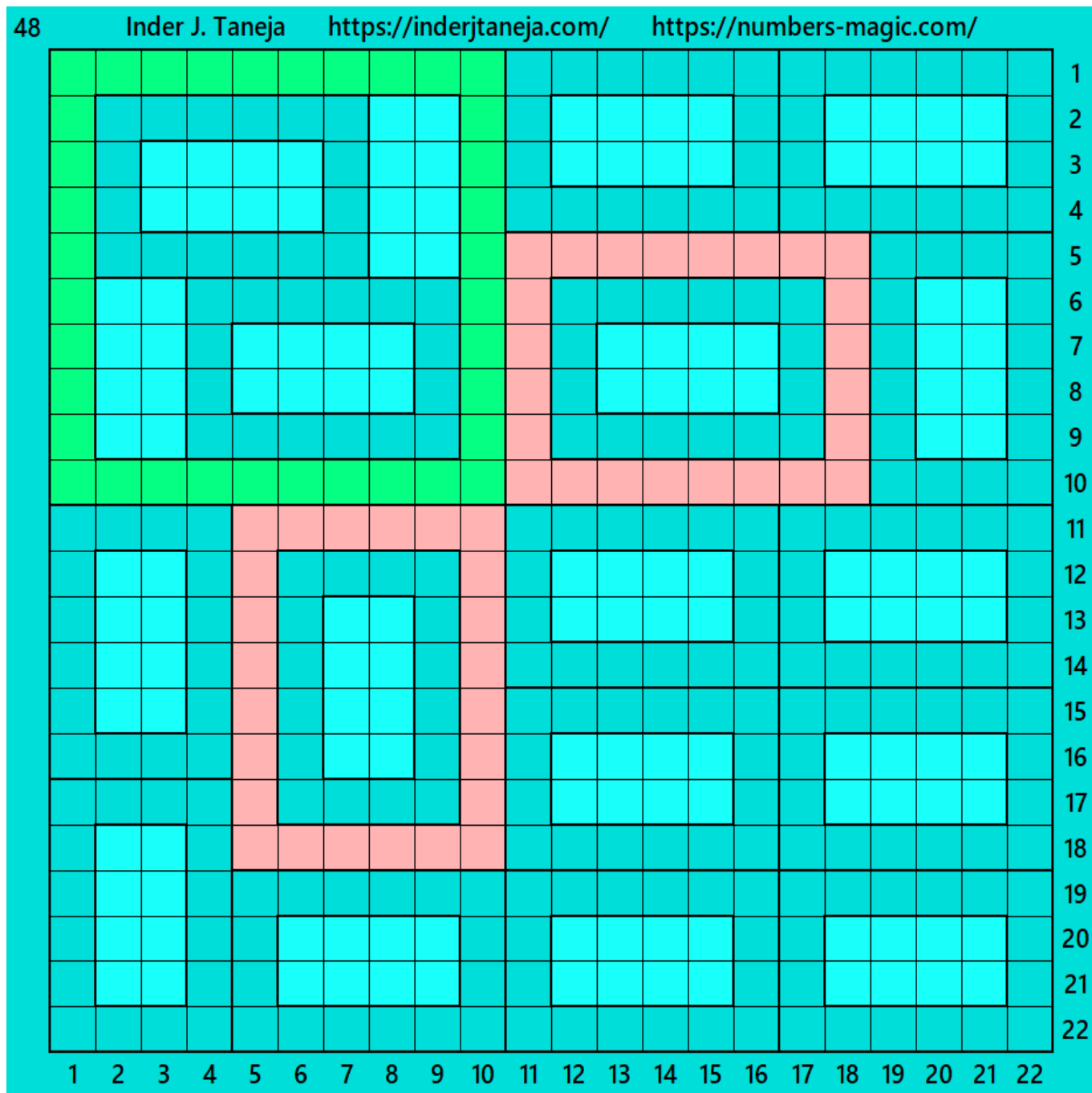


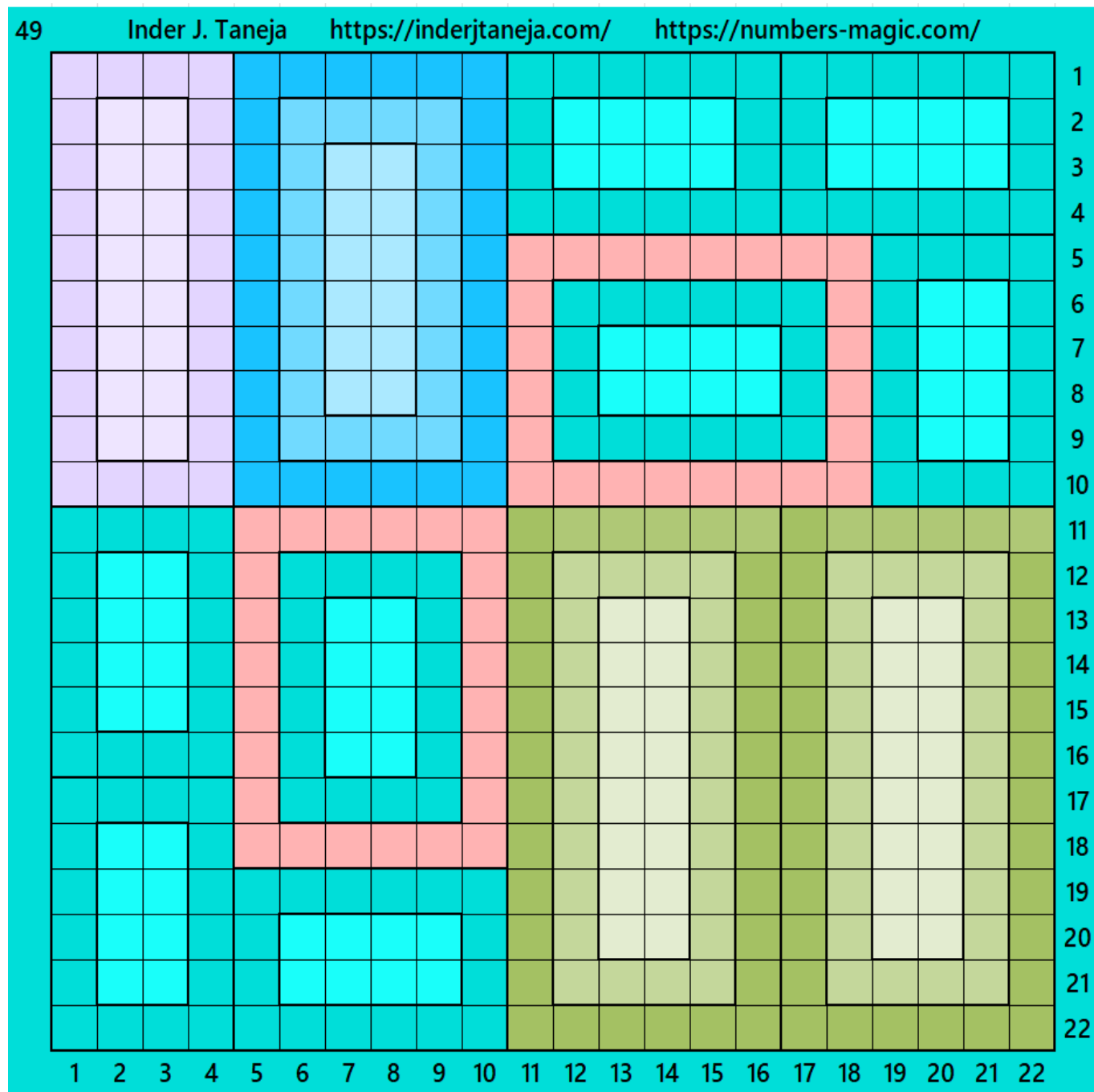












3 Appendix

Below are tables giving the existence of **bordered magic rectangles** for **half-sequential** numbers.

