Capped Accumulated Return Call Option

We propose a Monte Carlo (Gaussian MC and Quasi MC) pricing model for the product named capped-accumulated-return-call (CARC) with lock in feature.

Let $L_1 < L_2 < \ldots < L_p$ be an increasing series of lock in returns. Let L_k be the greatest lock in return such that the maximum of the partial accumulated returns is greater or equal than L_k. If L_k exists, then the final accumulated return will not be smaller than L_k. By its definition L_k is path-dependent. It acts as a path-dependent floor, as opposed to the global floor which is fixed.

Let $S_1, \cdots, S_M$ be M stocks in a given basket, $\mathrm{S_j(t)}$ be the price process of the *j*th stock and $1 \le j \le M$, and $\{\mathrm{t_0 < t_1 < \cdots < t_n}\}$ be a set of reset dates and $\mathrm{T \ge t_n}$ be a payoff settlement date.

The CARC with the multiple underlyings $\mathrm{S_1, \cdots, S_N}$ is a European type derivative security whose matured payoff at the settlement date is given by

$$\mathrm{N + N \times max\{R_c, R_f\}} \tag{1}$$

where $\mathrm{R_f}$ is the global floor of the return rate, N is the notional principal, and $\mathrm{R_c}$ is capped-accumulated-return and defined as

$$\mathrm{R_c = \prod_{i=1}^{n}(1 + R_{cap}^{(i)}) - 1} \tag{2}$$

where $\mathrm{R_{cap}^{(i)}}$ is the capped return-rate for each period described as follows: Define the actual period return-rate as

$$\mathrm{R_i = \frac{\overline{S}(t_i) - \overline{S}(t_{i-1})}{\overline{S}(t_{i-1})}, i = 1, \cdots, n}, \tag{3}$$

where

$$\bar{S}(t_i) = \sum_{j=1}^{M} w_j S_j(t_i) . \tag{4}$$

Here $w_j,\ j = 1,\cdots,M$ are the weights and

$$\sum_{j=1}^{M} w_j = 1 . \tag{5}$$

Then we define

$$\mathrm{R}_{\mathrm{cap}}^{(\mathrm{i})} = \min\{\mathrm{c}, \mathrm{R}_{\mathrm{i}}\}, \tag{6}$$

where c is the cap.

The new feature is represented by an increasing series of attributes LOCK_IN_RETURN. Let $L_1 < L_2 < \ldots < L_p$ be this series of increasing lock in returns. Define the jth partial accumulated return as follows: $R_c(j) = \prod_{i=1}^{j}(1 + R_{cap}^{(i)}) - 1$. In particular, the final accumulated return, R_{c}, is equal to the nth partial accumulated return: $R_c = R_c(n)$.

Let L_k be the greatest lock in return such that the maximum of the partial accumulated returns is greater or equal than L_k, that is $L_k := \max\{L_s \mid s = 1,\ldots,p, \max_{j=1,\ldots,n} R_c(j) \geq L_s\}$. In our simulation, L_k is path-dependent (just like R_{c}'s), as opposed to R_f which is fixed. If there is no such L_k, that is the set above is empty, we set $L_k = -\infty$.

We impose that the final accumulated return will not be less than L_k (see the payoff below).

Let t be the current value date, then the current value of CARC with lock in feature can be written

$$df(t,T) \times N \times \left[1 + E_t\left[\max\{\max\{R_c, L_k\}, R_f\}\right]\right] \qquad (7),$$

where $\mathrm{df}(\mathrm{t},\mathrm{T})$ is the discounting factor at the value date (note that the inner term is equal to $\max\{R_c, L_k, R_f\}$). The above formula is in a world that is forward risk-neutral with respect to a specific currency C_p.

As a result, the notional principal N is measured in the currency C_p, and the discounting factor should be calculated by a C_p zero curve given at the value date. If the underlying asset is measured in another currency C_U, assuming the option is a Quanto type transaction, the governing price dynamics of the underlying asset in the risk-neutral world of C_p should be written as

$$dS_t = (r^U - q - \rho\sigma_x\sigma_s)S_t dt + \sigma_s S_t dW_t \qquad (8)$$

where r^U is the short rate of C_U, q is the dividend yield of the asset, σ_s is the volatility of the asset price, σ_x is the volatility of the exchange rate between C_p and C_U, ρ is correlation coefficient between the asset price and the exchange rate, and W_t is the Wiener process. All these parameters are assumed deterministic (see https://finpricing.com/lib/FiZeroBond.html).