Credit Default Swap Index Curve Adjustment

The methodology of credit default swap index (CDSI) curve adjustment serves the purpose of making adjustment to the credit spread curve of each reference name in the index portfolio such that the market price of the index can be reproduced using these constituent curves. The adjusted index constituent curves are then used to price the standard collateral debt obligation (CDO) tranches based on the index portfolio.

Within the credit derivative modelling framework, the valuation of CDS index, by employing index constituent curves, has been previously approved for the CDSI option valuation model and found consistent with the method proposed by Bear Sterns and Morgan Stanley. The adjustment procedure involves iteration by multiplying all the constituent credit spread curves with a scaling factor.

The purpose of the model is to make adjustment to the credit spread curve of each reference name in the credit default swap (CDS) index portfolio such that the market price of the index can be reproduced using these individual curves. The adjusted index constituent curves are then used to price the standard collateral debt obligation (CDO) tranches based on the index portfolio, which is essential to build a base correlation curve.

Market quoted CDS indices, such as CDX or Tracer-X, are defined as the fair premium paid over a stream of risky coupons in exchange of default protection on standardized portfolios. Unlike a stock index it is not simply the weighted average of credit spreads for the reference names in the portfolio. A CDS index on a portfolio can be viewed as the b/e spread to an untranched basket credit default swap on that portfolio. As a result, an index position at a level $\bar{s}$ is equivalent to a portfolio of CDS on the reference names, where all CDSs are entered at the same spread $\bar{s}$. Individually, each CDS does not trade at b/e spread, but the portfolio as a whole does. Furthermore, market quoted index assumes a flat credit spread curve and 0.40 recovery rate.

Within the credit derivative modelling framework, pricing an index by employing constituent curves of the index portfolio has been previously approved and found consistent with the one proposed by Bear Sterns and Morgan Stanley.

For some reasons, such as quoting convention differences and liquidity, a direct pricing from constituent curves won't be the same as the market quoted index implied price. An adjustment is needed. The proposed method involves the following steps:

1. Find the present value (PV) of the index using the index curve.
2. Find the PV using the constituent curves.
3. Calculate the scaling factor=1-(PV using the constituent curves-PV using the index curve)/(PV01 Index X index spread).
4. Multiply the constituent curves by the scaling factor and then repeat steps 1~4 until the value calculated in step 1 and 2 matches.

We set the scaling factor range 0.9998~1.0002 be the criterion to stop iteration. That is, the iteration stops if the scaling factor falls within this range. Normally 3~4 iterations would meet this requirement.

The test of the model was conducted by running the submitted model to generate the adjusted curves and then these curves were employed to calculate the index price using the submitted model and previously approved CDSIO model with the same parameters, respectively. The criterion to stop iteration was also tested.

The index trade CDX4 is employed as the test trade with the detailed information as of May 23, 2005. In the CDSIO model, the b/e spread of the index is calculated using Monte Carlo simulation and number of paths is set to be 500,000.

First, the PVs of CDX4 calculated using adjusted constituent credit spread curves with the previously approved CDSIO template and submitted template, respectively. It is shown that the PVs calculated using both the submitted model and the CDSIO model are essentially the same as that of the market quoted index implies.

All the scaling factors and calculated PVs using constituent curves in every iteration step. It can be seen that 4 iterations would give us essentially no discrepancies, indicating that setting 0.9998 as the criterion is good enough.

The market quoted CDX4 @ May 23, 2005 is 88bps, which leads to a scaling factor smaller than one. In order to test the case of scaling factor larger than one, we leave everything unchanged and increase the index level to 98bps. The same set of results with scaling factor larger than one is shown in Table 3 and the same behaviour of iteration can be observed.

Market quoted CDS indices, such as CDX or Tracer-X, are defined as the fair premium paid over a stream of risky coupons in exchange of default protection on standardized portfolios. Unlike a stock index it is not simply the weighted average of credit spreads for the reference name in the portfolio. A CDS index on a portfolio can be viewed as the b/e spread to an untranched basket credit default swap on that portfolio. As a result, an index position at a level $\bar{s}$ is equivalent to a portfolio of CDS on the reference names, where all CDSs are entered at the same spread $\bar{s}$. Individually, each CDS does not trade at b/e spread, but the portfolio as a whole does.

A CDS index is defined as a set of M CDS trades, $\Omega = \{1, 2, \cdots, M\}$ (M=125 for CDX and M=100 for Tracer-X), in which each CDS is described by a reference name and a notional amount $Notl_i$. The entire notional of the index is $Notl_{index} = \sum_{i=1}^{M} Notl_i$. We assume the valuation date *t*, the option maturity date T, and the CDS index fee payments at interval $T_1 < T_2 < \cdots < T_n$ (maturity of CDS index).

Unlike an ordinary single name CDS, the CDS indices always have a fixed maturity. For example, as shown in the appendix, the 5-year “iBoxx_Series_III_NA” launched in March 2005 will mature in March 2010, and will continue trading, despite its decreasing maturity, until the maturity date.

Within the current credit derivatives framework a reduced form of default probability of a reference entity has been implemented and well maintained. Let the default time of i^{th} CDS reference entity in the index portfolio be τ_i, a deterministic risk-neutral hazard rate, $h_i(s)$, is defined such that the default probability between s and $s+ds$ $P[s < \tau_i < s + ds \mid \tau_i > s] = h_i(s)ds$. With this definition the default probability functions built upon the hazard rate are

(1) $S_i(t,s) = P[t < s < \tau_i] = e^{-\int_t^s h_i(u)du}$ = survival probability seen at time *t*,

(2) $f_i(t,s) = h_i(s)S_i(t,s)$ = default probability density function seen at time *t*.

Just like the Black model in the interest rate market, the volatility of the forward credit spread is not a market observable. Hence one purpose of the CDSO model is to provide a method to back out implied volatility information from the market.

It should be noted that, when we price the CDSIO using this model, the volatility should always be directly implied from the market. If there is no liquid option market for a given index, the volatility information could be found by proxy to other indices.

As well as the market implied volatility for the index, there exist the implied volatilities for single name CDS. The model has implemented a model to link two volatilities, by following a standard approach for the basket option.

The test of the model was conducted in two phases. In the first phase a test model, which is independent of the model, was implemented (see https://finpricing.com/lib/EqCallable.html). The closed form solution was implemented in the test model. The results of the test model and the model were compared. Then, based on the test model the underlying assumptions of the model and their implications in the pricing were assessed.

A CDX index trade, "**iBoxx_Series_III_NA**", is employed as the test trade with the detailed information as of March 24, 2005 provided in Appendix II. Another test trade with a homogeneous portfolio, flat credit curve, and small number of names (=20) is also used. The setup parameters of the CDSIO and USD interest rate curve are also included in the Appendix I.