Order of Magic Square	Bordered Magic Rectangles
6	4×6
8	6×8
10	$4 \times 10, 8 \times 10$
12	$6 \times 12, 10 \times 12$
14	$4 \times 14, 8 \times 14, 12 \times 14$
16	$6 \times 16, 10 \times 16, 14 \times 16$
18	$4 \times 18, 8 \times 18, 12 \times 18$
20	$6 \times 20, 10 \times 20, 14 \times 20, 18 \times 20$
22	$4 \times 22, 8 \times 22, 12 \times 22,$ $16 \times 22, 20 \times 22$
24	$6 \times 24, 10 \times 24, 14 \times 24,$ $18 \times 24, 22 \times 24$
26	$4 \times 26, 8 \times 26, 12 \times 26,$ $16 \times 26, 20 \times 26, 24 \times 26$
28	$6 \times 28, 10 \times 28, 14 \times 28,$ $18 \times 28, 22 \times 28, 26 \times 28$
30	$4 \times 30, 8 \times 30, 12 \times 30, 16 \times 30,$ $20 \times 30, 24 \times 30, 28 \times 30$
32	$6 \times 32, 10 \times 32, 14 \times 32, 18 \times 32,$ $22 \times 32, 26 \times 32, 30 \times 32$
34	$4 \times 34, 8 \times 34, 12 \times 34, 16 \times 34,$ $20 \times 34, 24 \times 34, 28 \times 34, 32 \times 34$

Order of Magic Square	Bordered Magic Rectangles
36	6 × 36, 10 × 36, 14 × 36, 18 × 36, 22 × 36, 26 × 36, 30 × 36, 34 × 36
38	4 × 38, 8 × 38, 12 × 38, 16 × 38, 20 × 38, 24 × 38, 28 × 38, 32 × 38, 36 × 38
40	6 × 40, 10 × 40, 14 × 40, 18 × 40, 22 × 40, 26 × 40, 30 × 40, 34 × 40, 38 × 40
42	4 × 42, 8 × 42, 12 × 42, 16 × 42, 20 × 42, 24 × 42, 28 × 42, 32 × 42, 36 × 42, 40 × 42
44	6 × 44, 10 × 44, 14 × 44, 18 × 44, 22 × 44, 26 × 44, 30 × 44, 34 × 44, 38 × 44, 42 × 44
46	4 × 46, 8 × 46, 12 × 46, 16 × 46, 20 × 46, 24 × 46, 28 × 46, 32 × 46, 36 × 46, 40 × 46, 44 × 46
48	6 × 48, 10 × 48, 14 × 48, 18 × 48, 22 × 48, 26 × 48, 30 × 48, 34 × 48, 38 × 48, 42 × 48, 46 × 48
50	4 × 50, 8 × 50, 12 × 50, 16 × 50, 20 × 50, 24 × 50, 28 × 50, 32 × 50, 36 × 50, 40 × 50, 44 × 50, 48 × 50

4 Author's Contribution to Magic Squares and Recreation Numbers

For author's contribution to **magic squares** and **recreation numbers** please see the links below:

- Inder J. Taneja, Magic Squares, <https://inderjtaneja.com/2019/06/27/publications-magic-squares/>
- Inder J. Taneja, Recreation of Numbers, <https://inderjtaneja.com/2019/06/27/publications-recreation-of-numbers/>

Acknowledgement

The **bordered magic rectangles** are constructed based on the software produced by H. White [1]. The author is thankful to H. White for his valuable help.

References

- [1] H. White, Bordered Magic Squares, <http://budshaw.ca/MagicRectangles.html>.
- [2] Inder J. Taneja, Magic Squares - <https://inderjtaneja.com/category/magic-squares/>.
- [3] Inder J. Taneja, Recreating Numbers and Magic Squares - <https://numbers-magic.com/>.

● Block-Wise Magic Squares

- [4] Inder J. Taneja, Block-Wise Constructions of Magic and Bimagic Squares of Orders 8 to 108, May 15, 2019, pp. 1-43, **Zenodo**, <http://doi.org/10.5281/zenodo.2843326>.
- [5] Inder J. Taneja, Block-Wise Equal Sums Pandiagonal Magic Squares of Order $4k$, **Zenodo**, January 31, 2019, pp. 1-17, <http://doi.org/10.5281/zenodo.2554288>.
- [6] Inder J. Taneja, Magic Rectangles in Construction of Block-Wise Pandiagonal Magic Squares, **Zenodo**, January 31, 2019, pp. 1-49, <http://doi.org/10.5281/zenodo.2554520>.
- [7] Inder J. Taneja, Block-Wise Equal Sums Magic Squares of Orders $3k$ and $6k$, **Zenodo**, February 1, 2019, pp. 1-55, <http://doi.org/10.5281/zenodo.2554895>.
- [8] Inder J. Taneja, Block-Wise Unequal Sums Magic Squares, **Zenodo**, February 1, 2019, pp. 1-52, <http://doi.org/10.5281/zenodo.2555260>.

