Commodity Futures Swaption Model

A commodity futures swaption (CFSn) is a linear portfolio of options on FCCF. Hence it suffices to consider one option on an FCCF. Suppose that the underlying FCCF has a maturity of settlement *T*fccf , a commodity principal *P*, average time points *t*1 < ¢ ¢ ¢ < *tn* <*T*fccf , average weights *f!*1*; ¢ ¢ ¢ ; !ng*, and an index ¯. Let *T* be a maturity of the option onthe FCCF where *T* < *t*1 and *KC*2comm be a strike of the option. Now let *VC*1(*t;* ¯) be the value at time *t* of the option on the ¯-FCCF. From the FCCF pricing formula (6), the payoff of the option at the maturity *T* becomes

$$\begin{aligned} V_{C_1}(T,\beta) &= P \cdot \frac{\mathrm{df}^{C_1}(T,T_{\mathrm{fccf}})}{f_{C_1}^{C_2}(T,T_{\mathrm{fccf}})} \cdot \left[\beta \cdot \left(\sum_{i=1}^{n} \omega_i \cdot f_{C_1}^{C_2}(T,t_i) \cdot F_{\mathrm{comm}}^{C_1}(T,\mathrm{I}(t_i)) - K_{\mathrm{comm}}^{C_2}\right)\right]^+ \\ &= P \cdot \frac{\mathrm{df}^{C_2}(T,T_{\mathrm{fccf}})}{S_{C_1}^{C_2}(T)} \cdot \left[\beta \cdot \left(\sum_{i=1}^{n} \omega_i \cdot f_{C_1}^{C_2}(T,t_i) \cdot F_{\mathrm{comm}}^{C_1}(T,\mathrm{I}(t_i)) - K_{\mathrm{comm}}^{C_2}\right)\right]^+ \end{aligned}$$

It is clearly to see that

$$V_{C_1}(T,1) - V_{C_1}(T,-1) = P \cdot \frac{\mathrm{df}^{C_2}(T,T_{\mathrm{fccf}})}{S_{C_1}^{C_2}(T)} \cdot \left(\sum_{i=1}^{n} \omega_i \cdot f_{C_1}^{C_2}(T,t_i) \cdot F_{\mathrm{comm}}^{C_1}(T,\mathrm{I}(t_i)) - K_{\mathrm{comm}}^{C_2}\right)$$

By the no-arbitragy pricing theory, we have

$$V_{C_1}(t,1) - V_{C_1}(t,-1) = P \cdot \frac{\mathrm{df}^{C_1}(t,T)}{f_{C_1}^{C_2}(t,T)} \cdot \left(\sum_{i=1}^{n} \omega_i \cdot f_{C_1}^{C_2}(t,t_i) \cdot F_{\mathrm{comm}}^{C_1}(t,\mathrm{I}(t_i)) - K_{\mathrm{comm}}^{C_2}\right)$$

which is called the put-call parity for options on FCCF. If we could find exact solution for either *VC*1(*t;* 1) or *VC*1(*t;*¡1), then we may get the exact solution for the other. However, in real life, we can get precise solution for neither of them. Only approximate solutions are available. In some cases, having an approximate solution for ¯ = 1, it is not a good idea to get the one for ¯ = ¡1 by using the put-call parity. Thus we have to consider both cases of ¯ = 1 and ¯ = ¡1 (see https://finpricing.com/lib/FiBondCoupon.html)

Besides the assumption proposed in the previous sections, it is also assumed that the commodity price and the currency exchange rate follow geometric Brownian motions with deterministic volatilities. The correlative coefficient between the commodity futures and the exchange rate is assumed to be constant. Applying Vorst's approximation, we have

$$V_{C_1}(t,\beta) \;\approx\; P\cdot\frac{\mathrm{df}^{C_1}(t,T_{\mathrm{fccf}})}{f^{C_2}_{C_1}(t,T_{\mathrm{fccf}})}\cdot\beta\cdot\Bigg\{\exp\left(\mu+\frac{v}{2}\right)\cdot\Phi\left(\beta\cdot\frac{\mu+v-\ln\left(K^{C_2}_{\mathrm{comm}}+K^*\right)}{\sqrt{v}}\right) - \left(K^{C_2}_{\mathrm{comm}}+K^*\right)\cdot\Phi\left(\beta\cdot\frac{\mu-\ln\left(K^{C_2}_{\mathrm{comm}}+K^*\right)}{\sqrt{v}}\right)\Bigg\}\ ,$$

where © is the standard normal cumulative distribution function, *K*¤, ¹, *v*, and details of derivations are given

Let *FC*(*t;* I) be the futures price corresponding to some futures contract with index I in the *C*-currency. The first result states that the futures price process *FC*(*t;* I) is a martingale in the *C*-risk-neutral world, i.e., for *s* < *t*,

$$F^C(s,\mathrm{I}) = \mathrm{E}^C_s[F^C(t,\mathrm{I})]\ .$$

Let *t* < *t0*, *XC* be a random payoff in *C*-currency which is measurable at time *t0* and *SC0C* is currency exchange rate between currencies *C* and *C0*. Then we have the second result that

$$\mathrm{E}^{C'}_t\left[S^{C'}_C(t')\cdot X^C\right] = f^{C'}_C(t,t')\cdot\mathrm{E}^C_t[X^C]\ ,$$

where *fC0C* is the forward exchange rate between currencies *C* and *C0*. This also tells us that if *X* is a martingale in the *C*-risk-neutral world up to a time *T0*, then it may not be a martingale in the *C0*-risk-neutral world. However, *fC0C* (¢; *t0*) ¢ *X* is a martingale in the *C0*-risk-neutral world for any given *t0* · *T0*.

Let us consider a European derivative with a matured payoff given by *VC*1(*T*) in the *C*1-currency world, where *T* is the maturity. then we have, for any time *t* < *T*,

$$V_{C_1}(t) = \mathrm{df}^{C_1}(t,T)\cdot\mathrm{E}^{C_1}_t[V_{C_1}(T)]\ .$$

Consider the above derivative in the $C2$-currency world. i.e., we have the derivative with the matured payoff given by $VC2(T) = SC2C1\ (T) ¢\ VC1(T)$. The third result tells us that, for any time $t < T$,

$$V_{C_2}(t) = \mathrm{df}^{C_2}(t,T) \cdot \mathrm{E}_t^{C_2}[V_{C_2}(T)] = S_{C_1}^{C_2}(t) \cdot \mathrm{df}^{C_1}(t,T) \cdot \mathrm{E}_t^{C_1}[V_{C_1}(T)] = S_{C_1}^{C_2}(t) \cdot V_{C_1}(t)$$