- [9] **Inder J. Taneja**, Block-Wise Magic and Bimagic Squares of Orders 12 to 36, **Zenodo**, February 1, 2019, pp. 1-53, <http://doi.org/10.5281/zenodo.2555343>.
- [10] **Inder J. Taneja**, Block-Wise Magic and Bimagic Squares of Orders 39 to 45, **Zenodo**, February 2, 2019, pp. 1-73, <http://doi.org/10.5281/zenodo.2555889>.

• Bordered Magic Squares

- [11] **Inder J. Taneja**, Nested Magic Squares With Perfect Square Sums, Pythagorean Triples, and Borders Differences, **Zenodo**, June 14, 2019, pp. 1-59, <http://doi.org/10.5281/zenodo.3246586>.
- [12] **Inder J. Taneja**, Symmetric Properties of Nested Magic Squares, **Zenodo**, June 29, 2019, pp. 1-55, <http://doi.org/10.5281/zenodo.3262170>.
- [13] **Inder J. Taneja**, General Sum Symmetric and Positive Entries Nested Magic Squares, **Zenodo**, July 04, 2019, pp. 1-55, <http://doi.org/10.5281/zenodo.3268877>.
- [14] **Inder J. Taneja**, Bordered Magic Squares With Order Square Magic Sums, **Zenodo**, January 20, 2020, pp. 1-26, <http://doi.org/10.5281/zenodo.3613690>.
- [15] **Inder J. Taneja**, Fractional and Decimal Type Bordered Magic Squares With Magic Sum 2020. **Zenodo**, January 20, 2020, pp.1-25. <http://doi.org/10.5281/zenodo.3613698>.
- [16] **Inder J. Taneja**, Fractional and Decimal Type Bordered Magic Squares With Magic Sum 2021, **Zenodo**, December 16, 2020, pp. 1-33, <http://doi.org/10.5281/zenodo.4327333>.
- [17] **Inder J. Taneja**, Inder J. Taneja, Block-Wise and Block-Bordered Magic Squares With Magic Sum 2022, **Zenodo**, December 28, 2021, pp. 1-38, <https://doi.org/10.5281/zenodo.5807789>

● Block-Bordered Magic Squares

- [18] **Inder J. Taneja**, Block-Bordered Magic Squares of Prime and Double Prime Numbers - I, **Zenodo**, August 18, 2020, pp. 1-81, <http://doi.org/10.5281/zenodo.3990291>.
- [19] **Inder J. Taneja**, Block-Bordered Magic Squares of Prime and Double Prime Numbers - II, **Zenodo**, August 18, 2020, pp. 1-90, <http://doi.org/10.5281/zenodo.3990293>.
- [20] **Inder J. Taneja**, Block-Bordered Magic Squares of Prime and Double Prime Numbers - III, **Zenodo**, September 01, 2020, pp. 1-93, <http://doi.org/10.5281/zenodo.4011213>.

● Block-Wise and Block-Bordered Magic Squares

- [21] **Inder J. Taneja**, Block-Wise and Block-Bordered Magic and Bimagic Squares With Magic Sums 21, 21^2 and 2021. **Zenodo**, December 16, 2020, pp. 1-118, <http://doi.org/10.5281/zenodo.4380343>.
- [22] **Inder J. Taneja**, Block-Wise and Block-Bordered Magic and Bimagic Squares of Orders 10 to 47. **Zenodo**, January 14, 2021, pp. 1-185, <http://doi.org/10.5281/zenodo.4437783>.
- [23] **Inder J. Taneja**, Bordered and Block-Wise Bordered Magic Squares: Odd Order Multiples, **Zenodo**, February 10, 2021, pp. 1-75, <http://doi.org/10.5281/zenodo.4527739>
- [24] **Inder J. Taneja**, Bordered and Block-Wise Bordered Magic Squares: Even Order Multiples, **Zenodo**, February 10, 2021, pp. 1-96, <http://doi.org/10.5281/zenodo.4527746>

● Block-Wise Bordered Magic Squares

- [25] **Inder J. Taneja**, Block-Wise Bordered and Pandiagonal Magic Squares Multiples of 4, **Zenodo**, August 31, 2021, pp. 1-148, <https://doi.org/10.5281/zenodo.5347897>.
- [26] **Inder J. Taneja**, Block-Wise Bordered Magic Squares Multiples of Magic and Bordered Magic Squares of Order 6, **Zenodo**, September 10, pp. 1-99 <https://doi.org/10.5281/zenodo.5500134>.
- [27] **Inder J. Taneja**, Block-Wise Bordered Magic Squares Multiples of 8, **Zenodo**, September 17, pp. 1-80, <https://doi.org/10.5281/zenodo.5514396>.
- [28] **Inder J. Taneja**, Block-Wise Bordered Magic Squares Multiples of 10, **Zenodo**, September 17, pp. 1-170, <https://doi.org/10.5281/zenodo.5514398>.
- [29] **Inder J. Taneja**, Block-Wise Bordered and Pandiagonal Magic Squares Multiples of 12, **Zenodo**, September 23, pp. 1-170, <https://doi.org/10.5281/zenodo.5523608>.
- [30] **Inder J. Taneja**, Block-Wise Bordered Magic Squares Multiples of 14, **Zenodo**, September 26, pp. 1-198, <https://doi.org/10.5281/zenodo.5528867>.

● Bordered Magic Rectangles

- [31] **Inder J. Taneja**, Different Styles of Magic Squares of Orders 6, 8, 10 and 12 Using Bordered Magic Rectangles, **Zenodo**, November 14, 2022, pp. 1-26, <https://doi.org/10.5281/zenodo.7319985>.
- [32] **Inder J. Taneja**, Different Styles of Magic Squares of Order 14 Using Bordered Magic Rectangles, **Zenodo**, November 14, 2022, pp. 1-40, <https://doi.org/10.5281/zenodo.7319787>.

- [33] **Inder J. Taneja**, Different Styles of Magic Squares of Order 16 Using Bordered Magic Rectangles, **Zenodo**, November 14, 2022, pp. 1-63, <https://doi.org/10.5281/zenodo.7320116>.
- [34] **Inder J. Taneja**, Different Styles of Magic Squares of Order 18 Using Bordered Magic Rectangles, **Zenodo**, November 14, 2022, pp. 1-85, <https://doi.org/10.5281/zenodo.7320131>.
- [35] **Inder J. Taneja**, Different Styles of Magic Squares of Order 20 Using Bordered Magic Rectangles, **Zenodo**, November 14, 2022, pp. 1-88, <https://doi.org/10.5281/zenodo.7320877>.
- [36] **Inder J. Taneja**, Few Examples of Magic Squares of Even Orders 6 to 18 Using Bordered Magic Rectangles, **Zenodo**, October 19, 2022, pp. 1-30, <https://doi.org/10.5281/zenodo.7225854>.
- [37] **Inder J. Taneja**, Few Examples of Magic Squares of Even Orders 20 to 30 Using Bordered Magic Rectangles, **Zenodo**, October 19, 2022, pp. 1-100, <https://doi.org/10.5281/zenodo.7225886>.

● Figured Magic Squares and Bordered Magic Rectangles

- [38] **Inder J. Taneja**, Figured Magic Squares of Orders 6, 10, 12, 14 and 16 Using Bordered Magic Rectangles, **Zenodo**, November 29, 2022, pp. 1-31, <https://doi.org/10.5281/zenodo.7377674>.
- [39] **Inder J. Taneja**, Figured Magic Squares of Orders 18 and 20 Using Bordered Magic Rectangles, **Zenodo**, November 29, 2022, pp. 1-87, <https://doi.org/10.5281/zenodo.7377689>.
- [40] **Inder J. Taneja**, Figured Magic Squares of Order 22 Using Bordered Magic Rectangles, **Zenodo**, November 29, 2022, pp. 1-61, <https://doi.org/10.5281/zenodo.7377706>.

[41] **Inder J. Taneja**, Figured Magic Squares of Order 24 Using Bordered Magic Rectangles, **Zenodo**, November 29, 2022, pp. 1-104, <https://doi.org/10.5281/zenodo.7377779>.

[42] **Inder J. Taneja**, Figured Magic Squares of Order 26 Using Bordered Magic Rectangles, **Zenodo**, November 29, 2022, pp. 1-88, <https://doi.org/10.5281/zenodo.7377794>.

• Creative Magic Squares

[43] **Inder J. Taneja**, Creative Magic Squares: Area Representations, **Zenodo**, June 22, pp. 1-45, 2021, <http://doi.org/10.5281/zenodo.5009224>.
